

# Chapter 1

## Quantum Mechanics: Some Questions

*...[quantum-mechanical] vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice.*

Bell (1987, page 160)

*... that today there is no interpretation of quantum mechanics that does not have serious flaws, and that we ought to take seriously the possibility of finding some more satisfactory other theory, to which quantum mechanics is merely a good approximation .*

Weinberg (2013, page 95)

### 1.1 On Being Principled... At Least on Sundays

Tied to our microscopic place in the immensities of the Cosmos, we are beginning to unfold its mysteries with remarkable precision. Being as gigantic as we are compared to the atomic and subatomic worlds, we have been able nevertheless to uncover an important fraction of its workings. We do not know yet what an electron is made of, but we know already many of its secrets (see e.g. Wilczek 2002).

The remarkable scientific, technological, philosophical, and even economic success of quantum mechanics is only the beginning. No physicist on Earth would question the numerically fitting description that quantum mechanics offers of the part of the world that pertains to its domains, which extend much beyond the atomic scale the theory originally was intended to cover, both towards the macroscopic and the ultramicroscopic. However, a nonnegligible portion of the practicing physicists would also acknowledge, either openly or reluctantly, that the mysteries of the quantum world have not been satisfactorily cleared or explained, after more than eighty years of successful existence of this most basic theory.

Such acknowledgment depends of course on what is meant by *explanation*. A historical example of what we have in mind follows from the Newtonian theory of

gravitation: the clarity, universality, simplicity and high precision of this theory made of it a grandiose paradigm; the theory reigned undisputed for over two centuries and became the ideological pedestal that supported the European Enlightenment. The universal gravitational force became the pivotal element to understand innumerable terrestrial and celestial facts, and a central element in the construction of a whole philosophy of nature. This occurred despite the known shortcomings of the theory in more than one essential aspect. Not only did it rest on the ageing concept of action at a distance, but the specific form of the force was selected ad hoc to lead to the Keplerian ellipses, introduced as a mere patch into the Newtonian system of mechanics, with no theoretical support or physical mechanism that would lead to it or explain it. From this more exacting point of view, one could say that the classical theory gives a precise and simple *description* of the facts, sufficiently good for *all practical purposes* (FAPP); but it hardly constitutes an *explanation* of what is going on in the real world. To find such an explanation the whole edifice of general relativity had to be put forth, allowing us to dispense with ad hoc elements or actions at a distance, and providing us instead with a causal rule. Indeed, general relativity *explains* the Newtonian theory.

Today we can calculate atomic transition frequencies to within a billionth part, and use refined applications of the quantum properties of matter and the radiation field to construct marvelous and powerful devices that have become emblematic of our civilization. However, have we really got an understanding of what is happening deep-down in the quantum world? A glance at the quantum literature dedicated to the discussion of its fundamental aspects is sufficient to reveal the vast spread of meanings and uncertainties that beset current quantum knowledge. Of course, if the number predicted by the theory, or the use that is made of it, is taken as its test, just as was the case with Newtonian gravitation and the extended pragmatic viewpoint it prompted, the conclusion is that there is no problem at all. But we may be a bit more demanding and ask, for instance, for the physical (rather than formal) explanation of atomic stability, the origin of uncertainty or the quantum fluctuations. Again, are wave-particle duality and quantum nonlocalities the final word? Do superluminal influences really exist?<sup>1</sup> In short: the quantum formalism describes its portion of Nature astonishingly well and we do not know why. It would be difficult to express this kind of feelings about the status of present-day quantum theory more lucidly than Bell did in 1976: quantum mechanics is a FAPP theory. And Maxwell (1992) rightly asks: what is beyond FAPP?

Since the creation of quantum mechanics (QM) there has been a flood of papers and essays discussing these and similar or deeper questions, and almost any conceivable (or inconceivable) argument or answer has been advanced, both from within physics and from the philosophy of science, ranging from a complete accord with quantum orthodoxy to a radical departure from it. Such extended and deep rumination has not been the endeavor of idle physicists and philosophers, since names such as Bohr, de

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<sup>1</sup> In statements about superluminal influences, it is difficult to know which kind of influences are being considered. Anyhow, detailed analysis shows that special relativity and quantum mechanics have still a peaceful coexistence (see e.g. Shimony 1978; Redhead 1983, 1987).

Broglie, Dirac, Einstein, Heisenberg, Landé, Popper, Schrödinger, do honor to an unending list of active participants.

Let us listen to some few big voices to get a better feeling of the magnitude of the quantum muddle, as Popper (1959) calls it. Feynman writes:

I think I can safely say that nobody understands quantum mechanics,

and goes on speaking of the [unsolved] mysteries of QM (Feynman et al. 1965). Referring to matter diffraction he asserts:

A phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the only mystery...

How does it really work? What machinery is actually producing this thing? Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it.

Gell-Mann (1981) in his turn qualifies:

In elementary particle theory one assumes the validity of three principles that appear to be exactly correct.

(1) Quantum mechanics, that mysterious, confusing discipline, which none of us really understands but which we know how to use. It works perfectly, as far as we can tell, in describing physical reality, but it is a 'counter-intuitive discipline', as social scientists would say. Quantum mechanics is not a theory, but rather a framework, within which we believe any correct theory must fit. (2) Relativity. (3) Causality.

In his turn Dyson (1958) observes:

...the student says to himself: 'I understand QM' or rather he says: 'I understand now that there isn't anything to be understood...'

And speaking about himself, he adds (Dyson 2007)

...the important thing about quantum mechanics is the equations, the mathematics. If you want to understand quantum mechanics, just do the math. All the words that are spun around it don't mean very much.

Despite the hundreds of books and of international conferences discussing both physical and philosophical problems of QM, the basic conundrums remain alive and as unresolved as they were eight decades ago. Fortunately nobody (to our knowledge) has blamed Bell of having been unable to understand QM, as was said about Einstein. He, Bell, solved the matter his own way: at the time of some lectures he explained that during the week he used the handy FAPP theory. The weekends however he would regain his principles and search for something better (quoted in Gisin 2002).

Experience shows that so far, neither physical nor philosophical arguments have been effective to get us out of the muddle. For the normal practicing physicist the philosophical arguments, when they have a meaning for science, are little more than an abstraction, an ethereal generalization of the truths already discovered by science. But if along its lines of reasoning, science has been unable to set foot on the profundities of the quantum world, we cannot expect philosophy to unfold them for

us. Something of revealing importance can thus be extracted from these persistent discussions: as long as the issues are debated and the differing points of view defended from *inside* quantum theory, no definite conclusion can be reached. What is required then is to gain a look onto QM from *outside* it, to get a wider and clearer perspective. The work presented here represents precisely a systematic attempt to look onto QM from outside it, with the help of a deeper physical theory. This provides us with the possibility of getting answers from a wider perspective than that obtained by just interpreting (or reinterpreting, or misinterpreting) the formalism.

In fact, many of the difficulties with QM arise as a result of the interpretation ascribed to its formalism. Though there have been claims that QM does not need interpretation,<sup>2</sup> the truth is that in no other place of physics do the theory and its formal content elicit such diverse and even contradictory meanings as in QM (see Sect. 1.2). And indeed, the formal apparatus of a theory is in general not enough to interpret it.<sup>3</sup> If “nobody understands quantum theory” it is difficult to hold that the theory speaks for itself. Apart from the immediate problem that represents the lack of consensus on the interpretation of QM, the critical point is that many interpretations of it, particularly the dominant one, jeopardize (when not simply do away with) some principles that have been pillars of the whole edifice of physics. Even if—or precisely because—the principles of scientific philosophy are a distillate of the most fundamental discoveries of science, *if* QM demonstrates that Nature (not a certain description of it) is incompatible with some of those principles, as might be realism, determinism, locality or objectivism, then the philosophical framework must of course be modified accordingly, instead of forcing us to attune physics to worn presuppositions. It could be that the advances of science demand a revision of what is taken at a given moment for a firmly established general outlook; history is full of experiences of this nature. The central concerns and theories of the philosophy of science should be consistent with scientific discovery, and are therefore subject to revision, just as happens with science itself. When the scientific case is clear, science philosophy must adapt to what science tells us. But that requires an absolutely convincing demonstration, since principles as realism, say, are just that, general principles extracted from a huge plurality of cases and circumstances, so their generality, universality, solidity and soundness are utterly confirmed. Convincing demonstrations, not a mere interpretation of the formal apparatus of QM, are thus required to abandon these solid principles.<sup>4</sup>

In the following section we present and comment on some of the most basic issues that beset QM, which originate when adopting a certain interpretation of the theory.

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<sup>2</sup> See e.g. Fuchs and Peres (2000), or Omnés (1994). Compare with, e.g. Bunge (1956), de Witt and Graham (1974), and Marchildon (2004).

<sup>3</sup> For example, a given system of linear differential equations can represent a mechanical, an acoustical, an electrical or an electromagnetic system, or even an analog computer as well. There is ample conceptual space to accommodate the interpretation.

<sup>4</sup> Virtually all science philosophers have received with approval the philosophical conclusions arrived at from (orthodox) quantum mechanics, despite its nonrealistic (even antirealistic) and subjective trends. Far from helping to drive quantum physics towards a more realistic conception, this of course has contributed to reinforce such trends.

By the same token, in this introductory chapter there is no attempt to resolve these issues or give answers to them. It is along the subsequent chapters, as we develop the theory, that we will be finding answers. This will allow us to summarize, in the final chapter, the insights afforded by the theory and discuss its outlook.

### 1.1.1 *The Sins of Quantum Mechanics*

Let us point out in brief some of the sins of QM—some venial, others capital—that are readily found and discussed in the scientific literature, particularly the one written under the spell of the orthodox interpretation. It may seem amazing that two discussions on the subject written by physicists (one of whom later became a recognized philosopher of science) published almost half a century apart (Bunge 1956; Laloë 2002), touch essentially upon the same fundamental questions, of course with an emphasis that corresponds to the given moment.

- QM is an indeterministic theory. Indeed, though the quantum dynamic laws evolve deterministically, the theory is unable to predict individual events. The most the theory can offer are probabilistic predictions, whence the specific outcome of an experiment cannot be determined in advance. In itself, indeterminism is not a regrettable property of a physical theory. The statistical theories of classical physics are indeterministic (or, for some people, they obey statistical determinism) and this is not considered a shortcoming. The reason is that in such cases the origin of such indeterminacy is clear. Recall for instance the statistical description of a classical gas; there is a distribution of velocities of the molecules that calls for a statistical description with no practical alternative. The distribution of velocities of the molecules is a direct consequence of the fact that there is a myriad of microstates compatible with the macroscopic state under scrutiny, all of them having equivalent possibilities corresponding to the initial conditions. In other words, the indeterminacy is a feature of the description, not of the system itself. By contrast, in the usual rendering of QM we have no more explanation for the statistical indeterminism than the indeterminism of the theory. For some this means quantum indeterminism is irreducible.<sup>5,6</sup>

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<sup>5</sup> Determinism must be clearly distinguished from causality, the latter referring to an ontological property of the system. The notion of indeterminism wavers in the literature from ontological to epistemic connotations, and from objective to subjective meanings. In this book we understand by (physical) determinism a property of the *description* of a physical system, not of the system itself, and thus of epistemological nature. Although many different meanings are ascribed also to causality, this term refers to a direct *genetic* connection among the elements of the description, i.e. to an ontological property of the underlying physical reality. We could say that causality refers to the hardware of nature, determinism to our software about it.

<sup>6</sup> Whether the indeterminism is ontic or merely manifests itself at the observational or descriptive level is a controversial issue, to which every decoder adds his own preferred interpretation (see Bunge 1956 for examples). Still, the attempts to construct a fundamental and deeper *deterministic* theory from which QM could emerge through an appropriate mechanism to generate indeterminism,

- QM has intrinsic limitations to its predictive power. As stated above, the predictions of QM are only probabilistic. The specific reading of the meter is beyond what QM can predict, yet Nature gives in each instance a well-defined unique answer; we are therefore faced with two possibilities: (a) the predictions of QM are incomplete, or (b) the predictions are complete and God plays dice.
- QM is a noncausal theory. One of the most conspicuous examples of noncausality in QM (which is also a towering manifestation of indeterminism) are the Heisenberg inequalities, which imply the existence of unavoidable (quantum) fluctuations. The cause for such fluctuations is alien to the theory (assuming that a cause must indeed exist), or is simply inexistent at all (assuming that no property of Nature escapes to the quantum description). There is a long list of schools and subschools, with different views on whether the Heisenberg inequalities refer to uncertainties (a measure of our ignorance), to (objective or ontic) indeterminacies, or to something else.<sup>7,8</sup> In any case, the widespread attitude is that no cause for quantum fluctuations is considered to be required, and even less, investigated; they can happily remain ‘spontaneous’.
- QM is not a legitimate probabilistic theory. Though the predictions of QM deal with probabilities, no formulation of QM is fully consistent with a genuine probabilistic interpretation (in the classical sense). The use of probability amplitudes instead of probabilities implies a distinctive probability theory by itself. For example, negative probabilities appear in QM not only in connection with phase-space distributions, but also as a result of the superposition principle. The amplitudes can interfere destructively and give rise to negative contributions to the probability densities, of a nonclassical nature. These results have led to a widespread acceptance of negative probabilities as a necessary trait of quantum theory.<sup>9</sup>

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speak to the existing conviction in some circles that quantum indeterminism demands explanation. For example, t’Hooft has envisioned a process of local information loss leading to equivalence classes that correspond to the quantum states (t’Hooft 2002, 2005, 2006).

<sup>7</sup> The textbook (and historical) explanation of the Heisenberg inequalities as a result of the perturbation of, say, the electron by the observation cannot be taken as the last word, at least because the inequalities follow (as a theorem) from the formalism without introducing observers and measuring apparatus.

Within the statistical interpretation of QM (see Sect. 1.2.2) they indeed refer to the product of the (objective) variances of two noncommuting dynamic variables in a given state (see e.g. Ballentine 1998, Sect. 8.4).

<sup>8</sup> The interpretative difficulties are even greater with the energy-time inequality, because this inequality (in its usual form) does not belong to the customary formal apparatus of the theory. There are of course various proposals to replace it (see e.g. Bunge 1970; Jammer 1974, Sect. 5.4). Also the introduction of a time operator has been explored by several authors (see e.g. Muga et al. 2008, in particular the contribution by P. Busch; see also Hilgevoord and Atkinson 2011).

<sup>9</sup> The acceptance of negative probabilities implies a fundamental change in the axioms of probability theory. Since “they are well-defined concepts mathematically, which like a negative sum of money ...should be considered simply as things which do not appear in experimental results” (Dirac 1942; see also Feynman 1982, 1987; d’Espagnat 1995, 1999; and the detailed discussion in Mückenheim et al. 1986, where they are called *extended probabilities*), they tend to be pragmatically accepted, even if this renders the meaning of probability obscure. Once this door is open, anything may step

- QM is a nonlocal theory. Nonlocality is a major issue for quantum physics. It is inherent to the structure of the theory, although subject to quite different connotations, some of which lead to the notion of action at a distance. Locality is a most fundamental physical demand; it pertains to the conceptual framework upon which theoretical physics is founded, yet it is apparently contravened by *all* quantum systems, not only multipartite ones, in which the entanglement introduces the well-known nonlocal correlations between the subsystems. Thus, to understand the origin and meaning of quantum nonlocality is a major task for a deeper understanding of present-day physics, one that has been put aside in favour of the development and expansions of its applications.
- QM is a theory of observables, not of *beables*. According to the more extended interpretation of QM, it is meaningless to speak of the value of a certain variable of a physical system until the corresponding measurement has been performed. Therefore the theory refers to measured variables (observables) and not to preexisting, objective, individual properties of the system (beables). This is clearly a shortcoming from a realist point of view.
- QM is a contextual theory. In quantum theory (Bell's) contextuality means that the result of measuring an observable *A* depends both on the state of the system *and* the whole experimental context. In particular, it depends on the result obtained in a previous (or simultaneous) measurement of another, *commuting* observable *B*. Thus the value attributed to *A* depends on the whole context.<sup>10</sup>
- QM requires a measurement theory. The pure states of the microworld are not realized in our everyday world. We need some means to reduce the former to mixtures when passing to the macroscopic level. Traditionally the assumed agent is the observation (measurement); thus the observer and his proxy break actively into the description in order to produce results.<sup>11</sup> It would not be an overstatement to say that the notion of measurement in QM raises more conceptual problems than those it is intended to solve.
- QM postulates a nonunitary evolution foreign to its formalism. In its usual interpretation, QM demands the collapse of the vector state (the projection onto a subspace associated with the observable under measurement) as a means to reduce all the possibilities encoded in the state into a single one, to account for the measurement process.<sup>12</sup> It is thus the observer who does the dirty task of suspending the uni-

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in; thus, for instance, imaginary probabilities have been considered to reconcile quantum theory with locality (Ivanović 1978).

In Khrennikov (2009) the probabilistic machinery of quantum mechanics is extended within a realist point of view, to the description of any kind of contextual contingencies, which leads to a theory that finds application in several fields of inquiry, including economics and psychology.

<sup>10</sup> We are referring to the use of the term 'contextuality' as e.g. in Bell (1985) or Svozil (2005). In particular, this property of a quantum systems is at the base of the response of (Bohr 1935) to the EPR 1935 argument (see Einstein et al. 1935).

<sup>11</sup> One should add that a theory of measurement (i.e., of *our* methods to interrogate nature) cannot be part of a *fundamental* (thus general) description of nature, because the former must be quite specific and detailed in every instance to have any predictive capacity.

<sup>12</sup> The notion of reduction or collapse of the wave function was introduced as a quantum postulate by von Neumann (1932) and Pauli (1933). There is no clear definition of the qualities of the perturbation

tary and causal evolution law to allow for the (nonunitary) collapse of the wave function.<sup>13</sup>

- QM risks becoming subjective with the entry into scene of the observer. The observer is an active intruder, the element that transforms the potential into the real; however, he/she is not part of the libretto. For some people this is an opportunity to add subjective elements to the interpretation.<sup>14</sup>
- QM requires a boundary between the observed and the observer, but the theory cannot define it. To avoid an infinite regression, the measuring instrument must be classical. Thus a part of the world is not described by QM, despite the fact that it is considered to be a fundamental theory, one that should apply to everything.<sup>15</sup> Since quantum theory should lead to the description of the macroscopic world as a limiting process, in principle it cannot refer to elements of the latter in its foundations; yet it does precisely that.
- QM deals with objects of undefined nature. The theory does not embody an objective strict rule of demarcation that distinguishes between corpuscular and wave entities. Worse, even: whether these objects exhibit a corpuscle- or a wavelike behaviour is controlled by the free undertakings of the observer. There is room for three quarks within a proton, but an electron may occupy the whole interferometer before hitting a single point on the screen.
- QM lacks of a space-time description. In particular, the notion of trajectory is foreign to QM, presumably prevented by the Heisenberg inequalities. Thus, QM describes what the atomic electrons do in the abstract Hilbert space, but says nothing about what they do in common three-dimensional space.<sup>16</sup>
- QM is a nonrealist theory. The usual quantum description averts realism from several sides, through the lack of a space-time description, incomplete causality,

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of the physical system that demarcate the two ways of evolution (the causal one and the collapse). Thus, “[T]he observed system is required to be isolated in order to be defined, yet interacting to be observed” (Stapp 1971). Within the single-system interpretation the collapse is avoided by means of the ‘many-worlds interpretation’ (or ‘relative-state formulation’) of QM (Everett 1957, from Everett’s thesis 1956), according to which the world splits into as many independent worlds as different results of the measurement can occur. We will not discuss here this (extreme, even if logical) interpretation.

<sup>13</sup> It is of course possible in principle to include the measurement apparatus in the Hamiltonian; a well known example of this is Bohm’s theory (see Chap. 8). This helps to express the measurement problem in more realistic terms. Another well-known example is van Kampen (1988).

<sup>14</sup> An argument against the observer, aimed at recovering objectivity in the quantum ‘potentialities’, has been advanced from cosmology. According to inflationary theory, the early classical inhomogeneities in the cosmic microwave background originated in earlier quantum fluctuations. This quantum-to-classical transition took place much before even galaxies existed. It follows that the measurement problem in cosmology is of a different kind (Perez et al. 2006; Valentini 2008).

<sup>15</sup> It is even applied to the universe as a whole; see e.g. Hartle and Hawking (1983). A well-grounded critique of the boundary, for the general public, is contained in Wick (1995).

<sup>16</sup> However, the possibility to construct quantum trajectories (by considering additional elements into the usual quantum description) has received special attention since the times of de Broglie. The best known example of quantum trajectory is perhaps the one afforded by Bohm’s theory (discussed in Chap. 8).



unexplained indeterminism, nonlocality... (see Sect. 1.3 for a discussion on realism and quantum mechanics).

## 1.2 The Two Basic Readings of the Quantum Formalism

### 1.2.1 *The Need for an Interpretation*

The pure theoretical skeleton of a physical theory, its *formalism*, says nothing about the world; it is devoid of empirical meaning. To attribute physical meaning to the abstract mathematical apparatus, a set of *semantic rules*, collectively known as *the interpretation*, is required. The interpretation assigns a concrete empirical meaning to the nonlogical terms in the theoretical model (such as mass, force, charge, electric field, and so on). Physically, the model normally does not resemble what it models; the conformity resides in the functioning.

Which is the meaning we should ascribe to the different elements in the quantum formalism, e.g. the wave function, solution of the Schrödinger equation for a given problem? The answer is left to our ingenuity. And this is where the real problem starts... It is not difficult to count a dozen different interpretations of the *same* theory: Copenhagen interpretation (Bohr, Heisenberg, etc., from 1926 on); ensemble interpretation (Einstein, etc., from 1926 on); de Broglie–Bohm theory (de Broglie 1927; Bohm 1952a, b); quantum logic (Birkhoff and von Neumann 1936); many worlds (Everett 1957); stochastic electrodynamics (Marshall 1936); stochastic mechanics (Nelson 1966); modal interpretations (van Fraassen 1972); propensities of smearing (Maxwell 1982); consistent histories (Griffiths 1984); quantum information (Wheeler 1983); transactional interpretation (Cramer 1986); zitterbewegung interpretation (Hestenes 1990); no-signaling plus some nonlocality (Popescu and Rohrlich 1994); relational quantum mechanics (Rovelli 1996); and so on. According to other authors, QM does not require an interpretation at all (Peres (2000)), or on the contrary, there is only one legitimate interpretation (Omnès 1994), or even any interpretation goes (Feyerabend 1978). We are further told that the description does not really describe the system, but merely our knowledge (or information) about it (Heisenberg 1958a, b, but see Marchildon 2004; Jaeger 2009); or that the theory is about measurements and observables and not about beables (see Bell 1976, 1985); or that the awareness of our knowledge ‘actualizes’ the wave function, thus promoting us from external passive bystanders into active (although involuntary) participators (Patton and Wheeler 1975), without being included however in the formal structure. A recent trend is to say that QM refers not to matter, but to bits of information (see e.g. Vedral 2010). And so forth...

Thus we have a nice formal description of the quantum world, empirically adequate for our purposes, but we still lack of a real understanding of that world. No wonder that there are expressed recognitions of the need of a fundamental and deep amendment of our present quantum image (see e.g. Delta Scan 2008; Stenger 2010).

## 1.2.2 A Single System, or an Ensemble of Them?

A most basic and crucial question for any interpretation of QM relates to the meaning of the wave function: does it describe the dynamics of a single particle, or does it instead refer to an ensemble of similarly prepared particles? The answer to this question distinguishes between the two mainstreams of the interpretation of quantum theory, the Copenhagen and the ensemble interpretations.<sup>17</sup>

The usual textbook standpoint on QM is based on some variant of the Copenhagen (or orthodox) interpretation (CI).<sup>18</sup> It might also be called the *customary*, *mainstream* or *regular* interpretation, although it is not so clear that the present-day practicing physicists (and physical and quantum chemists) adhere to it in their daily endeavours as tightly as such names may fancy. The founding fathers of the CI are of course Heisenberg (1930) and Bohr (1934), who were joined almost from the start by physicists like Pauli, Dirac (1930), Born (1971), von Neumann (1932), and Landau. One should bear in mind, however, that the name CI does not refer to a sharp set of precepts, since a wide range of tenets with respect to some of the central interpretative issues can be distinguished among its practitioners. Thus it encompasses a collection of variants of interpretation rather than a tight doctrine. In a broad sense one refers normally (but not necessarily) to any of the members of such collection as the *conventional interpretation*. The basic tenet of the CI of QM is that a pure state provides a description *as complete and exhaustive as possible of an individual system*. So, QM goes as far as is possible in the knowledge of Nature, and physicists must renounce once and for all the hope for a more detailed description of the individual; Nature imposes upon us a limitation to our knowledge. This assumption has enormous consequences, some of which will be discussed in the following section.

A very different outlook ensues from the ensemble (or statistical) interpretation (EI) of QM. According to this interpretation the wave function refers to a (theoretical) ensemble of similarly prepared systems, rather than to a single one. The earliest attempts to formulate an ensemble interpretation of QM are found in Slater (1929), Schrödinger (1932) and Fürth (1933). Other early advocates of this interpretation were Langevin (1934), Popper (1959), Einstein (1936, 1949), Landé (1955, 1965), Blokhintsev (1964, 1965) (the original Russian version of 1949 was the first systematic treatment of the ensemble interpretation of QM).<sup>19</sup> Being an intrinsically

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<sup>17</sup> An early introductory account of the different interpretations of QM and their variants can be found in Bunge (1956). More advanced expositions, also by professional philosophers of science, are found, among others, in Bunge (1973) and Redhead (1987). A more recent monograph by a physicist is Auletta (2000).

<sup>18</sup> Since this interpretation (as indeed all interpretations) contains in an essential way Born's (1926) probabilistic notion of the wave function, and in addition it was strongly influenced by Heisenberg, it would be more properly called *Copenhagen-Göttingen interpretation*. Wigner (1963) proposed to apply the term 'orthodox' more specifically to the view adopted by von Neumann, as reshaped by London and Bauer (1939).

<sup>19</sup> More recent advocates are Margenau (1958, 1978), Sokolov et al. (1962), Mott (1964), Marshall (1965), Lamb (1969, 1978), Belinfante (1975), Newton (1980), Santos (1991), de Muynck (2002), Laughlin (2005), Khrennikov (2009), Nieuwenhuizen (2005) (in Adenier et al. 2006), etc. For an

statistical description, for the advocates of the EI the description afforded by the wave function  $\psi$  is *neither complete nor exhaustive of the individual systems* that conform the ensemble (which in its turn gives significance to the different probabilities encoded in  $\psi$ ). Chance enters into the picture in a fundamental way; the wave function does not “represent things themselves, but merely the probability of their occurrence” (Einstein 1933, slightly adapted).

### 1.3 Is Realism Still Alive?

“Quantum mechanics demolishes the view that the universe exists out there” (Wheeler 1979).

Quantum mechanics, or a certain interpretation of it?

Such a view of QM is clearly nonrealist. This may not mean much to some, to others it may be unimportant, but to still others it may be of high significance, because philosophical realism is not a capricious free invention. As mentioned earlier, philosophers arrived at the notion of realism by distilling the works of creative scientists (and philosophers) along the centuries, and recognizing and extracting the essence of their diverse procedures. They have thus discovered that there are realist scientists, nonrealist scientists and anti-realist scientists, and that the large majority of creative natural scientists are (spontaneously or consciously) realist and work under the assumption (or conviction) that the world they are studying is not an illusion, but exists by itself. This is the essence of scientific realism: the belief in a real world, external to us, independent of our attention to it, a world in which we act, which acts upon us, and upon which we act to know more about it. A nonrealist negates either the reality of the external world or its independence from us, or both; an antirealist is more extreme and believes that the world is a result of our mental activity.<sup>20</sup> Along the centuries, science, with its remarkable development, has nourished and reinforced realism. Shortly stated, realism is a synthetic result of the scientific venture.

Further to the general defining attributes of scientific realism—external reality, independent from our deeds, and the possibility to know the world—realism in physics embodies other demands of general validity. An obvious one is causality, which lies at the basis of physical science. Another is the recognition that the phenomena occur in space and time, and thus should admit a space-time description. A

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important defense of the ensemble interpretation of QM see the old paper by Ballentine (1970), or his more recent books (1989, 1998); Ballentine takes, however, an indeterministic view. Home and Whitaker (1992) contains a detailed discussion, from a realist point of view, of the different versions of the ensemble interpretation of QM. Further, an interesting analysis is that of Rylov (1995) who demonstrates on general arguments that QM (including Dirac’s theory) necessarily refers to an ensemble of particles.

<sup>20</sup> It is not too difficult to find openly antirealistic views nourished by the conventional interpretation of QM. See e.g. Rigden (1986), Adler (1989). There are also some researchers that go as far as to consider that the universe itself is not real; see e.g. Henry (2005).

third one is that the causal relations are local, which means that there are no actions at a distance.<sup>21,22</sup>

Let us look at some of the features of QM as seen from the CI and the EI, to make clear the position of these interpretations with regard to realism. In doing so, we will touch upon some of the difficulties encountered in Sect. 1.1.1 and discuss them more at length.

As stated above, a most distinctive quality of QM is its indeterminism, which in some instances is taken as noncausality. In a situation commonly considered, a given observation can lead to one of a miscellany of possible results (e.g. a specific eigenvalue among a set of values). Which is the outcome is a matter of chance, and the CI grants that nothing, except chance, determines the result. The example of the decay of a single radioactive nucleus is illustrative: quantum theory can correctly assign a mean lifetime to the nucleus, but it cannot predict the precise moment or direction of the decay products. However, a nearby detector shows that such moment and such directions exist. The precise prediction escapes quantum theory. By considering the quantum description to provide the most complete attainable information about a given system, not unusually the CI declares that precise values of the physical variables cannot be predicted by QM simply because such variables do not have pre-existent values; they do not exist until a measurement is performed, until a precise value is recorded).<sup>23</sup> Thus, for example, for the conventional school, the position of the particle is materialized or brought into being, as it were, as a result of its measurement. The values of the dynamical variables are thus objectively undetermined prior to their measurement, and only probable values can be assigned to them; probabilities become irreducible. Since the nonexistent cannot be measured, it is the measurement itself which fixes the measured value, giving reality to it. It is here that the observer (or the observer's proxy) slips into the description; the realist fundamental principle that physics should refer to the world rather than to our knowledge of it (or information about it) is eroded, and with it the no less fundamental demand of a strictly objective rendering of the physical world. All this was clearly recognized

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<sup>21</sup> We are using here the term *realism* with the meaning of *gnoseologic realism* (Bunge 1985), i.e. ontologically as the belief in an external world, independent of our theories and observations, and epistemologically as the conviction that it is possible to know that world, part by part. However, in some places we use a restricted notion of physical realism which originates in the famous EPR 1935 paper, namely that if a value can be determined for a variable without disturbing the individual system, there exists an element of reality associated with it, even prior to the measurement. According to this notion, the individual systems are at all times in objectively real states (Deltete and Guy 1990), even if unknown, and should in principle be amenable to a space-time description.

<sup>22</sup> An introductory discussion of scientific realism by a realist can be seen in Boyd (1983). The author shows, in particular, how the educated (expressly in science) common sense is a good guide towards scientific realism.

<sup>23</sup> A word of caution is needed here. The measured value may or may not preexist, it suffices to consider that some feature or property related to the measured value preexists. The clearest example is perhaps the measurement of a spin with a Stern-Gerlach apparatus, which obviously may reorient the spin. Thus, a realist theory is compatible with both possibilities; it all depends on the nature of the measured variable. See Allahverdiyev et al. (2013).

(and accepted) by Bohr (1928) in his famous Como Lecture of September 1927, a characteristic sentence of which says:

...the finite interaction between the object and the measuring devices... implies... the necessity to renounce the classical idea of causality, and a radical revision of our attitude toward the problem of physical reality,

and by Heisenberg in denying the existence of an underlying quantum realm (Heisenberg 1958a, page 129):

...the idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them ... is impossible...

or further (Heisenberg 1958b, page 15):

... the natural laws formulated mathematically in quantum theory no longer deal with the elementary particles themselves but with our knowledge of them. Nor is it any longer possible to ask whether or not these particles exist in space and time objectively...

The role of the observer is not limited to bringing out a real physical variable out of a mere potentiality, it includes determining the very nature of the system. For instance, in an electron diffraction experiment the electron suffers a series of transformations from being a (more or less) localized entity (with corpuscle-like properties) to becoming a structure that fills a macroscopic volume (with wavelike properties) and vice versa. It seems difficult to bring to terms this series of transmutations with the idea of a reality independent of our undertakings.<sup>24</sup>

Along with the observer, a radical form of nonlocality is introduced into the theoretical framework: the collapse of the wave function—instantaneous over the whole space—determined by a local measurement. Indeed, the collapse, which is the theoretical counterpart of the changes on the individual system brought about by the active observer, becomes the inevitable mechanism by which a specific result is selected from among the various possibilities. The collapse disrupts the orderly causal development described by the evolution equation, introducing an abrupt fall to a lawlessly established state of a certain statistical mixture (these are the spooky actions at a distance, mentioned by Einstein to Born; see Born 1971). Thus two forms of evolution compete within the theory, and it is the observer—the ineluctable intruder—who determines with his actions which of them should operate. Of course, interpreting the collapse as merely a theoretical tool, without ascribing to it a sense of reality, becomes an acceptable pragmatic procedure. But this is not its usual grasp.<sup>25</sup>

<sup>24</sup> In a letter to *Physics Today* by Henry (2004, p. 14) discussing why physics understanding is so poor in the United States, the author ends by saying: “We know from quantum mechanics that nothing is real, except for the observations themselves.” Another typical example reads: “one cannot consider quantum properties as being ‘real,’ in the sense of ‘objective reality’” (Paul 2008).

<sup>25</sup> As is the case with other quantum paradoxes, the collapse of the wave function becomes understandable within the ensemble interpretation. The fact that an *individual* observation is made does not change the (original) ensemble, it only changes our knowledge by giving us an extra piece of information. We add this information to construct a *new* ensemble that corresponds to the updated situation, a quite normal statistical procedure. The ‘collapsed’ state vector describes the new situation.

Since according to the spirit of the Copenhagen interpretation it is meaningless to attribute any existence to a certain physical variable until it is measured,<sup>26</sup> the quantum variables have been transformed into *observables*. Hence, the standard adumbration of QM demands from us to assume that the theory is not about existing objects of nature, but about our measurements and observations on them. Bohr states it clearly (as reported by Petersen 1963):

There is no quantum world. There is only an abstract quantum mechanical description. It is wrong to think that the task of physics is to find out how Nature *is*. Physics concerns what we can say about Nature.

Heisenberg goes even farther (Heisenberg 1958b), by negating the reality of his very object of study:

...the atoms or the elementary particles are not as real [as any phenomena in daily life]; they form a world of potentialities or possibilities rather than one of things and facts.

Out of the frying pan into the fire, today we see a modern version of this idealistic vision of the world swiftly extending in connection with information, which argues that the building blocks that constitute the world are not matter and energy, but ... bits of information (see e.g. Vedral 2010; Boriboje and Brukner 2011, and references therein). A most fashionable formula for this was introduced by Wheeler (1990): “It from bit”, where ‘bit’ stands for the unit of information; according to this dictum, the material world emerges from the (qu)bits of quantum information, not conversely.

As for the possibility to construct a space-time description of quantum systems, the very idea was firmly negated by Heisenberg, Bohr and other founders of QM, who declared the quantum world to be nonvisualizable. Thus, the concept of trajectory was taken as untenable in quantum theory since it is contrary to Heisenberg inequalities (and to the wavelike properties, many would add).<sup>27</sup> The view of a nonvisualizable world helped to do away with the need to explain some of the quantum paradoxes (Jones 2008, particularly Chap. 16). By 1927 quantum trajectories were so insistently negated—with the exception of de Broglie and Einstein<sup>28</sup> (Bacciagaluppi and Valentini 2009)—that at the closure of the Solvay 1927 Congress Lorentz felt obliged to make a declaration of principles:

... I should like to preserve this ideal of the past, to describe everything that happens in the world with distinct images. I am ready to accept other theories, on condition that one is able to re-express them in terms of clear and distinct images.

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<sup>26</sup> The dictum “No elementary phenomenon is a phenomenon until it is a registered phenomenon” (Wheeler 1978, 1983) is a transparent revelation of the positivism that permeates usual quantum theory.

<sup>27</sup> We find trajectories in Feynman’s method of path integrals, but they are virtual and attain arbitrary velocities, and besides all possible trajectories are considered with equal amplitude, not only those (unknown) related to the actual motion followed by a given electron travelling from point *A* to point *B*.

<sup>28</sup> This was precisely one of the persistent arguments put forward by Einstein against the Copenhagen interpretation.

We are not longing for a past full of clear images, if that past is gone for ever. But, is it really gone? As Lorentz put it, we should be ready to accept the new theories, on condition that they are the result of transparent and definitive knowledge, free of free elections. Yet, by embracing the Copenhagen interpretation, we forsake the possibility not only of making precise predictions about individual trajectories, but entertaining that very notion.

The widespread conclusion that the violation of the Bell inequalities by QM demonstrates Nature's nonlocality represents one more argument against realism. As an example, van Fraassen (1989) contends that scientific realism is invalidated at the microlevel by the violation of Bell's inequalities, and therefore it cannot be valid more generally.<sup>29</sup> In fact, there is no need of these inequalities or any of their variants to demonstrate that QM corresponds indeed to a nonlocal *description*, as follows, for example, from Bohm's interpretation of QM. The point is that we must carefully distinguish between Nature being intrinsically nonlocal and a nonlocal rendering of the relevant portion of Nature.

To maintain a realist view of physics, either the definition of realism must be changed to accommodate for the new situation, or we must accept that QM cannot be the final tale. The standard lore purports the first alternative, which leads to consider that our current notion of realism is incompatible with science.<sup>30</sup> For example Stapp (1972) writes "If the statistical predictions of quantum theory are true, an objective universe is incompatible with the law of local causes." It is interesting to compare this with Einstein's contention (in Born 1971, page 221):

I cannot seriously believe in [quantum mechanics] because it cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance.

Clearly Einstein opted for the second alternative above, namely to admit that QM is not the final tale. As he expressed in Einstein (1949):

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<sup>29</sup> By contrast, Shimony (1989) contends that the formalism of QM may have to be modified so that the theory meets certain metaphysical constraints. He even suggests the need to modify QM to save physical realism. By way of example he points out a possible modification of the topology of space-time at a subquantum scale. He alerts the reader, remarking that "[t]his proposal is the antithesis of [his] attempt to draw philosophical consequences from scientific results, for it indicates rather a reliance on philosophical considerations to supply the heuristics for a scientific investigation." (page 34).

As can be surmised, the conceptual problems associated with the violation of the Bell inequalities have led some authors to even question QM as a fundamental theory of nature [see e.g. Howard (1989)].

<sup>30</sup> More precisely, that local realism and quantum theory are incompatible. This can be argued, as summarized by Ferrero (1987), as follows: It is possible to demonstrate that the following four statements are incompatible:

a) Realism; b) Locality; c-EPR) Quantum mechanics is a complete theory; c-Bell) Quantum mechanics accepts hidden variables (it is not a complete theory); d) Quantum mechanics is a valid theory of Nature.

a, b, d and c-EPR are the assumptions in the EPR paper;

a, b, d and c-Bell are the assumptions in the early derivation of Bell's theorem.

Thus, independently of the completeness of QM (i.e., of c-EPR or c-Bell), a, b and d are incompatible. In Bell 1971 the demand c-Bell was eliminated.

If in quantum mechanics we consider the psi-function as (in principle) a complete description of a real physical situation, we thereby imply the hypothesis of action-at-distance, a hypothesis that is hardly acceptable. If, on the other hand, we consider the psi-function as an incomplete description of a real physical situation, then it is hardly to be believed that, for this incomplete description, strict laws of temporal dependence hold.

By assuming that QM goes as far as possible in the knowledge of Nature, the CI forces us to admit a nonrealistic, irreducibly indeterministic, nonlocal and noncausal world. In contrast, once we concede that the quantum description is incomplete, the possibility of going beyond QM without having to renounce to realism opens in principle. A means to recover realism is thus offered by adhering to the ensemble interpretation. In particular, by recognizing that quantum theory is statistical and as such incomplete, the ensemble school allows for the possibility of understanding the indeterminism as due to such incompleteness, without necessarily assigning to it a more fundamental meaning, as could be that of an ontological property, or, perhaps, an irreducible indeterminism at the observational level. This leaves the door open to further studies at a deeper level, for the identification of the source of the indeterministic (or stochastic) behavior characteristic of quantum systems. For those who profess this credo this is a most important alternative. For a hard realist, who believes that each individual system has always a real state (may be unknown), and that among the tasks of physics an important one is to discover such real states, an *essentially* statistical theory cannot be taken as complete.

In an extended variant of the EI (also here there are variants, of course) the particle is assumed to have at each moment a set of well-defined, objectively real properties, even if these properties are not simultaneously described by the wave function.<sup>31, 32</sup> Thus for example, one thing is to say that the values of two variables associated with noncommuting operators cannot be simultaneously ascertained by resorting to  $\psi$ , and another one is to say that such values are not simultaneously defined, or simultaneously existent, even if distributed and unknown. Preexisting values thus may exist (Deltete and Deltete and Guy 1990), yet the wave function  $\psi$ —a catalog of all the different possible outcomes—can only assign to each of them a certain probability. In the example of the decay of a single radioactive nucleus the fact that

<sup>31</sup> There exists a widespread belief that if two quantities cannot be measured simultaneously, they do not exist simultaneously. This (positivist) identification of existing (being) and being observed (measured) is of course merely a point of view; it is not part of the postulates of QM.

<sup>32</sup> A simple example may be illustrative of the ambiguity of the quantum description. Consider the state vector of two spin 1/2 particles in the singlet state (referred to a certain direction  $z$ )

$$|00\rangle_z = \frac{1}{\sqrt{2}} (|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle).$$

A rotation of the system of reference to an arbitrary direction  $\hat{n}$  transforms this description into

$$|00\rangle_{\hat{n}} = \frac{1}{\sqrt{2}} (|\hat{n}_+\rangle|\hat{n}_-\rangle - |\hat{n}_-\rangle|\hat{n}_+\rangle).$$

Now the spins are referred to the arbitrary direction  $\hat{n}$ . Thus, the spins may be aligned in any direction whatsoever. In other words, the state vector gives absolutely no indication of the actual direction of the spins. From the ensemble point of view, the individual spin pairs are distributed uniformly in all directions.



a precise prediction escapes to QM, does not mean by necessity that there are no precise (although unknown) factors precisely determining the result. Thus, the EI advocate distinguishes between the capabilities of our theories or descriptions, and what happens in the real world, at the ontological level. A particular, but immediate consequence of this is that the notion of trajectory, though recognized as foreign to the quantum description, is not forbidden in principle.

From an ontological point of view, what the EI and CI schools claim is the preexistence or not of features that lead to the observed value (see, however, footnote 23). Thus, referring to the observables of the CI, Bell contends: observables are not beables (Bell 1987, particularly articles number 5 and 7).<sup>33</sup> The transition from beables to observables—from preexisting values to undefined or nonexistent values—is one most important issue of quantum theory, which remains nevertheless unstudied. Out of the blue the observer enters the scene, although the quantum-mechanical formalism does not provide tools to establish where that boundary between the observed and the observer lies, leaving room for an ambiguity and cloudiness that is totally strange to theoretical physics. Bell (1987, article 20) refers to this in unequivocal terms: “It is the toleration of such an ambiguity, not merely provisionally but permanently, and at the most fundamental level, that is the real break with the classical ideal. It is this rather than the failure of any particular concept such as ‘particle’ or ‘determinism’.”<sup>34</sup>

The pictures provided by the CI and the EI differ so widely—they in fact exclude each other—that at first glance it should be a simple matter to empirically demonstrate the fallacies behind one or the other. But almost eighty years have elapsed since the advent of quantum theory and the dichotomy remains, notwithstanding the endless discussions and enlightened studies on the subject.<sup>35</sup> The root of the difficulties is that the problem is deeply influenced by the personal philosophical stance. There coexist several general outlooks about the world, and each one of us adopts one or another, consciously or unconsciously to different degrees. This is an (apparently) free personal selection, more or less as (apparently) free is the selection of a religious credo. Add to that the characteristic positivistic standpoint that pervades textbooks, entangled with their scientific content. The physics student is normally unprepared to recognize the presence of this mixture, and less so to disentangle it, so that he ends up assimilating as established knowledge what is far from that.

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<sup>33</sup> Not surprisingly, other terms equivalent to beable have been proposed in the literature, such as ‘being’ or ‘existent’ (Shimony 1978; d’Espagnat 1984). Bell (1987, article 19) adds ‘beer’ as another one, personally suggested to him by Zumino.

<sup>34</sup> A strong contention against the pragmatic and nonrealist views associated with the observer and his (hers in his language) measurements, reigns in the whole little (big) book of Bell on the foundations of quantum mechanics (Bell 1987). He even says that there are words that should not belong to the lingo of theoretical physics and should be banned from it, such as ‘measurement’, ‘observation’, ‘observer’.

<sup>35</sup> Reviews or reprints of important work expressing differing views, as well as ample lists of references to papers dealing with this subject, can be found in de Witt and Graham (1974); Belinfante (1973); Jammer (1974); Nilson (1976); Wheeler and Zurek (1983); Cushing and McMullin (1989); Ballentine (1989, 1998); Omnès (1994, 1999); Home (1997); Auletta (2000); Bertlmann and Zeilinger (2002), etc. The list is endless.

For a realist the CI is implausible, to say it mildly (other more bellicose terms have been used), while a moderate orthodox considers the EI full of unnecessary metaphysics Fuchs and Peres (2000), or just dogmatic. For a more radical orthodox, the EI lacks the space needed to accommodate other elements demanded by his world view, such as the observer and perhaps his mind. The pragmatic (FAPP) physicist argues that the Copenhagen theory has been used successfully for many years without a single failure, which is a proof of its correctness, so we should derive from it our vision of the world and not the other way round. He therefore expects us to renounce our basic principles of physical thought in order to be able to understand physics (Tambakis 1994) on the basis of a ‘quantum syllogism’, an attitude similar in nature to that required to give theological support to the theory of the epicycles, as Jaynes (1993) put it. Further, not few physicists add that QM describes what can be described, and that importing into the quantum domain knowledge that originated in the classical world leads to contradictions and paradoxes (see e.g. Lévy-Leblond 1973), as Bohr alerted us since 1935.

It should be noted that, much as the strength of the EI lies in its essentially statistical nature, in it lies also its weakness. Indeed, the EI (as expounded e.g. in Ballentine 1970, 1989, 1998) is far from being free of difficulties on a very fundamental level. An immediate one is that the quantum-mechanical description is a very particular sort of statistical description, in terms, not of probabilities, but of amplitudes of probability, which have the peculiarity that they interfere among themselves. This is fundamental for QM; it is the basis for quantum interference and entanglement, two most important and characteristic features of the quantum systems. This superposition of amplitudes has at least two implications that go counter to the usual theory of probability: the occurrence of probabilities that depend on the context (contextuality, for short), and of negative probabilities, as remarked in Sect. 1.1.1. Moreover, and connected to the latter, the quantum description does not allow for a joint distribution for noncommuting variables, so it lacks of a true phase-space distribution of general applicability. The fact that joint probability distributions do not exist for noncommuting variables puts into question the very definition of correlations between them. It should therefore not be surprising to find results such as those of Gleason (1957), Bell (1966), Kochen and Specker (1967),<sup>36</sup> showing that even if each observable is considered as a classical random variable, two incompatible observables (noncommuting operators) cannot be viewed simultaneously as classical random variables defined on the same space of events, with *independence* from the specific context. The consequence of this is the nonexistence of a (context-independent) joint distribution of such variables (Suppes and Zanotti 1981). A particular sequel of such theorems is that any hidden-variables theory of QM is necessarily contextual.

Of course, such problems as negative probabilities and the lack of a phase-space description, being characteristic of the quantum formalism, are common to all inter-

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<sup>36</sup> The latter is the name by which the theorem of these authors is commonly known, although a similar result was presented somewhat earlier in Bell (1966). For this reason some authors refer to it under the fairer acronym BKS. There are not so many instances in which an almost simultaneous discovery by several authors is duly recognized—more often, science seems to have become a one-hundred meter steeplechase race.

pretations of QM. However, the problem becomes more acute for the EI, precisely because it sees QM as a statistical theory. The widespread lack of clarity about this topic has led to a series of objections against the ensemble interpretation of QM, with some authors claiming with conviction that such a formulation has been empirically disproved. About this there is still much to say.

## 1.4 What is this Book About?

Through the following eight chapters, a fundamental theory for quantum mechanics is constructed from first physical principles, disclosing quantization as an emergent phenomenon arising from a deeper stochastic process. The elements that sustain the pillars of the quantum-mechanical formalism are identified; hallmarks such as the mechanism responsible for atomic stability, the nature of quantum fluctuations, the origin and meaning of quantum nonlocalities, as well as other central features of quantum theory, are elucidated. All this is carried out within a comprehensive and self-consistent theoretical framework that reaffirms fundamental scientific principles such as realism, causality, locality, and objectivity. Thus, the theory developed in the present monograph hopefully may serve to show that those principles can survive their apparently unsurmountable adversities.

If one lesson can be drawn from the persistent but inconclusive enlightened studies on the meaning of the quantum laws, it is that the analysis of quantum theory from its inside leads to nowhere. Such studies may add richness, deepness and erudition to an interpretation, but the essentials remain the same. The virtue of the theory presented here is that it offers a perspective on the quantum world from outside it; one arrives at the quantum formalism from a distance, with a well-defined physical perspective. The interpretation comes from the physics, not the physics from the interpretation.

### 1.4.1 *The Underlying Hypothesis*

The fundamental hypothesis that is put to test and developed at length in this book is that every material system is an open stochastic system in permanent contact with the random zero-point radiation field (ZPF). The existence of an all-pervading ZPF follows quite naturally from the (classical) Maxwell equations, yet it is foreign to the classical realm, which graciously assigns zero energy to the field oscillators at zero temperature. The ZPF is taken here as the athermal component of the radiation field, as real as any other solution of the Maxwell equations.

The most significant conclusion drawn from the present theory is that the quantum phenomenon, rather than being an intrinsic property of matter or the radiation field, emerges from their interaction. A key element is found in the fluctuations of the ZPF, which correspond to the ‘vacuum fluctuations’ of quantum electrodynamics (QED). Vacuum fluctuations are commonplace in modern quantum theory, though

some of their consequences seem not to be fully appreciated. The fluctuations of the best known vacuum field, the electromagnetic radiation field, are commonly considered to be (totally or partially) responsible for several physical phenomena, such as spontaneous radiation from excited systems (see e.g. Dalibard et al. 1982), the Casimir forces (see, e.g., Davydov 1965; Ballentine 1989), and the Lamb shift (see e.g. Sokolov et al. 1962; Milonni 1994). But apart from serving to explain these quantum *corrections*, the vacuum field is mostly viewed as a nuisance, because it is responsible for several of the infinities that spoil the otherwise smooth quantum calculations.<sup>37</sup> Thus it is swept under the carpet as soon as possible (only to reenter through the back door) and reduced to a merely virtual field. In the theory presented here, rather than being a nuisance, the ZPF becomes central for the understanding of the behavior of atomic matter. Thus, far from being considered as merely the origin of some small corrections or effects to be added on top of the quantum pattern of matter, the ZPF is seen as the source of the quantum behavior of matter. This is the central premise of stochastic electrodynamics (SED), at least from the point of view of the present authors.

Naturally, since all vacuum fields may contribute in principle to the universal background noise, in line with our approach all of them could contribute to the fundamental stochastic behavior of matter on the microscopic level. However, at the scales to which QM is most frequently applied, or for systems basically of an electrodynamic nature, it is the electromagnetic vacuum that plays the pivotal role. At deeper levels or for systems of another sort, it may well be that other vacua become relevant; one can even speculate that all vacuum fields have similar statistical properties, so that a kind of *universality* holds, in the sense that the essential stochasticity of matter is basically independent of the nature of the dominant background field. One could also consider that the required random field is just a construct to simulate the effects of random fluctuations of the metric, and take these as the ultimate origin of the quantum phenomenon (a first heuristic approach to this idea has been given in Santos 2006).

### ***1.4.2 The System Under Investigation***

Our system of study is composed of a material charged particle (rather, an ensemble of them) embedded in the ZPF and having a dynamics that is initially described by a classical (stochastic) equation of motion. Due to the randomness of the system, the theory is statistical in essence. The system is then left to evolve. When, and if, it reaches a reversible regime in which detailed energy balance (i.e., at each frequency of the field) is attained in the mean between the field and matter, the radiative terms in

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<sup>37</sup> Interestingly, at present the zero-point fields are seen as possible sources of the conjectured dark energy. Even if for the moment this is not much more than a speculation (which carries its own problems), it brings to the fore the possible importance of zero-point fields (see e.g. Saunders and Brown 1991).

the dynamical equations for the mechanical subsystem become mere corrections that can be neglected in a first approximation. Under these conditions the evolution turns out to be controlled by the quantum equations. Two independent and complementary derivations of this fundamental result are presented, one in Chap. 4 (leading to the Schrödinger description), and another in Chap. 5 (leading to the Heisenberg formalism). The ensuing classical-to-quantum transition could in a way evoke the usual textbook derivations in quantum field theory that start from a classical field theory and at some point incorporate an extra-classical (quantum) demand. Of course the converse transition, from quantum to classical, is theoretical commonplace—although not always based on conclusive arguments. Yet our procedure differs profoundly in essence and scope from such formal methods; here no quantum demand is introduced (neither a priori nor a posteriori). The ZPF is the extra-classical physical entity that ultimately endows the system with its quantum properties, and in addition guarantees the internal consistency of the theory. The quantum is not the means, but the consequence.

The present theory should not be confused with a semiclassical theory, which treats matter quantum-mechanically but the field classically, or conversely (see e.g. Sokolov and Tumanov 1956). Quite the contrary, here we deal with an initially continuous radiation field (classical, but with its zero-point component) and a particle that initially satisfies classical equations of motion, and show that both end up being quantized.

As a prelude to the derivations in Chaps. 4 and 5, the phenomenological description of QM as a stochastic theory is discussed in Chap. 2, with the purpose of introducing the reader to some of the (old) methods that succeed in showing that it makes sense indeed to understand QM as a stochastic theory. In Chap. 3 we initiate the testing of our hypothesis, by analyzing the consequences of allowing for a zero-point contribution in the equilibrium radiation field. There it is shown that the ZPF has a decisive role in leading to the Planck distribution for the radiation in thermal equilibrium, and to the quantized spectrum for the oscillators of the field.

The treatment of matter and field as inseparable elements of a whole system makes it possible for the theory to go *beyond* QM in the most natural way. It provides the elements to study the radiation and absorption terms—a matter that is normally considered to belong to the domain of QED—which here appear as radiative corrections (neglected in the previous approximation) to the quantum-mechanical description. In Chap. 6 it is shown that indeed, these terms are responsible for the finite lifetimes of excited atomic states, as well as for the absolute stability of the ground state in the sole presence of the ZPF. A further radiative correction that appears quite naturally gives the Lamb shift for isolated atoms, and the corresponding shifts in more complex situations. Of particular interest is the discussion, in the same Chap. 6, related to the origin of the electron spin from the present perspective, as another consequence of the fluctuations imposed on the particle by the field, in this case, those that give rise to rotational motions. We are thus faced with one more element that cannot be predicted from within the Schrödinger realm, but can be unfolded by recognizing the presence and action of the ZPF. Moreover, being the spin of the charged particle

the support for its magnetic moment, it becomes clear that along with it, the theory determines the spin  $g$ -factor of the electron, predicting its correct value of 2.

When the theory is generalized to include systems of two particles, which is the subject of Chap. 7, a phenomenon expected in the present treatment appears, namely the emergence of correlations between (even otherwise noninteracting) nearby particles through common relevant modes of the vacuum field. The correlated motions of the particles attest to their entanglement, induced by the ZPF. Therefore, just as the ZPF may be capable of generating decoherence of the system, it also stands as the most important source of coherence in a significant class of bipartite systems. In particular, when the particles are identical and subject to the same external potential, our results disclose the mechanism underlying the Pauli exclusion principle. More generally, the vacuum field is exhibited as an important source of nonlocality: when this field is ignored, the consequences of its action appear as nonlocal. Nonlocality is further studied in Chap. 8, both for the single-particle case and for a pair of correlated (entangled) particles; these studies unfold the important role played by the so-called diffusive velocity, just the one due to the quantum fluctuations, in providing the quantum system with its characteristic nonlocal descriptive features. In addition, in Chap. 8 we make a brief detour to the causal interpretation of QM, which among interesting features provides an opportunity to glance at a hidden-variables description and to take a fresh look at quantum nonlocality.

Attention is paid in Chap. 9 to the undulatory properties of matter; the de Broglie wave is constructed and shown to originate in the radiation field around the moving particle. A well-defined physical wave is thus naturally associated to the moving corpuscle, yet both entities (particle and wave) are clearly distinguished from each other at all times. Further, a brief discussion is presented regarding the diffraction of electrons, which is explained by arguing that the electron diffraction pattern is but a trace of the pattern produced by the diffracted ZPF. A final section is devoted to a discussion on the relationship between atomic and cosmological constants, with the ZPF, of cosmic presence, acting as the bridge between these two realms of Nature. The final Chap. 10 contains an overview of the main results and implications for QM of the theory developed in the previous chapters. It further provides a brief account of several of its limitations and possible extensions, and ends with a brief discussion of SED in the broader context of theories of space-time metric fluctuations.

It should indeed be noted from the start that the treatment given here to the quantum problem corresponds to a restricted theory in several senses. An obvious one is that the entire discussion is nonrelativistic. Further, the dynamics that takes place during the transition from the original classical state—in which the system is far from equilibrium—to the final state—the quantum regime, controlled by the detailed balance of energy—still needs to be worked out in detail; surely such studies will reveal a rich physics that so far remains hidden. Moreover, the entire treatment is limited here to the description of the dynamics of the material part of the system, while the field is considered as basically (though not entirely!) unperturbed. This excludes by construction the possibility of a full quantum-electrodynamic description. Consequently, the calculation of those phenomena that correspond to QED is everywhere limited in this volume to the lowest significative order of approximation. Within these

limitations, nevertheless, the results derived are always the correct ones, appropriately coinciding with the corresponding predictions of either (nonrelativistic) QM or QED.

By looking at quantum theory from the perspective offered here, we hope that the reader will find a satisfactory explanation or answer to a number of the issues and puzzles mentioned in this chapter, and to others that may be boggling his mind. On the other hand, as discussed in the final chapter, it is clear that there are still many fundamental (and treacherous) facets to learn about the quantum world and its intriguing machinery. QM is a marvelous theory. Just because it is marvelous, it deserves to be better understood.

In concluding, we should note that the theory developed in this volume is an alternative, more advanced, complete and elaborate version of the previously developed theory of SED.<sup>38</sup> When it is necessary to distinguish between the traditional theory and the present version, the latter will be designated with LSED (the *l* stands for *linear*; see the explanation in Sect. 5.2). The theory offers substantial answer to a fundamental question posed by T. H. Boyer,<sup>39</sup> namely: which quantum problems can be explained using classical physics plus the ZPF? A large collection of papers published in the past half century by different authors (by Boyer himself, P. Claverie, D. C. Cole, H. M. França, T. W. Marshall, A. Rueda, E. Santos, ourselves and several others) provided the ground for the construction of the present version and anticipated some of the results derived here. Recent results obtained by some of these authors and others serve to legitimate or reinforce the ones presented here. We therefore wish, through the present work, to pay tribute to all those colleagues who have joined us in this exciting endeavour with the shared conviction that the quantum puzzle *can* be solved, and that the ZPF is a central part of the solution.

## Appendix A: The Ensemble Meaning of Probability

Considering that probability is a somewhat obscure subject, about which all sorts of debates have taken place, the following observations—due in essence to Brody (1975, 1993)—may be appreciated by some of our readers. The point is that several notions of probability coexist and are used in the physical literature, with their respective caveats. It would not be an overstatement to say that the personal grasp of the notion of probability plays an important role in the espousal of one or the other interpretation of QM. It therefore seems appropriate to give some precision to the meaning given to it in the present work.<sup>40</sup>

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<sup>38</sup> A comprehensive account of the results obtained in SED up to 1995 is contained in the book *The Quantum Dice*, by L. de la Peña and A.M. Cetto (1996), hereafter referred to as *The Dice*.

<sup>39</sup> We attribute this question to Boyer by inferring it from his papers. In a private communication he has expressed himself in similar terms. See however Boyer (2011).

<sup>40</sup> Among the many different perspectives on the subject within physics, the following cover a wide range of possibilities: Bunge (1970); Lucas (1970); Gillies (1973); Rédei and Szegedi (1989); Home

Apart from the formal or axiomatic (Kolmogorovian) probabilities and the subjective interpretation of probability,<sup>41</sup> there are two interpretations of probability popular among the practitioners of physics. One of them is the *frequentist* or *objective (empirical) interpretation*. According to this interpretation, proposed by Venn (1880), and developed by Reichenbach (1949) and von Mises (1957), among others, a series of observations is made and the relative frequency of an event is thus determined; its probability is taken as the value attained in the limit when the number of cases in the series tends to infinity. Here we are dealing with events (not with propositions as in the formal rendering, or with opinions or beliefs as is the case with the subjective interpretation), and the determination of the relative frequency is an empirical, objective (although approximate) process. There are however some problems that hamper a strict formulation of this probability: if experimental frequencies are used, the infinite limit is unattainable; if the relative frequency is a theoretical estimate, then the limit is probabilistic and the frequentist definition becomes circular. Again, the existence of the limit value should be assumed. Moreover, the theoretical structure lacks an experimental counterpart: why should the experimental relative frequencies correspond to the theoretical estimates? Notwithstanding such difficulties, this interpretation constitutes a widely used practical tool. As Bunge (1970) puts it: “All we have is a frequency *evaluation* of probability”.

Let us turn our attention to another important view on probability, much extended among physicists, namely the *ensemble interpretation*. We follow here the discussion on the subject by Brody (1975, 1975), particularly Chap.10), and start by recalling the usual concept of *ensemble*. Each theoretical model of reality should be in principle applicable to all cases of the same kind, i.e., to all cases where the properties of the system considered by the model are equal; the factors neglected by the model may vary or fluctuate freely, but in consistency with the applicable physical laws. The set of all these cases constitutes the ensemble of interest. The notion of ensemble as a set of theoretical constructs can thus be established without recourse to the concept of probability, and can be structured so as to possess a measure, which is then used to define averages over the ensemble. The ensemble concept of probability can then be introduced as follows. Let  $A$  be a property of interest and let  $\chi_A$  be the indicator function of  $A$ , i.e.,  $\chi_A(\omega) = 1$  if the member  $\omega$  of the ensemble has the property  $A$ ,

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and Whitaker (1992). See also *Interpretations of Probability* in the online Stanford Encyclopedia of Philosophy.

<sup>41</sup> The most extended subjective views of probability are the *individual* degree of acceptability of a proposition (de Finetti 1974), or its Bayesian version (Jeffreys (1939); Jaynes (1995); Caticha (2008) as a measure of the informed personal opinion. According to the Bayesian views, any evaluation of a probability is conditional to some evidence that partially entails it; thus, Keynes (1921) asserts that “the probability of the same statement varies with the evidence presented”. By contrast, the probability of decay of an atomic nucleus depends on the internal physical situation of the constituent nucleons, and is entirely independent of any personal information. This illustrates the different use of the concept of probability in physics and in other fields of knowledge. It should be considered that even if an assigned numerical probability is taken as depending on our degree of rational belief (or our degree of partial entailment), it contains some logical elements, since it is limited by rational constraints that ensure the possibility of using a mathematical apparatus (see Gillies (1973), Introduction).



$\chi_A(\omega) = 0$  otherwise. Then the probability of  $A$  is the expectation over the ensemble of  $\chi_A(\omega)$ ,

$$\Pr(A) = \int_{\Omega} \chi_A(\omega) d\mu(\omega), \quad (\text{A.1})$$

where  $\mu(\omega)$  is the measure function for the ensemble, usually normalized over  $\Omega$ , the range of the events  $\omega$ . It is possible to show that this definition satisfies all the axioms of Kolmogorov (1956), so that indeed the ensemble can become the basic tool for probabilistic theorization.

The experimental counterpart of this probability is the relative frequency as measured in an actual (and of course finite) series of experiments. If the relative frequencies thus measured do not correspond to the theoretical estimates, the ensemble (the measure) should be redefined until agreement is reached through the appropriate research work. Here there is no global recipe. Of course, as is the case with any other physical quantity, theoretical probabilities and their experimental values need not necessarily be exactly the same.

The ensemble definition of probability does not allow the application of the notion of probability to a singular case (there is no ensemble). Thus, for example, the philosophical problem of the probability of a given theory being true, becomes meaningless. To give meaning to the assertion about the probability of a single event, it must be translated into a statement about its relative frequency.

The most interesting aspect of the ensemble notion of probability is its direct correspondence with the concept used by physicists in their daily undertakings, so that we adhere to it in the present work, even though it is not entirely free of conceptual and philosophical problems—as any other interpretation of probability.

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