# An Auction-Bargaining Model for Initial Emission Permits

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**Abstract.** This paper studies the initial emission permits auction problem from the perspective of government' activities. In the traditional auction models, the basic assumption is that the government, i.e., the auctioneer, only pursues the maximum economic revenue. In this paper, we consider a hybrid auction-bargaining model, which gives new insights on how the government's economic and social goals effect the equilibrium strategies. For this model, we find a symmetric bidding strategy equilibrium for the firms in a sealed bid auction form, which is closely related to the classical results in the auction. Our most important finding is that, compared with the classical auction mechanism, the final trading price is based on not only firm's bidding strategy, but also the application quality of emission permits in the energy consumption market. The results also show that this auction-bargaining mechanism can alleviate distortion by excessive allowance in initial emission permits auction market and promote the social goals in both auction market and consumption market.

**Keywords:** emission permits, auction-bargaining, equilibrium.

## 1 Introduction

There is increasingly broad recognition that greenhouse gas emissions are contributing to changes to earth's climate. Emissions trading schemes (ETS) that $CO_2$ reductions are carried costly, are an important part of the policy response to this problem. The high potential costs of controlling pollutants by emission trading have led to growing interest in economic instruments. One critical issue in designing a tradable emission permit system is how the initial emission permits are distributed.

There are two different approaches existing in the initial emission permits schemes: the grandfathered approach and the auction approach, where the two mainly differ in the costs levied on the producers. Since Montgomery (1972) [1] showed that as long as permits markets were competitive, the initial emission permits allocation schemes might be irrelevant for emission abatement. There

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has been an ongoing debate about these two means. Most studies recognize an auction is preferred to grandfathering[2]. The reason is that auction allows reduced tax distortions, provides more flexibility in distribution of costs, provides greater incentives for innovation, and reduces the need for politically contentious arguments over the allocation of rents. Many important considerations are relevant to the implemented auction design. Firstly, there are a variety of auction formats, i.e., a sealed bid, a descending bid, an ascending bid or an ascending clock auction. Secondly, the pricing rule can be uniform, discriminatory or based on the Ausubel-Vickrey principle[3]. For example, the US Environmental Protection Agency (EPA) auctions for $SO_2$ permits are in the sealed bid discriminatory price format[4]. Cramton and Kerr [5] explained that when no bidder had significant market power, uniform pricing was nearly as efficient as Vickrey pricing and that among sealed bid auctions, a uniform price auction was probably the best. Betz [6] also proposed an ascending clock auction based on the policy frame work and theoretical as well as experimental findings in the literature, and this auction was later applied by the Australian government. There is broad consensus among economists specializing in auction design that a pay-as-bid auction is not best suited for emissions permits. Even in an idealized perfect competition setting in which all bidders lack market power, the pay-as-bid auction need not lead to an efficient or even an approximately efficient distribution of emission permits[7]. Moreover, many researches study which factors can be used to guide the successful auction design of emission permits[8]. These factors include the ability of the auction to elicit bids that reflect actual valuations by bidders[9], and restricting bidder opportunities for acting strategically in a conclusion way[10].

The above literature is based on the same assumption that the government aims at maximizing the profit to achieve the energy abatement goal. In some situations, a simple emission permits auction's efficiency can be very close to that of the optimal mechanism. Thus, it may be optimal for the government to employ a simple auction to reap most of the efficiency with low implementation costs[11]. However, in many countries there are such situations that grandfathered permits together with auction are more prevalent attributing to its political acceptability. These hybrid forms are perceived to potentially distort inter-firm competitiveness relations in initial emission permits auction market and lead to a lower equilibrium price[12]. There is a significant gap between the governments expected pay-off with a simple emission permits auction and that under the optimal mechanisms. Under such circumstances, the government may search for an intermediate solution to balance between economic goals and environmental goals. This paper introduces a novel hybrid auction mechanism, i.e., auction-bargaining mechanism to bridge the gap. Our proposed approach includes a sealed bid auction, followed by bargaining on payments to ensure that the emission permits transaction will finish.

This paper is also related to the work of [7] and [12], where they consider that the government can reach an environmental goal in an economically efficient way. Their discussion is based on the assumption that the total auctioning quantities of emission permits are exogenous to the auction models. In this paper, this assumption is relaxed and the self implement of energy goal in auction market is not available. Our attention will be focussed on not only the auction revenue in emission permits auction market, but also a further complication, one that will figure extensively in the application of emission permits in the energy consumption market(see Fig.1). This complication is that the winning firms should produce goods by emission permits at an acceptable level of quality, which is also called the quality threshold. This is particularly the case when a government agency is auctioning emission permits on behalf of the public. In the second phase, the government bargains with the chosen firm over the final price of the emission permits. The outcome of bargaining results in the final trading price that is based on the government's preference to economic revenue or social reward. It concludes that one of quality threshold, i.e., mandatory green standard in energy consumption market, impacts on the final trading price. Our chief finding is that the hybrid auction-bargaining generates alleviated impact on price violations in the emission permits auction market and promote the environmental goals.

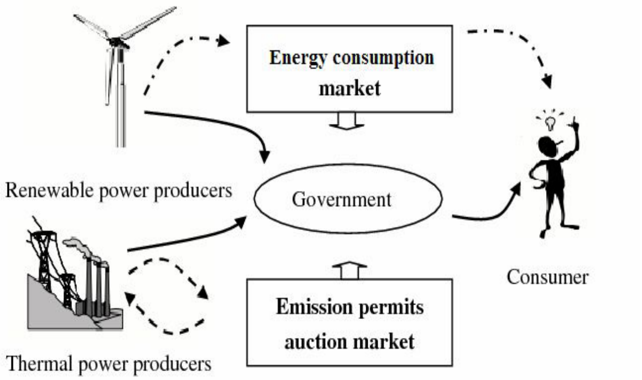


**Fig. 1.** The two markets included in the government's policy discussion

## 2 The Model

We next discuss our model and then proceed with the analysis. For the initial emission permits auction problem, we establish a two-stage dynamic game model, in which there are two main participants, i.e., government and firms. The time line of the game is as follows. In the auction stage, the government announces the mandatory green standard $\varepsilon$, and determines the temporary winner(s) based on the auction rules in the first stage. After receiving the invitation, the firms submit their bids according to their own abilities.

We will make a number of simplifications in order to compare various factors with equilibrium strategies in the hybrid auction-bargaining mechanism. The major ones, which we will hope to relax in later work, are that the maximum number of bidders is two and that these begin from the same position: thus we restrict ourselves to a symmetric case. It is supposed that the risk neutral firm

$i$ (i=1,2) wants to buy some units $k$ of emission permits for its production as in [13]. Since emission permits are homogeneous goods, the auctioning number of permits always does not influence the bidding strategies. For simplicity, we assume that $k = 1$. In order to allow for analytical solutions, the firm's quadratic abatement costs are assumed to be $\frac{c_i}{2}\varepsilon^2$, in which $c_i$ is the cost coefficient of each firm $i$(see details in [14]). Here, $c_i$ is the private information only known by each firm and is uniformly distributed over $[0, \overline{c}]$. Thus, firm $i$'s net profit is given by

$$u_i = g_i - \frac{c_i}{2}\varepsilon^2 - b_i = v_i - b_i \tag{1}$$

where $g_i$ is the profit received by firm $i$ from production based on purchased emission permits and $b_i$ ( $b_i \in [0, \overline{b}]$) is firm $i$'s bid. To keep the model simple, it also assumes that $g_i$ is uniformly distributed over $[0, \overline{g}]$. In this paper, we take the same assumption as [9] that each firm is truthfully to present his bid. Thus firm $i$'s valuation $v_i$ of auctioning emission permits satisfies $v_i = g_i - \frac{c_i}{2}\varepsilon^2$.

Different from previous literature, in this paper, the government not only considers the aspect of the revenue in the emission permits auction market, but also focuses on controlling the emission abatement in the energy consumption market. For example, there are environment instrument of mandatory green standard, which is to ensure a politically planned deployment of renewable energy technologies under liberalised market conditions[15]. Therefore, the benefit of the government $U_G$ can be defined as

$$U_G = \alpha R + (1 - \alpha)\varepsilon. \tag{2}$$

where $R$ denotes the economic revenue in emission permits auction market, $\alpha$ ($\alpha \in [0, 1]$) is the coefficient of weight, representing the government's preference to these two objects. Note that $\varepsilon$ as the quality threshold in the energy consumption market, which can directly bring environmental benefit. Of course, quality threshold has many dimensions. For the environmental governance problem, the government knows exactly what it wants, and can obtain perfect information on the quality achieved, then including quality considerations within an auction is relatively straightforward. Thus, we denote the quality threshold by the mandatory green standard denoted by $\varepsilon$, one kind of climate policies, which can reach lower emissions and develop future renewable industries. Thus, the specific function of $\varepsilon$ is not necessary for our two-stage model.

## 3 Auction Stage

The two-stage game can be solved by backward induction. We look for a subgame perfect Nash equilibrium (SPNE), defined by a set of strategies for the firms and the government. Firstly, we analyze the bidding strategies $b_i$ of firm $i$ ($i = 1, 2, \cdots, n$) in the auction under the condition of announced mandatory green standard $\varepsilon$ by the government. According to the assumptions about $c_i$ and $g_i$, we can compute the distribution function $F_i(v_i)$ of firm $i$'s valuation as follows.

$$F_i(v_i) = \begin{cases} \frac{v_i}{\bar{g}} + \frac{\varepsilon^2 \bar{c}}{4\bar{g}}, & 0 \leq v_i \leq \bar{g} - \frac{\varepsilon^2 \bar{c}}{2} \\ 1 - \frac{(v_i - \bar{g})^2}{\varepsilon^2 \bar{g} \bar{c}}, & \bar{g} - \frac{\varepsilon^2 \bar{c}}{2} < v_i \leq \bar{g} \\ 1, & \bar{g} < v_i \end{cases} \tag{3}$$

Notice that each firms' valuation about the auctioning emission permits is relative to mandatory green standard $\varepsilon$. With the increase of $\varepsilon$, the valuation of emission permits of each firm decreases.

Let the bidding strategy of firm $i$ be $\beta_i$, where $\beta_i(\cdot)$ is the function of firm $i$'s valuation $v_i$. If the cost information is the firm's private information and all the firms are all symmetric, the firms have the symmetric behaviors. Thus this assumption is available and do not influence our results. We achieve that $\beta_i(0) = 0$ and $\beta_i(\overline{g}) = \overline{b}$. Thus, the firm $i$'s valuation function is $\beta_i^{-1}$, i.e., the inverse functions of $\beta_i$. Set $\beta_i^{-1} = \phi_i(b)$. When firm $i$ chooses $b$ as its the equilibrium bidding strategy, $\phi_i(b)$ is the valuation of the auctioning emission permits.

**Lemma 1.** *When firm $i$ chooses $b$ as its the equilibrium bidding strategy, the probability of its success is*

$$Prob(firm\ i\ wins) = \prod F_{-i}(\phi_{-i}(b)) \tag{4}$$

**Proof.** These are $n$ firms strictly compete for the auction stage. If firm $i$ wins the game, the other firms denoted by $-i$ lose it. Thus, when the bidding strategy of firm $i$ is $b$ , the probability of its success can be denoted by

$$\begin{aligned} Prob(firm\ i\ wins) &= Prob(\beta_1(v_1) < b) \cdot Prob(\beta_2(v_2) < b) \cdots Prob(\beta_n(v_n) < b) \\ &= F_1(\phi_1(b)) \cdots F_{i-1}(\phi_{i-1}(b)) \cdot F_{i+1}(\phi_{i+1}(b)) \cdots F_n(\phi_n(b)) \\ &= \prod F_{-i}(\phi_{-i}(b)) \end{aligned} \tag{5}$$

**Theorem 1.** *The equilibrium bidding strategies $b_1^*$ and $b_2^*$ for firm 1 and firm 2 respectively are solutions of the following inverse bidding functions.*

$$\begin{cases} b_1^* = \{b | b + \frac{H_2(b)}{h_2(b)} - \phi_1(b) = 0\} \\ b_2^* = \{b | b + \frac{H_1(b)}{h_1(b)} - \phi_2(b) = 0\} \end{cases} \tag{6}$$

**Proof.** The first firm's expect profit $\pi_1$ is denoted by

$$\begin{aligned} \pi_1(v_1, b) &= Prob(firm\ 1\ wins)[v_1 - b] + Prob(firm\ 1\ loses) * 0 \\ &= Prob(firm\ 1\ wins)[v_1 - b] \\ &= F_2(\phi_2(b))[v_1 - b] \end{aligned} \tag{7}$$

In this paper we assume that the collusion behavior which will damage the benefit of the government is not available. Thus, the optimal bidding strategy $b^*$ must satisfy the condition as follows.

$$b^* \in argmax\{F_2(\phi_2(b))[v_1 - b]\} \tag{8}$$

Setting $\partial \pi_1(v_1, b)/\partial b = 0$, we find that

$$\frac{\partial \pi_1(v_1, b)}{\partial b} = F_2^{'}(\phi_2(b))[v_1 - b] - F_2(\phi_2(b)) = 0 \tag{9}$$

For simplicity, it assumes that $H_i(b) \equiv F_i(\phi_i(b))$ and $h_i(b) \equiv H_i^{'}(b)$. Solving the Eq.(9), we can attain the first firm's valuation function, i.e., inverse bidding function, as follows.

$$\phi_1(b) = b + \frac{H_2(b)}{h_2(b)} \tag{10}$$

As the same discuss as the first firm, the second firm's inverse bidding function is that

$$\phi_2(b) = b + \frac{H_1(b)}{h_1(b)} \tag{11}$$

In Fig.2, notice that the firm $i$'s valuation will decrease with $\varepsilon$ or $c_i$. If the firm estimates the emission permits will bring it more benefit, it would like to pay more for the emission. Otherwise, if the firm costs a lot to reach the announced quality threshold $\varepsilon$, it would like to pay less for the emission permits. Furthermore, we analyze the relation between $g_i$ and valuation. We have that $\frac{\partial F}{\partial g_i} < 0$. Therefore, the bidding strategy of the firms for the emission permits will increase with the firm's except benefit and decrease with the mandatory green standard.

## 4 Bargaining Stage

In this stage, we discuss the final trading price based on the government's equilibrium strategy, i.e., the equilibrium mandatory green standard $\varepsilon^*$. It assumes that the equilibrium trading price of the emission permits is $P$. Suppose that the distribution function of auctioning price $p$ is $G(p)$ and its density function is $g(p)$. In the emission permits auction, if there are only two firms, then $P = max(b_1, b_2)$, where $b_1$ and $b_2$ are independent from each other. Thus, $G(p)$ can be expressed by $G(p) = Prob(P \leq p) = Prob\{max(b_1, b_2) \leq p\} = F_1(\phi_1(p))F_2(\phi_2(p))$.

**Lemma 2.** *In the bargaining stage, the final trading price $p^*$ is*

$$p^* = argmax\{\alpha(\overline{b} - \int_0^{\overline{b}} G(p)dp) + (1 - \alpha)(1 - \frac{\overline{c}^2 \varepsilon^4}{16\overline{g}^2})\varepsilon\} \tag{12}$$

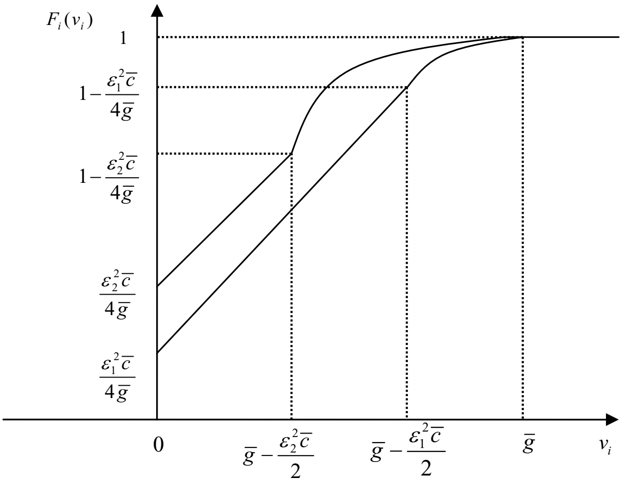


**Fig. 2.** Firm's valuation distribution as a function of $\bar{c}$ and $\varepsilon$ when $g_i$ is set as $\bar{g}$

**Proof.** When one of two firms wins the auction game, the government can achieve the expected revenue $ER$ from the auctioning activity in the emission permits auction market. $ER$ is given by

$$ER = \int_0^{\bar{b}} pg(p)dp = pG(p)|_0^{\bar{b}} - \int_0^{\bar{b}} G(p)dp = \bar{b} - \int_0^{\bar{b}} G(p)dp \qquad (13)$$

Except for the revenue from the auctioning activity, the government also pay attention to the reward from the mandatory emission standard, i.e., the reward of climate policies.

According to Eq.(3), if there is no firms participating into game, then $Prob(v_i \leq 0) = \frac{\varepsilon^2\bar{c}}{16\bar{g}^2}$. Moreover, if there is at least one participant in the auction, then the probability of this case is $1 - Prob(v_1 \leq 0)Prob(v_2 \leq 0) = 1 - \frac{\bar{c}^2\varepsilon^4}{16\bar{g}^2}$. Therefore, the mandatory emission standard $\varepsilon$ can bring the expected reward denoted by $MR$ for the government as follows.

$$MR = (1 - \frac{\bar{c}^2\varepsilon^4}{16\bar{g}^2})\varepsilon \qquad (14)$$

Furthermore, the expected benefit of the government can be achieved as follows.

$$U_G = \alpha(\bar{b} - \int_0^{\bar{b}} G(p)dp) + (1-\alpha)(1 - \frac{\bar{c}^2\varepsilon^4}{16\bar{g}^2})\varepsilon \qquad (15)$$

**Theorem 2.** *When $\alpha = 0$, the government's equilibrium strategy about green standard is $\varepsilon^* = \sqrt[4]{\frac{16\bar{g}^2}{5\bar{c}^2}}$*

**Proof.** In order to maximize the government's benefit function, we take the derivative of Eq.(15) as follows.

$$\frac{\partial U_G}{\partial \varepsilon} = \alpha(\overline{b}^{'}(\varepsilon) - G(\overline{b}(\varepsilon))\overline{b}^{'}(\varepsilon)) - (1-\alpha)(1 - \frac{5\overline{c}^2\varepsilon^4}{16\overline{g}^2}) \tag{16}$$

Set $\alpha = 0$. It means that the key concern of the government is to set mandatary green standard rate for social goals, and has little idea of the revenue of the auction. Thus, the government's equilibrium strategy about green standard is the optimal solution of Eq.(16).

**Theorem 3.** *When $\alpha = 1$, the government's equilibrium strategy about green standard is $\varepsilon^* = 0$.*

**Proof.** In the situation of $\alpha = 1$, the key concern of the government is the auction revenue of the emission permits auction. Since the higher green standard $\varepsilon$, the lower bidding price the firm will submit. Based on Eq.(16), for $b^{'}(\varepsilon) \leq 0$, the derivative $\frac{\partial U_G}{\partial \varepsilon} = \overline{b}^{'}(\varepsilon)[1 - G(\overline{b}^{'}(\varepsilon))] \leq 0$. Thus, when $\varepsilon^* = 0$, the government can get the maximum of expect revenue. The constraint of green standard becomes the incredible threat, so the valuation price of the firms will fully depend on the expect revenue of the emission permit auction, i.e., $v_i = g_i$. The same result also can be found in [7], when only considering the government's economic goals.

**Theorem 4.** *When $0 < \alpha < 1$, the government's equilibrium strategy about green standard is $\varepsilon^*$, which is the solution of*

$$\varepsilon^* = \{\varepsilon | \overline{b}(\varepsilon) - \int_0^{\overline{b}} G(p)dp + \varepsilon - \frac{\overline{c}^2\varepsilon^5}{16\overline{g}^2} = 0\} \tag{17}$$

**Proof.** Based on Eq.(15), we can get the optimal mandatory green standard $\varepsilon^*$. This means that the increasing of the government's total benefit is generated by economic revenue and social reward. The optimal point is that the increase of economic revenue is equal to the reduction of social reward. After designing the weight of the economic revenue and that of social reward about mandatory green standard, the government achieves its equilibrium strategy as follows.

$$\frac{\partial U_G^*}{\partial \alpha} = (\overline{b}(\varepsilon) - \int_0^{\overline{b}} G(p)dp) + (\varepsilon - \frac{\overline{c}^2\varepsilon^5}{16\overline{g}^2}) = 0 \tag{18}$$

The derivative $\partial U_G^*/\partial\alpha$ determines the optimal green standard $\varepsilon^*$. We need to choose the optimal $\alpha^*$ in Eq.(18) and then achieve $\varepsilon^*$. Notice that in the process of $\alpha$ increase from 0 to 1, the goal of choosing the weight is to achieve the target that the government both focus on maximizing the revenue and setting the green standard.

**Corollary 1.** If there is no bargaining stage, the final trading price of emission permits is only based on the private information of $c_i$. A lower equilibrium price always exists (see detail in [11]).

**Corollary 2.** If there are two stages, the optimal bidding strategy $b_i^*$ in the auction stage and the optimal mandatory green standard $\varepsilon^*$ in the bargaining stage constitute a subgame perfect Nash equilibrium.

## 5 Conclusion

For initial emission permits auction problem, this paper introduces a hybrid form, auction-bargaining mechanism, into the traditional emission permits auction market. We highlight the strategic interaction between the government's economic actions and social actions in the auction-bargaining model, which can explain how the government's economic and social goals effect the equilibrium strategies. We also find that mandatory green standard will be the equilibrium strategy in some circumstances. It concludes that this hybrid mechanism can alleviate distortion by excessive allowance in initial emission permits auction market. This result can be applied to a more general question regarding the choice between economic goals and social goals. We make a number of simplifications, e.g., the number of bidders and the distribution form, and future studies can relax these assumptions.

## References

1. Montgomery, W.D.: Markets in Licenses and Efficient Pollution Control Programs. Journal of Economic Theory 5(3), 395–418 (1972)
2. Rao, C.J., Zhao, Y., Li, C.F.: Asymmetric Nash Equilibrium in Emission Rights Auctions. Technological Forecasting & Social Change 79, 429–435 (2012)
3. Benz, E., Loschel, A., Sturm, B.: Auctioning of $CO_2$ Emission Allowances in Phase 3 of the EU Emissions Trading Scheme. Climate Policy 10(6), 705–718 (2010)
4. Porter, D., Rassenti, S., Shobe, W.: The Design, Testing and Implementation of Virginias NOx Allowance Auction. Journal of Economic Behavior & Organization 69(2), 190–200 (2009)
5. Genc, T.S.: Discriminatory Versus Uniform-price Electricity Auctions with Supply Function Equilibrium. J. Optim. Theory Appl. 140, 9–31 (2009)
6. Cramton, P., Kerr, S.: Tradeable Carbon Permit Auctions: How and Why to Auction not Grandfather. Energy Policy 30(4), 333–345 (2002)
7. Betz, R., Seifert, S., Cramton, P., Kerr, S.: Auctioning Greenhous Gas Emissions Permits in Australia. Aust. J. Agric. Resour. Econ. 54(2), 219–238 (2010)
8. Lai, Y.B.: Auctions or Grandfathering: the Political Economy of Tradable Emission Permits. Public Choice 136(1-2), 181–200 (2008)
9. Ausubel, L.M.: An Efficient Ascending-bid Auction for Multiple Objects. The American Economic Review 94(5), 1452–1475 (2004)
10. Mougeot, M., Naegelen, F., Pelloux, B., et al.: Breaking Collusion in Auctions through Speculation: An Experiment on CO2 Emission Permit Markets. Journal of Public Economic Theory 13(5), 829–856 (2011)
11. Yu, Y.: An Optimal Ad Valorem Tax/Subsidy with an Output-Based Refunded Emission Payment for Permits Auction in an Oligopoly Market. Environmental and Resource Economics 52(2), 235–248 (2012)

12. Sunnevag, K.J.: Auction Design for the Allocation of Emission Permits in the Presence of Market Power. Environmental and Resource Economics 26(3), 385–400 (2003)
13. Goeree, J.K., Palmer, K., Holt, C.A.: An Experimental Study of Auctions versus Grandfathering to Assign Pollution Permits. Journal of the European Economic Association 8(2-3), 514–525 (2010)
14. Jensen, S.G., Skytte, K.: Simultaneous Attainment of Energy Goals by Means of Green Certificates and Emission Permits. Energy Policy 31(1), 63–71 (2003)
15. Harstad, B., Eskeland, G.S.: Trading for the Future: Signaling in Permit Markets. Energy Policy 94(9), 749–760 (2010)