Effect of an Intervention on Conceptual Change of Decimals in Chinese Elementary Students: A Problem-Based Learning Approach

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In this chapter, we described a study that compared a problem-based learning (PBL) approach to a traditional approach for teaching decimal concepts to 76 Chinese fifth graders. This chapter started with a review of literature regarding conceptual change, challenges in teaching decimals to elementary students, the PBL in relation to self-efficacy, and the rationales for conducting the present study. Then, we elaborated the PBL approach as an intervention approach in an independent sample of fifth graders. Finally, we discussed implication of PBL in educational settings.

Decimal fraction learning is considered one of the cornerstones of mathematics education internationally (Stacey et al., 2001). In the United States, formal instruction of decimal fractions begins in fourth grade and continues throughout all secondary grade levels (National Council of Teachers of Mathematics (NCTM), 2000). The NCTM Standards require third to fifth graders to be able to understand and convert fractions, decimals and percentages. And students older than sixth graders should flexibly solve problems involving fractions, decimals and percentages. In China, decimals and fractions are also introduced to students at the elementary level beginning in fourth grade (Zong, 2006).

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A substantial number of studies have demonstrated that children have difficulties understanding decimals (Baturo, 1998; Hiebert & Wearne, 1983; Ni & Zhou, 2005; Resnick et al., 1989; Sackur-Grisvard & Leonard, 1985; Stacey & Steinle, 1998; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2004). Difficulty with fractions (including decimals and percent) has been identified as a pervasive problem and is a major obstacle preventing students from progressing in mathematics, including algebra (National Mathematics Advisory Panel, 2008). Even a considerable number of adults continue to hold such misconceptions (Putt, 1995; Silver, 1986; Stacey et al., 2001). Therefore, exploring how to help children develop their decimal knowledge is a priority for educational researchers.

In addition to the technical aspects of learning specific mathematics concepts, noncognitive variables play a role in student performance in mathematics. One such factor is students' self-efficacy. Bandura (1986) has argued that self-efficacy has a powerful impact on academic achievement. Research regarding mathematics self-efficacy has indicated that, in comparison to their counterparts with low self-efficacy, students with high self-efficacy demonstrate stronger persistence in difficult problem-solving situations and have better execution results in mathematics computation (Collins, 1982; Hoffman & Schraw, 2009). Thus, exploring self-efficacy in the context of mathematics learning has been of interest to educators and researchers.

A Conceptual Change Approach to Explain Children's Difficulties with Decimals

Rational numbers, including integers, terminating decimals, and repeating decimals, are numbers that can be expressed as a/b (both a and b are integers, and b can not be zero) (Vamvakoussi & Vosniadou, 2010). A single rational number, such as ½, can be represented in several ways (e.g., 5/10, 50/100, or 0.5), which all have the same value and are all alternative representations of the same rational number. In this chapter, we use the term "decimals" to refer to decimal representations of elements (i.e., a subset) of the set of rational numbers; we do not discuss decimals such as π that are not rational numbers.

The conceptual change approach has been recently used to explain students' persistent misconceptions regarding rational numbers (Vosniadou, 2007; Vosniadou & Verschaffel, 2004). Children's initial number frameworks are essentially natural numbers, which possess discreteness, whereas rational numbers have the feature of density, closely related to the concept of infinity (Hannula, Maijala, Pehkonen, & Soro, 2001; Malara, 2001; Merenluoto & Lehtinen, 2002). Natural numbers follow the successor principle (Vamvakoussi & Vosniadou, 2010) that all natural numbers are well ordered. Each natural number has a definite position in a sequence (e.g., 3 is the third number in the sequence of natural numbers), but rational numbers do not have this feature. When non-natural numbers, such as decimals and fractions, are introduced to students, their prior number frameworks based on natural numbers might hinder their understanding of the non-natural numbers.

Misconceptions. Previous literature has documented substantial information regarding the difficulties students usually encounter when they learn decimals. A common misconception is the notion that *longer is larger* (Moskal & Magone, 2000; Moss, 2005; Roche, 2005), in which students evaluate the value of a decimal number by comparing the number of its digits (e.g., 0.56 is larger than 0.8). A contrasting misconception is that *shorter is larger*, in which the students confuse decimals with fraction denominators (Steinle & Stacey, 2004). For example, in one study, children consistently judged that the larger number had fewer digits to the right of the decimal point; thus, 2.43 was larger than 2.897 (Sackur-Grisvard & Leonard, 1985). Another misconception, *multiplication makes bigger* (Fischbein, Deri, Nello, & Marino, 1985; Steffe, 1994), is true for natural numbers other than one, but is incorrect for decimal or fractional numbers less than one. Misconceptions can also arise in children's understanding of the density and infinity features of decimals. Finally, children often have difficulty with combining a string of digits into a single decimal quantity (Hannula et al., 2001; Malara, 2001; Merenluoto & Lehtinen, 2002; Resnick et al., 1989).

Children's misconceptions are often associated with over-generalization from their knowledge of natural numbers. For example, *longer is larger* and *multiplication makes bigger* may originate in children's experiences with comparing whole numbers. Children's difficulty with understanding the infinity feature (i.e., there are infinitely many decimal numbers between any two different decimals) of decimals can also be associated with their existing concept of whole numbers (Nunes & Bryant, 2007). In the domain of whole numbers, a number is a set of units of one, whereas in decimals, there is no minimum unit corresponding to ones. Instead, the minimum unit of decimals could be tenths, hundredths, thousandths, and so on. Children tend to intuitively generalize their mathematical reasoning skills regarding whole numbers to solving problems with decimals, which often leads to errors.

Existing Interventions for Teaching Decimals

The literature has documented programs that help children, sometimes identified as having had low achievement, to learn decimals. Two studies (Resnick, Bill, & Lesgold, 1992; Resnick, Bill, Lesgold, & Leer, 1991) emphasized the importance of helping low-SES African American parents to understand algebra and decimals and thus to provide their children with a better learning environment at home. Another study (Rao & Kane, 2009) helped children with intellectual disabilities to learn decimal calculation using a behavioral simultaneous prompting procedure in which the teacher delivered the target stimuli and the controlling prompt simultaneously; thus, the children did not have time to respond independently and therefore did not learn the task with errors.

Several existing interventions have taught students decimal concepts using representation techniques, such as using a number line and blocks to represent the place values. Hiebert and Wearne (Hiebert, 1988; Wearne, 1990; Wearne & Hiebert, 1988, 1989) conducted a group of studies using representation techniques. These studies emphasized using manipulatives (blocks) to promote conceptual understanding of decimals. The researchers taught students using place-value blocks in which different shaped blocks were named to represent different place values. Specifically, large cubes represented a unit, a flat block represented a tenth of a unit, and a long block represented a hundredth of unit. Results showed that fourth through sixth graders made notable improvement based on the intervention. Similarly, Swan (1983) used representation techniques (a number line model) to help students understand the meaning of decimal notation and found that the students made considerable progress. Woodward, Baxter, and Robinson (1999) also successfully used visual representations (e.g., wood block rectangles, squares, or cubes) to teach basic decimal concepts to children with learning disabilities.

One group of studies focused on the effects of exposing children to their misconceptions regarding problem solving, where decimals were used. For example, Swan (1993) compared two classes whose teachers had adopted two different teaching styles. One class was taught with a “positive-only” teaching style. First, the teachers explained the concepts and methods of using a number line; then, the students practiced using this method but were not asked to mark their work or diagnose errors. Another class was taught using a “conflict teaching style,” in which the teacher initially gave students problems that exposed them to misconceptions and taught students a method using a number line; then, the teacher led a discussion of the students’ errors and misconceptions. The “conflict teaching style” resulted in significantly more progress in children’s achievement than the “positive-only style.” The results suggested that exposing children to their misconceptions helped them to overcome their errors. Another study by Pierce, Steinle, Stacey, and Widjaja (2008) revealed the importance of identifying college students’ difficulties with decimals. In this study, nursing students were given a decimal problem-solving test that identified the students’ misunderstandings of particular items. Next, the teacher used various models to illustrate the place value and base ten concepts, and students were encouraged to ask questions and provide responses. This study found significant improvement by the students on a delayed post-intervention test, and the researchers concluded that it was necessary to expose students to their errors and plan for remediation of students’ misconceptions before teaching procedure rules.

Similarly, Huang, Liu, and Shiu (2008) revealed the effectiveness of exposing students to incorrect examples to facilitate their conceptual understanding of decimals. Sixth graders were exposed to incorrect examples when learning the meaning of decimals (e.g., in 5.4, saying the .4 represents 4 ones instead of 4 tenths). After 4 weeks, these students performed better than students who were not presented with incorrect examples.

In summary, existing interventions have suggested the importance of exposing students to their mathematical misconceptions and errors. Researchers have reported that providing incorrect examples or examining students’ own mistakes can promote deeper reflection on correct concepts (VanLehn, 1999) and increase students’ frequency of choosing correct strategies (Siegler, 2002). Based on these findings, it is plausible to assume that PBL would be effective for improving children’s conceptual understanding of decimals.

Problem-Based Learning and Self-Efficacy

PBL is a student-centered instructional strategy in which students learn through solving problems in groups and making reflections on their problem solving experiences. PBL is rooted in constructivist theories of learning that stress the importance of learners being engaged in constructing their own knowledge (Mayer, 2004; Palincsar, 1998). In PBL, students work in groups and are challenged with open ended and ill-defined problems. PBL is highly student-centered: Students are encouraged to explore the solutions and direct the problem solving process by themselves, and teachers only serve as facilitators (Hmelo & Guadial, 1996; Quntana, et al.). PBL is reported to be effective in enhancing content knowledge and fostering the development of communication, problem-solving, and metacognitive skills (Hmelo-Silver, Duncan, & Chinn, 2007). PBL has been shown to be effective in various empirical studies as described by Hmelo-Silver et al. For example, "there is an extensive body of research on scaffolding learning in inquiry- and problem-based environments (Collins, Brown, & Newman, 1989; Davis & Linn, 2000; Golan, Kyza, Reiser, & Edelson, 2002; Guzdial, 1994; Jackson, Stratford, Krajcik, & Soloway, 1996; Reiser, 2004; Toth, Suthers, & Lesgold, 2002" (p. 100, Hmelo-Silver, Duncan, & Chinn, 2007). Theory based and empirically validated strategies for effectively scaffolding students during PBL have been developed by many researchers (Hmelo-Silver, 2006; Hmelo-Silver, Duncan & Chinn 2007; Reiser et al., 2001). PBL is often used to assist learning of complex tasks. Complex tasks often require scaffolding to help students engage in sense making, self-management of their problem-solving processes, and facilitate students to articulate their thinking and reflect on their learning experiences (Quintana et al., 2004). Scaffolding helps to reorganize complex tasks and reduce cognitive load by structuring a task in a way that allows the learners to focus on relevant aspects of the task (Hmelo-Silver, 2006).

Many challenging tasks require both adequate skills and self-efficacy, which is about one's beliefs about whether or not one can successfully complete a task (Bandura, 1986). The relationship between PBL and self-efficacy has gained increasing attention. For example, self-efficacy was a significant predictor of science achievement in middle school students in a computer-enhanced PBL environment (Liu, Hsieh, Cho, & Schallert, 2006). For adult learners, specific instructional strategies (i.e., authentic problems of practice, collaboration, and reflection) used in PBL were reported to improve levels of self-efficacy in undergraduate computer science students (Dunlap, 2005). For educators, those with higher scores of self-efficacy demonstrated a significantly higher use of a PBL approach, direction instruction, and communication skills in mathematics teaching (Ordonez-Feliciano, 2010).

However, there are no studies dealing with the application of PBL to the instruction of decimal fractions in Chinese elementary students with consideration of students' self-efficacy. We were particularly interested in the PBL approach in Chinese students partially due to the fact that traditional Chinese mathematics instruction often follows a curriculum-centered approach with relatively large student–teacher ratios, making few opportunities available for students to be exposed to a PBL environment. Given that previous studies have found that exposing students to their misconceptions or errors and to challenging problems was effective in enhancing their conceptual understanding of decimals, the purpose of the present study was to investigate the following questions: (a) Does a PBL approach outperform a traditional instructional approach to enhance conceptual change in decimal computation? (b) Does a PBL approach outperform a traditional instructional approach to promote metacognition, measured by explicit interpretation of strategy use? and (c) Does a PBL approach lead to a higher level of self-efficacy and academic interest than a traditional instructional approach?

Method

Design

This study utilized a quasi-experimental design to compare pretest and posttest measures. The independent variable was the instructional method, consisting of a PBL approach in the experimental group and a traditional instructional approach in the control group.

Participants and Setting

The instructors were two experienced mathematics education teachers. One investigator majoring in educational psychology was on site for training, progress monitoring, and data collection. Each classroom had one experienced teacher as the lead teacher. The two classes of students had mathematics classes at different time periods on each day, so the investigator was able to observe classroom activities and collected data both in the control group and the experimental group for similar amounts of time to avoid the Hawthorne Effect. The participants were 76 fifth graders at an elementary school in the urban area of Beijing in Mainland China. The students were in two parallel classes, which were chosen because they were equivalent in terms of the students' performance in mathematics. Both classes followed the same mathematics curriculum, had a similar pace (teaching unit by unit according to textbook), used the same curriculum-based exams (designed by curriculum committees at the school), and the two teachers had comparable teaching experiences (i.e., years of teaching mathematics, teaching similar students at similar schools). Both teachers were new to the two groups because data collection started in the beginning of the school year. One class ($n=38$) received experimental PBL instruction that emphasized problem-based scenarios for teaching and students' own computation errors and prior experiences for discussion and problem analysis. The other class ($n=38$) received traditional instruction that emphasized curriculum-centered lecture and use of demonstration examples from the textbooks. All students were with normal intellectual abilities and were enrolled in regular classroom settings. Students' prior whole number and decimal number knowledge was similar due to the highly uniform school instruction implemented in Chinese schools.

Dependent Measures

Decimal computation test. To quantitatively measure students' conceptual change in decimals, we developed two sets of decimal computation tests. The measurement instruments used in this study involved a pretest (eight items in total) and a posttest (ten items in total). The pretest and posttest involved computation of both decimal and whole number problems. The pretest had three items that involved whole number computation only. Of the other five items relevant to decimals in the pretest, three of them involved decimal computation only and two of them involved mixed computation. The posttest had four items that involved whole number computation only. Of the other six items relevant to decimals in the posttest, four of them involved decimal computation only and two of them involved mixed computation. Each test included pairs of corresponding decimal and whole number items, as explained in Table 1. The computations included addition, subtraction, multiplication, and division. These items were chosen from *Beijing Compulsory Education Curriculum Reform Experimental Materials of Mathematics in Elementary School-Volume IX* (Lu & Yang, 2005). The pretest and posttest items were not identical due to consideration of the curriculum taught during the 22 classes. The pretest functioned as a placement test to examine whether the experimental group had similar prior knowledge as the control group. The posttest functioned as a summative test to measure whether students had mastered designated computation skills after receiving 22 classes of formal instruction. The difficulty levels of the pretest and posttest were consistent with curriculum content. Cronbach's alpha was .63 for the pretest and 0.72 for the posttest. Because of the limited testing time, we only designed eight items for the pretest and ten items for the posttest. The relatively low reliabilities might be attributable to the number of testing items we had. Sample items are listed in Table 1.

Qualitative measure of students' conceptual understanding of decimal division. Students' conceptual change was qualitatively measured by an open-ended question on the posttest to examine students' conceptual understanding of decimal division. The open-ended question asked, "Currently, there is a student who does not understand decimal division. Please elaborate your procedures of problem solving. For example, tell this student what to do first, what to do as a second step, and then what else."

Self-efficacy survey. A self-report questionnaire was developed based on Qin (2003) and Zhang (2005) to explore (a) social self-efficacy, (b) academic self-efficacy, and (c) academic interest. The questionnaire utilized a 6-point Likert scale ranging from *completely unlike me* to *completely like me*. The subtest of social self-efficacy included six items, with 36 points as the highest score. The subtest of academic self-efficacy included seven items, with 42 points as the highest score. The subtest of academic interest included seven items, with 42 points as the highest score.

Table 1 Sample problems in probes

Measures	Sample items	
Computation tests (*All items included*)		
Pretest computation	10.1÷0.2 (decimal)	0.9+2.32 (decimal)
	15×0.8 (mixed)	5.85×0.60 (decimal)
	101÷2 (whole number)	9+232 (whole number)
	585×60 (whole number)	100−2.56×5+32.5÷10 (mixed)
Posttest computation	1.21÷0.2 (decimal)	0.9+3.25 (decimal)
	5.58−0.9 (decimal)	7.8×0.60 (decimal)
	120÷20 (whole number)	9+325 (whole number)
	78×60 (whole number)	2.56×5+32.4÷10−4.85 (mixed)
	2.5×18−0.67+0.5÷5 (mixed)	558−9 (whole)
Sample items of self-efficacy and *interest survey*		
Students are asked to rate on a Likert scale (i.e., 1–6, 1 stands for completely disagree and 6 stands for completely agree) according to each item		
Social self-efficacy questionnaire	Sample A: I can successfully interpret my thoughts to my classmates	
	Sample B: When other students talk with me, I do not know what I should talk about with them	
Academic self-efficacy questionnaire	Sample A: If I have sufficient time, I can learn mathematics well	
	Sample B: I can learn math even if some contents are very difficult	
Academic interest questionnaire	Sample A: I like math class more than I do other subjects	
	Sample B: The problems discussed in math class are very interesting	

Note: In the pretest computation, 10.1 ÷ 0.2 (decimal) corresponds with 101 ÷ 2 (whole number); 0.9 + 2.32 (decimal) corresponds with 9 + 232 (whole number); 5.85 × 0.60 (decimal) corresponds with 585 × 60 (whole number). In the posttest computation, 1.21 ÷ 0.2 (decimal) corresponds with 120 ÷ 20 (whole number); 7.8 × 0.60 (decimal) corresponds with 78 × 60 (whole number); and 0.9 + 3.25 (decimal) corresponds with 9 + 325 (whole number). Between pretest and posttest computation, 10.1 ÷ 0.2 (decimal division) corresponds with 1.21 ÷ 0.2; 101 ÷ 2 (whole number division) corresponds with 120 ÷ 20; 0.9 + 2.32 (decimal addition) corresponds with 0.9 + 3.25; and 5.85 × 0.60 (decimal multiplication) corresponds with 7.8 × 0.60; 9 + 232 (whole number addition) corresponds with 9 + 325. There are two mixed (whole number and decimal number) computation problems in pretest and posttest, respectively

The internal consistency coefficients were 0.71, 0.92, and 0.92 for the three subtests, respectively. The survey was administered to students during both the pretest and the posttest. Sample items are listed in Table 1.

Coding and Scoring

Decimal computation test. First, a graduate assistant who was unaware of the purpose of the study scored the decimal computation test using an answer key. Specifically, items on the test were scored as correct or incorrect, with one point awarded if the correct answer was given. In the pretest, there were five items involving decimal computation. In the posttest, there were six items involving decimal computation. We calculated students' total scores on the pretest and posttest and also calculated their decimal computation scores in pretest and posttest, respectively.

Second, as there were paired whole number and decimal items in each of the two tests, the students' answers were classified into four categories: (1) whole number computation is correct, and decimal computation is also correct; (2) whole number computation is correct, but decimal computation is incorrect; (3) whole number computation is incorrect, and decimal computation is also incorrect; and (4) whole number computation is incorrect, but decimal computation is correct. The second category of responses indicated that the students were unable to correctly apply whole number computation rules to decimal computation.

Third, we coded for errors to examine the mistakes students made during their problem solving. The investigators used a coding system to categorize seven types of computation errors in both groups: aligning place value, carrying, displacement of decimal point, carelessness, mnemonics, computation order, and missing values (see Table 2). Most Chinese textbooks have a student version and an instructor version, and the instructor version provides details such as exercise items, solutions, and common types of errors. The classification of computation errors was designed based on the types of errors suggested by the instructor version of the mathematics textbook utilized in the school.

Qualitative measure of students' procedural understanding of decimal division. There was one open-ended question in the posttest to qualitatively examine students' procedural understanding of decimal division. Students' levels of awareness of the strategies they used were categorized into three types, including missing or inaccurate (i.e., incorrect answers), nonessential (i.e., answers regarding general computation rules that applied to whole numbers but did not apply to decimal numbers), and essential answers (i.e., answers that were essential for decimal computations or answers showing correct examples or decimal computation rules) (see Table 2).

Self-efficacy survey. Scores on negatively worded items were reverse coded. A higher score indicated a higher level of self-efficacy. Students were rated as "0" when they chose *completely unlike me* and were rated as "6" when they chose *completely like me*.

Inter-rater agreement. Another graduate assistant rescored 30 % of the tests. Inter-rater agreement was computed as the percentage of the number of agreements divided by the total number of rated items. Inter-rater agreement was 95 %.

Procedures

Following the pretest assessment, one intact class became the PBL group and the other intact class became the control group. Because the two participating classes had identical curriculum, similar class schedules and similar instructional approaches, the selection of the PBL group and control group was totally random. Students in the PBL group received the intervention during five classes per week for

Table 2 Coding scheme for computation errors and awareness of strategy use

	Descriptors	Examples
Coding for different types of computation errors		
Aligning	Lining up the decimal points as below	
	0.9	0.9
	+3.25	*+3.25*
	4.15	3.34
Carrying	Students made mistakes during carrying numbers between different unit positions	0.9
		+3.25
		3.15
Displacement	Students placed the decimal point at wrong place after calculation	$7.8 \times 0.60 = 0.468$
Carelessness	Due to carelessness, students made mistakes like miscopying of numbers, omission, or skipping of calculation steps	"I accidentally put 0.12 as 0.18"
Mnemonics	Students retrieved incorrect multiplication facts	"3 times 7 is 22"
Computation order	Students did not calculate according to computation order, such as (1) calculating from left to right, (2) calculation in parenthesis should be done first, (3) exponents or radicals should be done next, (4) multiplication and division should be done in the order in which it occurs, and (5) addition and subtraction should be done in the order in which it occurs	In the example of "$2.56 \times 5 + 32.4 \div 10 - 4.85 = ?$," the student did not calculate multiplication and division before they calculated addition and subtraction
Missing	During pretest measures, students had not yet learned decimal division, thus students chose to give up on some of the items	"Can I skip this problem?"
Coding for conceptual understanding of decimal computation		
Missing or inaccurate	Students provided inaccurate answers or did not provide any answers	"Well, I am not supposed to explain that"
		"Let me think about it"
		"That is good"
Nonessential	Students provided only general computation rules that applied to whole numbers but did not apply to decimal numbers	"Decimal division is pretty much like division of whole numbers"
		"You compute it like division of whole numbers, then add a decimal point afterwards"
Essential	Students provided answers that were essential for decimal computations, used correct examples, or mentioned important decimal computation rules	"To divide by a decimal, multiply that decimal by a power of 10 great enough to obtain a whole number"
		"When we multiply the divisor, we also need to multiply the dividend"

4½ weeks during the school day, for a total of 22 classes. Each session of class lasted approximately 40–45 min. The control group continued to have their regular mathematics classes (i.e., five classes per week for 4½ weeks), for a total of 22 sessions. The major teaching content for both classes was decimal multiplication and division, which covered two units of the textbook. The complete content for that semester included seven units.

Teacher training. The instructor for the experimental group had utilized a PBL approach for more than 5 years and was very familiar with PBL. She received 1 week of additional training on the PBL approach before the intervention started. The purpose of the training was to help the teacher to conduct the intervention in the designated manner and to train the teacher to be proactive. The teacher relearned the PBL approach, had opportunities to practice how to teach students using the PBL approach, and received feedback from the investigator during the training. The investigator developed the teaching scripts (see Appendices 1 and 2), which were studied by the instructor of the experimental group to prepare for teaching the lessons. For the control group, the investigator observed classroom activities and collected data. For the experimental group, the investigator was on site for observation, progress monitoring (i.e., making sure the teacher was following the teaching scripts), and data collection. The investigator spent a similar amount of time in each classroom for observation and data collection.

Assessment conditions. Assessment conditions refer to the pretest assessment prior to the intervention and the post-intervention assessment. Pre- and post-intervention tests were administered using paper and pencil for all students. Experimenters did not provide any prompting or feedback regarding the accuracy of students' solutions. Students were provided with sufficient time to complete the test and the survey.

Experimental group. The experimental group adopted a PBL approach. The students began with specific problem scenarios and the teacher provided them with opportunities to reveal their computation errors and prior experiences. The teacher encouraged the students not only to explain the patterns of computation errors, but also to analyze the causes of computation errors. The teacher in the PBL group encouraged an open learning atmosphere and supported the students' reliance on prior learning experiences to guide their learning behaviors.

The instructional materials included projectors, experiment record sheets, and reminder cards. The reminder cards provided hints to the students when the problems were presented; for example, after the computation, the reminder cards helped the students to self-check the computational procedure, such as "I have checked the placement of the decimal point." Therefore, reminder cards were considered an effective method to monitor students' metacognition (Tong & Zhang, 2004).

The teacher gradually faded out the use of reminder cards as the instruction progressed. Specifically, at the beginning of the instruction, the teacher provided the students with complete reminder cards. The teacher determined the instructional framework and distributed the reminder cards to every student in the classroom. After discussion, the students summarized the types of computation errors made by all of the students in the class and noted the computation errors on the reminder cards according to a scaffolding framework. As the instruction progressed to approximately halfway through the intervention, the teacher provided the students with partially completed reminder cards. Each student analyzed only computation errors that he or she made and then noted the types of errors on the reminder cards. Each student could individualize his or her reminder card.

For both new lecture and review classes, the PBL curriculum followed similar procedures, including class preparation, instruction, and PBL. The focus was to analyze the students' prior experiences and design a PBL environment to motivate the students to think through problems and work out solutions. During the instructional procedure, the focus was to facilitate group discussion, analyze problems, and guide students to come up with solutions to solve problems. The initial PBL sessions helped students to identify errors and analyze prior experiences. The later PBL sessions emphasized exercises tapping into metacognition, such as analyzing types of computation errors, discussing the rationales for errors, and self-revising computation errors. Appendices 1 and 2 present examples for a new lecture and a review class. Appendix 3 presents a flowchart of the PBL approach guiding our intervention.

Control group. The teacher in the control group closely followed the instructional guidelines used for the regular curriculum. Traditional Chinese mathematics instruction focuses on a curriculum-based teaching approach. Due to a relatively large student–teacher ratio (e.g., 40 or 50:1), lecture that closely follows the curriculum is often the main teaching method. Due to the mandatory teaching content specified by the Ministry of Education of the People's Republic of China, teachers often closely follow guidelines in the curriculum as a typical practice. Although the teacher of the control group had opportunities to question students, few opportunities were available for small-group discussion, close interaction between the teacher and students, and students' reflection on their own errors and prior experiences. The 4½-week instructional activities included new lectures and review classes for decimal multiplication and decimal division. For the new lectures, the teacher introduced new concepts based on the textbook, started with demo exercises, explained rules of computation, asked the students to complete exercises, and provided students with opportunities to ask questions. During the review classes, the teacher primarily relied on demo items in the textbook to explain computation errors, and the discussion of patterns of computation errors was based on teaching experience rather than on actual computation errors that occurred during the students' exercises. Thus, the discussion of computation errors was not specific, and the teacher did not provide the students with opportunities to reflect on their own computation errors. The most frequently used method was to discuss classical computation errors addressed by the textbook as examples. Although there were opportunities for teacher–student interaction, most of the demo items had fixed answers, which were not likely to challenge students' higher levels of reasoning. The teacher typically gave students general praise but did not provide specific feedback.

Treatment Fidelity

In addition to the first investigator who was on site for data collection and progress monitoring, a second investigator independently observed ten treatment sessions in the experimental group to assess fidelity or quality of implementation of specific performance indicators. Treatment fidelity checklists are provided in Appendix 4. Half of the sessions were new lectures and half of the sessions were review classes. The observation sessions were equally distributed throughout the intervention period. The teacher used a teaching script to guide the teaching strategy during each class session. In addition, for each session, the first and third author used a checklist, which listed the key instructional components, to evaluate teachers' adherence to the assigned instructional condition type. The second investigator judged the adherence of the instructor's teaching based on the presence or absence of the features listed on the fidelity checklist. The overall treatment fidelity was .92 for the sessions observed.

Results

Pretreatment Group Equivalency

We used ANOVA tests to examine pretreatment group equivalency on the decimal computation test, self-efficacy questionnaire, and academic interest survey. Results indicated no significant difference between the two groups on the total computation test, $F(1, 74)=0.044$, $p=0.835$; the decimal computation test (i.e., decimal computation in computations involved decimals only and mixed numbers), $F(1, 74)=0.633$, $p=0.429$; the social self-efficacy questionnaire, $F(1, 73)=0.048$, $p=0.828$; the academic self-efficacy questionnaire, $F(1, 73)=3.783$, $p=0.056$; or the academic interest survey, $F(1, 73)=3.633$, $p=0.061$ (see Table 3).

We also compared computation errors made by the two groups of students during the pretest. Both groups appeared to make the most frequent computation errors in aligning place value, carrying, and displacement of the decimal point. In both groups, a large number of students chose to skip the questions because they had not learned decimal division prior to the intervention. The chi-square test indicated nonsignificant differences in the distribution of the seven computation errors with the exception of displacement of the decimal point, $\chi^2=5.775$, $p=0.038$ (see Table 6), with better performance in the control group.

Quantitative Measure of Students' Conceptual Change in Decimals

Total computation. We performed an ANOVA test (with the pretest difference as a covariate) on the posttest scores to assess the effects of instruction on students' total computation performance. Results indicated a significant difference between groups at posttest, $F(1, 74)=10.063$, $p=0.002$ (see Table 3), favoring the PBL group.

Table 3 Means, standard deviations, paired samples *T*-test, and ANOVA by treatment conditions

	M		SD		*N*			Paired samples *T*-test		ANOVA
Test	PBL	Control	PBL	Control	PBL	Control	ES	PBL	Control	*F*/Sig.
Pretest-total	4.32	4.24	1.69	1.60	38	38	0.05			.044/.835
Posttest-total	8.32	7.08	1.53	1.87	38	38	0.73	NA	NA	10.063/.002
Pre-De	2.39	2.21	0.99	1.03	38	38	0.18			.633/.429
Post-De	4.42	3.45	1.46	1.33	38	38	0.69	NA	NA	9.215/.003
Pre-SSE	27.24	27.49	4.71	5.18	38	37	−0.05			.048/.828
Post-SSE	31.68	27.84	4.13	5.76	38	37	0.77	−6.141/.000	−.455/.652	35.723/.000
Pre-ASE	30.24	32.54	4.88	5.37	38	37	−0.45			3.783/.056
Post-ASE	32.61	33.03	4.95	4.51	38	37	−0.09	−5.070/.000	−.709/.483	2.30/.134
Pre-AC	27.76	30.59	6.29	6.57	38	37	−0.44			3.633/.061
Post-AC	34.55	30.57	6.85	5.69	38	37	0.63	−6.203/.000	.031/.976	18.950/.000

Note: *Pretest-Total*, *Posttest-Total* pretest, posttest test total scores on all computation items, *Pre-De* all decimal computation pretest (including decimal computation in mixed computation), *Post-De* all decimal computation posttest (including decimal computation in mixed computation), *Pre-SSE* social self-efficacy pre-survey, *Post-SSE* social self-efficacy post-survey, *Pre-ASE* academic self-efficacy pre-survey, *Post-ASE* academic self-efficacy post-survey, *Pre-AC* academic interest pre-survey, *Post-AC* academic interest post-survey, *ES* effect size by Cohen's *d*, which is calculated as the two conditions' mean differences divided by the pooled standard deviation (Hedges & Olkin, 1985)

Table 4 Univariate analysis of variance of posttest decimal computation in two groups

Source	Type III sum of squares	df	Mean square	*F*	Sig
Corrected model	23.375[a]	2	11.687	6.125	.003
Intercept	132.046	1	132.046	69.200	.000
Pre-De	5.361	1	5.361	2.810	.098
Group	16.104	1	16.104	8.439	.005
Error	139.296	73	1.908		
Total	1,339.000	76			
Corrected total	162.671	75			

Note: [a]R squared = .144 (adjusted R squared = .120); *Pre-De* all decimal computation pretest (including decimal computation in mixed computation), *Post-De* Dependent variable, all decimal computation posttest (including decimal computation in mixed computation)

Decimal computation. An ANOVA test on the posttest scores revealed that the experimental group outperformed the control group on decimal computation (i.e., the decimal computation and mixed items), $F(1, 74) = 9.215$, $p = 0.003$ (see Table 3). Due to the fact that the pretest (which served as a placement test) and posttest (which served as summative evaluation) did not have the same question items, within-group comparison of pretest and posttest scores for each group could not be conducted. We conducted univariate analysis of variance to further control for differences in pretest decimal computation performance (termed Pre-De in Table 3). The analysis results in Table 4 indicated a significant main effect caused by group difference (i.e., experimental group vs. control group) and a nonsignificant main effect of pretest decimal computation performance.

Students' Self-Efficacy and Academic Interest

Self-efficacy. An ANOVA test (with the pretest difference as a covariate) on the post-survey of self-efficacy revealed significantly higher social self-efficacy in the experimental group, $F(1, 73) = 35.723$, $p = 0.000$. Although the control group reported relatively higher academic self-efficacy, the test did not indicate a significantly higher score than the score of the experimental group, $F(1, 73) = 02.30$, $p = 0.134$ (see Table 3). In terms of within-group comparison, we conducted a paired samples t test. The control group did not show significant improvement on either social self-efficacy or academic self-efficacy after 22 sessions of classes. The experimental group showed significant improvement on both social self-efficacy ($p = .000$) and academic self-efficacy ($p = .000$) after receiving the entire intervention.

Academic interest. An ANOVA test (with the pretest difference as a covariate) on the post-survey of academic interest indicated significantly higher academic interest in the experimental group over the control group, $F(1, 73) = 18.950$, $p = 0.000$ (see Table 3). We also conducted a paired samples t test to examine within-group

Table 5 Qualitative analysis of students' conceptual understanding of decimal computation

Type of answers	Control	MA	Total	χ^2/Sig.
Missing	26	14	40	15.857/.000
Nonessential	11	10	21	
Essential	1	14	15	
Total	38	38	76	

improvement. The control group did not show significant improvement on academic interest from pretest to posttest measures, whereas the experimental group demonstrated significant improvement.

Qualitative Measure of Students' Conceptual Change in Decimals

The posttest included an open-ended item that asked, "Currently, there is a student who does not understand decimal division. Please elaborate your procedures of problem solving. For example, tell this student what to do first, what to do as a second step, and then what else." Approximately two thirds of the students in the control group chose to give up, and another one third of the students provided answers showing no conceptual understanding of decimal division (e.g., using whole number rules for decimal computation). Only one student in the control group provided an answer showing a conceptual understanding of decimal division. In contrast, 14 students in the experimental group explained essential features associated with computation of decimal division. We used the coding scheme listed in Table 2 to classify the narrative responses provided by the students, including missing or inaccurate, nonessential, and essential answers. The Monte Carlo chi-square test revealed significant differences in the distribution of the three types of answers in the two groups, $\chi^2 = 15.857$, $p = 0.000$ (see Table 5). Students in the experimental group were more likely to explicitly describe their computation procedures and demonstrated understanding of unique features of decimal computation that differ from whole number computation (see Table 5).

Students' Computation Errors

Given that the pretest and posttest instruments did not consist of the same number of testing items, a comparison of absolute numbers of computation errors on the pretest and posttest measures was not conducted. There were no significant differences among the computation errors between the experimental group and the control group during the pretest measures, with one exception (more errors occurred on displacement of the decimal point for the experimental group). In other words, prior to the treatment, students in the experimental group had similar or slightly worse

Table 6 Analysis of pre- and-posttest computation errors

	Pretest		Chi-square	Posttest		Chi-square
Error type	Control	MA	χ^2/Sig.	Control	MA	χ^2/Sig.
Aligning	15(12)	27(19)	3.619/.282	23(18)	11(6)	7.664/.006
Carrying	15(12)	15(13)	.252/1.000	28(20)	13(9)	2.427/.119
Displacement	14(12)	24(22)	5.775/.038	22(19)	12(10)	5.573/.018
Carelessness	4(4)	7(6)	1.151/.734	13(12)	8(6)	1.891/.169
Mnemonics	5(4)	9(8)	2.023/.523	20(12)	5(5)	7.649/.006
Computation order	5(5)	8(8)	.835/.361	4(4)	3(3)	.642/.423
Missing	86(37)	66(32)	7.060/.173	NA	NA	NA/ NA

Note: Numbers within the parentheses indicate the number of students who made the errors

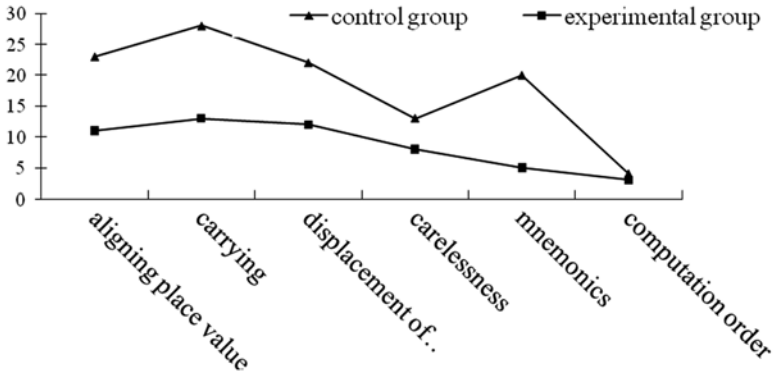


Fig. 1 Comparison of number of posttest computation errors in each group

computation skills than did those in the control group. Students in the experimental group did not have a better computation foundation before the treatment. For every single type of computation error made during the posttest, those in the control group made more errors than did those in the experimental group. The students in the control group had relatively more computation errors on aligning, carrying, displacement of the decimal point, and mnemonics. Computation errors made by the students in experimental group were primarily errors on aligning, carrying, and displacement of the decimal point. The experimental group had fewer students who made computation errors and as a group made fewer total computation errors (see Table 6).

The data in Figs. 1 and 2 present the total number of items with computation errors and the total number of students who made computation errors in the two groups. The trends in the two figures consistently indicate that students in the experimental group made fewer computation errors and had fewer students who made errors. For each type of computation error, a chi-square test was performed with a 2 (pretest, posttest) × 2 (control group, experimental group) contingency table on the number of errors the students made of that type. The results showed significant

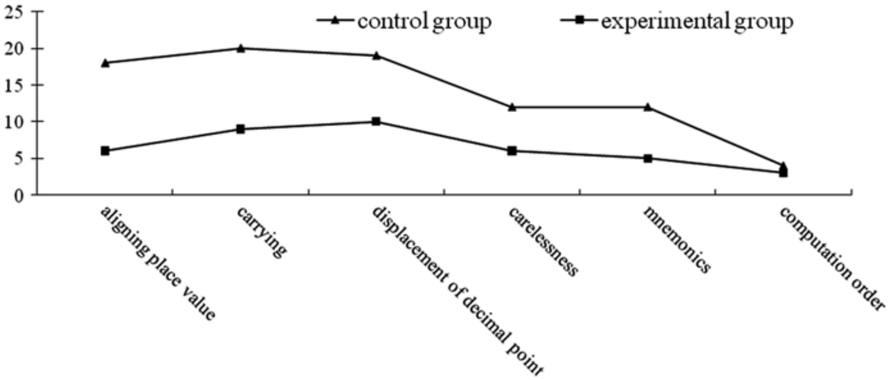


Fig. 2 Comparison of number of students in each group who made errors on posttest

differences in the distribution of aligning, displacement of the decimal point, and mnemonics, $\chi^2=7.664$, $p=0.006$; $\chi^2=5.573$, $p=0.018$; $\chi^2=7.649$, $p=0.006$ (see Table 6), for which students in the experimental group had significantly fewer errors. In terms of carrying, carelessness, and computation order, there were no significant differences between the two groups, $p=0.119$, $p=0.169$, $p=0.423$ (see Table 6).

Analysis of Relations Between Whole Number and Decimal Computation

Some computation rules for whole numbers are similar to those for decimals; however, other computation rules are different. The students' answers to the paired whole number and decimal test items were classified into four categories: (1) whole number computation is correct, and decimal computation is also correct; (2) whole number computation is correct, but decimal computation is incorrect; (3) whole number computation is incorrect, and decimal computation is also incorrect; and (4) whole number computation is incorrect, but decimal computation is correct. The second category of responses indicated that the students were unable to correctly apply rules of whole number computation to decimal computation. Thus, the second type of error tapped into our research interest regarding the relations between whole number computation and decimal number computation. If students demonstrated that their whole number computation was correct but decimal computation was incorrect, we assumed that the students did not achieve conceptual change. A chi-square test was conducted to compare differences between the two groups in this type of error pattern for addition, subtraction, multiplication, and division, respectively (see Table 7). It appeared that the two groups of students significantly differed in this type of error on multiplication. Students in the control group tended to make more

Table 7 Posttest mistakes in applying rules of whole number to decimal computation for each operation

Second category of response	Control	MA	χ^2/Sig.
Division	12	8	1.09/.297
Multiplication	17	6	7.54/.006
Addition	5	4	0.13/.723
Subtraction	4	8	1.58/.208

such errors on multiplication than did the experimental group. For division, addition, and subtraction, the two groups of students did not show significant differences in this type of error, $p=0.297$, $p=0.723$, $p=0.208$, respectively (see Table 7).

Discussion

The purpose of this study was to evaluate and compare the effectiveness of a PBL instructional approach and a traditional instructional approach for teaching decimal multiplication and division to Chinese fifth-grade elementary students. We examined students' conceptual change in decimal computation both quantitatively and qualitatively. The results showed that the students in the experimental group had a higher accuracy rate on computation and were more likely to explain their computation procedures and principles of computation strategically.

PBL and Improvement in Computation Skills

The findings revealed a significant intervention effect for computation skills in the experimental group when compared to the control group. In other words, PBL outperformed a traditional instructional approach in enhancing students' computation skills involving both whole numbers and decimal numbers.

Effects on Enhancing Students' Self-Efficacy and Academic Interest

This study also examined the intervention effects on students' self-efficacy and academic interest. The PBL approach primarily improved the students' social self-efficacy, whereas it had little impact on their academic self-efficacy compared to the traditional approach. One interpretation might be that the experimental group had ample opportunity for teacher–student and student–student interactions. The mathematics class was no longer a competitive environment in which the students needed to compete to answer the questions. If there was a disagreement, the students had opportunities to share differences and express their ideas and suggestions, which may have resulted in a higher level of willingness to collaborate among these students

(Hmelo, Gotterer, & Bransford, 1997). This might explain the higher level of social self-efficacy in the experimental group. The limited impact on academic self-efficacy might be due to the intervention duration of only 1 month on one instructional unit. During an intervention with a relatively short duration, it might be difficult to change students' overall impressions and attitudes toward mathematics learning. In addition, the learning of decimals was a relatively challenging unit, and the students might have experienced some level of anxiety. Thus, it might be unrealistic to expect a rapid change in students' academic self-efficacy after a 1-month intervention.

In terms of academic interest, the students in the experimental group reported a higher level of academic interest than did those in the control group. The PBL approach emphasized student-centered instruction during the choice of problem situations, collaboration and discussion in class, and reflections on solutions. The teachers were facilitators of learning. The focus of the class was to maintain the students' interest and provide more opportunities for self-exploration. The instruction included interesting and challenging problem situations (examples provided in Appendix 5), and the students were able to freely express their opinions and experience a sense of accomplishment after they solved the problems. As a result, they reported a higher level of academic interest. Although similar problem situations provided by the textbooks might be available to students in the control group, no efforts were made to give students opportunities for self-exploration and self-reflection.

Effects on Enhancing Students' Metacognition

Vosniadou (1999) emphasized the importance of metacognition during children's mathematical problem-solving processes. In the present study, the measure of students' awareness of their strategy use was to examine students' metacognition. The results indicated that students in the experimental group were more likely to explicitly describe their own computation procedures and were more likely to discover essential features of the computation procedures. It appeared that the PBL approach not only guided the students to explore the rationales for computation, but also provided opportunities for the development of students' metacognition. The error clinic was designed for review classes. The teachers provided reminder cards to guide the students to externalize the metacognitive procedures, such as analyzing and exploring the rationales of computation errors, and self-revising computation procedures (Alan & Hennie, 1990; Tong & Zhang, 2004).

Effects on Conceptual Change in Decimals

When presented with the open-ended prompt regarding decimal division, 14 students in the experimental group were able to explicitly explain essential features of decimal division. Only one student in the control group provided an answer showing a conceptual understanding of decimal division. This could be due to the fact that students in the control group were given few opportunities in class to self-reflect on computational procedures and errors, and thus they might have a lower level of metacognition when asked to verbalize procedures and essential features of decimal computation. Chinese students are traditionally trained to execute computations or work out problems, but are not provided with ample opportunities to verbally elaborate their understanding of specific concepts or computations. The PBL approach appeared to help students develop their metacognition.

Moreover, in addition to being able to elaborate on decimal computation, it is also important to examine whether students can actually execute the computation procedure correctly. If a student can only explain how to do decimal computation, but fails in actual computation, then the student may not have achieved conceptual understanding. The posttest revealed that the experimental group had significantly fewer students making computation errors and that the group as a whole made significantly fewer computation errors than did the control group, which suggests that PBL had a positive impact on conceptual change.

The analysis of types of pretest computation errors revealed that students in both the experimental group and the control group frequently made mistakes of aligning place value, carrying, and displacement of the decimal point. During the posttest, the experimental group had fewer students making mistakes and as a group made fewer mistakes on the three types of computation errors. The three types of computation errors revealed essential differences between whole number computation and decimal computation. In whole number computation, the last place is the units place; thus, students automatically aligned place value based on the last digit (units place), instead of the same place value (e.g., units place to units place, tens place to tens place). When the students were conducting decimal computation, they also mechanically aligned the place value based on the last digit of the decimal numbers. The decimal point is unique for decimal numbers. Sometimes students arbitrarily placed the decimal point and randomly deleted "0" after the decimal point. The errors related to aligning place value indicated the need for students to understand the computation rules of both whole numbers and decimal numbers. Although strategies of borrowing and carrying in whole numbers and decimal numbers are not considerably different from each other, the introduction of decimal point concepts results in increased cognitive workload and increased use of working memory. The use of reminder cards in the experimental group helped the students to divide complex computations into smaller steps, which might have decreased their computation errors. For computation errors that were unique to decimal numbers, the intervention showed a positive impact. In contrast, for general computation errors that did not differ between whole numbers and decimal numbers, such as carelessness and computation order, the intervention did not show as much impact because the students could directly transfer computation knowledge and skills from whole numbers to decimal numbers.

In the posttest, the researchers designed some whole number computations and decimal computations with identical digits, with the only difference being the decimal point placed in decimal computation (e.g., 7.8×0.60 vs. 78×60). Some students correctly completed the whole number computation, but made errors on the corresponding decimal computations. Because of the introduction of the decimal point, some students made errors by directly transferring whole number computation rules to decimal computation. In the multiplication computation in particular, some students disregarded the differences between the computation rules for whole numbers and those for decimal numbers, such as deleting the redundant "0" after a decimal point or arbitrarily applying whole number computation rules to decimal computation (Markovits & Even, 1999). The control group made significantly more second category of responses (i.e., whole number computation is correct, but decimal computation is incorrect) on multiplication than did the experimental group. For division, addition, and subtraction, the two groups of students did not show significant differences with respect to this type of error. This suggests that the decimal point is a challenging concept and that decimal computation, particularly in decimal multiplication, is difficult to master. Prior knowledge of whole number computation might interfere with decimal computation, and so the students' computation errors varied.

In short, the analysis of computation errors and how students explicitly explained decimal division indicated that students tended to rely on prior knowledge and computation rules to work out decimal computation. PBL helped the students to deal explicitly with rationales of computation rules and to differentiate between whole number and decimal computations. To some degree, this approach promoted conceptual change regarding some erroneous conceptions of computation rules.

Limitations and Conclusions

This study has implications for educational practitioners and future researchers. However, there are a number of limitations of the study that suggest caution in generalizing the results. First, the students were not randomly assigned to two groups, although they shared many commonalities and showed similar performance on most measures during our pretests. Second, the number of problems in the decimal computation tests was relatively small, which might explain the relatively low internal consistencies for the pretest and posttest. There was not the same number and type of items in the pretest as in the posttest, although the difficulty level of items in the pretest and posttest was similar, according to the textbook we referred to. Third, some variables could not be controlled, such as students' prior beliefs about learning mathematics and about decimal and whole number computation, teachers' beliefs about mathematics learning and decimal computation, and teachers' knowledge about students' misconceptions. Fourth, ideally, learning behavior is better assessed by using a variety of methods (e.g., qualitative and quantitative methods) to provide a relatively comprehensive view of an individual's learning behavior. Because of limited resources, we were unable to videotape teacher–student interactions and were unable to provide a systematic qualitative analysis of changes in learning behaviors. Fifth, the traditional PBL approach is often utilized in small group settings. Due to the reality of the Chinese school system, it was impossible to have a very small student–teacher ratio to conduct the PBL. Thus, our study primarily relied on group discussions and activities that could take place simultaneously with all students. Although this was not an ideal way to implement the PBL, it provided insights for future Chinese teachers who might implement a similar approach in large classroom settings. Finally, the intervention was implemented intensively over a 1-month period, and there was no longitudinal follow-up to examine the persistence of the treatment effect.

This study revealed the importance of exposing students to their mathematics errors. Computation errors helped students to discover computation problems and provided opportunities for conceptual change. The PBL approach is driven by challenging, open-ended, ill-defined, and ill-structured PBL (as in the examples provided in Appendix 5). Our conclusions are in line with prior research findings where students were exposed to challenges and were guided to reflect on their misconceptions.

This study also has implications for teachers' roles during instruction. During PBL instruction, teachers serve only as facilitators. In contrast, during traditional instruction, teachers often emphasize the teaching of computation rules instead of the conceptual understanding of decimals. Discussion of computation errors is not encouraged in traditional instruction, which might result in students' resistance to disclosing their computation errors, and some students might hide exercise books to avoid sharing them with other students. When new learning content is introduced, it is important to allow students the opportunities to reflect on errors and causes, to enhance metacognition, and to promote the construction of new knowledge.

Appendix 1: Teaching Scripts for Teaching New Decimal Division

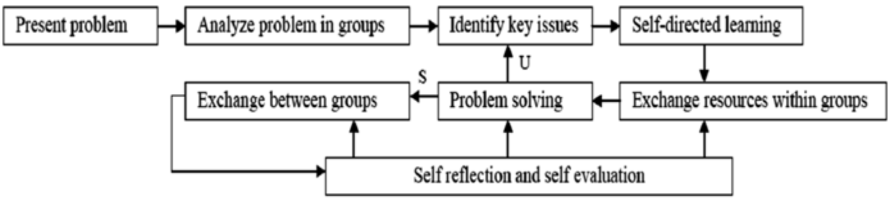
1. Introduction to the problems and divide students into five groups to solve the problems.
   "We have successfully overcome the decimal in multiplication. Now, it occurs in computation of division. We have new challenges now. Does anybody have any ideas to solve the problem? Now, let us divide the class into five groups and we will work on five division problems, including whole number divided by decimal, decimal smaller than 1, and decimal larger than 1. We want to see which group can come up with more solutions. When we explain the solutions, you need to tell us the procedures to reach the solution. What types of principles do you use to solve the problem?"
2. Encourage students to solve the problems with their own problem-solving methods. Based on previous experience, students are asked to create hypotheses, such as dealing with decimal division like division for whole numbers, ignoring the decimal point, and following rules of division for whole numbers first and then placing the decimal point.
3. Guided practice
   Students are asked to report their problem-solving methods to the class, and the teacher guides the students to reflect on these methods. The teachers guide the students to differentiate the differences between these questions. They encourage the students to reflect on the principles of multiplication and division that they have learned in their previous classes. The teachers ask the students to use multiplication to verify the results for division. The students are asked to raise questions and summarize the principles for decimal division.

4. Students modify their results and report their results to the class.
5. Transfer and application.
   Provide additional problems for students to practice, including two-digit decimals.

Appendix 2: Teaching Scripts for Reviewing Previous Contents (Decimal Division Error Clinic)

1. Introduction to the problems.
   The students form five groups to examine errors in the worksheet. Say: “Thank you for joining error clinic. Today, we will focus on examining errors in decimal division. It is hoped that we will all be able to solve different challenges in decimal division.” Then, the teacher distributes the worksheets.
2. Encourage students to solve the problems and explain the errors that they found, such as errors due to misunderstanding of principles or careless errors.
3. Guided practice.
   The teacher guides students to draw conclusions about their problem-solving methods, help students make their reminder cards, and encourage students to reflect on their problem-solving experiences. Ask the students to summarize the causes of mistakes, such as assuming decimal division is similar to division of whole numbers.
4. Students modify their results, establish their own reminder cards, and report their results to the class.
5. Transfer and application.
   Ask the students to give an example of mistakes they made in decimal computation during the previous week.

Appendix 3: PBL Procedure



A flowchart of the PBL procedure. S=successful; U=unsuccessful.

Appendix 4: Treatment Fidelity Checklists

New lectures	Yes/No
1. Provide problem-based learning situations at the beginning of classes	
2. Small group discussion	
3. Students form hypotheses and try to work out solutions	
4. Students report discussion results group by group	
5. Teacher provides feedback and analyzes differences and relations between different problems	
6. Students break into small groups for further discussion	
7. Students report further discussion results	
8. Application and transfer	

Review classes	Yes/No
1. Provide problem-based learning situations and welcome students to error clinic	
2. Small group discussion to find own mistakes	
3. Teacher provides guidance	
4. Teacher provides reminder cards, uses scaffolding, and coaches students to be able to fill out reminder cards	
5. Students report their discussion report and report what is on reminder card	
6. Application and transfer, students create problems for others	

Appendix 5: Sample Problems from the Curriculum

Samples for ill-defined and ill-structured problems

Directions: Please check the following computation procedures and see whether they are right. Please correct computations that were executed incorrectly.

7.下面各题计算的对吗？把不对的改正过来。

$$\begin{aligned} &50.4\times 1.9-1.8 \\ &=50.4\times 0.1 \\ &=5.04 \end{aligned} \qquad \begin{aligned} &3.76\times 0.25+25.8 \\ &=0.094+25.8 \\ &=25.894 \end{aligned}$$

Samples for ill-defined and ill-structured problems

Directions: Are the following computation procedures right? If no, please check them.

下面的计算对吗？如果不对，错在哪里？

$24\div 15=16$ $\qquad$ $1.26\div 18=0.7$

$$\begin{array}{r} 16 \\ 15\overline{)\,24} \\ \underline{15} \\ 90 \\ \underline{90} \\ 0 \end{array} \qquad \begin{array}{r} 0.7 \\ 18\overline{)\,1.26} \\ \underline{126} \\ 0 \end{array}$$

Samples for challenging problem

Directions: (1) In a parking lot, the parking rate is 2.50 Yuan/h if it is within 1 h. (2) After the first hour, the parking rate is 2.50 Yuan/0.5 h. Uncle Li paid 12.5 Yuan, then how many hours did he park in the parking lot?

(Sources: Lu & Yang, 2005)

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