# Chapter 9
# The Zero-Point Field Waves (and) Matter

*Students should not be taught to doubt that electrons, protons and the like are particles... The waves cannot be observed in any way than by observing particles.*

Mott (1964)

*The electron is either here, or there, or somewhere else, but wherever it is, it is a point charge.*

Feynman et al. (1965)

So far in our exploration of the fundamentals of QM we have paid null attention to the very concept that gave rise to wave mechanics, the de Broglie wavelength and the associated undulatory behavior of matter—certainly one of the most mysterious properties of the quantum world. The notion of the quantum corpuscle as something that possesses intrinsic wave properties, which preclude the possibility of describing it as a localized entity, is widely extended. What we intend to show in the present chapter, by contrast, is that the fundamental wave properties associated with quantum particles can be understood without renouncing the notion of localized corpuscles.

Of course, formal manipulations of the results obtained in previous chapters allow to ascertain the wave content of quantum mechanics and eventually arrive at de Broglie's wavelength. But such procedure would appear to reduce it to a mere mathematical artifact, without providing a clue about its physical content, and more deeply, without throwing light about the nature of the de Broglie wave. It seems therefore obligatory to pay closer attention to this most significant entity.

In line with the spirit of the theory exposed in the present volume, the ZPF should be expected to play an important role in the elucidation of de Broglie's wave, and more generally in the explanation of the undulatory properties of matter. It would be even surprising if the ZPF did not in some way or another impress its wave properties on the particles embedded in it. However, such possibility has only been occasionally explored within SED; therefore, this chapter contains the results of some of the initial

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investigations into the territory of the quantum waves, which obviously deserves further exploration.[1]

## 9.1 Genesis of de Broglie's Wave

We recall that in de Broglie's theory of matter waves,[2] an oscillation of Compton's frequency

$$\omega_C = \frac{m_0 c^2}{\hbar} \tag{9.1}$$

is associated with a corpuscle at rest ($m_0$ stands for the rest mass). If the particle moves with respect to the laboratory with velocity $v$ along some axis, the frequency $\omega$ in this latter reference frame is Doppler-shifted according to the formula

$$\omega = \gamma\omega_C(1+\beta), \tag{9.2}$$

with

$$\gamma = \left(1-\beta^2\right)^{-1/2}, \quad \beta = v/c. \tag{9.3}$$

The shift $\gamma\beta\omega_C$ in Eq. (9.2) can be rewritten as ($m = \gamma m_0$)

$$\gamma\beta\omega_C \equiv \omega_B = 2\pi c\frac{mv}{h} = \frac{2\pi c}{\lambda_B}, \tag{9.4}$$

and therefore

$$\lambda_B = \frac{h}{mv} = \frac{h}{p}. \tag{9.5}$$

Equation (9.5) is the well-known expression for the de Broglie wavelength, which originates in the Doppler shift of the frequency $\omega_C$. In de Broglie's theory, a physical

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[1] Previous versions of the material presented in the first part of this chapter can be found in de la Peña and Cetto (1992, 1994), Cetto and de la Peña (1955a, b), and *The Dice*.

[2] Detailed, first-hand expositions of de Broglie's theory can be found in de Broglie (1926, 1956, 1963). Modern presentations by one of its advocates made in Selleri (1990). A most elaborate development of a variant of de Broglie's theory for the relativistic electron is the *geometrical mechanics* developed by Synge (1954). An informed historical discussion of de Broglie's work up to the 1927 Solvay conference is given in Bacciagaluppi and Valentini (2009). MacKinnon (1976) presents a detailed analysis and improvement of de Broglie's derivation in his thesis. Another detailed discussion of de Broglie's phase waves is presented in Espinosa (1982).

wave with the wavelength $\lambda_B$ becomes a central entity, directly related with the moving particle; yet the nature of such wave remains unspecified.[3]

Within the quantum formalism it is customary to introduce the expression (9.5) as a means to assign wave properties to the quantum corpuscle. Practical applications of the de Broglie wavelength are contained in almost any textbook, largely in the form of restrictions on $\lambda_B$ associated with atomic stationarity conditions, from which (quantized) spectra are extracted. De Broglie's wavelength appears also in connection with particle diffraction patterns, notably the electron equivalent of Young's double-slit experiment, and in the optics of electron microscopy. However, discussions on the nature and origin of the de Broglie *wave* (not just the wavelength $\lambda_B$) are found only rarely. In the following sections we dig into such matters, with the intention to throw some light on the concept of de Broglie's wave.

### *9.1.1 The de Broglie 'Clock'*

The first point that deserves attention in any attempt to understand the de Broglie wave relates to the physical origin of the oscillations of frequency $\omega_C$ associated with the particle in its rest frame, which constitute a sort of 'clock' in de Broglie's theory. In this regard we recall that according to QED (see e.g. Milonni 1994, Chap. 11), the interaction of an electron with the electromagnetic vacuum dresses the particle and endows it with an effective size, estimated between $(\lambda_C r_c)^{1/2} = \lambda_C(\alpha/2\pi)^{1/2} \simeq \lambda_C/30$ and $\lambda_C$, where $r_c = e^2/(mc^2) = (\alpha/2\pi)\lambda_C$ is the classical electron radius and $\lambda_C$ is the Compton wavelength

$$\lambda_C = \frac{2\pi c}{\omega_C} = \frac{h}{m_0 c}. \tag{9.6}$$

In terms of $\lambda_C$, Eq. (9.5) takes the form

$$\lambda_B = \frac{\lambda_C}{\gamma\beta} = \lambda_C\sqrt{\frac{c^2}{v^2} - 1}, \tag{9.7}$$

which means that for nonrelativistic motions $\lambda_B$ is usually much larger than Compton's wavelength.

From the point of view of SED, it is also natural to consider the charged particle immersed in the vacuum field as endowed with an effective size of the order of the Compton wavelength $\lambda_C$.[4] As a result, the particle decouples from the components

[3] In Surdin (1979) it is proposed to consider that de Broglie's wave is of electromagnetic nature, in some undefined way associated with the electromagnetic ZPF.

[4] A crude way to reach the same conclusion is the following. From the Heisenberg inequality one obtains $\sigma_x^2 \geq (\hbar^2/4\sigma_p^2)$, whence the minimum dispersion in the position variable determines an

of the radiation field with wavelengths smaller than $\lambda_C$ (and frequencies larger than $\omega_C$), so that the Compton frequency appears as a cutoff frequency. Any specific model for the charge with structure (real or effective) would be arbitrary at this stage, but also unnecessary, since our present purpose is limited to the introduction of the appropriate cutoff, which we accept to be of the order of $\omega_C$.

The characteristic equation of motion for a free particle with structure (real or effective) acted on by the ZPF and radiation reaction has complex roots, giving rise to oscillations of a very high frequency.[5] This frequency is determined basically by the size of the particle rather than by the details of its structure, so the phenomenon is quite general; for an (effective) radius of order $\lambda_C$ the frequency is of the order of $\omega_C$. In a classical context, these high-frequency oscillations are transient, related to initial motions, momentary disturbances and the like. However, when the particle is in permanent interaction with the random background field, as is the present case, things change essentially. The electromagnetic environment not only puts the particle into resonance and makes it radiate, but it is also constantly knocking the particle, so that the high-frequency oscillations become continuously renewed and acquire a permanent (though fluctuating) character. It is appealing to identify these fine oscillations of frequency $\omega_C$ with the zitterbewegung, of which we have here an informal rendering.

In short, even if the particle is initially conceived of as pointlike—which sounds somewhat extreme for a physical, rather than mathematical element— it behaves as an object with some structure that performs, in addition to any other motion, a sustained oscillation with a frequency of about $\omega_C$. In this way the vacuum field provides the physical sustenance for the de Broglie clock.

Because of its oscillating behavior, the particle at rest is continuously radiating at the frequency $\omega_C$, a process that in a stationary state must be compensated by absorption from the vacuum field. This means that the particle interacts intensely with the modes of frequency $\omega_C$, as measured in its proper frame, and that these modes sustain the jitter. The specific mechanism of this interaction is irrelevant for the kinematics that follow; what is important is that the particle interacts selectively with a narrow band of modes of the field of frequencies around $\omega_C$.

Let us assume for simplicity that the particle motion is restricted to one dimension, along some axis $\hat{\boldsymbol{x}}'$.This means that in its proper frame (denoted with $S'$) the components of the ZPF of interest are the two plane waves of frequency $\omega_C$ travelling in opposite directions. The resulting (standing) wave is thus the superposition

$$\varphi'(x', t') = e^{-i(\omega_C t' - \boldsymbol{k}'_+ \cdot \boldsymbol{x}' + \theta_+)} + e^{-i(\omega_C t' - \boldsymbol{k}'_- \cdot \boldsymbol{x}' + \theta_-)} + \text{c.c.}, \tag{9.8}$$

(Footnote 4 continued)
effective radius $a \sim (\sigma_x)_{\min}$. Such minimum value is achieved for the largest $\sigma_p^2$, which in the nonrelativistic regime can be limited by $m_0^2 c^2$. This results in $a \sim (\hbar/m_0 c)$.

[5] A detailed discussion can be seen in de la Peña et al. (1982), and *The Dice*, Sects. 3.4 and 7.3.3. In this latter it is shown that the selfcorrelation of the position coordinate of a harmonic oscillator contains a permanent oscillatory contribution of a frequency determined by the cutoff (Eq. 7.101), and with a value that is not too far from the Compton frequency.

where

$$\boldsymbol{k}'_{\pm} = \pm k_C \hat{\boldsymbol{x}}', \quad k_C = \omega_C / c, \tag{9.9}$$

and $\theta_{\pm}$ are statistically independent random phases, in accordance with the results of Chap. 4. In the laboratory frame (denoted by $S$) where the particle is seen to move with velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}'$, the frequency $\omega_C$ and the wave vectors $\boldsymbol{k}'_{\pm}$ transform in such a way that the phases appearing in (9.8), being a relativistic scalar, remain the same. This means that if $\omega_{\pm}$ and $\boldsymbol{k}_{\pm}$ stand, respectively, for the frequency and the wave vector as seen in $S$ of the plane waves traveling in the positive and negative direction along the axis $\hat{\boldsymbol{x}}'$, then

$$\omega_C t' - \boldsymbol{k}'_{\pm} \cdot \boldsymbol{x}' = \omega_{\pm} t - \boldsymbol{k}_{\pm} \cdot \boldsymbol{x}. \tag{9.10}$$

The expressions for the frequencies $\omega_{\pm}$ and the wave vectors $\boldsymbol{k}_{\pm}$ read (see, e.g., Jackson 1975, Sect. 11.3)

$$\begin{aligned} \omega_{\pm} &= \gamma\omega_C \left(1 \pm \beta\right), \\ \boldsymbol{k}_{\pm} &= \pm\gamma k_C \left(1 \pm \beta\right) \hat{\boldsymbol{x}} = \pm k_{\pm} \hat{\boldsymbol{x}}, \end{aligned} \tag{9.11}$$

and the standing wave $\varphi'(x', t')$ in $S'$ has therefore the following form in $S$,

$$\varphi_v(x, t) = e^{-i(\omega_+ t - k_+ x + \theta_+)} + e^{-i(\omega_- t + k_- x + \theta_-)} + \text{c.c.} \tag{9.12}$$

In terms of the frequencies

$$\omega_A = ck_A = \tfrac{1}{2}\left(\omega_+ + \omega_-\right) = \gamma\omega_C, \tag{9.13a}$$

$$\omega_B = ck_B = \tfrac{1}{2}\left(\omega_+ - \omega_-\right) = \gamma\beta\omega_C = \beta\omega_A, \tag{9.13b}$$

Equation (9.12) becomes

$$\varphi_v(x, t) = 4\cos\left(\omega_A t - k_B x + \theta_1\right) \cos\left(\omega_B t - k_A x + \theta_2\right), \tag{9.14}$$

with $\theta_{1,2} \equiv \frac{1}{2}\left(\theta_+ \pm \theta_-\right)$. This result, to which we shall return below, represents the standing wave of the ZPF that activates the de Broglie clock, as seen from the laboratory frame.

### 9.1.2 Energy, Frequency and Matter Waves

In order to relate $\varphi_v(x, t)$ with the de Broglie wave, let us resort to the relativistic expression for the energy

$$\mathcal{E}^2 = m_0^2 c^4 + c^2 p^2. \tag{9.15a}$$

From Eqs. (9.1) and (9.4) we obtain

$$\hbar\omega_C = m_0 c^2, \tag{9.15b}$$
$$\hbar\omega_B = cp, \tag{9.15c}$$

which together with (9.13a) and (9.13b) allows us to recast Eq. (9.15a) as

$$\mathcal{E}^2 = \hbar^2\left(\omega_C^2 + \omega_B^2\right) = \hbar^2\omega_C^2\left(1 + \gamma^2\beta^2\right) = \hbar^2\omega_C^2\gamma^2 = \hbar^2\omega_A^2. \tag{9.16}$$

It follows that

$$\mathcal{E} = \hbar\omega_A = \hbar\gamma\omega_C, \tag{9.17}$$

and the relation for the energy (9.15a) becomes equivalent to

$$\omega_A^2 = \omega_B^2 + \omega_C^2. \tag{9.18}$$

Formula (9.17) exhibits the energy as a manifestation of a vibration of very high frequency, so that energy and frequency become two aspects of the same reality, as is strongly expressed by Eq. (9.18). This suggests that all forms of energy are essentially the same thing, namely vibrations (energy *is* motion!). Under the consideration that $\omega_B$ refers to an electromagnetic wave, the successive discoveries by Planck (captured in the quantum relation $\mathcal{E} \sim \omega$), by Einstein (Eq. (9.15a)) and by de Broglie become integrated into the general law (9.18), which is simultaneously relativistic and quantum. In addition, this equation shows that de Broglie's frequency can be understood as a measure of the deviation of the actual frequency of vibration of the particle in the laboratory ($\omega_A$) from its reference value (the Compton frequency $\omega_C$), i.e., $\omega_B = \left(\omega_A^2 - \omega_C^2\right)^{1/2}$.

Taken together, Eqs. (9.15c) and (9.17) associate the wave number $k_B = p/\hbar$ and the frequency $\omega_A = \mathcal{E}/\hbar$ with a particle having momentum $p$ and energy $\mathcal{E}$. Such quantities are thus the natural ones to characterize a 'matter' wave associated with the moving corpuscle. The dispersion relation for such wave is therefore given by the relation $\mathcal{E} = \mathcal{E}(p)$, whence from Eqs. (9.15a) and (9.17) it follows that the group velocity $v_{\mathrm{g}}$ of the matter wave is

$$v_{\mathrm{g}} = \frac{\partial\mathcal{E}}{\partial p} = v. \tag{9.19a}$$

On the other hand, the phase velocity is just

$$v_{\mathrm{p}} = \frac{\mathcal{E}}{p} = \frac{mc^2}{p} = \frac{c^2}{v}. \tag{9.19b}$$

Notice that the *sole* specification of the wave number and the frequency of the matter wave ($k_B$ and $\omega_A$, respectively) could suggest to identify it with a simple wave of the form $\cos(\omega_A t - k_B x + \alpha)$ (with $\alpha$ constant). However, such wave does not comply with the above expression for $v_g$. This stresses the importance of the correct dispersion relation, and clearly indicates that the matter wave must be more complex than a simple oscillation.

### 9.1.3 The de Broglie Wave

We see that the intimate connection between energy and frequency not only brings in the notion of a matter wave associated with the moving corpuscle, but also determines its group and phase velocities. Two immediate questions arise, about its identification and about its physical reality. Is the matter wave simply a mathematical artifact, conveniently put in correspondence with the physical corpuscle, or is it a truly physical wave? In this section we briefly tackle this issue.

From the above discussion we know that the matter wave is not simply $\cos(\omega_A t - k_B x + \alpha)$, but this wave modulated so that there is a wave traveling with velocity $v$; hence it must be a wave of the form

$$\cos(\omega_A t - k_B x + \alpha) \times f(x - vt). \tag{9.20}$$

In Eq. (9.14) we have precisely this kind of wave. Indeed, with $\omega_B / k_A = v$, $\varphi_v(x, t)$ is found to have just the structure of (9.20),

$$\varphi_v(x, t) = 4 \cos\left(\omega_A t - k_B x + \theta_1\right) \cos\left[k_A\left(x - vt\right) - \theta_2\right]. \tag{9.21}$$

Taking a snapshot of (9.21) at $t = 0$ gives

$$\varphi_v(x, 0) = 4 \cos\left(k_A x - \theta_2\right) \cos\left(k_B x - \theta_1\right). \tag{9.22}$$

Since $k_B = \beta k_A < k_A$, $\varphi_v(x, 0)$ represents a rapid spatial oscillation with an amplitude that is modulated by a wave of wavelength $\lambda_B = 2\pi / k_B$; that is, the wavelength of the (spatial) modulation (envelope) *is precisely de Broglie's* $\lambda_B$. Let us now assume that instead of a snapshot we take a video with the position fixed at $x = 0$; this gives

$$\varphi_v(0, t) = 4 \cos\left(\omega_A t + \theta_1\right) \cos\left(\omega_B t + \theta_2\right). \tag{9.23}$$

The fact that $\omega_B = \beta\omega_A < \omega_A$, implies that the amplitude of the higher-frequency wave (the carrier) is modulated by an oscillation of frequency $\omega_B$. In other words, the frequency of the (temporal) envelope *coincides with the de Broglie frequency*.

We are now in a position to identify the whole structure $\varphi_v(x, t)$ with the 'matter wave', or de Broglie wave. Recognizing the origin of $\varphi_v(x, t)$ in the ZPF, we conclude that the de Broglie wave represents a physically real wave, as 'seen' from the laboratory. Since the spatial modulation of $\varphi_v(x,t)$ travels with velocity $v$, to an observer in $S$ it appears to keep company to the particle, as if surrounding and 'guiding' it along its motion—thus calling to mind the idea behind the guidance formula in de Broglie's theory. Both entities, particle and wave, appear thus as an indissoluble couple, yet each of them has a well-defined and complementary nature; in particular, the particle remains always a corpuscle, a nonextended object (though with some structure), in contrast with the always extended $\varphi_v(x,t)$. Notice that, even though from this perspective the particle is an intrinsically localizable object, its specific position within the matter wave's wavelength is not determined.

Consideration of the ZPF seems thus to be a natural means to incorporate not only the de Broglie wavelength, but also the de Broglie *wave*, into the narrative of quantum mechanics.[6] An additional relation between $\lambda_B$, the vacuum field, and the dynamics of the particle, can be obtained rewriting Eq. (9.4) in the form

$$\omega_B \lambda_B = 2\pi c. \tag{9.24}$$

This relation characterizes an electromagnetic wave in vacuum, with de Broglie's wavelength and with a linear momentum equal to $p_B = \hbar\omega_B/c$, which, according to Eq. (9.15c), $\hbar\omega_B = cp$, coincides with the momentum $p$ of the particle,

$$p = p_B. \tag{9.25}$$

Consequently, while the particle travels 'sitting' on the de Broglie wave, it bears the same momentum as the ZPF modes of frequency $\omega_B$; such modes thus acquire special relevance for the moving particle. In this sense it is natural to associate the ZPF modes of wavelength $\lambda_B$ also to the moving corpuscle—bearing in mind, however, that de Broglie's wavelength $\lambda_B$ does actually originate in the background field. De Broglie's formula should then be recast in the form

$$\lambda_B = \frac{h}{p_B}, \tag{9.26}$$

representing a genuine wave formula written in terms of parameters pertaining to a wave only, *without reference at all to the particle*. From this perspective, it is via the condition (9.25) that the wave property is *transferred* to the particle, so that $\lambda_B = h/p$. That the modes of the ZPF having frequency $\omega_B$ (and wavelength $\lambda_B$) turn out to be of particular importance for the dynamics of the particle will be further discussed in Sect. 9.3, in relation with matter diffraction.

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[6] Or rather, into the ontology of quantum mechanics. We see in the wave *function* of quantum mechanics an abstract object that lives in a mathemathical configuration space. By contrast, the de Broglie wave associated with the ZPF modulations should be understood as a real wave in three-dimensional space. They are therefore two objects of an entirely different nature.

## 9.2 An Exercise on Quantization à la de Broglie

In this section we resort to the de Broglie wave constructed above to show by means of an example how it can be applied to analyze some properties of stationary, bounded, one-dimensional quantum motions. With this aim let us consider a benchmark case and examine the stationary description of a particle trapped in an infinite square potential well of width $a$. In this case there is no net flux and the particles will be performing periodical back and forth motions inside the box. In order to construct the de Broglie description for this situation, one must take into account not only the $\varphi_v(x,t)$, representing the wave associated with a particle that travels in the $+x$ direction with velocity $v$, but also the reflected wave $\varphi_{-v}(x,t)$ that travels in the $-x$ direction with the same speed. We therefore take the superposition

$$\varphi(x,t) = \varphi_v(x,t) + \varphi_{-v}(x,t). \tag{9.27}$$

As follows from Eq. (9.11), with the substitution $v \to -v$ the frequency $\omega_\pm$ becomes $\omega_\mp$, and similarly for $k_\pm = \omega_\pm/c$. We shall assume that the phases $\theta_\pm$ in Eq. (9.12) are the same in both components (they both refer to the same wave). Taking all this into account, Eq. (9.27) reads

$$\varphi(x,t) = e^{-i\theta}[e^{-i(\omega_+ t - k_+ x)} + e^{-i(\omega_+ t + k_+ x)} + e^{-i(\omega_- t - k_- x)} + e^{-i(\omega_- t + k_- x)}] + \text{c.c}, \tag{9.28}$$

which reduces to

$$\varphi(x,t) = 4\left[\cos(\omega_+ t + \theta)\cos k_+ x + \cos(\omega_- t + \theta)\cos k_- x\right]. \tag{9.29}$$

This standing wave inside the well is consistent with the condition of zero flux velocity. Unlike the de Broglie wave, the superposition $\varphi(x,t)$ does not travel with the particle, but reflects the periodicity of the motion. Further, since $\varphi(x,t)$ corresponds to a stationary situation, it means that it is periodic in $x$ with period $a$,

$$\varphi(x,t) = \varphi(x+a,t). \tag{9.30}$$

This stationarity condition applied to Eq. (9.29) leads to

$$k_\pm = \frac{2\pi}{a} n_\pm, \quad n_\pm = 0, 1, \ldots \tag{9.31}$$

Notice that for $v \neq 0$ we have $k_+ > k_-$ (see Eq. (9.11)), whence $n_+ > n_-$. From here and Eqs. (9.13a), (9.13b) it follows that

$$\tfrac{1}{2}(k_+ - k_-) = k_B = \tfrac{\pi}{a}(n_+ - n_-) \equiv \tfrac{\pi}{a} n, \quad n = 1, \ldots, \tag{9.32a}$$

$$\tfrac{1}{2}(k_+ + k_-) = k_A = \tfrac{\pi}{a}(n_+ + n_-) \equiv \tfrac{\pi}{a} N, \quad N = 1, \ldots \tag{9.32b}$$

Equation (9.32b), together with (9.15c), gives

$$p = \hbar k_B \rightarrow p_n = \frac{\hbar\pi}{a} n, \tag{9.33}$$

whence

$$n\lambda_B = 2a. \tag{9.34}$$

One can recognize here the well-known statement that the well can accommodate an integer number of half-de Broglie's wavelengths under stationarity, in agreement with usual phenomenology. Notice that the result arises as a consequence of imposing the stationarity condition on the *wave* $\varphi(x, t)$ that reflects the periodicity of the *corpuscle's* motion. Equations (9.33) and (9.34) mean that the dynamics and the de Broglie wave have become conformed to the geometry of the system.

Notice that Eq. (9.33) follows also from (9.25), under conditions of stationarity of the standing waves of the ZPF inside the well. In other words, the quantization implied by Eq. (9.33) can be seen as a result of the presence of the vacuum field and the identification $p = p_B$, a relation that plays thus the role of a quantization rule.

Let us now turn to Eq. (9.32b), which together with $\lambda_A = 2\pi/k_A$ gives

$$N\lambda_A = 2a. \tag{9.35}$$

According to this expression, also an integer number of half-wavelengths $\lambda_A$ must be accommodated inside the well to attain stationarity. However, since $k_B/k_A = \beta = n/N$, in the nonrelativistic regime $n \ll N$. Comparison between Eqs. (9.34) and (9.35) thus indicates that the wave with $\lambda_A$ inside the well has many more nodes that the wave with $\lambda_B$. In terms of the de Broglie wave, this is explained by recalling that at any given time, $\varphi_v(x, t_0)$ represents a rapid oscillation of wavelength $\lambda_A$ modulated by an oscillation of wavelength $\lambda_B \gg \lambda_A$ (cf. Eq. (9.22)). Physically, this reflects the fact that the particle inside the box is not simply performing a uniform motion with (mean) velocity $v$ (like a classical particle would do), but that such motion is superposed to a vibration at the high frequency $\omega_A \sim \omega_C$. As mentioned earlier, this oscillation, the zitterbewegung, constitutes an echo—the laboratory frame—of de Broglie's clock.

The above results can be somewhat completed to get a more detailed picture of what is happening inside the well. The formula for the energy $\mathcal{E}_n$ associated with the smooth motion of the particles follows directly from Eq. (9.33),

$$\mathcal{E}_n = \frac{p_n^2}{2m} = \frac{\pi^2\hbar^2}{2ma^2} n^2, \quad n = 1, \ldots \tag{9.36}$$

Since the particles are being perfectly reflected at the walls of the well, it has sense to define the period of the (mean) motion in state $n$ as

$$\tau_n \equiv \frac{2a}{v_n} = 2\pi n \frac{\hbar}{2\mathcal{E}_n}, \tag{9.37}$$

which suggests to introduce a *mechanical* frequency $\omega_n^{\text{mec}}$ such that

$$\omega_n^{\text{mec}} \tau_n = 2\pi n. \tag{9.38}$$

With this definition, Eq. (9.37) gives

$$\mathcal{E}_n = \frac{\hbar}{2}\omega_n^{\text{mec}}, \quad \omega_n^{\text{mec}} = \frac{\pi^2 \hbar}{ma^2} n^2, \quad v_n = \frac{\pi\hbar}{ma} n = v_1 n. \tag{9.39}$$

In state $n$ the particle surveys in the mean $n$ times the distance that corresponds to the fundamental state $n = 1$. This result relates the index $n$ in $p_n$ to the number of complete cycles performed by the component $n$ during the time that the slowest component (for $n = 1$) completes one cycle.

The first relation in (9.39) looks akin to the substance of SED, and states that the energy of the particle in the state $n$ coincides with the energy of the modes of the ZPF of frequency $\omega_n^{\text{mec}}$. Further, direct calculation gives

$$\lambda_{Bn} \omega_n^{\text{mec}} = 2\pi v_n, \tag{9.40}$$

with $\lambda_{Bn} = h/p_n$. This relation defines a geometric wave inside the well, moving with the particle; it has the de Broglie wavelength and the mechanical frequency $\omega_n^{\text{mec}}$. This latter can be related with the de Broglie frequency by a comparison of Eqs. (9.24) and (9.40), resulting in

$$\omega_n^{\text{mec}} = \beta_n \omega_{B_n}. \tag{9.41}$$

For nonrelativistic motions, we verify that the de Broglie frequency is very high compared with the frequency of the dominant motion. In contrast, the de Broglie wavelength is considerably larger than the Compton wavelength (see Eq. (9.7)).

Unlike the resonance frequencies studied in Chap. 5, which are the frequencies of transition between states, the $\omega_n^{\text{mec}}$ $(n = 1, 2, ...)$ are related directly to the *permanence* in the respective state $n$. They remain hidden to QM, not being part of its ontology, its epistemology, or its semantics. Equation (9.39) together with Bohr's rule shows that there exists a relation between the transition and the permanence frequencies,

$$\omega_{nm} = \hbar^{-1}(\mathcal{E}_n - \mathcal{E}_m) = (\omega_n^{\text{mec}} - \omega_m^{\text{mec}})/2. \tag{9.42}$$

To the extent to which the definition $\mathcal{E}_n = \hbar\omega_n^{\text{mec}}/2$ has a meaning, such relationship may be significant. The factor 2 is specific of the present example, of course.

## 9.3 Undulatory Properties of Matter

Because of their wave-like properties, quantum corpuscles are often not 'seen' as particles—or waves—but as another sort of entity, such as 'wavicles' (Eddington 1928), 'microparticles' (Blokhinsev 1953/1964), 'quantons' (Bunge 1967, 1973), 'smearons' (Maxwell 1981), 'wavelets' (Barut 1993), and what not.[7] A few examples, taken from among scores of them, of the kind of contrasting points of view to which the consideration of the wave properties of matter is prone to lead, may be seen in Diner et al. (1983), Agazzi (1988), and Combourieu and Rauch (1992). Leaving aside particular details, what the existence of so many and diverse approaches evinces is that the undulatory properties of matter are among the most perplexing and least understood aspects of the quantum world.

To pay attention to the wave-like properties of matter, let us appeal to one of the simplest and at the same time most revealing quantum experiments, that of electrons passing through two parallel slits made on a screen. The amazing result is well known, and popularized by the Tonomura et al 1989 experiment, which has been seen by many thanks to the web (www.hitachi.com/rd/portal/research/em/movie.html; see also Bach et al. 2013). It is important to draw attention to this experiment (and earlier ones, such as those described in Jönsson (1961), and Matteucci and Pozzi (1978), since all of them reveal that a *single* electron does not give rise to the diffraction pattern: it merely produces a spot (seemingly at random) on the screen. The diffraction pattern, a wave-like phenomenon, results from the addition of tens of thousands of events, and hence depicts the statistical distribution of electrons on the screen. The Schrödinger equation, which refers to the wave properties of particles, describes just this statistical behavior. It cannot provide in general a detailed description of the wanderings of an individual electron. It is devised to describe the multitude, not what each and every electron is doing. And it certainly does not provide a physical *explanation* for the diffraction pattern.

Let us now look from the present SED perspective at the problem of particle diffraction by the pair of parallel slits. One should start by considering that the the ZPF is not immune to the presence of the slits. The Casimir effect, as well as the cavity effects on atomic lifetimes and energy levels, are well-known instances that remind us that ZPF must satisfy the same boundary conditions as any other electromagnetic field in the presence of matter [see e.g. Boyer (1980) and references therein; Cetto and de la Peña (1988a, b)]. And indeed, Fig. 9.1 shows an image of the ZPF diffracted by two parallel slits, opened on an infinite, totally reflecting plate; the wavelentgth of the field modes has been chosen to be of the order of the distance between slits. This is the kind of field that the electrons 'feel' when traveling in the neighborhood of the screen. The partially reorganized electric forces act on the particles, and one

[7] The term *wavelet* refers to localized nonspreading solutions of massless wave equations that move like massive quantum particles. Wavelets are seen as a bridge between classical point particles and the waves of QM; the mass of the particle is determined by the internal frequency of the wavelet, much as the 'internal clock' in the Broglie's theory.

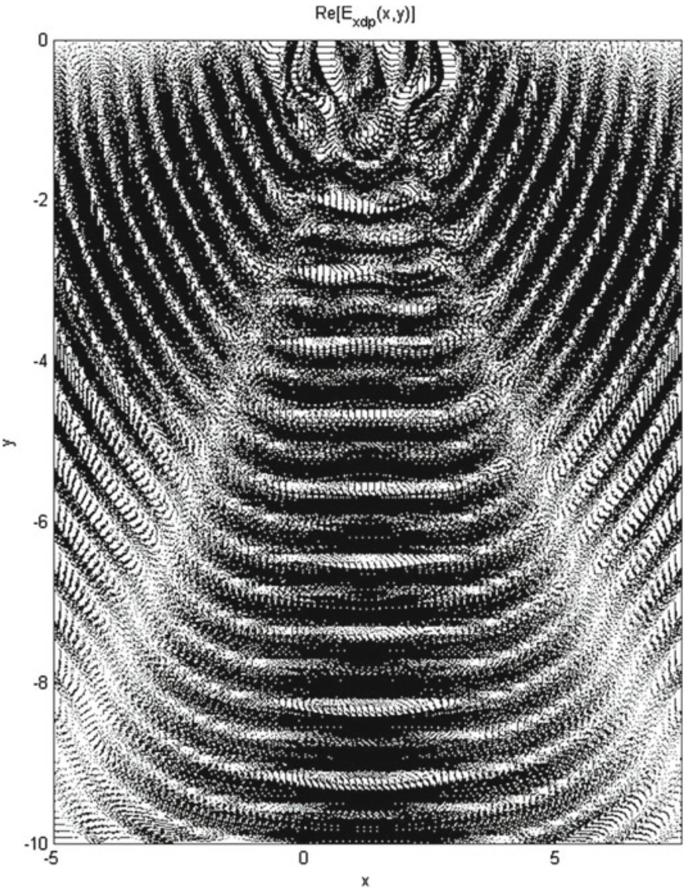


**Fig. 9.1** Contour map of the real part of the $E_x$ component of the zero-point field of wavelength $0.9l$, diffracted by two slits separated a distance $1.1l$, pierced on a conducting plate; $l$ is the width of the slits. Reprinted from Avendaño and de la Peña (2005) with permission from Elsevier

should therefore expect to get on the screen a footprint of the diffracted field, traced out by the electrons.

The detailed dynamics of the particles travelling in a diffracted field like the one in figure reffig1 needs still to be worked out. However, according to the discussion in the previous section, one may reasonably assume that the electrons with a momentum $p$ will be particularly affected by the diffracted modes that satisfy the condition (9.25), $p_B = p$ (i.e., the modes of wavelength $\lambda_B = h/p$), and guided by them towards the screen along the preferred directions determined by Bragg's law. This would give shape to an interference pattern superimposed on the noisy background. Thus, the particle needs to 'know' nothing about the existence of the slits: it is the background field what carries the required information and operates accordingly *on* the particles.

The picture that emerges reminds us of the image suggested by J. Clauser some time ago: "If a bunch of surfers pass through a breakwater with two entrances, you'll see the two-slit pattern later on the beach in surfer flesh!" (quoted in Wick (1995), p. 116). And indeed, for over 80 years we have been observing interference patterns in electron flesh. The electrons maintain their corpuscular identity all along the experiment, and there is no need of particle self-interference. Since the diffracted field exists even in the absence of the electrons, it might be possible to put this explanation to experimental test. The observation of the diffracted field with independence from the presence or absence of the electrons would demonstrate that it is the field, not matter, what is diffracted. An initial exposition of these matters is given in Avendaño and de la Peña (2010). An example of the kind of results that such an explanation can afford is shown in Fig. 9.2. This figure shows some preliminary results obtained again by numerical calculation (Avendaño and de la Peña 2005, and work in preparation), for the trajectories followed by electrons in the double-slit experiment. The fluctuating component of the diffracted field has been suppressed to highlight the guiding effect of the field. The momentum $p$ of the particles has been selected according to the law $p = p_B$ and the kinetic energy of the electrons has been assumed to remain constant, i.e., the particles are deflected without changing their speed. Even if in the real situation the trajectories may be not as smooth, the figure offers a clear image of the behavior one may expect for the particles under the action of the diffracted ZPF. In particular, it is distinctly seen that on its way to the distant screen, each particle crosses a single time a single slit, and behaves as a localized corpuscle all along its journey.[8]

Particle diffraction patterns have been obtained experimentally also with neutrons and other neutral particles, as is well known from crystallography and has been confirmed by the famous experiments by Rauch and colleagues [Rauch et al. (1974); a detailed review is Greenberger (1983)]. This means that the SED explanation should not be restricted to charged particles. As suggested in Chap. 4, a possible answer to this observation is that all known particles, including the neutral ones, have electromagnetic interactions. An interaction with the ZPF through the coupling of the electric dipole moment, the magnetic moment, or any other multipole, is able in principle to lead to results that are similar to the ones obtained from electric charge coupling, although details such as the relaxation times may vary; such differences, however, are irrelevant for the performed experiments, which proceed very slowly in comparison.

The double-slit experiment affords an opportunity to give a more precise meaning to the assertion that the Schrödinger equation predicts *only* the wavelike behavior of quantum corpuscles. That such statement requires qualification can be illustrated with the aid of Fig. 9.3. This figure shows the numerical solution of the Schrödinger equation for the problem of two ideal slits, at different distances from the plane

[8] The results obtained with this numerical experiment are similar to those obtained by Couder and Fort (2006) in their macroscopic Young-type experiment, showing clearly that the bouncing droplet goes through either one of the two slits but the associated wave passes through both slits, and the interference of the resulting waves is responsible for the trajectory of the walker.

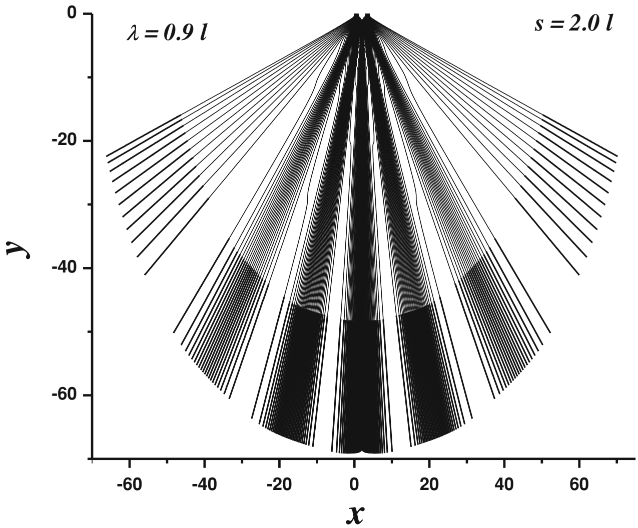


**Fig. 9.2** Trajectories followed by electrons in a realistic simulation of a two-slit experiment. The particles are uniformly distributed in the beam behind the slits. The diffracted modes of the field have momentum $p_B$ and the momentum of the particles is $p$, with $p = p_B$. Figure courtesy of J. Avendaño, adapted from Avendaño and de la Peña (2010)

that contains the slits. The solution shows that sufficiently close to the slits the Schrödinger equation predicts a corpuscular behavior of the particles, whereas the wavelike behavior corresponds to the Fraunhofer (far) region. For intermediate distances both aspects are simultaneously manifested. Here we have an example of coexistence of corpuscular and wavelike manifestations, which reminds us of Einstein's reading of Eq. (3.71) as an expression of these two complementary aspects in the case of light.

## 9.4 Cosmological Origin of Planck's Constant

Let us make a detour from the line of inquiry followed so far in this chapter, and direct our attention to another interesting question directly related with the ZPF, namely: what fixes the scale $\hbar$ of the ZPF fluctuations? Being the ZPF of cosmological origin, it sounds natural to assume that $\hbar$ should be linked in any way to other universal constants.

To find an answer to this question let us consider a world made of just harmonic oscillators, representing both matter and the modes of the zero-point radiation field. Such a crude model should be appropriate for the purpose of performing an order-of-magnitude estimate of certain quantities of interest for our present intent. Since the thermal (photonic) background radiation is of no interest here, we assume that all

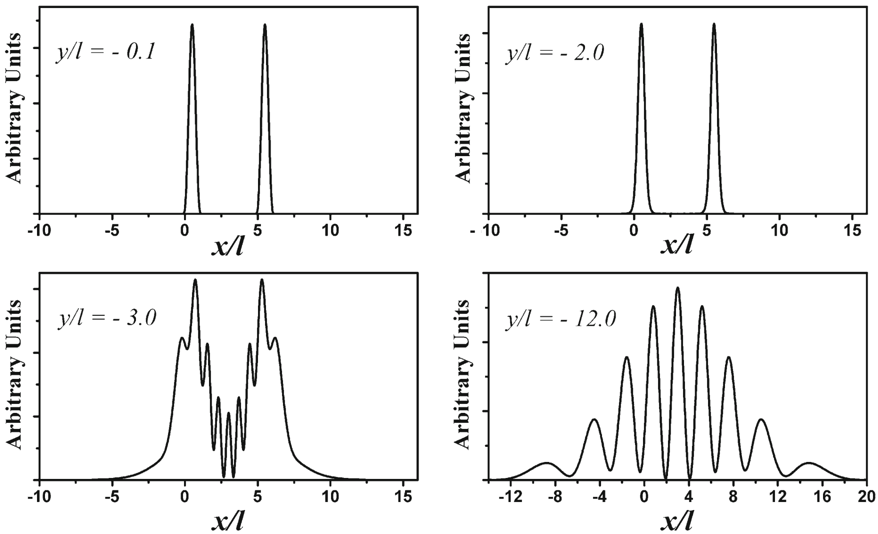


**Fig. 9.3** Numerical solution of the Schrödinger equation for two slits, shown as function of the distance from the plane of the slits. At very short distances the solution resembles the one that corresponds to particles, whereas at very far distances a Fraunhofer diffraction pattern is observed. The distance from the slits to the screen is given by $-y$. Figure courtesy of J. Avendaño

elementary oscillators of a given frequency, irrespective of their nature, are in their ground state. According to the image developed in this book, the charged (matter) oscillators (electric dipoles) are radiating and contributing to the background field, but they are also absorbing energy from the field. The random field so regenerated should coincide with the vacuum field at each frequency under the assumption of a universe in equilibrium. We herewith espouse a sort of cosmological principle associated with SED, or, if preferred, a kind of *electromagnetic Mach principle*: the field produced at a given point by all dipoles in the Universe should equal the random field acting at that point on the particles themselves. This requirement establishes a relationship between cosmological and atomic constants; in other words, it establishes the scale of quantum fluctuations. Planck's constant becomes thus determined by cosmological parameters (de la Peña and Cetto 1984, 1997).[9]

A parallel reasoning, but dealing entirely with the gravitational field, has been discussed by Calogero (1997) in an interesting essay restricted to order-of-magnitude considerations. In that work, the identification of the unavoidable gravitational fluctuations with the quantum fluctuations of atomic systems is shown to lead to a relationship between atomic and cosmological constants. One should recall also a similar attempt made in Puthoff (1991) within SED, also on the basis of a self-regenerating model. The basic idea in Puthoff's paper is much in line with the one studied here,

[9] Recently we have become aware of a similar proposal by Mavrychev (1967), in which the author reaches a comparable result.

although in our rough estimate we refrain from resorting to any specific cosmological model. Still we leave aside any problem related with the infinite gravitational effects of the zero-point field, just as is done in QED (and in cosmology) with all vacuum fields, simply because nobody knows yet how to solve this so-much studied and poorly understood problem (see e.g. Weinberg 1989). By equating the radiation field predicted by the model at a given point with the corresponding component of the zero-point field, Puthoff obtains a prediction for the baryonic mass density of the Universe, which establishes a relation between atomic constants and the Hubble constant. This relation happens to correspond essentially to the one discussed in Weinberg (1972) book (Sect. 16.4), which is usually taken as a numerical coincidence, of unknown origin and meaning.

Consider, then, the radiating dipole $\alpha$ ($\alpha = 1, 2, \ldots, N$) of frequency $\omega$ located at the position $\boldsymbol{r}_\alpha$; we take the origin of coordinates at our place. The Fourier amplitude of the electric field produced by this oscillator at the origin is

$$\boldsymbol{E}_\alpha(\omega) = -k^2 \boldsymbol{n}_\alpha \times (\boldsymbol{n}_\alpha \times \boldsymbol{p}_\alpha) \frac{e^{ikr_\alpha}}{r_\alpha}, \quad k = \frac{\omega}{c}, \tag{9.43}$$

where $\boldsymbol{p}_\alpha = (e/2)(\boldsymbol{q}_{0\alpha} + i\dot{\boldsymbol{q}}_{0\alpha}/\omega)$ is the (complex) amplitude of the dipole moment $\boldsymbol{p}_\alpha e^{-i\omega t}$, and $\boldsymbol{n}_\alpha = \boldsymbol{r}_\alpha / r_\alpha$ is the unit vector in the direction of $\boldsymbol{r}_\alpha$. Since the mean energy of the oscillator is $\hbar\omega/2$, one has $\left\langle \boldsymbol{q}_{0\alpha}^2 + \dot{\boldsymbol{q}}_{0\alpha}^2/\omega^2 \right\rangle = \hbar/m\omega$, where the average is taken over the set of oscillators of frequency $\omega$, so that we write

$$\boldsymbol{p}_\alpha = \frac{e}{2}\sqrt{\frac{\hbar}{m\omega}} \boldsymbol{B}_\alpha \tag{9.44}$$

and consider the components $B_{i\alpha}$ of $\boldsymbol{B}_\alpha$ to be statistically independent complex random variables with zero mean and second moments given by

$$\left\langle B_{i\alpha} B_{j\beta}^* \right\rangle = \delta_{\alpha\beta}\delta_{ij}, \quad \left\langle B_{i\alpha} B_{j\beta} \right\rangle = 0. \tag{9.45}$$

With these assumptions the mean square of Eq. (9.43) is (omitting the index $\alpha$)

$$\langle |\boldsymbol{E}(\omega)|^2 \rangle = \frac{\hbar e^2}{4mc^4} \frac{\omega^3}{r^2} \langle |\boldsymbol{n} \times (\boldsymbol{n} \times \boldsymbol{B})|^2 \rangle = \frac{\hbar e^2}{2mc^4} \frac{\omega^3}{r^2}, \tag{9.46}$$

since $\langle |\boldsymbol{n} \times (\boldsymbol{n} \times \boldsymbol{B})|^2 \rangle = \left\langle \boldsymbol{B} \cdot \boldsymbol{B}^* - (\boldsymbol{n} \cdot \boldsymbol{B})(\boldsymbol{n} \cdot \boldsymbol{B}^*) \right\rangle = 2$. We have taken into account that the field amplitudes produced by statistically independent oscillators are uncorrelated.

To evaluate the average energy content $\langle \mathcal{E}(\omega) \rangle$ of the radiation field of frequency $\omega$ at the origin, we integrate Eq. (9.46) over a spherical volume of radius $R$, assuming an isotropic and homogeneous distribution of oscillators, of which there are $n(\omega)$ of frequency $\omega$ and a total number $N = \sum_\omega n(\omega)$:

$$\langle \mathcal{E}(\omega) \rangle = \frac{n(\omega)}{4\pi} \int_V \langle |\boldsymbol{E}(\omega)|^2 \rangle dV = \frac{\hbar e^2 \omega^3 R}{2mc^4} n(\omega). \tag{9.47}$$

The cosmological postulate asserts that this energy should correspond to the ZPF energy of a mode of frequency $\omega$, i.e., $\hbar\omega/2$ ; one thus obtains

$$n(\omega) = \frac{mc^4}{e^2 \omega^2 R}. \tag{9.48}$$

To estimate the total number of oscillators we integrate over all frequencies, using the rule $V^{-1} \sum_\omega \to (2\pi^2 c^3)^{-1} \int d\omega\, \omega^2$, which gives

$$N = \sum_\omega n(\omega) = \frac{mc^4}{e^2 R} \sum_\omega \frac{1}{\omega^2} \to \frac{mcV}{2\pi^2 e^2 R} \int_0^\Omega d\omega = \frac{mc\Omega V}{2\pi^2 e^2 R}. \tag{9.49}$$

Since the integral is divergent we have introduced a cutoff frequency $\Omega$ for the material oscillators. Indeed, the material oscillators are transparent at arbitrarily high frequencies; one can consider a cutoff around the pair-creation (Zitterwebegung) frequency $\Omega = 2mc^2/\hbar$ as physically meaningful (see also the discussion in Sect. 9.1.1), so that (9.49) becomes

$$\frac{N}{V} = \frac{m^2 c^2}{\pi^2 \alpha \hbar^2 R}, \tag{9.50}$$

where $\alpha = e^2/\hbar c$ stands for the fine-structure constant. Here $V$ must be taken as the volume of the visible part of the Universe, as this is the part that contributes to the radiation field, and thus $N/V$ is to be identified with the cosmological density of charged particles, which multiplied by $m_N$ (the nucleon mass or any typical baryon mass) gives for the baryonic density of the Universe the estimate

$$\rho \simeq \frac{m^2 m_N c^2}{\pi^2 \alpha \hbar^2 R}. \tag{9.51}$$

Before proceeding further let us add a couple of remarks with regard to this expression. Firstly, we have not taken into account any absorption process, the reason being that we are dealing with the ZPF, which is not absorbed by matter in equilibrium with it. Of course it is scattered by matter, but for a uniform and homogeneous universe the final distribution remains the same. Therefore Eq. (9.51) needs *no* correction from Thomson scattering.

The second comment refers to the simplicity of the model. The intention here is to make a qualitative test of the SED cosmological principle, and for such purpose the present rough estimate should suffice. For example, a somewhat more realistic model would take into account the expansion of the Universe, which produces a redshift, so that instead of the original frequency $\omega$ radiated when the Universe had a scale factor $R(t)$, the red-shifted frequency

$$\omega_0 = \frac{R(t)}{R_0}\omega \tag{9.52}$$

should be used, where the subindex 0 refers to the present moment and place of observation. Thus, if $\nu(\omega)$ represents the spatial density of oscillators of local frequency $\omega$ at a distance $r$ from us, instead of Eq. (9.47) one should write

$$\langle \mathcal{E}(\omega) \rangle = \frac{e^2\hbar}{2mc^4}\omega_0^3 R_0^3 \int_0^{R_0} \frac{\nu(\omega_0 R_0/R(t))}{R^3(t)r^2} r^2 dr, \tag{9.53}$$

where, using Weinberg's (1972) notation, one must put $dr = (\sqrt{1-kr^2}/R(t))dt$. To go further one would have to specify the cosmological model; however, any reasonable choice for $R(t)$ would only change the numerical factors, without altering the essential contents of Eq. (9.51). Thus, up to such numerical factors we take the former result (9.51) as a reasonable relation among the relevant constants of nature.

Let us try to draw some conclusion from Eq. (9.51). For this purpose we first introduce an auxiliary (representative) mass defined as

$$\bar{m} = \left(\frac{m^2 m_N}{\pi^2 \alpha}\right)^{1/3} \backsimeq 30m. \tag{9.54}$$

where $m$ is the mass of the electron. Equation (9.51) can then be rewritten in the form (we put $R = R_0$, and add the subindex 0 to mark the present values of the cosmological parameters)

$$\frac{\rho_0 R_0^3}{\bar{m}} = \frac{\bar{m}^2 c^2 R_0^2}{\hbar^2} = \left(\frac{R_0}{\lambda_{\bar{m}}}\right)^2, \tag{9.55}$$

where $\lambda_{\bar{m}}$ is the Compton wavelength (divided by $2\pi$) associated with the mass $\bar{m}, \lambda_{\bar{m}} = \hbar/\bar{m}c$. We recognize in each side of Eq. (9.55) one of the 'large numbers' of cosmology, which are ($H_0$ is the present value of Hubble constant, $H_0 = c/R_0$, and $G$ stands for the gravitational coupling constant)

$$N_1 = \frac{\hbar c}{G m_N^2} \sim \frac{1}{6}10^{39}, \tag{9.56}$$

$$N_2 = \frac{mc^2}{\hbar H_0} = \frac{mcR_0}{\hbar} = \frac{R_0}{\lambda_m} \sim \frac{1}{3}10^{39}, \tag{9.57}$$

$$N_3 = \frac{\rho_0 c^3}{m_N H_0^3} = \frac{\rho_0 R_0^3}{m_N} \sim 10^{79}. \tag{9.58}$$

Except for the differences in the masses, Eq. (9.55) reads

$$N_3 = N_2^2, \tag{9.59}$$

which is one of the well-known numerical coincidences among these large numbers. The surprising content of this expression is that it relates cosmological parameters with Planck's constant, which is a highly nontrivial result (remember that Weinberg (1972) qualifies Eq. (9.59) as mysterious). The second independent relation among these numbers, which can be taken to be $N_1 N_2 \backsimeq N_3$, does not involve Planck's constant and can be obtained from cosmological models, such as the Friedmann model.

We conclude that the SED Cosmological Principle, namely that the energy of the vacuum fluctuations corresponds to the energy radiated by all dipoles of the Universe in a self-regenerating process, seems to hold and serves to explain the relation $N_3 = N_2^2$ up to a constant factor of at most a few orders of magnitude.

Let us now recast Eq. (9.51) in a different form. In terms of the dimensionless gravitational coupling constant $\alpha_G = Gmm_N/\hbar c$ it reads

$$\alpha_G R_0 \simeq \frac{3\pi}{8}\alpha\lambda_m, \tag{9.60}$$

which we write simply as

$$\alpha\lambda_m \simeq \alpha_G R_0, \tag{9.61}$$

where the value of the common length $l = \alpha\lambda_m = e^2/mc^2 = r_0$ equals the classical electron radius. It seems interesting to observe that Eq. (9.61) can be extended to include nuclear forces by taking the coupling constant of order 1, $\alpha_N \simeq 1$, and a characteristic lenght $R_N \simeq \hbar/m_\pi c$, ($m_\pi$ is the pion mass), which gives a numerical value $\simeq \alpha\lambda_m/2$, so that

$$\alpha_G R_0 \simeq \alpha\lambda_m \simeq \alpha_N R_N \simeq r_0. \tag{9.62}$$

Equation (9.61) explains why Calogero's gravitational arguments and the present electromagnetic ones lead to equivalent results. This is another form of saying that it should be feasible to represent the effects of the zero-point field as a fluctuating metric field, a possibility that was already studied by Einstein himself (1924). It is interesting to observe that Eq. (9.59) (or Eq. (9.61)) cannot be obtained solely from the usual quantum formalism; it is within the conceptual frame of SED where the cosmological principle leading to Eq. (9.59) finds its natural place.[10]

To end this detour, we note that Eq. (9.55) can be written in the form

$$\hbar = \left(\frac{\bar{m}^3 c^2}{\rho_0 R_0}\right)^{1/2}. \tag{9.63}$$

[10] For some enriching comments of differing nature on the ZPF see Ibison (2003), and Dasgupta and Roy (2007).

This suggests speculating that the fluctuations of matter density at cosmological scales may produce local fluctuations on the value of $\hbar$. Whether this has any sense at all is a question that we leave to cosmologists.

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