Exploring the Impact of Knowledge of Multiple Strategies on Students' Learning About Proportions

Rozy Vig, Jon R. Star, Danielle N. Dupuis, Amy E. Lein, and Asha K. Jitendra

Proportional reasoning is widely considered a major goal of mathematics education in the middle grades, where problems involving the use of proportional reasoning are most frequently encountered (Common Core State Standards Initiative, 2010; National Council of Teachers of Mathematics, 2000; National Research Council, 2001). The core of proportional reasoning, which involves multiplicative thinking, is foundational for more advanced mathematics (e.g., algebra, geometry, trigonometry, and calculus) encountered in high school and college (National Mathematics Advisory Panel, 2008). The development of proportional reasoning among students is a complex process that progresses gradually over many years (Lamon, 1999; Lesh, Post, & Behr, 1988). In spite of the centrality and promise of proportional reasoning in the middle grades, students experience great difficulty with this content domain (Lamon, 2007; Lobato, Ellis, & Zbiek, 2010; NRC, 2001). As an illustration consider a simple missing value proportion problem, $2/25 = n/500$. According to the National Assessment of Educational Progress (2009), 52 % of eighth-grade students failed to choose the correct answer of $n = 40$ from among a list of multiple-choice options.

In response to such student difficulties, a great deal of research has explored the teaching and learning of proportions (Behr, Harel, Post, & Lesh, 1992; Boyer, Levine, & Huttenlocher, 2008; Fujimura, 2001; Fuson & Abrahamson, 2005; Lamon, 2007; Lesh et al., 1988; Litwiller & Bright, 2002; Pitta-Pantazi & Christou, 2011; Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005). Most prominently, the Rational Number Project (e.g., Behr et al., 1992; Cramer, Post, & Currier, 1993; Harel & Behr, 1989; Lesh, Behr, & Post, 1987) has exerted a major influence on scholarship, curriculum, and policy around the teaching and learning of fractions,

R. Vig • J.R. Star (✉)
Harvard Graduate School of Education, 442 Gutman Library, 6 Appian Way,
Cambridge, MA 02476, USA
e-mail: jon_star@harvard.edu

D.N. Dupuis • A.E. Lein • A.K. Jitendra
University of Minnesota, Minneapolis, MN, USA

J.A. Middleton et al. (eds.), *Large-Scale Studies in Mathematics Education*,
Research in Mathematics Education, DOI 10.1007/978-3-319-07716-1_4

ratios, and proportions. While the peak of research on rational numbers may have been in the 1980s and early 1990s, work on proportional reasoning continues. More recently scholars have explored teacher knowledge of proportional reasoning (see, for example, Berk, Taber, Gorowara, & Poetzl, 2009), the role of multiple representations and/or technology in supporting students' understanding of proportional reasoning (see, for example, Fujimura, 2001), and the broader application of proportional reasoning to STEM curricula (see, for example, Bakker, Groenveld, Wijers, Akkerman, & Gravemeijer, 2014).

In this chapter, we revisit an issue that first emerged in the work of the Rational Number Project but has not been carefully explored in some time—namely, the strategies that students use when solving simple proportion problems. Our interest is in learning more about how students approach proportion problems, whether these approaches may have changed since this issue was last explored over 20 years ago, and whether strategy use has an impact on students' future learning about proportion.

Theoretical Background

Proportional reasoning refers to the ability to understand (interpret, construct, and use) relationships in which two quantities (ratio or rates) covary and to see how changes in one quantity are multiplicatively related to change in the other quantity. The presence of a multiplicative relationship between quantities and also within quantities is considered a defining feature of a problem that requires proportional reasoning (Behr et al., 1992). Typically, a proportion is defined as a statement of equality between two ratios. An example and commonly seen task relating to proportions in the elementary and middle school mathematics curriculum is to find the value of z that makes a proportion such as $3/9=6/z$ a true statement.

Of the many strategies that could be used to solve this kind of proportion problem, three (see Table 1) have been discussed at length in the literature (e.g., Post, Behr, & Lesh, 1988). The first is known as the cross-multiplication strategy (or CM), which involves multiplication across a problem's diagonals. For the problem $3/9=6/z$, CM could be used to rewrite the proportion as $3z=9(6)$, and solve for z to

Table 1 Strategies for solving simple proportion problems

Cross-multiplication strategy	Equivalent fractions strategy	Unit rate strategy
Solve for z: $\frac{3}{9}=\frac{6}{z}$	Solve for z: $\frac{3}{9}=\frac{6}{z}$	Solve for z: $\frac{3}{9}=\frac{6}{z}$
$3\cdot z=9\cdot 6$	$\frac{3}{9}\cdot\left[\frac{2}{2}\right]=\frac{6}{z}\ z=18$	$3\cdot[3]=9$
$3z=54$ $z=54\div 3$	$9\cdot[2]=18$	$6\cdot[3]=18$
$z=18$	$z=18$	$z=18$

yield an answer of $z = 18$. The second strategy is referred to here as the equivalent fractions strategy (or EF); EF involves examining the two ratios in a proportion and using their equivalence to solve for an unknown.[1] For the problem $3/9 = 6/z$, EF could be used to determine 2 as the multiplicative constant needed to arrive at an equivalent fraction; since 3 times 2 is 6, it follows that 9 times 2 is 18, so $z = 18$. Finally, we refer to a third strategy as the unit rate strategy (or UR); UR involves examining the multiplicative relationship *within* the quantity, determining the scalar multiple within a ratio or rate and using it to arrive at the missing value.[2] In the problem $3/9 = 6/z$, one could employ UR by noticing that 3/9 has a scalar multiple of 3, meaning that the denominator is three times as large as the numerator. The value of z can then be determined by multiplying 6 by 3 to arrive at 18. Note that the unit rate strategy (as we define it) does not require the explicit identification of a unit rate (in this case, 1/3)—only that the idea of a unit rate is implicitly used to determine the missing value of the variable.

It is worth noting that, while each of these strategies, if executed correctly, can yield the correct answer, one can argue that certain strategies may be easier than other strategies for particular problems. For example, for the problem $4/5 = 8/x$, EF might be considered easier than UR, since (especially for elementary and middle school students) using the multiplicative relationship between 4 and 8 to determine a solution (4 times 2 is 8, so 5 times 2 is 10; $x = 10$) is easier than using the multiplicative relationship between 4 and 5 (4 times 1.25 is 5, so 8 times 1.25 is 10). Conversely, for the problem $5/15 = 9/x$, UR (5 times 3 is 15, so 9 times 3 is 27; $x = 27$) is arguably easier than EF.

The existing literature on how students approach simple proportion problems such as the ones above suggests that students tend to rely heavily on the cross-multiplication strategy (Cramer & Post, 1993; Stanley, McGowan, & Hull, 2003). The consensus among many mathematics educators is that such a reliance on CM is problematic, primarily because of the belief that students often do not understand what they are doing when they perform the CM algorithm (e.g., Lesh et al., 1988). Furthermore, some have characterized CM as a conceptually opaque or even conceptually vacuous algorithm, in that multiplication across a diagonal is generally not considered a valid mathematical operation, and the algorithm does not make clear why it is permissible to perform this action for CM (e.g., Lesh et al., 1988).

In response to these types of concerns, many mathematics educators and researchers have advocated (1) delaying or even eliminating formal instruction in cross multiplication as a strategy for solving proportion problems and (2) teaching more intuitive strategies for proportion problems first (Cramer & Post, 1993; Ercole, Frantz, & Ashline, 2011; Lesh et al., 1988; Stanley et al., 2003). This de-emphasis on cross multiplication and advocacy of strategies such as EF has been steadily

[1] The equivalent fraction strategy is sometimes referred to as the factor-of-change method and involves attending to the multiplicative relationship between two ratios (Ercole et al., 2011).

[2] The unit rate strategy is sometimes referred to as the factor-of-change method (or scalar method) and involves the attending to the multiplicative relationship within each ratio (Ercole et al., 2011).

increasing since the Rational Number Project initially proposed it in the late 1980s. The Rational Number Project reports evidence in support for these recommendations; for example, when instruction is delayed on cross multiplication, students tend to use the unit rate strategy most frequently (Cramer & Post, 1993; Post et al., 1988).

The extent to which the two suggestions above have been implemented into practice is unclear. As noted above, the majority of work that explored students' strategies for solving simple proportion problems occurred in the 1980s and early 1990s, as part of the Rational Number Project. Given the significant changes that have occurred in US elementary and middle school mathematics curricula in the past 20 years, we were interested in revisiting the issue of how students approach these types of problems today. Despite apparent consensus for the two suggestions above, we are not aware of any recent studies that document changes since the 1980s and 1990s in how students approach simple proportion problems. As a result, it is worth noting that we began this study expecting to find that students continue to rely heavily or exclusively on cross multiplication for solving simple proportion problems.

In this chapter, we consider the following questions. First, do students continue to rely upon CM as a primary strategy for solving proportion problems, or have the past decades of de-emphasis of CM and advocacy of strategies such as UR and EF had an impact? Second, if students no longer rely as exclusively on CM, what potential impact might this have on their learning about proportional reasoning more generally? These questions were explored within the context of a larger research project investigating the impact of a curriculum unit on ratio, proportion, and percent word problems on student learning, as described below.

Method

As part of a study evaluating a 6-week curriculum unit on ratio, proportion, and percent problem solving, students were administered a pretest that evaluated their knowledge of strategies for solving simple proportion problems. Elsewhere we report the results of the larger study (Jitendra, Star, Dupuis, & Rodriguez, 2013); here our interest is in the strategy profile of students as demonstrated on the pretest and the relationship between students' strategy profiles and their future learning from the intervention.

Participants

Participants were 430 seventh-grade students drawn from 17 classrooms at three middle schools in two suburban school districts. Of the 430 students, 208 (48 %) were male, 200 (47 %) were eligible to receive free or reduced priced lunch, 37 (9 %) were English language learners, and 50 (12 %) received special education services. There were 216 (50 %) Caucasian students, 124 (29 %) African American students, 57 (13 %) Hispanic students, 23 (5 %) Asian students, and 9 (2 %) American Indian students. The mean student age was 12.5 years (SD = 0.4 years). Within the larger study, all 430 students were in classrooms that implemented the intervention.

The two districts used either *Math Thematics Book 2* (Billstein & Williamson, 2008), a reform-oriented curriculum developed with funding from the National Science Foundation or *Math Course 2* (Larson, Boswell, Kanold, and Stiff (2007)), a more "traditional" mathematics curriculum.

Intervention

The complete details of the 6-week intervention are described elsewhere (Jitendra et al., 2013). In brief, the intervention contained 21 scripted lessons where students were introduced to the concepts of ratio, proportion, and percent and were taught strategies for how to solve ratio, proportion, and percent word problems. Each lesson required students to make use of schematic diagrams, multiple solution strategies, and metacognitive strategies. In prior studies, this intervention had been found to be effective (e.g., Jitendra et al., 2009), and the present study was designed to build on and extend existing work on the intervention's efficacy.

Measures

Students completed a 45 min pretest before the intervention and then a 45 min posttest at the conclusion of the intervention. The common questions on the pretest and posttest were taken or adapted from state, national, and international standardized tests and had been used in prior studies investigating the efficacy of the intervention.

The posttest was designed to measure students' learning from the intervention; all problems related to ratio, proportion, and percent problem solving. The posttest contained 3 open-response questions and 21 multiple-choice questions. As an example, one of the posttest multiple-choice problems was, "A machine uses 2.4 L of gasoline for every 30 h of operation. How many liters of gasoline will the machine use in 100 h?" Possible responses were 7.2, 8.0, and 8.4 L.

With respect to the pretest, of interest here are four problems that appeared only on the pretest and were designed to assess students' knowledge (prior to the intervention) for solving simple proportion problems (see Table 2). Within the 45 min pretest, students were given 15 min to complete these four problems. Problems 3 and 4 were considered as "prompted" items, in that students were shown one strategy (CM) and prompted to try to solve the simple proportion problem in a different way. The assumption guiding the inclusion of these two problems was that students knew and would rely upon CM; the items sought to determine whether students knew any other strategies for approaching this type of problem. We predicted that students would have trouble on problems 3 and 4, due to a lack of knowledge of

Table 2 Pretest problems on strategy use

Item		Type
1. Solve for x. Show your work and circle your answer.		Unprompted
$\frac{3}{5}=\frac{x}{15}$		
Describe how you solved the problem in 1–2 sentences		
2. Solve for y. Show your work and circle your answer		Unprompted
$\frac{2}{8}=\frac{3}{y}$		
Describe how you solved the problem in 1–2 sentences		
3. Miguel was asked to solve the problem for x:		Prompted
$\frac{2}{7}=\frac{x}{21}$		
Here is his solution:	Please solve this same problem again but in a different way. Show your work below	
$\frac{2}{7}=\frac{x}{21}$ $7x=2\cdot 21$ $7x=42$ $\frac{7x}{7}=\frac{42}{7}$ $x=6$		
Describe how you solved the problem in 1–2 sentences		
4. Ayana was asked to solve the problem for y:		Prompted
$\frac{5}{20}=\frac{2}{y}$		
Here is her solution:	Please solve this same problem again but in a different way. Show your work below	
$\frac{5}{20}=\frac{2}{y}$ $5y=40$ $\frac{5y}{5}=\frac{40}{5}$ $y=8$		
Describe how you solved the problem in 1–2 sentences		

strategies other than CM. Problem 3 was designed to suggest the use of EF, given the relationship between the two denominators of the problem (7 and 21). Problem 4 was designed to suggest the use of UR, given the relationship between the numerator and denominator of the given ratio (5 and 20).

In contrast, problems 1 and 2 were unprompted in that students could use whatever strategy they wanted for solving the proportion problems. We expected most students to use CM for problems 1 and 2. Problem 1 was designed to suggest the use of EF, while problem 2 was designed to suggest the use of UR. Problems 1 and 2 appeared on a single page of the pretest and problems 3 and 4 were placed on the next page. Students were instructed not to work backwards and to complete the test in the order that the problems were presented.

Strategy Coding

Posttests and the four pretest problems of interest were scored by two independent coders, who met to resolve all disagreements. For the scoring of the four pretest problems, students' strategies were coded based on which of the three strategies described above were used. Students' written mathematical work (in the "show your work below" box of each problem), as well as students' description "in 1–2 sentences" were used in determining a strategy code.

A student's strategy was coded as EF when the student indicated the (horizontal) relationship between the two denominators and used this relationship to find the value of the unknown. For example, for problem 1, $3/5 = x/15$, one student wrote, "5 goes into 15 three times, but I need to times the numerator by 3 too—which is 9". The UR code was given when a student's work indicated awareness of the (vertical) relationship between the numerator and denominator of one of the ratios in the proportion and used this relationship to determine the unknown. For example, for problem 2, $2/8 = 3/y$, one student wrote, "$y = 12$, because you divide $2/8 = 4$ and then you do 3 times $4 = 12$". The CM code was given when a student multiplied across the diagonals of the problem. For example, for problem 1, one student wrote, "Well what I did was multiply 3 times 15 and I got 45 so what I did was times 9 times 5 and I get 45." Arithmetic errors in executing the strategy were not taken into consideration in the strategy coding, as we were primarily interested in capturing student strategy and not the correctness of the solution.

In addition to codes for CM, UR, and EF, we also coded for the presence of common erroneous ways that UR and EF could be applied—when additive rather than multiplicative reasoning was used. For EF, we used the code "mal-EF" to indicate when a student made use of an additive relationship between denominators to determine the unknown. For example, on problem 1, $3/5 = x/15$, one student noted that 15 was 10 more than 5, noting, "I added ten on the bottom. So I added ten on the top." Similarly, we used the code "mal-UR" to note when a student used an additive relationship between numerator and denominator to find the unknown. For example, again on problem 1, one student noticed that 5 was 2 less than 3 and wrote, "I just thought of the pattern and just subtracted 2 from 15, which was 13." We did not use a mal-CM code, as we found no instances in which students applied the CM strategy in an erroneous way.

In addition, an OC or "other correct" code was used to indicate a correct strategy other than CM, UR, or EF. For example, on problem 2, $2/8 = 3/y$, one student multiplied each ratio by the reciprocal of $3/y$ which results in the equivalent equation $2y/24 = 1$. The student went on to explain, "after I multiplied by the reciprocals I got $y = 12$." The code OI or "other incorrect" was used when students had a decipherable strategy that involved steps that were not mathematically permissible. For example, again on problem 2 ($2/8 = 3/y$), one student multiplied the two numerators to arrive at a new numerator (2 times 3 = 6), then multiplied all three of the given numbers in the problem to arrive at a new denominator (2 times 3 times 8 = 48), and then reduced the resulting fraction (6/48) to arrive at the missing value (6/48 = 3/24 so $y = 24$). Note that the "incorrect" in OI refers to the steps of the strategy, rather than to the correctness of the answer, just as the "correct" in OC refers to the steps of the strategy rather than the correctness of the answer. Finally, we coded as "none" any instances where students arrived at an answer without showing any work or left the problem blank. In addition, for problems 3 and 4 (the prompted items where the problem is solved using CM and students are asked to use a different strategy), students received a code of "none" when they used CM to solve these problems—in essence, copying over the strategy that was already provided in the problem statement.

Results

Due to missing data, below we report the results based on the 423 students who completed the pretest and the 414 students who completed both the pretest and the posttest. We begin by reporting on students' strategy use at pretest. We then examine the relationship between strategy profiles at pretest and students' scores at posttest.

Strategy Use at Pretest

Recall that we expected (based on the literature) that students would rely on CM as their preferred strategy for these problems, and that the pretest problems were designed with the assumption that many students knew CM already. As shown in Table 3, these expectations were completely off base. Only 27 students (6 %) used CM on problems 1 and 2 on the pretest. (Recall that CM was illustrated on the prompted problems 3 and 4 and thus students could not use CM on these problems.) Of these 27 students, only 8 (2 %) used CM for both problems 1 and 2. Students' use of UR was small (12 % of students), but it is noteworthy that there were almost twice as many students who used UR as used CM.

To our surprise, EF was very widely used by students in our sample. Seventy-seven percent of students showed knowledge of the EF strategy. Almost all of these students used EF on both problems 1 and 3 (60 % of the total sample), with a few

Table 3 Pretest strategy use and posttest mean scores

	n	%	*M*	SD
Used EF	327	77	19.93	5.12
Used UR	52	12	22.27	4.66
Used CM	27	6	18.42	5.54
Did not use EF, UR, or CM	77	19	13.55	5.31
Used exactly one (EF, UR, or CM)	277	67	19.34	5.16
Used multiple strategies (at least two of EF, UR, or CM)	60	15	21.93	4.74

students using EF for only problem 1 (9 % of the total sample) or only problem 3 (8 % of the total sample). Note that problems 1 and 3 were the ones that were designed to be optimal for EF—where the problem numbers made EF easily applicable. Clearly EF was the preferred strategy for most students; furthermore, when the numbers in the problem indicated that EF would be possible, the majority of students consistently used EF.

Students' reliance on EF can also be seen in the prevalence of mal-EF—the strategy where students try to use EF but erroneously reason additively rather than multiplicatively. On problems where the relationship between denominators in the simple proportion problem was obviously multiplicative (such as problems 1 and 3), students overwhelmingly used EF correctly; only 3 students (1 %) used mal-EF on problem 1, for example. But on problems where the relationship between denominators was not overtly multiplicative, many students attempted to determine the additive relationship between denominators to solve for the unknown: 31 % of students used mal-EF on problem 2. Similarly, while only 1 student used mal-EF on problem 3, 36 students (9 %) used mal-EF on problem 4.

Students' interest in using EF whenever possible (even if this meant using a mal-adaptive version of EF, mal-EF) is further illustrated by examining all students who used EF on at least one problem, to see which of these students also used mal-EF on at least one problem. Almost half of EF users (43 %) used mal-EF on at least one problem. In addition, recall that our assessment was also designed to examine students' knowledge of multiple strategies, but the predominance of EF was the clear take-away. Most students (66 %) only used one strategy (of the three strategies of interest here—CM, UR, and EF) on the four pretest questions. For almost all (63 % of all students, or 95 % of one-strategy students) of these students, this one strategy was EF.

An additional goal of the pretest was to explore students' knowledge of which strategies were most appropriate for a given problem. As noted above, some problems were designed to potentially elicit EF while others hoped to elicit UR. Our original aim was to determine not only whether students knew strategies other than CM but also whether they were able to select the most appropriate strategy for a given problem. Because students rarely used CM, and because EF was so widely used, it was no longer of interest (or even feasible) to look for which students knew the most appropriate strategy for a given problem.

Relationship Between Strategy Profile and Posttest Performance

In addition to exploring students' strategy profiles on the pretest, a second focus of the present analysis is the nature of the relationship between students' strategies at pretest and their performance on the posttest. As such, a series of independent-samples t-tests and a one-way ANOVA were conducted. There are three main findings.

First, students with knowledge of at least one strategy (EF, UR, or CM) on the pretest scored higher on the posttest than students who did not exhibit knowledge of EF, UR, or CM at pretest, $t(412) = 9.52$, $p < .001$. Given the predominance of EF, one might interpret this result as suggesting that students who knew EF outscored those who did not know EF. A direct examination of this possibility indicated that it was indeed the case: Students who knew EF scored higher than those who did not know EF, $t(335) = 2.03$, $p = .043$.

Second, although only a few students used CM on the pretest, these students performed *no worse* on the posttest than those students who knew EF. There was no difference between posttest scores of students who knew CM ($M = 18.42$) and those who did not use CM but did use EF and/or UR ($M = 19.92$), $t(335) = 1.42$, $p = .157$. Not only did the literature's prediction about students' overreliance on CM not hold in our sample, but those students who did use CM learned as much as those who did not use CM. It is also interesting to note that students who used UR did better on the posttest ($M = 22.27$) than those who did not use UR but did use EF and/or CM ($M = 19.36$), $t(335) = 3.77$, $p < .001$.

Finally, students who used more than one strategy on the pretest (typically, EF plus one other strategy) outperformed students who only knew one strategy, who in turn scored higher than those who did not use any strategies on the pretest (see Table 3), $F(2, 411) = 52.89$, $p < .001$. Although only a few students used more than one strategy on the pretest (15 %), these students did quite well on the posttest.

Discussion

Our aims in this study were to explore students' strategies for solving simple proportion problems and to determine whether and how knowledge of one or more strategies impacted students' learning from our intervention. There were four main results. First and surprisingly, students relied quite heavily at pretest on EF. Our review of the literature suggested that either CM would be used/known by most students, or that (when instruction on CM was delayed) UR would be the most common strategy (Cramer & Post, 1993; Post et al., 1988). However, the majority of students in our study either knew only EF or knew EF in addition to one or more other strategies.

To better understand why so many students in this district were using EF, we informally talked to teachers and also examined the math texts that were in use at the elementary and middle schools in the district. Although (judging from the textbooks and teachers' reports) students had not received any prior instruction in how to solve simple proportion problems, we found that the text's treatment of equivalent fractions in fourth grade provided the foundations for the EF strategy. Our results suggest that many students were able to recall their work with equivalent fractions in fourth grade as they attempted to solve unfamiliar simple proportion problems in our study in the seventh grade. Furthermore and somewhat anecdotally, these districts were geographically relatively close to the University of Minnesota, home of several key members of the Rational Number Project, and apparently received professional development for many years that was consistent with the Rational Number Project suggestions about delaying formal instruction on CM. Regardless of the reasons, we find it noteworthy that claims made in the past about students' overreliance on cross multiplication may now (in some districts) be a bit dated. Perhaps due to the greater diversity and types of curricula in use in elementary schools, EF now appears to be the strategy of choice, at least for students in the districts that were included in the present study.

Second, while EF was the preferred strategy for students at pretest, results indicate that the widespread use of EF brought its own set of challenges. A central concern noted in the literature about CM is that students often do not know conceptually what they are doing and thus seem to be blindly following the CM algorithm. Another related concern is that CM fails to emphasize the proportionality that is central to thinking about and solving simple proportion problems. Many scholars view EF as improving on both of these concerns: EF may be better connected to conceptual knowledge (related to fractions and ratios), and EF appears to foreground proportionality. However, our results suggest that, for many students, EF brings challenges of its own. In particular, many students in our sample overgeneralized EF—in the interest of applying EF as often as possible, many students erroneously used EF additively rather than multiplicatively. It is certainly encouraging that (a) these students seemed to spontaneously apply a strategy that they learned for working with equivalent fractions to proportion problems and (b) these students appear to see the similarities between proportion problems and equivalent fractions. However, the frequency of overgeneralization—where students attempted to apply EF where the problem numbers made it difficult to do so, and then erroneously modified EF so that was applied additively, is problematic.

Third, the results of the current study show that prior knowledge of one or more solution methods can have a positive impact on students' ability to learn from an instructional intervention for proportional reasoning. This result is consistent with a growing body of research in mathematics education and psychology that suggests that students' learning is enhanced when they have the opportunity to learn multiple methods and compare and contrast them (e.g., Rittle-Johnson & Star, 2007). Finally, students who used CM performed no worse on the posttest than those who did not use CM but did use EF and/or UR.

Taken as a whole, these results suggest that much has changed in the many years since the Rational Number Project began investigating students' strategies for simple proportion problems. If the two districts in the present study are indicative of national trends, we do not see the same reliance on cross multiplication as earlier studies might have predicted—equivalent fractions was clearly the strategy of choice. While some mathematics educators might find the prevalence of EF to be an encouraging sign, it is also the case that students' difficulties with solving simple proportion problems persist. Clearly more work is needed to better understand the nature of students' difficulties with solving simple proportion problems—decreasing reliance on cross multiplication as a default strategy may not have been sufficient to significantly advance student understanding of this important mathematical topic.

References

Bakker, A., Groenveld, D., Wijers, M., Akkerman, S., & Gravemeijer, K. (2014). Proportional reasoning in the laboratory: An intervention study in vocational education. *Educational Studies in Mathematics, 86*(2), 211–221.

Behr, M., Harel, G., Post, T., & Lesh, R. (1992). Rational number, ratio and proportion. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 296–333). New York: Macmillan.

Berk, D., Taber, S. B., Gorowara, C., & Poetzl, C. (2009). Developing prospective elementary teachers' flexibility in the domain of proportional reasoning. *Mathematical Thinking and Learning, 11*(3), 113–135.

Billstein, R., & Williamson, J. (2008). *Math thematics: Book 2* (new edition). Evanston, IL: McDougal Littell.

Boyer, T., Levine, S. C., & Huttenlocher, J. (2008). Development of proportional reasoning: Where young children go wrong. *Developmental Psychology, 44*(5), 1478–1490.

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Cramer, K., & Post, T. (1993). Connecting research to teaching proportional reasoning. *Mathematics Teacher, 86*(5), 404–407.

Cramer, K., Post, T., & Currier, S. (1993). Learning and teaching ratio and proportion: Research implications. In D. Owens (Ed.), *Research ideas for the classroom* (pp. 159–178). New York: Macmillan.

Ercole, L. K., Frantz, M., & Ashline, G. (2011). Multiple ways to solve proportions. *Mathematics Teaching in the Middle School, 16*(8), 482–490.

Fujimura, N. (2001). Facilitating children's proportional reasoning: A model of reasoning processes and effects of intervention on strategy change. *Journal of Educational Psychology, 93*(3), 589–603.

Fuson, K. C., & Abrahamson, D. (2005). Understanding ratio and proportion as an example of the apprehending zone and conceptual-phase problem-solving models. In J. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 213–234). New York: Psychology Press.

Harel, G., & Behr, M. (1989). Structure and hierarchy of missing value proportion problems and their representations. *Journal of Mathematical Behavior, 8*(1), 77–119.

Jitendra, A. K., Star, J. R., Dupuis, D., & Rodriguez, M. (2013). Effectiveness of schema-based instruction for improving seventh-grade students' proportional reasoning: A randomized experiment. *Journal of Research on Educational Effectiveness, 6*(2), 114–136.

Jitendra, A., Star, J. R., Starosta, K., Leh, J., Sood, S., Caskie, G., et al. (2009). Improving seventh grade students' learning of ratio and proportion: The role of schema-based instruction and self-monitoring. *Contemporary Educational Psychology, 34*(9), 250–264.

Lamon, S. J. (1999). *Teaching fractions and ratios for understanding*. Hillsdale, NJ: Lawrence Erlbaum.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Towards a theoretical framework for research. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629–666). Reston, VA: National Council of Teachers of Mathematics.

Larson, R., Boswell, L., Kanold, T. D., & Stiff, L. (2007). *Math course 2*. Evanston, IL: McDougal Littell.

Lesh, R., Behr, M., & Post, T. (1987). Rational number relations and proportions. In C. Janiver (Ed.), *Problems of representations in the teaching and learning of mathematics* (pp. 41–58). Hillsdale, NJ: Lawrence Erlbaum.

Lesh, R., Post, T., & Behr, M. (1988). Proportional reasoning. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 93–118). Reston, VA: National Council of Teachers of Mathematics.

Litwiller, B., & Bright, G. (2002). *Making sense of fractions, ratios, and proportions*. Reston, VA: National Council of Teachers of Mathematics.

Lobato, J., Ellis, A., & Zbiek, R. (2010). *Developing essential understanding of ratios, proportions, and proportional reasoning for teaching mathematics: Grades 6-8*. Reston, VA: National Council of Teachers of Mathematics.

National Assessment of Educational Progress. (2009). U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, Mathematics Assessment. Retrieved from http://nces.ed.gov/nationsreportcard/itmrlsx/search.aspx?subject=mathematics

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

National Research Council. (2001). Adding it up: Helping children learn mathematics. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.), *Mathematics learning study committee, center for education, division of behavioral and social sciences and education*. Washington, DC: National Academy Press.

Pitta-Pantazi, D., & Christou, C. (2011). The structure of prospective kindergarten teachers' proportional reasoning. *Journal of Mathematics Teacher Education, 14*(2), 149–169.

Post, T., Behr, M., & Lesh, R. (1988). Proportionality and the development of pre-algebra understandings. In A. Coxford & A. Shulte (Eds.), *The idea of algebra, K-12* (pp. 78–90). Reston, VA: National Council of Teachers of Mathematics.

Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology, 99*(3), 561–574.

Stanley, D., McGowan, D., & Hull, S. H. (2003). Pitfalls of overreliance on cross multiplication as a method to find missing values. *Texas Mathematics Teacher, 11*, 9–11.

Van Dooren, W., De Bock, D., Hessels, A., Janssens, D., & Verschaffel, L. (2005). Not everything is proportional: Effects of age and problem type on propensities for overgeneralization. *Cognition and Instruction, 23*(1), 57–86.