Methodological Issues in Mathematics Education Research When Exploring Issues Around Participation and Engagement

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Background

In common with many other countries, the government in England is committed to increasing the number of STEM (science, technology, engineering and mathematics) professionals as it sees this as crucial for England to be able to compete in an increasingly competitive global economy (Department for Business, Innovation and Skills, 2009). While the number of students choosing to study mathematics after the age at which it is no longer compulsory (16 in England) has been rising in recent years, there is still a problem with the relatively low proportion of English students, compared with other countries, who continue with mathematics in post-compulsory education (Royal Society, 2011). There have been a number of pieces of research that have concluded that this is at least in part due to the high levels of disaffection of many students taking secondary mathematics courses (e.g. Brown, Brown, & Bibby, 2008; Nardi & Steward, 2003). A shortage in the number of students undertaking post-compulsory mathematics has implications for the number who can go on to do careers that require mathematics, including specialist mathematics teacher training courses, which impacts the availability of good quality mathematics teaching for school students.

Existing research has demonstrated the importance of gender, as well as prior attainment, socio-economic status and ethnicity, on whether students continue with post-compulsory mathematics (e.g. Noyes, 2009). Feminist-inspired work has looked at why girls too often conclude that mathematics is not for them. In a qualitative study of young people in schools and colleges Mendick (2006) drew on theorisations of masculinities by Connell (1995), amongst others, and concluded that to understand gender difference we need to start from social context, processes and actions and see gender difference as relational. More recently, the importance of mathematical relationships in education has been stressed by Black, Mendick,

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and Solomon (2009) and others. For instance, the importance of pedagogy was investigated by Palmer (2009) who found that female teacher-education students became much more positive about mathematics after they undertook a course that adopted a feminist post-structural approach based on critical pedagogy and deconstructive theory.

A number of the factors that influence engagement with mathematics are to do with schooling, which is susceptible to a range of influences: changes in schools, changes in subject teachers, introduction of new learning plans. The family, though, is an important influence that is resistant to changes that take place in the school environment. Noyes (2003) qualitative study found that students' family backgrounds played a key role in how students identified with mathematics. Those students for whom the family habitus resonated with the culture of the school benefited more from school than did students for whom mathematics and the learning culture at home did not so resonate. This identification is partly the result of such cultural forces and an individual's relationship with their school, but it is the individual's affective response, both conscious and unconscious, that ultimately attracts, or fails to attract, each person to the subject (cf. Boaler, 2009; Middleton & Jansen, 2011).

The Context of This Study

This chapter aims to identify the factors that relate to students' intended choices with respect to mathematics in schools in England, using a mixed methods longitudinal approach. The data are drawn from the Understanding Participation rates in post-16 Mathematics And Physics (UPMAP) project which was conducted from 2008 to 2011. The quantitative element of the study, part of which we draw on here, surveyed students aged 12–13 (year 8) and 14–15 (year 10). Throughout we highlight the methodological issues that surfaced in our study; indeed, we have structured the paper as a sort of narrative of the various stages of the analysis, recounting the different methodological decisions we made at each stage.

Whilst there is a considerable literature in mathematics education pertaining to extrinsic factors affecting choices and achievement, comparatively little has been reported on the relationship between intrinsic factors, such as personality, attitudes to mathematics and achievement in mathematics, and their relationships to subject choice, achievement and post-16 participation. Accordingly, we designed student questionnaires to include items derived from established psychological constructs. Given that the focus of the study was to find factors that influence post-16 participation in mathematics and/or physics it was a deliberate part of the sampling to over-represent in our sample schools which were above average in either or both of mathematics and physics attainment and post-16 participation. In addition, given our research agenda, we targeted classes that contained students who were said by the teachers to be of above average or average attainment in mathematics and physics/science. This focus was intentional because, although all barriers to participation are important, we are particularly interested in factors that affect the choices of those students who have the opportunity, including fulfilment of attainment criteria, to study mathematics (or physics) post-16.

Student questionnaires were designed following a review of the literature (Reiss et al., 2011) that considered factors that may influence post-compulsory participation rates. Alongside questions related to intentions to continue to study mathematics post-16, the survey included mathematics-specific items to determine attitudes to the subject, attitudes to lessons, self-concept, perceptions of teachers, support for learning, intrinsic and extrinsic motivation for learning, personality and mathematical understanding. A factor analysis using principal components affirmed some of the constructs though also led to minor changes in others. Cronbach's alphas were used to assess the internal consistency of all constructs, which were found to have fair to high reliability (.6–.9). All of the items within each construct were scored so that a high score represents strong agreement.

This chapter also draws on qualitative data to provide further insights into the statistical findings and suggest new analyses. We use extracts from semi-structured interviews undertaken with three 15-year-old girls. Each interview was conducted by one of the authors and was around 30 min in length.

Introduction to Findings

For all of our surveys and for each year group (year 8 and year 10) we used factor analysis to determine the underlying dimensions of the constructs. For the mathematics surveys we found there were three mathematics-specific constructs related to motivation and values: intrinsic value, extrinsic social gain motivation and extrinsic material gain motivation (these constructs are explained below). In addition, there were seven mathematics-specific constructs which were related to perceptions of learning and students' mathematics education: home support for achievement in mathematics; perceptions of mathematics teachers; emotional response to mathematics lessons; perceptions of mathematics lessons; mathematics self-concept; advice-pressure to study mathematics and social support in mathematics learning. Items were on a six-point Likert scale with scores above three representing agreement/more favourable answers. Our year 8 survey also obtained data on four underlying personality dimensions: competitiveness (a measure of how competitive in life students are), self-direction (whether students report they can change what is going to happen to them), emotional stability (whether students report they are generally happy or upset) and extroversion. The surveys can be downloaded from www.ioe.ac.uk/UPMAP and information about the specifics of instrument design is available in Reiss et al. (2011).

We used a six-point Likert item that asked students whether they were intending to continue with mathematics post-16; this item was used as the dependent variable within our multi-level analysis. For the sake of brevity, 'intention to participate' refers to expressed intentions to continue with a mathematics course at post-compulsory education (i.e. after the age of 16). A high score (4, 5 or 6) represents a stated intention to continue with mathematics post-16 with 6 being 'strongly agree'; the other end of the scale (1, 2 or 3) represents disinclination to continue, with 1 being 'strongly disagree'. Table 1 indicates the overall mean response (4.36) for the year 8 students with statistically significant differences between boys and girls ($t = 5.508$, $p < .001$) in favour of boys with an effect size of .155 ($p < .001$).

Table 1 Year 8 students' perceptions of mathematics

Overall perceptions of maths	All students[a]				Boys			Girls			Comparison (boys vs. girls)		
Item	N	Corr.	M	SD	N	M	SD	N	M	SD	*t*	*df*	Effect size Cohen's d
I intend to continue to study maths after my GCSEs (key variable of interest)	*5,154*	*1*	*4.36*	*1.42*	*2,294*	*4.49*	*1.42*	*2,825*	*4.27*	*1.42*	*5.508*	*4,904.349*	*0.155****
Self concept (original construct)	*5,342*	*0.444****	*4.09*	*1.01*	*2,369*	*4.30*	*0.99*	*2,938*	*3.91*	*0.99*	*14.489*	*5,076.146*	*0.400****
Intrinsic perceived value of mathematics (original construct)	*5,270*	*0.516****	*4.13*	*0.90*	*2,324*	*4.27*	*0.92*	*2,913*	*4.03*	*0.86*	*9.600*	*4,830.896*	*0.269****
Extrinsic material gain motivation (original construct)	*5,334*	*0.572****	*4.84*	*0.80*	*2,364*	*4.93*	*0.83*	*2,935*	*4.78*	*0.78*	*6.643*	*4,905.685*	*0.185****
Extrinsic social gain motivation (original construct)	*5,168*	*0.363****	*3.08*	*1.13*	*2,273*	*3.17*	*1.19*	*2,863*	*3.01*	*1.07*	*4.760*	*4,606.280*	*0.135****
I am good at maths	5,169	0.451***	4.52	1.15	2,298	4.77	1.09	2,836	4.31	1.16	14.487	5,016.390	0.404***
I do not need help with maths	5,196	0.204***	3.83	1.48	2,291	4.08	1.50	2,872	3.63	1.42	11.006	4,786.352	0.310***
To be good at maths, you need to be creative	4,850	0.267***	2.87	1.36	2,151	3.08	1.44	2,670	2.71	1.27	9.336	4,313.640	0.274***
I think maths will help me in the job I want to do in the future	5,151	0.506***	4.60	1.42	2,283	4.77	1.39	2,833	4.45	1.43	8.094	4,949.433	0.227***
Being good at maths makes you popular	4,659	0.235***	2.19	1.22	2,045	2.34	1.30	2,584	2.08	1.13	7.132	4,070.883	0.215***
Maths is important in making new discoveries	4,805	0.391***	4.20	1.33	2,152	4.35	1.36	2,623	4.07	1.29	7.253	4,494.904	0.212***
People who are good at maths get well-paid jobs	4,797	0.326***	4.65	1.06	2,140	4.75	1.06	2,627	4.56	1.06	6.321	4,566.066	0.184***
To be good at maths, you need to work hard	5,127	0.251***	5.01	1.08	2,265	5.12	1.05	2,830	4.92	1.09	6.553	4,927.604	0.184***

Those who are good at maths are clever	4,928	0.165***	4.27	1.36	2,179	4.40	1.35	2,718	4.17	1.37	5.840	4,685.001	0.168***
I think maths is an interesting subject	5,182	0.568***	3.95	1.42	2,299	4.08	1.44	2,850	3.85	1.40	5.897	4,857.858	0.166***
Maths is interesting	5,119	0.517***	3.76	1.53	2,252	3.90	1.57	2,834	3.65	1.49	5.589	4,723.161	0.159***
Maths teaches you to think logically	4,844	0.390***	4.79	1.06	2,150	4.88	1.08	2,664	4.72	1.04	5.034	4,529.416	0.146***
In maths, it is interesting to find out about the laws that explain different phenomena	4,994	0.434***	4.07	1.45	2,209	4.18	1.46	2,756	3.97	1.43	5.071	4,683.308	0.145***
Maths improves your social skills	4,776	0.301***	3.15	1.46	2,127	3.23	1.52	2,619	3.08	1.40	3.411	4,380.065	0.100***
These days, everybody needs to know some maths	5,129	0.279***	5.09	1.01	2,262	5.14	1.01	2,835	5.05	1.00	3.248	4,846.272	0.092**
I think maths is a useful subject	5,245	0.515***	5.19	0.97	2,324	5.24	1.01	2,887	5.15	0.94	3.171	4,815.377	0.089**
Maths helps you in solving everyday problems	5,111	0.335***	4.75	1.19	2,260	4.80	1.21	2,818	4.71	1.18	2.416	4,775.948	0.068*
Being good at maths impresses people	4,780	0.335***	3.85	1.35	2,105	3.88	1.40	2,644	3.84	1.32	1.042	4,379.670	0.031
There is only one right way to solve any maths problem	4,823	−0.067***	4.69	1.41	2,138	4.70	1.49	2,656	4.68	1.34	0.413	4,338.214	0.012

Notes: *N* number; *M* mean; *SD* standard deviation; comparisons between girls and boys
[a]Shaded area is a separate set of analyses (correlations (Corr.)) between items that explore all students' perceptions of mathematics with students' intentions to continue to study maths post-16; unshaded area explores gender issues amongst items; ***significant at .001; **significant at .01; *significant at .05

Multi-level Findings: Intention to Participate in Mathematics Post-16 Amongst Year 8 Students

To ascertain which factors were the most important in explaining intended mathematics participation, multi-level modelling (MLM) procedures were used to establish which combinations of factors were best able to explain the variation in year 8 students' intentions to study mathematics post-16. These findings represent our initial key results which helped create new avenues of research and lines of enquiry. At this stage we felt that reliance on MLM procedures was appropriate for the sort of data we were analysing, given that MLM enabled us to recognise the nature of student responses by including students as one of the levels within our nested multi-level model. Students' intentions to continue with mathematics post-16 are likely to be influenced by factors operating at a number of levels and for the data we collected we were able to explore influences at the individual student level and at the school level. The variance in MLM procedures is therefore partitioned out between the student and school levels. The standard errors are smaller than those obtained using traditional regression techniques and so MLM procedures are less likely to have type 1 errors.

We began our analysis of data from year 8 students' questionnaires by fitting a variance components model for the outcome measure 'intention to study mathematics post-16'; the intra-school correlation demonstrated that around 7 % of the variation in students' intention to study mathematics post-16 is attributable to differences between secondary schools with the rest of the variation reflecting differences between students. Given this low intra-school correlation, our analytical approach focused more on exploring student level factors; we began with more basic models that evolved as we explored the importance of various influences on intended participation. The chi-square likelihood ratio test and the deviance statistic were used to establish whether the addition of new (statistically significant) variables provided better model fit than earlier models.

Our Initial Construct-Based Multi-level Analysis

The final model presented in Table 2 went through a number of stages; we will refer to findings from earlier models to create a more complete picture about what we tested and which student level variables were removed in the final model. To maximise sample sizes, scores on constructs from the survey were divided into quartiles; this allowed us to maximise the number of students within the models as it enabled us to retain students for whom we had scores for some but not all items within a construct.

Controlling for Only Background Characteristics

Student background characteristics were the first variables we controlled for, primarily because of the known influence of prior attainment, gender, ethnicity and socio-economic status on actual participation. We did find an independent influence of the

Table 2 Estimates of fixed effects on year 8 England students' intentions to study mathematics post-16

Parameter	Estimate	Std error	df	*t*	Sig.	Effect size
Intercept	5.766	0.103	1,161.470	56.244	0.001	
Gender	0.064	0.053	1,243.693	1.217	0.224	0.062
Maths self-concept (comparison group: top quartile)						
(Bottom quartile)	–0.038	0.072	1,703.892	–0.527	0.598	–0.036
(Lower middle quartile)	–0.241	0.077	1,686.183	–3.146	0.002	–0.231
(Upper middle quartile)	–0.588	0.084	1,685.733	–7.010	0.001	–0.564
Emotional response to maths lessons (comparison group: top quartile)						
(Bottom quartile)	–0.052	0.078	1,703.868	–0.665	0.506	–0.049
(Lower middle quartile)	–0.183	0.079	1,703.452	–2.317	0.021	–0.176
(Upper middle quartile)	–0.316	0.088	1,700.024	–3.591	0.001	–0.303
Advice-pressure to study maths (comparison group: top quartile)						
(Bottom quartile)	–0.170	0.076	1,702.531	–2.225	0.026	–0.163
(Lower middle quartile)	–0.351	0.084	1,703.920	–4.151	0.001	–0.336
(Upper middle quartile)	–0.964	0.081	1,698.910	–11.913	0.001	–0.925
Intrinsic value of maths (comparison group: top quartile)						
(Bottom quartile)	0.011	0.078	1,703.367	0.136	0.892	0.010
(Lower middle quartile)	–0.163	0.082	1,703.986	–1.989	0.047	–0.157
(Upper middle quartile)	–0.463	0.094	1,701.561	–4.946	0.001	–0.444
Extrinsic prospects (comparison group: top quartile)						
(Bottom quartile)	–0.313	0.081	1,703.610	–3.880	0.001	–0.300
(Lower middle quartile)	–0.530	0.082	1,701.668	–6.501	0.001	–0.509
(Upper middle quartile)	–1.044	0.096	1,703.619	–10.875	0.001	–1.002
Random-effects parameters						
Variance (Level 2)	0.008	0.007				
Variance (Level 1)	1.087	0.037				
Deviance (–2 × log restricted-likelihood)	5,093.092					

first three background characteristics although they are not reported in the final model in Table 2 because all of these lost significance when we began to control for students' attitudes and perceptions of their mathematics education. However, and somewhat surprisingly, the analysis indicated that even at this initial stage there was no influence of free school meal status (a measure of social deprivation—about one in six school children are entitled to receive meals at school without paying for them because of low household income). In line with other existing research we found that girls were less likely than boys to express intentions to continue with mathematics post-16. Students of Asian heritage were more likely than those of other ethnicities to express intentions to continue with mathematics post-16. However, once we accounted for more of the survey measures no ethnicity effects were statistically significant and so this measure was removed from the final model. Gender effects also lost statistical significance in later models but we retained gender as a control primarily because, as our analysis will indicate, when these same students were in year 10, gender differences were significant even after accounting for a range of survey measures. This finding, that as students progress through secondary school the gap in future mathematics aspirations widens between boys and girls, is important.

Controlling for Personality Traits

Within the UPMAP project we were interested in the association between intended participation and the psychological traits of students. The four core constructs for which we collected data were competitiveness (indicates that students have a tendency to self-enhancement); emotional stability (measures a state of composure and calmness); extroversion (measures a tendency to gain gratification through social interactions with others) and locus of control (which measures the extent to which students feel they have an influence over issues that impact them). Some of our earlier work found that girls with high intentions to study mathematics had statistically significantly higher competitive personalities than (a) girls with low intentions and (b) boys, whether boys had high or low intentions to study mathematics (Mujtaba & Reiss, under revision), with similar findings when exploring such trends in post-compulsory physics intentions (Mujtaba & Reiss, 2013c). Within our multi-level analysis we expected to find the association between competitiveness and intended mathematics participation (which we did initially); however, once we included mathematics-specific measures of motivation (see below), the influence of any general underlying personality trait, including competitiveness, was not significant.

Measures of Motivation and Support for Learning (Non-mathematics Specific)

We tested for the importance of our construct ‘general motivations and aspirations towards learning’ and ‘home support for achievement in general’ with mathematics aspirations. Initial findings indicated that both were positively associated with mathematics aspirations. This is hardly surprising (cf. Eccles, 2009; Schunk, Pintrich, & Meece, 2010). However, once we included (in later models, see Table 2) mathematics-specific measures of motivation and support for learning, these more general measures lost statistical significance. These findings demonstrate that without the inclusion of mathematics-specific measures we might have concluded that a general emphasis on learning in the home and at school will boost mathematics aspirations.

Inclusion of Measures That Explore Student Perceptions of Their Mathematics Education

To substantiate the impact of students’ perceptions of their mathematics education (mathematics self-concept, perception of teachers, lessons and emotional response to lessons) on future mathematics aspirations, such measures were included in the model prior to the inclusion of constructs that tapped into students’ attitudes towards mathematics-specific issues (e.g. extrinsic material gain motivation, intrinsic value, self-concept) and encouragement in mathematics learning and choice (e.g. advice-pressure to study mathematics, social support in mathematics learning, home support for achievement in mathematics). There was a statistically significant independent influence of ‘perceptions of mathematics teachers’ although this lost significance in later models once we controlled for other measures of students’ mathematics education. The only constructs that measure perceptions of mathematics education that continued to have an influence in the final model were ‘emotional response to mathematics lessons’ and ‘mathematics self-concept’.

Mathematics-Specific Measures of Motivation and Support for Learning

The constructs ‘social support in mathematics learning’ and ‘extrinsic social gain motivation’ (a measure which explores students’ desire to continue with mathematics for social gain) were associated with intended participation. We also found there was an association between ‘home support for achievement in mathematics’ (a construct which measures support that students derived from the family in raising mathematics attainment). However, once we introduced ‘extrinsic material gain motivation’ (which identifies students wanting to continue with mathematics for some tangible reward, such as future career prospects) and ‘advice-pressure to study mathematics’ (a construct which measures the encouragement students receive from a range of people about continuing with mathematics) the constructs ‘social support in mathematics learning’, ‘extrinsic social gain motivation’, and ‘home support for achievement in mathematics’ did not have an independent influence in demonstrating their association with future mathematics aspirations; these measures were subsequently removed.

The Final Model

In the final model the following five constructs were found to be significantly associated with the post-16 mathematics intentions of year 8 students: ‘mathematics self-concept’, ‘emotional response to mathematics lessons’, ‘advice-pressure to study mathematics post-16’, the ‘intrinsic value’ students accord to mathematics and ‘extrinsic material gain motivation’. The largest effect size (ES) out of all of the measures we tested within our models (and whilst controlling for the influence of other measures) was for ‘extrinsic material gain motivation’ (ES = 1.002), followed by ‘advice-pressure to study mathematics’ (ES = .925), ‘mathematics self-concept’ (ES = .564), ‘intrinsic value of mathematics’ (ES = .444) and positive ‘emotional response to mathematics lessons’ (ES = .303). As expected, boys were more likely to express intentions to participate in mathematics than girls, though at year 8, once we controlled for other survey responses, the influence of gender lost significance. However, we decided to retain gender in the final model (Table 2) to help illuminate issues around gender with further analysis (as discussed below).

Other Considerations

Our modelling explored the influence of a range of predictors and it was apparent that the non-inclusion of ‘extrinsic material gain motivation’ would have led to a conclusion that ‘other factors’ were important in explaining intended participation. Our initial modelling found that students’ perceptions of their ‘home support for achievement in mathematics’ was a better predictor than the construct ‘home support for achievement in general’, the latter being a measure of support from the family for all types of learning, which lost significance once mathematics-specific measures were introduced. In addition, general measures of students’ motivation for learning (our constructs ‘general motivations’ and ‘aspirations towards learning’) or personality-based measures of general motivation in life, such as ‘competitiveness’, were not significant predictors of intended participation in post-16 mathematics once we used mathematics-specific measure of motivation in our model. We found ‘extrinsic material gain motivation’ to be more precisely related to intention to study mathematics post-16 than any of our other measures of motivation in education (or life in general, as in the competitiveness measure). Without having mathematics-specific measures, we would not have been able to come to such conclusions and could easily have suggested that support for learning from the home and students’ own motivation towards learning were not important in intended mathematics choice.

The Importance of Looking Beyond What the Immediate Findings Suggest

It is also worth emphasising the importance of the order in which variables are introduced in the steps of multi-level modelling (MLM). The results from the construct-based MLM analysis indicated that ‘perception of mathematics lessons’ and ‘perception of mathematics teachers’ lost statistically significant association with year 8 students’ intentions to continue with mathematics post-16 once the constructs ‘extrinsic material gain motivation’ and ‘advice-pressure to study mathematics’ were introduced.

Does this mean that in school there should be less of a focus on the teacher–student relationship and on how students perceive their mathematics lessons and more of an emphasis on creating an awareness about the material gain of a post-16 mathematics qualification? We do not think that this would be an appropriate conclusion. For one thing, at least part of the influence of mathematics teachers and lessons may be absorbed by such constructs as ‘self-concept’ and ‘extrinsic material gain motivation’. It is rare for any attitude to exist in isolation from another. Although the constructs that measure the influence of teachers and lessons were not as strong/effective predictors of intended mathematics participation as other measures in our final construct-based multi-level analysis, we wondered whether there might be individual items within these constructs that have a strong effect on intended participation in mathematics. We reasoned there was a possibility that the importance of specific items might have been lost, once these were combined with other items within an overall construct. Such thinking was further influenced by the fact we did find within our qualitative work, as discussed below, that perceptions of mathematics teachers and mathematics lessons were very important in the decision making of some students. This led us to go back to some of our original constructs and analyse at the item level to help bridge the findings between the qualitative and quantitative work. In this next section we discuss some of the findings from the qualitative work which helped us to re-think how we ought to approach our survey analysis and the conclusions we were drawing before we return to deconstructing our constructs via an item-level analysis.

The Emergence of the Importance of Teachers via Qualitative Work

The qualitative element of this chapter focuses on interviews with 15 year olds (year 10 students). Although this section supports some of the key quantitative findings reported in Table 2 it also brings new insights, namely the importance of teachers in student choice. There is now a considerable body of evidence to suggest that the quality of teaching is a major determinant of student engagement and feelings of success in all school subjects. However, subject choices are not made solely on the quality of teaching. A substantial amount of research on subject choice has established that students are more likely to study subjects that they see as interesting and useful and ones in which they expect to do well in (Eccles, 1994; Mujtaba & Reiss, 2013a, 2013b), factors that may correlate with teaching quality but are not entirely contained within this. Students' feelings of success at mathematics (mathematics self-concept) can also contribute to their perceptions of mathematics and to intended subject choice.

Our qualitative work indicated that for some students a close, supportive relationship with mathematics teachers was important in future mathematics intention. The extracts below support the quantitative work by drawing out the role of self-concept, extrinsic material gain motivation and the intrinsic value of mathematics whilst also indicating that teachers' encouragement to some extent may have underpinned mathematics self-concept and students' intended choices. The analysis of three student interviews exemplifies the importance of teachers in students' decision-making processes; the students were specifically chosen from the larger pool of interviewees to portray three very different ways that teachers can have an influence on students' feelings about mathematics. In the first case, the teacher serves a role in connecting the student with mathematics; the student had a very weak relationship with mathematics prior to this teacher's long-term support and this encouragement eventually led to a choice to take an academic course in post-16 mathematics (A-Level mathematics). In the second case, the student already had a strong attachment to and self-concept in mathematics with an intention to continue with mathematics post-16; the teacher served simply to encourage and reinforce the student's mathematics choice. In the final example, the student had an attachment with mathematics which developed from the home; the teacher and class environment unfortunately served to break that mathematics attachment. All three students are female and were interviewed approximately at the same time.

Alice in Yellow-Wood School

Alice attended a semi-rural low socioeconomic status school. Her parents were both employed, her mother as an accountant and father as a landscaper/builder. Alice was one of two non-identical twin girls and said she had learning difficulties and low expectations until she reached secondary school:

> I have always struggled in previous years because I had a learning difficulty when I was younger. I couldn't read properly and I was always really slow at processing things in my mind and when I was a child my parents were told that I would never be able to learn.

Prior to year 9 Alice was not particularly fond of mathematics and struggled with it. However, her twin sister stated that mathematics was one of her easiest subjects and she intended to continue with it. In year 10, Alice's relationship with mathematics became linked to her relationship with her mathematics teacher (Mrs. S) who was also the Head of Mathematics at Alice's school. In a separate interview we undertook with Mrs. S, she acknowledged the importance of student performance in mathematics, more so than in other subjects. She also stated that though she felt she and her colleagues were under pressure to maximise student attainment in mathematics, the department strove to develop a culture where having rounded students who learnt to value mathematics as an end in itself, rather than simply increasing attainment, was seen as the objective. Perhaps this explains why a number of students within this school, a higher proportion than in other schools, said that they enjoyed mathematics and wanted to work hard at it, without necessarily intending to continue with it post-16.

Prior to year 10 Alice was a student of below-average attainment in mathematics who had not intended to continue with the subject. By the time of her year 10 interview she was considering doing mathematics post-16, which coincided, thanks to Mrs. S, with an increased confidence in her mathematical ability. The extracts below were chosen because they signify the importance of Mrs. S to Alice and mathematics, and how such encouragement translated into an increased self-concept and a more positive relationship Alice had with mathematics:

> Because having Miss S I've actually developed a load of skills in maths. I know a lot more than I thought I would know before and my grades have actually increased than what they were before. I went from down from a C grade to … [meaning an increase from a C grade to a B grade] and I've found it fun as well because my teacher isn't boring and I've managed to get on with the homework and I am pushing myself in maths because I come to see Miss S if I'm struggling and that … It's really organised and so you're never sort of stuck with what you're doing and she really goes through it really clearly so it's sort of a step-by-step guide but not in a patronising way. Like, if you get stuck she will definitely come and help you—she doesn't ignore you—she comes straight over. It's just a lot she does really, it's

> really helpful. Previous teachers I had are quite good, but I've never really got on with the style that they worked with. Like, they have taught it well but I've never felt confident enough to go and see them if I was stuck on something. Whereas with Miss S you have that confidence to say 'I don't get this, can I have the help, please'.

In many ways Alice's interview demonstrated how she felt she had found someone who believed in her educational capabilities. Alice held onto that attachment to support her through her schooling:

> I feel I am doing and achieving the most when it comes to maths … it wasn't the case before. But after these 2 years with Miss S I have improved.

The extract below indicates how her teacher's encouragement helped her overcome what had appeared to be an on-going problem with an aspect of mathematics (percentages); overcoming this problem clearly had a role in increasing her mathematics self-concept. Alice was asked what her most memorable mathematics lesson was:

> I've never grasped doing percentages—no matter how hard I tried—but Miss S just explained it in the way she does and I finally got it and I think that's just been probably the best time at maths because when you finally know something, after not knowing it for so long, it is so much better isn't it? And it just made me feel really good.

Nearly all other interviewees either gave a bland answer to the same question ('What is your most memorable mathematics lesson?') such as 'There's nothing I find particularly memorable for maths lessons' (a male student from her school) or talking about something unrelated to mathematics or a lesson which was different from the normal mathematics lessons such as 'In year 8 we went into the Tom Smith Hall and played all different maths games and Splat and everything like that'.

Given the encouragement Alice received, she chose to study mathematics at year 12, though she subsequently dropped the subject after finding the lessons difficult. In her interview what came across was her intrinsic liking of mathematics and how that relationship with mathematics developed through a teacher. There was no evidence from her interview that she was intending to choose mathematics because of the extrinsic material gain of the subject.

Sandy in Yellow-Wood School

Sandy was in the same school as Alice. Her mother was an administrator and her father a surveyor; both graduates. Sandy was also taught by Mrs. S, and also chose mathematics at year 12. Although the extracts below lend some support to the quantitative findings in Table 2, they also highlight how important individual relationships with teachers and perceptions of teachers are in subject choice.

In her year 10 interview, Sandy talked about the importance of her mathematics teacher's encouragement in her intention to continue with mathematics: 'We had parents evening and my maths teacher said I could be perfectly capable studying maths, that I'll be a good student, I was encouraged by that'. However, it was also evident that she was aware of the material gain of having a post-16 qualification in mathematics, a sentiment expressed by the great majority of students who continued with mathematics at year 12: ‘Maths and physics are quite hard to take, but I just want the best available options, keep the door open for later in life.’ Although Sandy did not especially express how important encouragement from her mathematics teacher was, she did indicate how important it was to ‘like’ and ‘be liked’ by teachers when deciding what subjects to continue with. When discussing influences, she noted that hers included:

> Probably relationships with the teachers and how the school works because like if there’s a subject that you’ve been put off from the lower years you’re not going to want to continue with it … because sort of year 7 and 8 I was really good at art and I took it in year 9 but the teachers were just awful teachers, I didn’t like them at all they didn’t like me and so then I didn’t bother because there was no way that we were going to get on with at GCSE [the examinations sat by the great majority of school students in England at age 16 in year 11].

This issue of personal relationships and their importance to choice is an issue we examined when we decided to explore whether certain individual items within the ‘perceptions of teachers’ construct were more important in explaining future mathematics aspirations and gender differences in perceptions (Table 3). Supporting the key findings of the quantitative multi-level analysis (see Table 1), Sandy was also very aware of the material gain of having a mathematics qualification and indicated that she was probably going to continue with mathematics after compulsory education (as she indeed did):

Because I like maths and I like physics and I believe they will give me the greatest gateway for work after I go to university and I’m just generally interested in them … I suppose because I’ve always been quite good at it [mathematics] and again it’s logical as well apart from when I thought I don’t like it anymore there were some proofs that weren’t very good but now I just generally enjoy it.

Elira in Cherry Blossom School

Just as teachers were important in encouraging students to continue with mathematics or build their self-concept and relationship with mathematics, teachers could also damage the relationship students had with mathematics. A prime example of this was Elira who attended a high-attaining Church of England school that had a high proportion of minority ethnic students. She was a second generation Muslim from Kosovo[1] who came to England at the age of three with refugee status. In Kosovo her mother was a doctor although it took quite a few years until she managed to do further training and find work as a gynaecologist in England. Her father graduated in physics or geology (Elira could not recall), though the only jobs open to him in England entailed unskilled work. By the time of Elira’s interview he had managed to create a business in buying and renting out homes in Albania and Bulgaria as well as owning restaurants in England. Her parents worked very hard to

[1] Her background is raised because she has raised it, which was distinct from other interviewees who largely did not indicate their cultural or religious heritage.

Table 3 Year 8 students' perceptions of their mathematics teachers

Overall perceptions of maths	All students[a]				Boys			Girls			Comparison (boys vs. girls)		
Item	N	Corr.	M	SD	N	M	SD	N	M	SD	*t*	*df*	Effect size Cohen's d
Students' perception of teacher (original construct)	*5,293*	*0.277****	*4.62*	*0.95*	*2,342*	*4.62*	*0.97*	*2,917*	*4.61*	*0.97*	*0.211*	*4,937.741*	*0.006*
My maths teacher is good at explaining maths	5,072	0.226***	4.63	1.37	2,220	4.73	1.34	2,820	4.55	1.34	4.559	4,835.753	0.129***
My maths teacher has high expectations of what the students can learn	4,737	0.191***	5.08	0.95	2,108	5.14	0.94	2,597	5.03	0.94	4.132	4,525.251	0.121***
My teacher thinks that I should continue with maths beyond my GCSEs	2,640	0.425***	4.89	1.25	1,220	4.97	1.20	1,400	4.83	1.20	2.998	2,603.995	0.117**
My maths teacher wants us to really understand maths	5,008	0.177***	5.29	0.89	2,190	5.34	0.87	2,788	5.26	0.87	3.239	4,784.903	0.092**
My maths teacher does not only care about students who get good marks in maths	4,731	0.138***	4.72	1.46	2,075	4.65	1.52	2,626	4.78	1.52	−3.043	4,266.580	0.090**
My maths teacher believes that all students can learn maths	4,902	0.164***	5.27	0.88	2,156	5.30	0.89	2,714	5.24	0.89	2.434	4,605.209	0.070*
I like my maths teacher	5,193	0.234***	4.25	1.57	2,295	4.19	1.61	2,864	4.29	1.61	−2.441	4,824.092	0.069*
My maths teacher seems to like all the students	4,684	0.185***	4.12	1.56	2,067	4.07	1.60	2,588	4.16	1.60	−2.027	4,319.770	0.060*
My maths teacher is interested in what the students think	4,756	0.206***	4.59	1.33	2,088	4.61	1.33	2,636	4.57	1.33	1.199	4,476.392	0.035
My maths teacher does not let us get away with not doing our homework	4,996	0.027	5.06	1.27	2,191	5.09	1.30	2,775	5.05	1.30	1.033	4,583.120	0.030
My maths teacher believes that mistakes are OK as long as we are learning	4,957	0.178***	5.01	1.13	2,173	4.99	1.16	2,753	5.01	1.16	−0.606	4,562.587	0.017
My maths teacher marks and returns homework quickly	4,869	0.158***	4.44	1.43	2,133	4.46	1.42	2,705	4.43	1.42	0.589	4,623.098	0.017
My maths teacher treats all students the same regardless of their maths ability	4,860	0.163***	4.64	1.44	2,146	4.65	1.45	2,683	4.63	1.45	0.426	4,549.322	0.012
My maths teacher sets us homework	5,139	0.071***	5.40	0.94	2,252	5.40	0.94	2,855	5.39	0.94	0.300	4,821.925	0.008
My maths teacher is interested in me as a person	3,927	0.225***	3.60	1.53	1,743	3.61	1.58	2,159	3.60	1.58	0.154	3,636.737	0.005

Notes: *N* **number;** *M* **mean;** *SD* **standard deviation; comparisons between girls and boys**

[a]Shaded area is a separate set of analyses (correlations (Corr.)) between items that explore all students' perceptions of their teachers with students' intentions to continue to study maths post-16; unshaded area explores gender issues amongst items; ***significant at .001; **significant at .01; *significant at .05

ensure that the family were able to rebuild their lives in England and instilled the same emphasis on hard work within Elira. Her mother was the deciding force behind Elira's year 9 subject choices and the pervasiveness of that influence is apparent when she talked about her future subject choices:

> And it's kind of—I want to have something in common with her, in a way … she thinks I'm her in a way, she thinks I'm more academic … my mum wants me to do sciences like physics, chemistry, coz she's a doctor … so that is quite a big influence in my life and she kind of encourages me … but, at the same time, I personally like and enjoy my subjects like maths, physics, chemistry, biology, I enjoy them.

Her interview suggested that her relationship with most subjects was through relative performance. However, her interview also suggested that she was considering mathematics because of its material gain:

> And I was thinking of taking maths because it's like a really important subject most jobs look for that … You need maths. It's like there and it looks good on your CV if you got an A or something.

Furthermore, choice in mathematics was also tied in with Elira's mathematics self-concept (not dissimilar to other students) and also her parents' expectations:

> My personal achievement will be to get an A or an A*. If I get a B I would probably be upset but I will still continue it. I don't think I'll continue if I got a C, I would just think I was kind of not good at it … my parents don't accept anything under an A; they'll be like "What are you doing?". And so they're strict on education … they're like "You get anything underneath an A you know you're not gonna go out …."

Elira's 'mathematics identity' stemmed from her earlier life experiences when her parents tested her mathematics knowledge to help strengthen her mathematics competencies. Her interview also indicated that such testing left her feeling quite anxious about mathematics as a child and she recalled thinking 'Oh God, don't make me get it wrong'. Nevertheless, as a 15-year-old she was able to identify positively with mathematics. However, in year 10 Elira's relationship with mathematics began to crumble. In the following extract she contrasts her mathematics lesson and teacher with that of physics:

> I like my teacher as well [in reference to physics], he's quite—it makes it interesting—and then the class actually reacts well to the lesson, and in maths, for example, our class is usually noisy, no-one concentrating, it's kind of hard to control them even though we're supposed to be one of the top sets—second top—it's still kind of—it kind of distracts the whole class.

Although Elira stated 'I think I'm quite good at maths', she also notes that:

> I was kind of not concentrating at all and everything; just talking and kind of being noisy … my mocks I got a D. It's kind of hard to find it fun in our class coz our class is really bad—even our Head of Year had to come and shout at us coz the grades we were getting weren't acceptable for the standard we're all supposed to be working on, and our ability. And it's like no one cares about it and it kind of influences everybody else.
>
> Interviewer: Why do you think nobody cares about it?
>
> Elira: Because no one does the work—Sir tries to explain, everybody's talking, no-one listens, it's kind of hectic in the room. We're always noisy. Even in exams we talk. And it's kind of hard to control the class. Some people in the class are kind of rude to Mr. W as well. And they go 'Oh, Sir, you're being unfair, we don't know this, we don't know this' but if they listened then obviously they would.

In a later interview in year 11, Elira gave a detailed account of how in year 10 she felt the maths lessons were so awful that she was unable to learn anything. Disruptive students continued to make the working life of the teacher difficult and, according to Elira, the entire class got left behind in mathematics since the norm became discussing anything other than mathematics. Her anxiety with mathematics became more pronounced as she was encouraged by her parents to be good at it and to continue with it. However, as she felt she did not do well at GCSE she did not continue with mathematics at A-Level.

Deconstructing What Our Original Constructs Actually Measured: Perceptions of Mathematics Teachers, Mathematics and Mathematics Lessons

Students' Perceptions of Their Mathematics Teachers

The MLM analysis (Table 2) indicated that the 'perceptions of teachers' construct did not have an independent statistically significant influence in explaining year 8 students' intentions to continue with mathematics post-16 after controlling for a range of other student level factors. Such findings are inconsistent with our qualitative research where encouragement and support from teachers were important in enhancing or severing students' relationship with mathematics. To see if findings from the two separate strands of our project could be aligned, we decided quantitatively to deconstruct what we meant by 'perception of teachers' and therefore conducted a series of item-level analyses.

Our perception of teachers construct explored two key dimensions: encouragement in learning and personal relationships. Students (as a group) reported positive perceptions of their teachers as indicated by their scores on the individual items; the mean for the actual construct 'perceptions of teachers' was also fairly high (4.62). Preliminary work suggested that particular items within constructs might be of especial significance. We decided to include an item which was a part of our original 'advice-pressure to study mathematics' construct, namely 'my teacher thinks that I should continue with maths post-16', on the grounds that teacher advice seems likely to be of importance, and analyse this item along with the remaining items that created the construct 'perception of teachers'. The means in Table 3 indicate that students were most positive about their teachers setting them homework (mean of 5.40); this was followed by their teachers really wanting them to understand maths (5.29) and teachers believing that all students can learn maths (mean of 5.27). These findings somewhat mirror results we found with year 10 physics students (see Mujtaba & Reiss, 2013b); two of these items were the top two most positive responses: teachers really wanting them to understand physics (mean of 4.93), teachers believing that all students can learn physics (mean of 4.90). Year 8 students were least positive about their mathematics teachers being interested in them as people (mean of 3.60) and liking all students (mean of 4.12); these findings also mirror those we found with physics (see Mujtaba & Reiss, 2013b) where we found means 3.33 and 3.80, respectively.

Table 3 also shows how boys and girls responded to each item and whether gender differences were statistically significant. In total, eight of the 15 items showed statistically significant differences between the responses of boys and girls, as well as some of the items having stronger associations with intended participation than others. However, the overall construct ‘perceptions of teachers’ indicated that there was no statistically significant difference between girls and boys. This is rather worrying given that we used the ‘perceptions of teachers’ construct to explore associations with year 8 students’ intended post-16 participation in mathematics and could have concluded that this construct was not important in explaining intended participation or gender differences in participation. There is a possibility that there are particular items within this overall construct that are individually better able to explain intended participation and that their effect(s) are masked by being immersed in an overall construct.

If we continued simply to use this construct to explore gender differences in students’ perceptions of their mathematics teachers without looking at individual-level items our findings would have also missed issues that can help explain gender differences in perceptions of mathematics teachers. Of the eight statistically significant items, the largest effect size in gender differences was for ‘my teacher is good at explaining maths’ (ES = .129); the remaining effect sizes were between .121 and .060. We found within the physics analysis that the item ‘my teacher thinks that I should continue with physics post-16’ had the strongest effect size in explaining gender differences at year 10 (ES = .337), although the effect size was almost three times as strong as that found for mathematics (ES = .117).

On average, boys responded more positively than girls about their mathematics teachers. Boys felt to a greater extent than girls that their mathematics teachers: encouraged them to continue with maths post-16 ($t = 2.998$, $p < .001$); had high expectations of what students can learn ($t = 4.132$, $p < .01$); wanted students to really understand maths ($t = 3.239$, $p < .01$); were good at explaining maths ($t = 4.559$, $p < .001$) and believed all students could learn maths ($t = 2.434$, $p < .05$). The only item for which girls were more positive than boys was ‘my maths teacher doesn’t only care about students who get good marks’ ($t = 3.043$, $p < .01$).

When we looked at personal relationships with mathematics teachers, girls were more likely to report that they liked their maths teacher ($t = 2.441$, $p < .05$) and that their teacher seemed to like all students ($t = 2.027$, $p < .05$).

Intention to Participate and Perceptions of Teachers

A correlation analysis between the items that explored year 8 students’ perceptions of their teachers and their intentions to participate in mathematics post-16 further revealed important findings about items that were originally clustered together within an overall construct (see Table 3). The original construct ‘perceptions of teachers’ was only weakly correlated with intended participation (.277). The correlations in Table 3 demonstrate that students’ perceptions of their teachers personally encouraging them to continue with mathematics post-16 (which, as noted above, was originally analysed as part of the ‘advice-pressure to study mathematics’ construct) is the most strongly associated item with intended participation—more so than items that measure students' perceptions around encouragement in doing mathematics homework. The four strongest correlations between students' perceptions of their teachers and their intention to continue with mathematics post-16 were 'my teacher thinks I should continue to study maths after the age of sixteen' (.425); 'my maths teacher is good at explaining maths' (.227); 'I like my maths teacher' (.226) and 'my maths teacher is interested in me as a person' (.225). This finding reflects what we found with physics. The correlation between year 10 intended participation and 'my teacher thinks I should continue to study physics after the age of sixteen' was .493 and this correlation was also set apart from the rest of the items that explored perceptions of physics teachers (see Mujtaba & Reiss, 2013b). The findings within the year 8 mathematics survey item-based analysis demonstrate that there are a handful of important issues about teachers which are very important in their associations with intended participation and explaining gender differences in participation, that such findings are not apparent when using an overall construct and that there are similarities in findings with the item-based physics analysis.

Students' Perceptions and Emotional Response to Their Mathematics Lessons

The means for the original constructs 'perceptions of lessons' and 'emotional response to lessons' used in the MLM analysis were positive: 4.11 and 4.00, respectively. The MLM analysis indicated that 'perceptions of lessons' despite initially having a significant association, lost statistical influence in explaining year 8 students' intended participation after controlling for a range of other constructs. In our analysis of individual items (see Table 4) we found that the items in these two constructs collectively explored relevance of mathematical concepts, intrinsic value of mathematics lessons, self-concept in mathematics as impacted by mathematics lessons, and emotional response to lessons.

The overall means in Table 4 demonstrate that collectively the students responded positively to items asking them about their mathematics lessons, but with some areas of concern. Collectively, boys and girls were most positive about 'when I am doing maths, I don't get upset' (5.18); 'when I am doing maths, I am learning new skills' (4.73) and seeing the relevance of maths lessons (4.55). They were least positive about looking forward to maths classes (3.41)—a finding which mirrors that for physics in our work reported elsewhere—and not being bored in maths lessons (3.41).

For the large majority of items in Table 4 there were statistically significant differences in responses between boys and girls, with boys responding more positively to questions around mathematics lessons. The effect sizes for gender differences in student perceptions of mathematics lessons were generally larger than those for perceptions of mathematics teachers (Table 2). Again, similar to the findings in physics, we found that the largest statistically significant gender difference was in response to the item, 'thinking about your maths lessons, how do you feel you compare with the others in your group?' (ES = .321), followed by 'I do well in maths tests' (ES = .318). These were followed by 'when I am doing maths, I always know what I am doing' (ES = .238), 'when I am doing maths, I do not get upset' (ES = .196)

Table 4 Year 8 students' perceptions of their mathematics lessons

Overall perceptions of maths	All students[a]				Boys			Girls			Comparison (boys vs. girls)		
Item	N	Corr.	M	SD	N	M	SD	N	M	SD	*t*	*df*	Effect size Cohen's d
Emotional response to maths/physics (original construct)	*5,293*	*0.333****	*4.00*	*0.98*	*2,338*	*4.09*	*0.99*	*2,921*	*3.93*	*0.96*	*6.080*	*4,936.096*	*0.169****
Perceptions of maths/physics lessons (original construct)	*5,302*	*0.557****	*4.11*	*0.98*	*2,347*	*4.17*	*1.01*	*2,921*	*4.06*	*0.94*	*4.042*	*4,858.115*	*0.113****
Thinking about your maths lessons, how do you feel you compare with the others in your group?	5,017	0.322***	3.50	1.09	2,241	3.69	1.11	2,744	3.34	1.05	11.225	4,681.128	0.321***
I do well in maths tests	5,231	0.410***	4.32	1.20	2,325	4.52	1.17	2,873	4.15	1.19	11.439	5,010.675	0.318***
When I am doing maths, I always know what I am doing	5,243	0.364***	3.58	1.34	2,314	3.75	1.34	2,897	3.44	1.32	8.520	4,923.437	0.238***
When I am doing maths, I don't get upset	5,204	0.126***	5.18	1.20	2,296	5.31	1.15	2,876	5.07	1.24	7.075	5,058.054	0.196***
In my maths lessons, my teacher explains how a maths idea can be applied to a number of different situations	5,067	0.371***	3.79	1.28	2,268	3.91	1.34	2,767	3.70	1.23	5.784	4,652.427	0.165***
When I am doing maths, I don't daydream	5,218	0.288***	3.66	1.63	2,307	3.81	1.67	2,878	3.55	1.59	5.557	4,823.160	0.156***
I enjoy my maths lessons	5,218	0.473***	3.75	1.51	2,319	3.84	1.54	2,866	3.67	1.48	3.996	4,870.835	0.112***
When I am doing maths, I am learning new skills	5,226	0.399***	4.73	1.11	2,307	4.80	1.13	2,887	4.68	1.08	3.833	4,844.078	0.108***
I look forward to maths classes	5,260	0.479***	3.41	1.48	2,330	3.50	1.51	2,896	3.35	1.45	3.463	4,890.205	0.097***
When I am doing maths, I am not bored	5,215	0.362***	3.41	1.56	2,306	3.49	1.60	2,877	3.36	1.52	2.803	4,811.937	0.079**
I can see the relevance of maths lessons	5,117	0.440***	4.55	1.28	2,282	4.60	1.33	2,804	4.51	1.24	2.433	4,717.279	0.069*
I find it difficult to apply most maths concepts to everyday problems	5,173	0.082***	3.74	1.39	2,297	3.78	1.45	2,843	3.71	1.33	1.991	4,714.571	0.056*
When I am doing maths, I pay attention	5,219	0.368***	4.31	1.23	2,309	4.33	1.26	2,877	4.30	1.21	1.000	4,859.442	0.028
In my maths lessons, I have the opportunity to discuss my ideas about maths	5,215	0.330***	4.26	1.35	2,314	4.26	1.39	2,868	4.25	1.31	0.192	4,806.804	0.005

Notes: *N* **number,** *M* **mean,** *SD* **standard deviation; comparisons between girls and boys**

[a]Shaded area is a separate set of analyses (correlations (Corr.)) between items that explore all students' perceptions of their lessons with students' intentions to continue to study maths post-16; unshaded area explores gender issues amongst items; ***significant at .001; **significant at .01; *significant at .05

and 'in my maths lessons, my teacher explains how a maths idea can be applied to a number of different situations' (ES = .170).

Amongst the perceptions of teacher items, the item 'my teacher is good at explaining maths' had a larger effect size in explaining gender differences than the majority of other items (ES = .129). Taking this finding with the effect size of 'my teacher explains how a maths idea can be applied to a number of different situations' (ES = .165) demonstrates how important it is for teachers to explain mathematics in a way that engages girls and aids their learning and understanding of mathematics. In order to emphasise our point we refer to the very similar patterns with the physics analysis. Amongst the perceptions of teacher items, 'my teacher is good at explaining physics' (ES = .237) had an effect size in line with an item clustered within lessons: 'my teacher explains how a physics idea can be applied to a number of different situations' (ES = .265) (see Mujtaba & Reiss, 2013b).

Boys were more likely to report that their teacher explained how maths ideas can be applied to a number of different situations ($t = 5.784$, $p < .001$); they saw the relevance of maths lessons ($t = 2.443$, $p < .05$) and they found it easy to apply most maths concepts to everyday problems ($t = 1.991$, $p < .05$). These items were a part of the 'perceptions of lessons' construct.

Boys were also more positive about looking forward to their maths classes ($t = 3.463$, $p < .001$) and enjoying their maths lessons ($t = 3.996$, $p < .001$); and gave more favourable answers about doing well in their maths tests ($t = 11.439$, $p < .001$); and doing better in their maths lessons than their peers ($t = 11.225$, $p < .001$).

Finally, boys were more positive about 'when I am doing maths, I always know what I am doing' ($t = 8.520$, $p < .001$); I am learning new skills ($t = 3.883$, $p < .001$); I am not bored ($t = 2.803$, $p < .01$); I don't get upset ($t = 7.075$, $p < .001$) and I do not daydream ($t = 5.557$, $p < .001$).

Intention to Participate and Perceptions of Mathematics Lessons

The actual constructs 'perceptions of lessons' and 'emotional response to lessons' were moderately correlated with intended participation (.557 and .333, respectively, see Table 4). We would have expected the associations to be the other way around given that in the final MLM model 'emotional response to lessons' had a statistically significant independent influence in explaining intended post-16 mathematics participation. These associations alone suggest again that our original lessons constructs possibly needed further refinement. Other than the associations with the original constructs, the three strongest item-level associations between intended post-16 participation and these cluster of mathematics lessons items were 'I look forward to maths classes' ($r = .479$); 'I enjoy my maths lessons' ($r = .473$) and 'I can see the relevance of maths lessons' ($r = .440$). It is interesting to note that these were the three strongest associations found with the year 10 physics analysis (Mujtaba & Reiss, 2013b). The item that had the smallest association with intended participation was 'I don't find it difficult to apply most maths concepts to everyday problems' ($r = .082$)—again mirroring our findings with physics.

Students' Perceptions of Mathematics

Items explored five areas concerning students' perceptions of mathematics: usefulness of mathematics (a part of the extrinsic material gain and social gain motivation constructs); self-concept in mathematics; liking of mathematics; mathematics and social skills; and doing mathematics. Overall student means (see Table 1) indicate that students' responses about mathematics were generally positive though there were some aspects of mathematics that they were not positive about or did not agree with. Students were most positive about or in agreement with 'I think maths is a useful subject' (mean 5.15) and least positive about 'being good at maths makes you popular' (mean 2.35)—this latter finding again mirrored that for physics.

Table 1 demonstrates that there were statistically significant differences in responses between boys and girls for the great majority of items, with year 8 boys responding more positively to questions about their perceptions of mathematics. The findings lend support to existing research that some (but certainly not all) girls typically feel disengaged from mathematics and this may be related to the way it is taught. This is possibly related to (some) girls not feeling there are a range of ways to learn mathematics. The largest significant difference in responses between boys and girls was for the item 'I am good at maths' (ES = .404), followed by 'I don't need help in maths' (ES = .310); both of these findings mirror those found for physics with their respective effect sizes being .583 and .548 (Mujtaba & Reiss, 2013b).

In addition, these are the largest effect sizes reported even when including items that explored perceptions of mathematics teachers and mathematics lessons (see Tables 3 and 4). The next four strongest effect sizes (ranging from .274 to .212) were still larger than the effect sizes found for any of the perception of teacher items: 'I think maths will help me in the job I want to do in the future' (ES = .227); 'to be good at maths you need to be creative' (ES = .274); 'being good at maths makes you popular' (ES = .215) and 'maths is important in making new discoveries' (ES = .212).

Boys were more positive that maths is a useful subject ($t = 3.171$, $p < .001$); is more likely to help them get into jobs they want to do in the future ($t = 8.094$, $p < .001$); teaches individuals to think logically ($t = 5.034$, $p < .001$); helps individuals to solve everyday problems ($t = 2.416$, $p < .01$); is important in making new discoveries ($t = 7.253$, $p < .001$) and that people who are good at maths get well-paid jobs ($t = 6.321$, $p < .001$).

Boys were more likely to report that they are good at maths ($t = 14.487$, $p < .001$) and do not need help with maths ($t = 11.006$, $p < .001$). They were more positive about maths being an interesting subject ($t = 5.897$, $p < .001$); finding maths interesting ($t = 5.589$, $p < .001$); everyone needing to know some maths ($t = 3.248$, $p < .01$); maths being a useful subject ($t = 3.171$, $p < .001$) and that it is interesting to find out about the laws of maths that explain different phenomena ($t = 5.071$ $p < .001$).

Boys were more likely to report that maths makes individuals popular ($t = 7.132$, $p < .001$) and improves social skills ($t = 3.411$, $p < .01$). Finally, boys were more likely to report that 'to be good at maths individuals need to be creative' ($t = 9.336$, $p < .001$); 'to be good at maths you need to work hard' ($t = 6.553$, $p < .001$) and 'those who are good at maths are those who are clever' ($t = 5.840$, $p < .001$).

Correlations Between Perceptions of Mathematics and Intended Participation in Mathematics

A correlation analysis was conducted between the items that explored students' perceptions of mathematics and their intention to participate in it post-16. Table 1 demonstrates that for the sample as a whole the three strongest associations between intended participation and perceptions of mathematics were for the items: 'I think maths will help me in the job I want to do in the future' (a part of the 'extrinsic material gain motivation' construct) (.506)—with the associated effect size for gender difference being .227; 'I think maths is an interesting subject' (.568)—with the associated effect size for gender difference being .166 and 'I think maths is a useful subject' (a part of the 'extrinsic material gain motivation' construct) (.515)—with the associated effect size for gender difference being .089.

In Mujtaba and Reiss (2013c) we found that boys and girls who intended to continue with mathematics post-16 had similar levels of 'extrinsic material gain motivation', though they differed in other perceptions of their mathematics education. The correlations between items measuring extrinsic material gain motivation and intended participation in mathematics are not surprising; despite two of the items from the 'extrinsic material gain construct' being the most strongly associated items with intended participation, the gender differences are not as strong as those found in other areas of students' perceptions of their mathematics education. These findings suggest that the differences between boys and girls are in their experiences of their mathematics education rather than girls not appreciating the value of mathematics as much as boys. Table 1 also shows the correlations and gender differences for the original constructs. As can be seen from some of the self-concept items, some are more strongly associated with intended participation than others (for example 'I am good at maths' versus 'I do not need help with maths'). We will discuss this further in the concluding section. The actual 'self-concept' construct was moderately correlated with intended mathematics participation (.444), along with some of the other original constructs: extrinsic social gain motivation (.363), extrinsic material gain motivation (.572) and intrinsic value of mathematics (.516).

Multi-level Re-analysis to Explore the Importance of Students' Perceptions on Intended Post-16 Mathematics Participation (Using Items from the Survey Rather than Constructs)

Finally, a further set of multi-level models were run in a series of stages which had particular conceptual relevance, this time driven by the analysis reported above which included findings from the qualitative work. This final set of analyses tested for items from the year 8 student survey and used survey data that the same students filled out in year 10 (age 15). Table 5 shows the final, best fit model and highlights a number of key messages:

1. Our original construct 'extrinsic material gain motivation', found to be an important construct associated with intended participation at year 8, continues to be important in explaining intended participation at year 10 (even whilst using an item-level analysis).

Table 5 Item-based analysis: estimates of fixed effects on year 10 England students' intentions to study mathematics post-16

Parameter	Estimate	Std error	df	*t*	Sig.	Effect size
Intercept	6.346	0.180	736.308	35.271	0.001	
Gender	–0.162	0.082	683.859	–1.965	0.050	–0.154
'I think maths will help me in the job I want to do in the future' (comparison group: Strongly agree)						
(Strongly disagree)	–1.007	0.220	800.304	–4.570	0.001	–0.961
(Disagree)	–1.029	0.165	806.611	–6.220	0.001	–0.983
(Slightly disagree)	–0.650	0.178	806.738	–3.653	0.001	–0.621
(Slightly agree)	–0.655	0.125	806.730	–5.252	0.001	–0.625
(Agree)	–0.355	0.097	806.950	–3.641	0.001	–0.338
'My teacher thought that I should continue with maths after my GCSEs' (comparison group: Strongly agree)						
(Strongly disagree)	–0.874	0.261	806.536	–3.353	0.001	–0.835
(Disagree)	–0.340	0.216	806.647	–1.580	0.115	–0.325
(Slightly disagree)	–0.483	0.205	803.362	–2.353	0.019	–0.461
(Slightly agree)	–0.305	0.136	806.885	–2.250	0.025	–0.292
(Agree)	–0.109	0.108	805.839	–1.010	0.313	–0.104
'My friends thought that I should continue with maths after my GCSEs' (comparison group: Strongly agree)						
(Strongly disagree)	–0.714	0.212	805.560	–3.373	0.001	–0.682
(Disagree)	–0.910	0.195	806.288	–4.664	0.001	–0.869
(Slightly disagree)	–0.467	0.178	803.045	–2.630	0.009	–0.446
(Slightly agree)	–0.322	0.140	804.960	–2.300	0.022	–0.308
(Agree)	0.166	0.123	803.267	1.356	0.175	0.159
'I was advised by my family that maths would be a good subject to study after my GCSEs' (comparison group: Strongly agree)						
(Strongly disagree)	–1.337	0.228	800.843	–5.869	0.001	–1.276
(Disagree)	–1.126	0.213	806.288	–5.294	0.001	–1.075
(Slightly disagree)	–0.804	0.198	806.394	–4.066	0.001	–0.767
(Slightly agree)	–0.478	0.139	806.826	–3.435	0.001	–0.457
(Agree)	–0.287	0.100	805.704	–2.864	0.004	–0.274
'I look/looked forward to maths classes' (comparison group: Strongly agree)						
(Strongly disagree)	–0.093	0.185	802.169	–0.504	0.614	–0.089
(Disagree)	–0.404	0.173	806.842	–2.337	0.020	–0.386
(Slightly disagree)	–0.117	0.162	806.502	–0.721	0.471	–0.112
(Slightly agree)	–0.001	0.148	806.688	–0.008	0.993	–0.001
(Agree)	–0.103	0.146	804.504	–0.707	0.480	–0.098

(continued)

Table 5 (continued)

Parameter	Estimate	Std error	df	*t*	Sig.	Effect size
'When I am/was doing maths, I got upset' (comparison group: Strongly disagree)						
(Strongly agree)	–0.200	0.184	805.048	–1.089	0.276	–0.191
(Agree)	–0.450	0.203	806.505	–2.212	0.027	–0.429
(Slightly agree)	–0.135	0.162	804.089	–0.834	0.405	–0.129
(Slightly disagree)	0.052	0.154	801.152	0.336	0.737	0.050
(Disagree)	–0.051	0.092	806.745	–0.549	0.583	–0.048
'I am good at maths' (comparison group: Strongly agree)						
(Strongly disagree)	–0.001	0.262	805.774	–0.005	0.996	–0.001
(Disagree)	–0.634	0.242	802.824	–2.623	0.009	–0.605
(Slightly disagree)	–0.220	0.218	806.667	–1.013	0.311	–0.210
(Slightly agree)	–0.307	0.140	806.812	–2.186	0.029	–0.293
(Agree)	–0.256	0.116	804.896	–2.218	0.027	–0.245
'I need/needed help with maths' (comparison group: Strongly disagree)						
(Strongly agree)	–0.532	0.189	806.975	–2.812	0.005	–0.507
(Agree)	–0.431	0.160	806.852	–2.695	0.007	–0.411
(Slightly agree)	–0.346	0.146	803.227	–2.377	0.018	–0.330
(Slightly disagree)	–0.154	0.153	804.823	–1.002	0.317	–0.147
(Disagree)	–0.144	0.135	806.657	–1.070	0.285	–0.138
Random-effects parameters						
Variance (Level 2)	0.028	0.018				
Variance (Level 1)	1.097	0.056				
Deviance (–2 × log restricted-likelihood)	2,560.182					

2. Students' views of their lessons and teachers are also important in explaining intended participation. This was missed by our construct-level analysis.
3. Gender becomes an important predictor for intended participation in year 10, whilst at year 8 for the same students the differences between boys and girls were not statistically significant.
4. Students' perceptions and experiences in year 10 are more important in explaining intended participation than in year 8.

The items that formed the original constructs which explored perceptions of mathematics (e.g. extrinsic material gain motivation and self-concept) were added towards the end of the model steps, primarily because it was predicted (given earlier multi-level findings and the associations reported in Table 1) that items from such constructs would wipe away the significant influence of teachers and lessons. We wanted to see what, if any, items were associated with year 10 students' mathematics aspirations in both the preliminary and final model.

Our original construct-based multi-level analysis indicated that underlying personality traits lost significance once more fine-grained measures of mathematics-specific measures were introduced in the models. Given such findings we did not include these (non-mathematics-specific) measures within this analysis. For the same reason, we omitted any non-mathematics-specific items that measured general attitudes/perceptions of learning, support and encouragement.

In this item-based multi-level analysis we tested students' survey responses as year 8 and as year 10 learners of mathematics as predictors of mathematics aspirations in year 10. We found that the students' year 10 survey responses about their mathematics education and support they received were better predictors of year 10 aspirations than the earlier year 8 responses; therefore, in the final model only the year 10 survey measures remain. The final model in many ways supported, built on and shed further light on what we found earlier with the construct-based multi-level analysis when the students were in year 8. Table 4 shows that as year 8 learners of mathematics, the construct 'advice-pressure to study mathematics' (which was a summed score of a range of influences students received) was a strong predictor of year 8 students' mathematics aspirations; some of the items which formed this construct also appear as important predictors of these students' aspirations when they were in year 10 (Table 5).

With respect to the items which formed the 'perception of teachers' construct, prior to the inclusion of items from 'self-concept' or 'extrinsic material gain motivation', we found that the 'my maths teacher is good at explaining maths' and 'my maths teacher is interested in me as a person' both had significant independent influences. However, neither of these items were significant predictors in the final model once we controlled for the items that measured 'advice-pressure to study mathematics' and 'extrinsic material gain motivation'. The item 'my teacher thought that I should continue with maths after my GCSEs' (which was originally a part of the 'advice-pressure to study mathematics' construct) had a significant independent influence in explaining year 10 students' mathematics aspirations, which concurs with the findings from the qualitative work. More generally, it is now clear that the influence of teachers is very important (also taking into account findings from Table 3). Furthermore, our original construct of 'perceptions of teachers' was subsequently found to be composed of a number of distinct sub-constructs. For example, the associations between both students' mathematics teacher being 'interested in them as a person' (.237) and students 'liking their mathematics teacher' (.238) with mathematics aspirations were much stronger than when compared to the items that tapped into homework (.057–.167).

Encouragement (most importantly by teachers and families) appears to be associated with raised mathematics aspirations, as evidenced by both our construct-based and item-based analyses. These findings have implications for policy and practice. In order to increase mathematics aspirations, teachers (given that schools generally have little influence on families) need not only to encourage students but to place an emphasis on the 'extrinsic material gain' of having a post-16 mathematics qualification. In addition, the bivariate item-level analysis and the qualitative work revealed that personal relationships with teachers are important in encouraging students' future mathematics aspirations. Teachers could enhance students' aspirations by actively creating more meaningful relationships with their students within their teaching (cf. Rodd, Reiss, & Mujtaba, 2014).

It was worth separating out and exploring the individual influence of each item that created the overall 'advice-pressure to study mathematics' construct. This was a construct developed and piloted (by ourselves) that proved to be of great value to the research. It was clear that the construct showed a large effect size in explaining year 8 students' intended participation (Table 2). We hope that this construct and the various items within it will prove useful for future studies, both qualitative and quantitative, in exploring mathematics aspirations and in enabling teachers and family members to boost post-compulsory mathematics participation. In the final model of the item-based analysis there was an item which indicated that family influence to continue with mathematics post-16 was quite important, which was in line with the findings within our qualitative work (e.g. Elira). Again, this effect was masked in the original analysis when all of the items formed one overall construct—'advice-pressure to study mathematics'.

Two of the items that were a part of the original mathematics self-concept construct were found to have a strong independent influence on mathematics aspirations: 'I am good at maths' and 'I don't need help with maths'. In fact, 'I am good at maths' had as strong a correlation with intended participation (.460) as the mathematics 'self-concept' construct (.455). Again, these findings support the construct-based analysis which indicated the importance of self-concept. We find it interesting that these two particular items were also uncovered as being important in a similar item-based multi-level modelling analysis when exploring factors that influence year 10 students' physics aspirations (Mujtaba & Reiss, 2013b).

Methodological Conclusions

Methodologically, this chapter reaches three principal conclusions. First, mathematics-specific measures are better predictors of intended participation in mathematics than more general measures. While hardly surprising, the use of mathematics-specific measures proved vital in helping this research discover more about the factors that shape future aspirations in mathematics. In particular, the mathematics-specific measure of extrinsic material gain motivation was more tightly related to future mathematics aspirations than any of the other measures used within our models that measure motivation.

Second, our work clearly demonstrates that research questions ought to guide and help conceptualise a measure whilst taking into account how students may respond differently to the various items within a construct. We conclude that, valuable as construct-based analyses are, researchers ought, at the very least, to complement such analyses by selected analysis at the level of items.

Third, while it is hardly unusual to combine quantitative and qualitative work within a single study, our work shows the benefit of the two approaches when they truly interdigitate. In the analyses reported above we began with quantitative analyses, then turned to qualitative work and then returned to a new set of quantitative analyses, drawing both on our first sets of quantitative analyses and on our qualitative work. The resulting conclusions are, we believe, more robust than had we relied on only quantitative or qualitative work—a conclusion reinforced by our observation that many of our final mathematics-specific findings are similar to those of our physics-specific investigation (Mujtaba & Reiss, 2013b).

References

Black, L., Mendick, H., & Solomon, Y. (Eds.). (2009). *Mathematical relationships in education: Identities and participation*. New York: Routledge.

Boaler, J. (2009). *The elephant in the classroom: Helping children learn and love maths*. London: Souvenir Press.

Brown, M., Brown, P., & Bibby, T. (2008). 'I would rather die': Reasons given by 16-year-olds for not continuing their study of mathematics education. *Research in Mathematics Education, 10*(1), 3–18.

Connell, R. W. (1995). *Masculinities*. Cambridge, England: Polity Press.

Department for Business, Innovation and Skills. (2009). *Higher ambitions: The future of universities in a knowledge economy*. London: Department for Business, Innovation and Skills.

Eccles, J. (1994). Understanding women's educational and occupational choices: Applying the Eccles et al. model of achievement-related choices. *Psychology of Women Quarterly, 18*, 585–609.

Eccles, J. (2009). Who am I and what am I going to do with my life? Personal and collective identities as motivators of action. *Educational Psychologist, 44*(2), 78–89.

Mendick, H. (2006). *Masculinities in mathematics*. Maidenhead, England: Open University Press.

Middleton, J. A., & Jansen, A. (2011). *Motivation matters and interest counts: Fostering engagement in mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Mujtaba, T., & Reiss, M. J. (under revision). *Girls in the UK have similar reasons to boys for intending to study mathematics post-16*.

Mujtaba, T., & Reiss, M. J. (2013a). A survey of psychological, motivational, family and perceptions of physics education factors that explain 15 year-old students' aspirations to study post-compulsory physics in English schools. *International Journal of Science and Mathematics Education, 12*, 371–393.

Mujtaba, T., & Reiss, M. J. (2013b). Inequality in experiences of physics education: secondary school girls' and boys' perceptions of their physics education and intentions to continue with physics after the age of 16. *International Journal of Science Education, 35*, 1824–1845.

Mujtaba, T., & Reiss, M. J. (2013c). What sort of girl wants to study physics after the age of 16? Findings from a large-scale UK survey. *International Journal of Science Education, 35*, 2979–2998.

Nardi, E., & Steward, S. (2003). Is mathematics T.I.R.E.D? A profile of quiet disaffection in the secondary mathematics classroom. *British Educational Research Journal, 29*(3), 345–367.

Noyes, A. (2003). Mathematics learning trajectories: Class, capital and conflict. *Research in Mathematics Education, 5*(1), 139–153.

Noyes, A. (2009). Exploring patterns of participation in university-entrance level mathematics in England. *Research in Mathematics Education, 11*(2), 167–183.

Palmer, A. (2009). 'I'm not a "maths-person"!' Reconstituting mathematical subjectivities in aesthetic teaching practices. *Gender and Education, 21*(4), 387–404.

Reiss, M. J., Hoyles, C., Mujtaba, T., Riazi-Farzad, B., Rodd, M., Simon, S., et al. (2011). Understanding participation rates in post-16 mathematics and physics: Conceptualising and operationalising the UPMAP Project. *International Journal of Science and Mathematics Education, 9*(2), 273–302.

Rodd, M., Reiss, M., & Mujtaba, T. (2013). Undergraduates talk about their choice to study physics at university: What was key to their participation? *Research in Science & Technological Education, 31*(2), 153–167.

Rodd, M., Reiss, M., & Mujtaba, T. (2014). Qualified, but not choosing STEM at university: Unconscious influences on choice of study. *Canadian Journal of Science, Mathematics, and Technology Education, 14*, 330–345.

Royal Society. (2011). *A 'state of the nation' report on preparing the transfer from school and college science and mathematics education to UK STEM higher education*. London: The Royal Society.

Schunk, D. H., Pintrich, P. R., & Meece, J. L. (2010). *Motivation in education: Theory, research, and applications* (3rd ed.). Upper Saddle River, NJ: Pearson Education.