Modeling Packet Traffic with the Use of Superpositions of Two-State MMPPs

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Abstract. The aim of this paper is to use the superposition of two-state Markov Modulated Poisson Processes to replicate the statistical nature of internet traffic over several time scales. This paper characterizes of network traffic using Bellcore data and LAN traces collected in IITiS PAN. The fitting procedure for matching second-order self-similar properties of real data traces to that of two-state MMPP's has also been described.

Keywords: two-State MMPPs, Markov Modulated Poisson Processes.

1 Introduction

The growing variety of IP networks services and applications has been resulting in an increase of requirement to make reliable measurement of packet flows and to describe them though appropriate traffic models.

Traditionally, the traffic intensity, has been regarded as a stochastic process, was represented in queueing models by short term dependencies [1]. However, the analysis of measurements shows that the traffic establishes also long-terms dependencies and has a self-similar character. The problem of self-similarity has been described in Sect. 2. This feature of a traffic was observed on various protocol layers and in different network structures [2–7].

Several models have been introduced for the purposes of modeling self-similar processes in the network traffic area. Pioneering work in [6] motivated other researchers [8] to model network traffic using fractional Brownian Motion. On-Off source model [9] provides an opportunity for different model based on Stable Levy Motion [10] depending on order of consideration of limits. Other models of traffic sources use chaotic maps [11], α-stable distribution [12], fractional Autoregressive Integrated Moving Average (fARIMA) [13] and fractional Levy Motion [14] for modeling of network traffic. All above mentioned traffic models are based on non-Markovian approach. The advantage of these models is that

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they give a good description of the traffic with the use of few parameters. Their drawbacks consist in the fact that they do not allow the use of traditional and well known queueing models and modeling techniques for computer networks performance analysis.

Some researchers use Markov based models to generate a self-similar traffic over a finite number of time scales [15–20]. This approach makes possible the adaptation of traditional Markovian queueing models to evaluate network performance. In our research we have made a fitting of a superposition of two state Markov Modulated Poisson Process (MMPP) proposed in [21] to real traffic data. We have used Ethernet traffic data of Bellcore Morristown Research and Engineering facility which is available for research purpose [22] and data captured in our Institute LAN [23].

The rest of this article is organized as follows. Section 2 briefly describes the mathematics underlying the theory of self-similarity. Section 3 describes the MMPP traffic model used in our study. Section 4 explains the fitting procedure for matching second-order self-similar properties of observed data traces. Obtained results are presented in Sect. 5. Conclusions about the work are drawn in Sect. 6.

2 Self-similarity of Internet Traffic

The term self-similar was introduced by Mandelbrot [24] for explaining water level pattern of river Nile observed by H. Hurst. This term was also known as *Hurst Effect*. The degree of self-similarity is expressed by *Hurst parameter*, denoted by H.

A real valued stochastic process:

$$X = \{X(t)\}_{t \in R}$$

is self-similar with $H > 0$, if for any $a > 0$,

$$\{X(at)\}_{t \in R} \stackrel{d}{=} \{a^H X(t)\}_{t \in R}$$

where $\stackrel{d}{=}$ denotes equality in finite dimensional distribution sense [25]. Above definition evidently shows that self-similar processes are scale invariant.

Mathematically, the difference between short-range dependent processes and long-range ones (self-similar) is as follows [26]:

For a short-range dependent process:
- $\sum_{r=0}^{\infty} \mathrm{Cov}(X_t, X_{t+\tau})$ is convergent,
- spectrum at $\omega = 0$ is finite,
- for large m, $\mathrm{Var}\,(X_k^{(m)})$ is asymptotically of the form $\mathrm{Var}\,(X)/m$,
- the aggregated process $X_k^{(m)}$ tends to the second order pure noise as $m \to \infty$;

For a long-range dependent process:
- $\sum_{r=0}^{\infty} \mathrm{Cov}(X_t, X_{t+\tau})$ is divergent,

- spectrum at $\omega = 0$ is singular,
- for large m, $\mathrm{Var}\,(X_k^{(m)})$ is asymptotically of the form $\mathrm{Var}\,(X)m^{-\beta}$,
- the aggregated process $X_k^{(m)}$ does not tend to the second order pure noise as $m \to \infty$,

where the spectrum of the process is the Fourier transformation of the autocorrelation function and the aggregated process $X_k^{(m)}$ is the average of X_t on the interval m:

$$X_k^{(m)} = \frac{1}{m}(X_{km-m+1} + \ldots + X_{km}) \qquad k \geq 1 \ .$$

Estimation of Hurst parameter is the most frequently used method to check if a process is self-similar: for non-self-simlar processes $H = 0.5$; for $0.5 < H < 1$ process is self-similar; the closer H is to 1, the greater the degree of persistence of long-range dependence. Hurst parameter H can be estimated by various methods. One of the method is called *Aggregated Variance* [27, 25], where aggregated sequence is created (as discussed above). This method is based on the analysis of variance-time plot. The variation of aggregated self-similar process is equal to:

$$Var(X_k^{(m)}) = Var(X)m^{-\beta}$$

so the log-log plot of $\frac{Var(X_k^{(m)})}{Var(X)}$ versus m is a line with slope β.

Another technique of estimation is called $R\!-\!SPlot$, which is based on *Central Limit Theorem*. Both the above techniques are time-domain based; in frequency domain one can estimate H using *Periodogram* in log-log scale.

As mentioned earlier, many empirical and theoretical studies have shown the self-similar characteristics of the network traffic. These features have a great impact on a network performance. They enlarge mean queue lengths at buffers and increase packet loss probability, reducing this way the quality of services provided by a network [28]. That is why it is necessary to take into account this feature when you want to create a realistic model of traffic sources [16].

3 MMPP Model of Packet Traffic Source

Markov chains and Markov-modulated processes (MMP) are well-known modeling techniques which are successful in wide variety of fields. These models are often motivated by the idea of capturing the long-range dependence (LRD) which is seen in real internet traffic and replicating the the Hurst parameter H which characterizes LRD [17].

Two-state Markov Modulated Poisson Process (MMPP) is also known as the Switched Poisson Process (SPP). The superposition of MMPP's is also an MMPP which is a special case of Markovian Arrival Process (MAP).

A MAP is defined by two square matrices $\mathbf{D_0}$ and $\mathbf{D_1}$ such that $\mathbf{Q} = \mathbf{D_0} + \mathbf{D_1}$ is an irreducible infinitesimal generator for the continuous-time Markov chain (CTMC) underlying the process, and $D_0(i,j)$ (respectively $D_1(i,j)$) is the rate of hidden (respectively observable) transitions from state i to state j [29]. Two-state MAP is a Markovian arrival process with square matrices as follows:

$$\mathbf{D_0} = \begin{bmatrix} -\sigma_1 & \lambda_{1,2} \\ \lambda_{2,1} & -\sigma_2 \end{bmatrix}$$

$$\mathbf{D_1} = \begin{bmatrix} \mu_{1,1} & \mu_{1,2} \\ \mu_{2,1} & \mu_{2,2} \end{bmatrix}$$

where $\lambda_{i,j} \geq 0$, $\mu_{i,j} \geq 0$, for all i, j. The diagonal elements of matrix $\mathbf{D_0}$ are $\sigma_1 = \lambda_{1,2} + \mu_{1,1} + \mu_{1,2} > 0$ and $\sigma_2 = \lambda_{2,1} + \mu_{2,2} + \mu_{2,1} > 0$ such that underlying continuous-time Markov chain Matrix $\mathbf{Q}$ has no absorbing states.

Following the model proposed in [21], a LRD process (used in our study) can be modeled as the superposition of d two-state MMPPs. The i-th MMPP $(1 \leq i \leq d)$ can be parameterized by two square matrices:

$$\mathbf{D_0^i} = \begin{bmatrix} -(c_{1i} + \lambda_{1i}) & c_{1i} \\ c_{2i} & -(c_{2i} + \lambda_{2i}) \end{bmatrix}$$

$$\mathbf{D_1^i} = \begin{bmatrix} \lambda_{1i} & 0 \\ 0 & \lambda_{2i} \end{bmatrix} \quad .$$

The element c_{1i} is the transition rate from state 1 to 2 of the i-th MMPP and c_{2i} is the rate out of state 2 to 1. λ_{1i} and λ_{2i} are the traffic rate when the i-th MMPP is in state 1 and 2 respectively. The sum of $\mathbf{D_0}^i$ and $\mathbf{D_1}^i$ is an irreducible infinitesimal generator $\mathbf{Q}^i$ with the stationary probability vector:

$$\overrightarrow{\pi}_i = \left(\frac{c_{2i}}{c_{1i} + c_{2i}}, \frac{c_{1i}}{c_{1i} + c_{2i}} \right)$$

The superposition of these two-state MMPPs is a new MMPP with 2^d states and its parameter matrices, $\mathbf{D_0}$ and $\mathbf{D_1}$, can be computed using the Kronecker sum of those of the d two-state MMPPs [30]:

$$(\mathbf{D}_0, \mathbf{D}_1) = \left(\oplus_{i=1}^{d} \mathbf{D_0}^i, \oplus_{i=1}^{d} \mathbf{D_1}^i \right) \quad .$$

Let N_t^i be a number of arrivals from the i-th MMPP in time slot $(0, t]$. The variance time for this MMPP can be expressed as:

$$Var\{N_t^i\} = (\lambda_i^* + 2k_{1i})t - \frac{2k_{1i}}{k_{2i}}(1 - e^{-k_{2i}t})$$

where:

$$\lambda_i^* = \frac{c_{2i}\lambda_{1i} + c_{1i}\lambda_{2i}}{c_{1i} + c_{2i}}$$

$$k_{1i} = (\lambda_{1i} - \lambda_{2i})^2 \frac{c_{1i}c_{2i}}{(c_{1i} + c_{2i})^3}$$

$$k_{2i} = c_{1i} + c_{2i}$$

The second-order properties are determined by this three entities: λ_i^*, k_{1i} and k_{2i}. The covariance function of the number of arrivals in two time slots of size Δt is expressed by [21]:

$$\gamma_i(k) = \frac{(\lambda_{1i} - \lambda_{2i})^2 c_{1i} c_{2i} e^{-((c_{1i} c_{2i})(k-1)\Delta t)}}{(c_{1i} + c_{2i})^4} * \left(1 - 2e^{-((c_{1i}+c_{2i})\Delta t)} + e^{-((c_{1i}+c_{2i})2\Delta t)}\right) =$$

$$= \frac{k_{1i}}{k_{2i}} e^{-(k_{2i}(k-1)\Delta t)} * \left(1 - 2e^{(-k_{2i}\Delta t)} + e^{(-k_{2i}2\Delta t)}\right) \approx$$

$$\approx \frac{(\Delta t)^2 (\lambda_{1i} - \lambda_{2i})^2 c_{1i} c_{2i} e^{-((c_{1i}+c_{2i})(k-1)\Delta t)}}{(c_{1i} + (c_{2i})^2} = (\Delta t)^2 k_{1i} k_{2i} e^{-(k_{2i}(k-1)\Delta t)} \;.$$

4 Fitting Procedure for the Self-similar Covariance Structure

There are two approaches for fitting the family of MAP to observed data: moment-based approach and likelihood-based approach [31]. The main advantage of moment-based approaches is to reduce computational cost. In such approaches one determines the MAP model parameters in order to fit theoretical moments to empirical ones obtained from observed traffic.

The article [21] proposed a fitting method for a superposition of two-state MAPs (described in Sect. 3) based on Hurst parameter as well as the moments.

For real traffic traces the covariance structure of the counting process is well described by the asymptotic covariance [21]:

$$\mathrm{cov}(k) = \psi_{\mathrm{cov}} k^{-\beta}$$

where ψ_{cov} jest an absolute measure of the variance, $\beta = 2 - 2H$ and k is the lag. The parameters ψ_{cov} and β should be estimated from the real data traces. The objective of the fitting is to achieve:

$$\gamma(k) = \sum_{i=1}^{d} \gamma_i(k) \approx \psi_{\mathrm{cov}} k^{-\beta}$$

where $1 \le k \le 10^n$ and n denotes the number of time scales the model demonstrate self-similar behavior. The fitting procedure requires the following input parameters:

λ^* – the mean rate of the process to be modeled,
n – number of time scales,
d^* – number of active MMPP's,
$H = 1 - \frac{\beta}{2}$ – the Hurst parameter,
ρ – lag 1 correlation.

The modulating parameters of the MMPP's have been chosen logarithmically with a factor a:

$$c_{1i} = c_{2i} = a^{1-i} c_{11}$$

for $i = 1, \ldots, d$. The smallest time scale should relate to the packet level so the fundamental rate has been assumed relative to this time scale: between 1 and 10 [21]. To achieve this, the modulating parameters $c_{11} = c_{21}$ have been initially chosen in the range $[0.25, 0.75]$ (most often to the value 0.4). The parameter a is dependent on the number of active MMPP's and number of time scales:

$$a = 10^{\frac{n}{d-1}} \ .$$

As was mentioned in Sect. 3, the second-order properties of a superposition of d MMPPs are determined by this three entities: λ_i^*, k_{1i} and k_{2i}. The arrival intensities λ_{1i} and λ_{2i} are only involved in k_{1i} through the quantity:

$$(\lambda_{1i} - \lambda_{2i})^2 \ .$$

It is only possible to interpret the superposition of d MMPP's as a superposition of d Interrupted Poisson Process (IPP's) and a Poisson process [21]. With this interpretation, the IPP's have arrival intensity:

$$\lambda_i^{\mathrm{IPP}} = \lambda_{1i} - \lambda_{2i} \ .$$

The Poisson intensity could be determined as:

$$\lambda_p = \sum_{i=1}^{d} \lambda_{2i} \ .$$

The fitting procedure requires the following steps:

1. give to the variable β the value: $2 - 2H$,
2. give to the number of IPP's d the value: d^*,
3. give to the variable k_{21} the value: $c_{11} + c_{21}$ (see Sect. 3),
4. give to the variable d_0 (the number of ϕ_j's which were set to 0) the value 0,
5. give to the intensity ϕ_d the value 1,
6. give to the variable i the value 1,
7. give to the logarithmic spacing parameter a the value:

$$10^{\frac{n}{d-1}} \ ,$$

8. give to the variable D the value:

$$a^{i\beta} - \sum_{j=0}^{i-1} (\phi_{d-j})^2 e^{1-a^{-(i-j)}} \ ,$$

9. **if** $D < 0$ **then**:
   give to the intensity ϕ_{d-i} the value 0,
   increment a value d_0 by one,
   if $d^* > d - d_0$ **then**: increment a value d by one and **go to** the step 4,
   else: give to the intensity ϕ_{d-i} the value $\sqrt{D}$,
10. increment a value i by one,
11. **if** $i <> d$ then: **go to** the step 8,
12. **for** $i = 2$ to d **do** $k_{2i} = a^{1-i} k_{21}$,
13. **if not** $(k_{2i} < 1 \rho < 0.5)$ **then**: if necessary adjust k_{2i} and/or ρ,
14. give to the variable η the value:

$$\frac{\sqrt{4\rho\lambda^*}}{\sqrt{\sum_{i=1}^{d} \phi_i^2 k_{2i}^{-2} \left((1 - e^{-k_{2i}})^2 - 2\rho(k_{2i} - (1 - e^{-k_{2i}})\right))}} ,$$

15. give to the variable L the value:

$$\eta \sum_{i=1}^{d} \frac{\phi_i}{2},$$

16. **if** $\lambda^* < L$ **then**:
   give to the Poisson intensity λ_P the value 0,
   for $i = 1$ to d **do**:
      give to the model parameter c_{1i} the value:

$$\frac{L^2}{\lambda^{*2} + L^2} k_{2i} \ ,$$

      give to the model parameter c_{2i} the value $k_{2i} - c_{1i}$,
      give to the arrival intensity λ_i^{IPP} the value:

$$\phi_i \frac{\lambda^{*2} + L^2}{\lambda^* \sum_{i=1}^{d} \phi_i},$$

   else:
   give to the Poisson intensity λ_P the value $\lambda^* - L$,
   for $i = 1$ to d **do**:
      give to the model parameters c_{1i} and c_{2i} the value $0.5 k_{2i}$,
      give to the arrival intensity λ_i^{IPP} the value $\eta\phi_i$.

5 Results

As was mentioned in Sect. 4 fitting algorithm requires five input parameters. Three of them: λ^* (the mean rate of the process), ρ (lag 1 correlation) and the Hurst parameter H should be estimated from the real data traces. Real traces used in our study comprises of Ethernet traffic data of Bellcore Laboratory [22] and data captured on the gateway of the Institute of Theoretical and Applied Informatics (IITiS) of the Polish Academy of Science in Gliwice (Poland).

Bellcore Laboratory data was already interpreted in multiple studies [6, 32, 25]. A dedicated hardware has been built for measuring each packet arrival and the measurements were performed without losses and with high precision. The large part of collected data is available by Internet [22]. In this study we use the file: OctExt.TL. It contains the first million of external arrivals gathered during 35 hours.

Other data set used in our study has been collected during the whole May 2012 on the Internet gateway of our Institute [23]. The traffic approximately stands for the few dozen office users (researchers), mainly working Monday–Friday 8AM–4PM. During May 1–3, there are national holidays in Poland, so the traffic can be smaller. IP packets were limited to 64 bytes – for most cases the they contain all headers, plus a few bytes of the transport protocol payload. The local DNS traffic is not visible, because of specific setup of the network.

The Hurst parameter was estimated by the *Aggregated Variance* method (see Sect. 2). Figure 1 presents the normalized variance of the aggregated series as a function of time scale in log-log coordinates. The slope of IITiS curve (estimated by the least squares method) is equal to -0.42, which gives the Hurst parameter equal to 0.79. The slope of Bellcore curve is equal to -0.3, which gives the Hurst parameter equal to 0.85. For comparison, the same plot is also drawn for the Poisson process. This line has the slope -1, which gives the Hurst parameter equal to 0.5 (non-self-similar process).

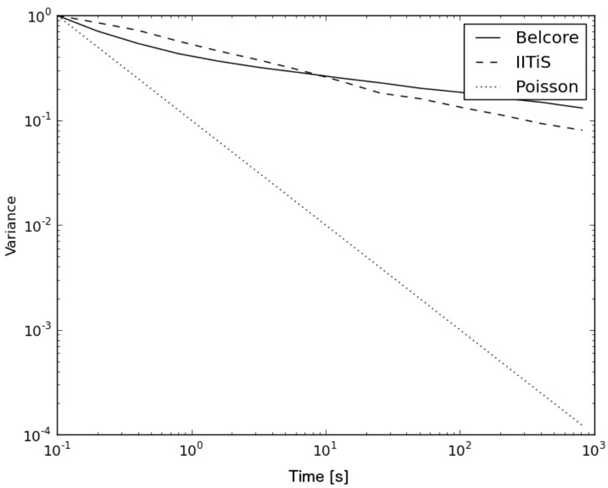

Fig. 1. Variance-time plot; log-log scale (IITiS data, May 2012)

As was mentioned earlier, the superposition of two-state MMPP's can be applied to model traffic exhibiting LRD traffic over a number of time scales. For LRD processes, the autocovariance decays hyperbolically. Any asymptotically second-order self-similar process exhibits LRD properties. Each MMPPs models a specific time scale of data, the volume of traffic modeled by each of the single two-state sources can be associated with the volume of traffic showing variability on a given time scale [21]. Figure 2 shows how the autocorrelations of the three IPP's behave as a function of lag k. The parameters defining this model are:

$$\lambda_1^{\mathrm{IPP}} = \lambda_2^{\mathrm{IPP}} = \lambda_3^{\mathrm{IPP}} = 6.0$$
$$c_{11} = c_{21} = 10^{-2}$$
$$c_{12} = c_{22} = 10^{-4}$$
$$c_{13} = c_{23} = 10^{-6}$$

Figure 3 shows the autocorrelations of the five IPP's. The parameters defining this model are:

$$\lambda_1^{\mathrm{IPP}} = \lambda_2^{\mathrm{IPP}} = \lambda_3^{\mathrm{IPP}} = \lambda_4^{\mathrm{IPP}} = \lambda_5^{\mathrm{IPP}} = 6.0$$
$$c_{11} = c_{21} = 10^{-2}$$
$$c_{12} = c_{22} = 10^{-\frac{7}{2}}$$
$$c_{13} = c_{23} = 10^{-5}$$
$$c_{14} = c_{24} = 10^{-\frac{13}{2}}$$
$$c_{15} = c_{25} = 10^{-8}$$

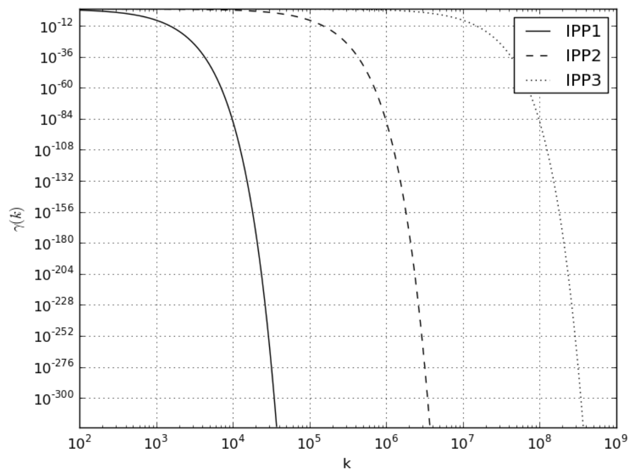

Fig. 2. Autocorrelation of the number of arrivals in a time unit (three IPP's)

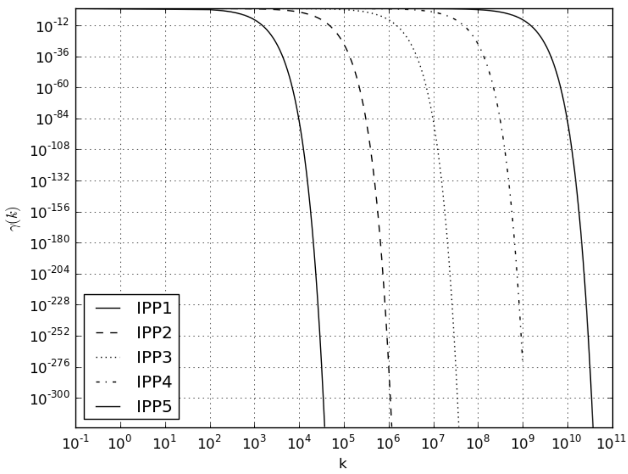

Fig. 3. Autocorrelation of the number of arrivals in a time unit (five IPP's)

Tables 1, 2 and 3 present the parameters obtained from the fitting approach described in Sect. 4. The input parameters defining the asymptotic second-order self-similarity were selected as an example.

Table 1. Obtained parameters of source fitted to second-order self-similarity with input parameters: $d = 5$, $n = 6$, $\lambda^* = 3.5$, $H = 0.6$ and $\rho = 0.6$

	λ_i^{IPP}	c_{1i}	c_{2i}
IPP_1	27.646	7.241×10^{-1}	7.590×10^{-2}
IPP_2	6.944	2.290×10^{-2}	2.400×10^{-3}
IPP_3	1.746	7.241×10^{-4}	7.590×10^{-5}
IPP_4	0.434	2.290×10^{-5}	2.400×10^{-6}
IPP_5	0.119	7.241×10^{-7}	7.590×10^{-8}
Poisson	$\lambda_p = 0$		

The fitting procedure described in Sect. 4 was also applied to the traces of IP traffic measured at Bellcore and IITiS. The parameters of the superposition of two-state MMPP's was fitted to these obtained from real data traffic. Table 4 gives the parameters defining the model fitted using the set of descriptors obtained from Bellcore trace. The superposition of four MMPP's is sufficient to model asymptotic second-order self-similarity of the counting process over five time-scales. Table 5 gives the parameters obtained for IITiS data traffic traces. For these trace we have used also the superposition of four MMPP's.

Table 2. Obtained parameters of source fitted to second-order self-similarity with input parameters: $d = 5$, $n = 6$, $\lambda^* = 3.5$, $H = 0.75$ and $\rho = 0.6$

	λ_i^{IPP}	c_{1i}	c_{2i}
IPP_1	13.634	6.797×10^{-1}	1.203×10^{-1}
IPP_2	5.701	2.150×10^{-2}	3.803×10^{-3}
IPP_3	2.475	6.797×10^{-4}	1.203×10^{-4}
IPP_4	0.933	2.150×10^{-5}	3.803×10^{-6}
IPP_5	0.540	6.797×10^{-7}	1.203×10^{-7}
Poisson	$\lambda_p = 0$		

Table 3. Obtained parameters of source fitted to second-order self-similarity with input parameters: $d = 5$, $n = 7$, $\lambda^* = 3.5$, $H = 0.9$ and $\rho = 0.5$

	λ_i^{IPP}	c_{1i}	c_{2i}
IPP_1	3.628	5.431×10^{-1}	2.569×10^{-1}
IPP_2	2.571	2.162×10^{-2}	1.022×10^{-2}
IPP_3	1.913	8.60×10^{-4}	4.071×10^{-4}
IPP_4	1.363	3.427×10^{-5}	1.621×10^{-5}
IPP_5	1.424	5.431×10^{-7}	2.569×10^{-7}
Poisson	$\lambda_p = 0$		

Table 4. Obtained parameters of source fitted to the correlation structure of Bellcore data (input parameters: $d = 4$, $n = 5$, $\lambda^* = 6.3$, $H = 0.85$ and $\rho = 0.15$)

	λ_i^{IPP}	c_{1i}	c_{2i}
IPP_1	2.062	4×10^{-1}	4×10^{-1}
IPP_2	1.469	8.618×10^{-3}	8.618×10^{-3}
IPP_3	0.427	1.857×10^{-4}	1.857×10^{-4}
IPP_4	0.602	4×10^{-6}	4×10^{-6}
Poisson	$\lambda_p = 4.020$		

Table 5. Obtained parameters of source fitted to the correlation structure of IITiS data (input parameters: $d = 4$, $n = 5$, $\lambda^* = 9.85$, $H = 0.79$ and $\rho = 0.0213$)

	λ_i^{IPP}	c_{1i}	c_{2i}
IPP_1	1.086	4×10^{-1}	4×10^{-1}
IPP_2	0.508	8.618×10^{-3}	8.618×10^{-3}
IPP_3	0.194	1.857×10^{-4}	1.857×10^{-4}
IPP_4	0.126	4×10^{-6}	4×10^{-6}
Poisson	$\lambda_p = 8.893$		

6 Conclusions

This paper illustrates how a superposition of two-state MMPP's can be fitted to a rate and variance time curve of measured traffic traces. The fitting algorithm for matching asymptotic second-order self-similarity was described in detail. This article also shows how to obtain the set of descriptors from real traffic traces using Bellcore Morristown Laboratory data and IITiS traffic trace. Our future works will focus on the fitting to additional real traffic descriptors besides second-order properties of the counting process. We will also try to use other methods of calculating the Hurst parameter of real data.

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