ISRM-Suggested Method for Determining the Mode I Static Fracture Toughness Using Semi-Circular Bend Specimen

M. D. Kuruppu, Y. Obara, M. R. Ayatollahi, K. P. Chong, and T. Funatsu

1 Introduction

Rock fracture mechanics can be used to identify and predict the imminent failure of rock mass structures thereby providing guidelines to improve the stability and the safety of these structures. Another application is for the exploitation of mineral resources by adopting techniques such as mechanical mining, blasting and hydraulic fracturing. In fracture processes which are not associated with high strain rates, the mode I plane-strain static fracture toughness gives the critical value of the stress intensity factor leading to the onset of crack growth in that mode (Liu 1983). Some of the applications of fracture toughness include index of fragmentation processes like those used in tunnel boring, a

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M. D. Kuruppu (🖂)

Curtin University, Locked Bag 30, Kalgoorlie, WA 6433, Australia e-mail: M.Kuruppu@curtin.edu.au

Y. Obara

Graduate School of Science and Technology, Kumamoto University, 2-39-1 Kurokami, Kumamoto, 860-8555, Japan

M. R. Ayatollahi

School of Mechanical Engineering, Iran University of Science and Technology, Narmak, 16846, Tehran, Iran

K. P. Chong

K. P. Chong

Department of Mechanical Engineering, George Washington University, Washington DC, 20052, USA

T. Funatsu

Institute for Geo-Resources and Environment, AIST, Central 7 1-1-1 Higashi, Tsukuba, Ibaraki 305-8567, Japan modelling parameter in processes such as rock cutting and hydraulic fracturing and for the stability analysis of civil, mining and earthen structures (Whittaker et al. 1992).

A number of standard methods have been proposed to determine the mode I fracture toughness of rock. They include those based on short rod (SR) specimen, chevron bend (CB) specimen and cracked chevron notched Brazilian disk (CCNBD) specimen (ISRM 2007). The semi-circular bend (SCB) specimen has been widely used for fracture toughness determination of geomaterials owing to inherent favourable properties such as its simplicity, minimal requirement of machining and the convenience of testing that can be accomplished by applying three-point compressive loading using a common laboratory load frame (Chong and Kuruppu 1984; Chong et al. 1987; Lim et al. 1993, 1994; Ayatollahi and Aliha 2007; Aliha et al. 2012; Karfakis and Akram 1993; Obara et al. 2006, 2007a, b, 2009; Molenar et al. 2002). As geomaterials are weak in tension, fracture tests should preferably be conducted with compressive loading in such a way that tensile fractures are induced. The CB and CCNBD specimens used for the standard methods as well as the SCB specimen satisfy those requirements. Use of more than one type of specimen is regarded as appropriate when it is required to measure the fracture toughness of anisotropic materials in different material directions of a rock sample (Chong et al. 1987). An ISRM-suggested method for mode I static fracture toughness determination of rock and other geomaterials using SCB specimen is herein presented.

2 Scope

This test method is intended to measure the mode I static fracture toughness K_{Ic} under slow and steady loading where dynamic effects are negligible. However, another suggested method developed by the ISRM should be followed if the loading rate is high, as in the case of explosive fragmentation of rock (Zhou et al. 2012). The geometry of the test specimen is designed to use standard core material. A minimum

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specimen diameter D_{\min} is suggested to be used in order to satisfy the minimum size requirement as explained in Sect. 7. If the rock material is known to be anisotropic, the core axis should be oriented either parallel or perpendicular to any anisotropic feature, such as a bedding plane. If required, the remaining material from mode I fracture toughness tests performed using CB and SR methods can be used to find fracture toughness in orthogonal directions (Chong et al. 1987). For example, for sedimentary rocks that exhibit transversely isotropic material properties, a combination of tests performed using SR, CB, CCNBD and/or SCB specimen with cores taken perpendicular and parallel to bedding planes, will give the complete information of fracture toughness. Alternatively, SCB specimens themselves can be made from cores such that the notch directions are either aligned or perpendicular to the bedding planes.

The advantages of using the SCB specimens are (a) material requirement per specimen is small, (b) machining is relatively simple and (c) only the maximum compressive load is required to determine the fracture toughness.

3 Specimen Preparation

The circular disks required to make the SCB specimen are prepared by sawing or slicing standard rock cores using a high-precision diamond tool. The geometry of the SCB specimen is shown in Fig. 1. The specimen diameter (D = 2R) should be related to the average grain size in the rock by a ratio of at least 10:1 or should be at least 76 mm and the minimum specimen thickness shall be the larger of 0.4D or 30 mm. Caution should be exercised to minimise the micromechanical damage of the specimens as it can affect the fracture toughness. Water or other coolant should be used while machining, in order to avoid heat damage that can alter the fracture toughness. Slow drilling is recommended in case that the cores are drilled from a large rock sample.

Each of the circular disks should be sawn into two halves which may be carried out using the same cutting tool used to make the circular disks. The final operation of introducing a notch should be performed using a thin cutting blade of thickness $\leq 1.5 \pm 0.2$ mm, or preferably, using a diamond-impregnated fine wire saw that will produce a straight notch of the required length. The radius of the notch tip should be less than the average grain size of the rock material. The notch length should be such that $0.4 \leq a/R \leq 0.6$.

The plane surface along the thickness direction should be flat to 0.01 mm. The plane of the notch shall not depart from perpendicularity to the plane surface in the thickness direction by more than 0.5° . The dimensions of the test specimen are given in Table 1.

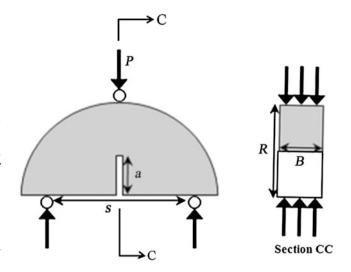


Fig. 1 SCB specimen geometry and schematic loading arrangement (R radius of the specimen, B thickness, a notch length, s distance between the two supporting cylindrical rollers, P monotonically increasing compressive load applied at the central loading roller of the three-point bend loading)

When slicing a core, the plane of the resulting disks should not deviate from the perpendicularity to the core axis by more than 0.5° .

When cutting a disk into two halves to form two semicircular disks, care must be given not to deviate the cutting plane from a diametral plane by more than 0.2 mm. Also, the perpendicularity to the plane of the disk should be assured to be within 0.5° .

The specimens must be marked with a reference that gives the details of its orientation with respect to any directions of material anisotropy (e.g. inclination of the notch plane to bedding planes). Specimens of the same sample should have identical notch orientation.

The notch length should be measured as an average taken on both the semi-circular planar surfaces which are perpendicular to the core axis. The two readings should be within 2 % of each other.

The thickness should be uniform and shall not deviate by more than 0.2 mm.

The dimensions of the specimens should be measured to the nearest 0.1 mm. The required dimensions are the radius R, the thickness B and the notch length a.

If the thickness of the saw blade used to cut the disks into semi-circular specimens is greater than 0.05*D*, where *D* is the disk diameter, then the values of the measured radius R'and the measured notch length a' should be corrected as shown in Fig. 2 (i.e. the corrected radius $R = R' + \Delta r$ and corrected notch length $a = a' + \Delta r$). Note that the radius measurement shall be taken aligned with the notch direction.

 Table 1
 Recommended geometrical dimensions of SCB specimen (see Fig. 1)

 Descriptions
 Values of service

Descriptions	Values or range
Diameter (D)	Larger of 10× grain size or 76 mm
Thickness (B)	Larger of 0.4D or 30 mm
Crack length (a)	$0.4 \le \frac{a}{R} (= \beta) \le 0.6$
Span length (s)	$0.5 \le rac{s}{2R} \le 0.8$

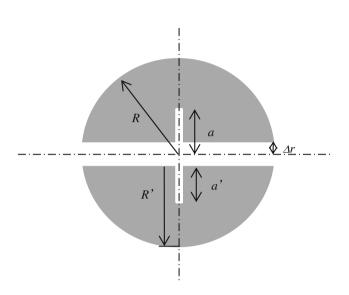


Fig. 2 Correction for *a*, *R* when the thickness of the saw blade is not negligible (i.e. $2\Delta r \ge 0.05D$, where Δr is the half thickness of the saw blade used for cutting)

The specimen should be stored after specimen preparation for an appropriate period of time that is sufficient to achieve the desired conditions (e.g. moisture content). The conditions of storage, moisture adjustment or drying, as well as any macroscopically noticeable features of the specimen surfaces, shall be reported with fracture toughness results.

The tensile strength of the material should be known (or measured) (ISRM 2007).

4 Experimental Setup

The test should be performed using a standard compressive or universal test frame commonly available in most rock mechanics laboratories. While a servo-hydraulic test system is preferable, a mechanically driven compressive testing machine may be adequate if the capacity and the precision of the load measurement is as given below. The load frame should be equipped with a system to record the load, the axial displacement and any other measuring signal of interest.

The load application is performed via a conventional three-point bend fixture. The specimen is to be placed on the two bottom loading cylindrical rollers which are kept



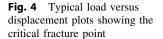
Fig. 3 SCB specimen loading fixture

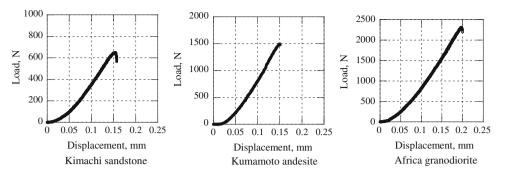
apart at a predetermined distance commensurate with the size of the specimen as shown in Fig. 3. The rollers shall be placed on the bottom loading plate so that they can rotate and move apart slightly when the specimen is loaded, thus permitting roller contact at supports offering no frictional resistance. A suitable span length should be selected within the range of span (s) to diameter ratio (D), s/D of 0.5 < s/D ≤ 0.8 . The parallel positioning of the two bottom support rollers should be ensured. It may help to mark the positions of the two bottom support rollers on either side of the semicircular faces of the specimen prior to its positioning on the support rollers. These positions should be drawn symmetrical to the plane of the notch. A top loading cylindrical roller is attached to the top loading plate so that the load application occurs symmetrically between the two bottom support rollers. A suitable recess made on the top loading plate may be required to hold the roller in position.

The diameter of the rollers should be chosen in relation to the specimen diameter. A ratio of 1:20 is recommended (i.e. 10 mm diameter rollers are used for testing 200 mm diameter specimens). However, the minimum diameter of the rollers used for testing specimens <100 mm diameter should be 5 mm.

Alignment of the notch plane with the loading direction should be carefully controlled.

The load frame should be equipped with a load cell having a resolution of 0.01 kN or greater. A linear variable displacement transducer (LVDT) set up between the top and bottom loading roller positions is the preferred arrangement for measuring the displacement. Crack opening displacement measurement by a clip gauge is also useful (Karfakis and Akram 1993). A successful test is usually associated with a





monotonically increasing and continuous load-displacement graph. Moreover, the load versus displacement behaviour reveals the degree of nonlinearity of the rock material.

If testing is required to be performed at conditions other than the ambient, then the specimen may be kept inside an environment chamber that will provide those conditions. For example, moisture content measured by water vapour pressure may be set at a predetermined level and maintained until reaching saturation under that condition (Obara et al. 2010). Temperature may be set at a predetermined level and maintained until the specimen is heated uniformly (Funatsu et al. 2004; Kuruppu and Seto 2001). They may be controlled independently from the axial load application that would perform the three-point bend loading of the specimen.

5 Testing Procedure

The minimum data required during testing is the peak load $P_{\rm max}$ and any other environmental conditions, if applicable. However, a continuous measurement of the load and the displacement between the top and bottom loading roller positions during the test is recommended in order to verify that the load has increased continuously with increasing displacement until reaching the point of fracture as shown in Fig. 4 (Kataoka et al. 2010, 2011). It is appropriate to gather data at a rate of four data sets per second if digital data acquisition is used.

The testing should be done at a constant displacement rate of not greater than 0.2 mm/min to avoid any dynamic effect (Backers and Stephansson 2012; Khan and Al-Shayea 2000). Data acquisition should begin prior to closing the gap between the specimen and the top loading roller and continue until the specimen fails completely.

When the gap is closed and a small load is applied, the test may be stopped momentarily to check the alignment of support/loading rollers and that they are in touch with the specimen along the entire thickness of the specimen. After the test is completed, the two parts of the broken specimen should be kept for further observation of failure mode. The results shall be considered invalid if the plane of the cracked ligament deviates from the notch plane by more than 0.05*D*.

The number of specimens tested per sample should be determined by practical considerations. A minimum of five specimens are recommended. All specimens of the sample ought to be tested subjected to the same conditions.

6 Calculations

Mode I fracture toughness K_{Ic} shall be determined using the observed peak load P_{max} such that:

$$K_{\rm Ic} = Y' \frac{P_{\rm max} \sqrt{\pi a}}{2RB} \tag{1}$$

where,

$$Y' = -1.297 + 9.516(s/2R) - (0.47 + 16.457(s/2R))\beta + (1.071 + 34.401(s/2R))\beta^2$$
(2)

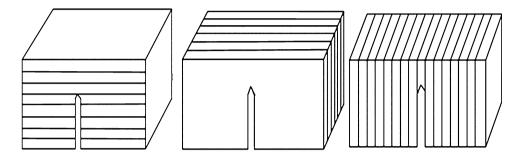
and $\beta = a/R$. Equation (2) gives the non-dimensional stress intensity factor Y' derived using the finite element method while assuming plane-strain conditions. Further details about the numerical analyses performed for deriving Eq. (2) can be found in the Appendix. Table 2 gives some of the values calculated for Y'. Equation (2) is valid for $\beta \ge 0.2$. However, a relatively deep notch is required for the bending effect to produce a strong mode I stress field near the tip of the notch. Hence, a normalised length β in the range $0.4 \le \beta \le 0.6$ is recommended to be used.

The suggested range of s/2R is $0.5 \le s/2R \le 0.8$. For testing strong materials, it is preferable to use a value approaching 0.8. However, this may not be practical for specimens made of weak geomaterials in which case a value at the lower end of the range should be used.

Table 2Non-dimensionalstress intensity factor Y'

s/2R	$\beta = 0.4$	eta=0.5	$\beta = 0.6$
0.5	2.905	3.679	4.819
0.6	3.748	4.668	6.022
0.7	4.592	5.657	7.224
0.8	5.436	6.645	8.427

Fig. 5 Principal crack orientations with respect to bedding planes (*left* to *right* arrester, divider and, short transverse configurations)



Some of the previously published suggested methods are meant to determine two levels of fracture toughness of rock. Level I is based on the maximum failure load and level II further incorporates a nonlinearity correction to take any non-linear material behaviour into account. However, this suggested method only addresses level I fracture toughness.

As described in Sect. 2 the SCB specimen can be used to determine the fracture toughness of sedimentary rock in which major planes of anisotropy can be found. Those rocks mostly exhibit transversely isotropic properties. For complete characterization, specimens with their notches aligned in three mutually perpendicular directions should be tested (Fig. 5). One possibility is to use three sets of SCB specimens having the notches oriented in each of the arrester, divider and short transverse directions. However, it may be more practical to use a combination of fracture toughness test specimens. While it is up to the user to decide which combination of specimens is to be employed, one combination is to use the straight edge cracked round bar in bending (SECRBB), SCB and centrally cracked Brazilian disk (CCBD) specimens made with their notches aligned to form divider, arrester and short transverse configurations, respectively. These specimens must be made with cores taken in the direction of bedding planes.

7 Size Effects

As geomaterials tend to form relatively large process zones prior to fracture, certain minimum specimen sizes need to be satisfied to achieve the requirements for linear elastic fracture mechanics, according to which the concept of K_{Ic} is defined. The process zone is largely affected by the grain size of the material; those consisting of relatively small grains have small size requirements and vice versa (Ouchterlony 1990; Kuruppu and Chong 2012; Bazant 1984).

Chong et al. (1987) suggested the following size requirement for the SCB specimen:

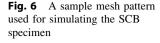
$$D \ge 2.0 \left(\frac{K_{\rm lc}}{\sigma_t}\right)^2 \tag{3}$$

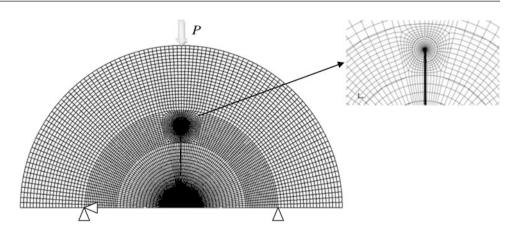
where σ_t is the tensile strength of the material. However, this may be a conservative estimate as the size requirements applicable for chevron notched CB and SR specimens are much lower (Ouchterlony 1989). No definitive size requirement can be given for SCB specimens. One way to determine the size requirement for a particular material is by comparing $K_{\rm Ic}$ values deduced using a number of specimens of different diameters *D*. The smallest specimen diameter $D_{\rm min}$, that generates $K_{\rm Ic}$ value consistent with larger diameter specimens, will be the minimum size required to give a valid fracture toughness test.

8 Reporting of Results

The report should include the following:

- 1. Source of specimens as precisely as possible (e.g. material, location, date and orientation).
- 2. Lithological description of the rock type including grain size.





- 3. Dimensions of the specimens. Any particular observations about macroscopic appearance of the specimen surface.
- 4. If applicable, the orientation of the notch with respect to the specimen anisotropy (e.g. direction of bedding planes, etc.).
- 5. History and environment of test specimen storage (e.g. temperature, water vapour pressure).
- 6. Conditions at the time of test (e.g. temperature, water vapour pressure).
- 7. Details of the test equipment and test procedure used, particularly if the method employed deviated from the suggested method and the reasons for such deviation.
- 8. Record of all signals measured, loading rate and any other relevant parameters not included in this list.
- 9. The calculated value of fracture toughness of each specimen.
- 10. Description of the broken specimens after testing. If there are fractures other than the near-symmetric split of the specimens then the results of those specimens will not be valid.
- 11. The average value of mode I fracture toughness of each sample disregarding any invalid results. Statement of any associated environmental conditions.

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Appendix: Details of Numerical Analysis Used for Deriving Eq. (2)

The SCB specimens of different crack lengths were simulated and analyzed using eight-node plane-strain elements in the finite element code Abaqus Unified FEA (2012). The loading, the boundary conditions and a typical finite element mesh used for the simulations are shown in Fig. 6.

Singular elements with nodes at quarter-point positions were used for the first ring of elements around the crack tip. In the circular partitions surrounding the crack tip where the contour integrals are calculated, the mesh was biased toward the crack tip. The stress intensity factors $K_{\rm I}$ were extracted directly from ABAQUS which makes use of the *J*-integral method to compute the stress intensity factors. The numerical results showed that there was negligible variation in the *J*-integral values calculated for successive contours surrounding the crack tip.

Using a fixed arbitrary load *P*, the stress intensity factor $K_{\rm I}$ was determined for each set of β and $\frac{s}{2R}$, and the nondimensional stress intensity factor Y' was calculated from

$$Y'(\beta, \frac{s}{2R}) = \frac{2RBK_{\rm I}}{P\sqrt{\pi a}} \tag{A1}$$

then Eq. (2) was derived by fitting a second order polynomial to the numerical results obtained for Y'. Tutluoglu and Keles (2011) recently reported limited numerical results for Y' in the SCB specimen. As shown in Table 3, very good agreement exists between the present results and those reported by Tutluoglu and Keles (2011). Table 3 can also be considered as validation for the finite element results obtained in this study, particularly for the ranges $0.4 \le \beta \le 0.6$ and $0.5 \le s/2R \le 0.8$, as suggested in Sect. 6.

It is noteworthy that a number of investigators have presented mode I stress intensity factors of the SCB specimen (Chong et al. 1987; Lim et al. 1994; Basham 1989). For instance, Lim et al. (1994) extracted the stress intensity factors of the SCB specimen from finite element analysis and suggested a fifth order polynomial for Y' as.

$$Y' = \frac{s}{2R} \left(2.91 + 54.39\beta - 391.4\beta^2 + 1210.6\beta^3 - 1650\beta^4 + 875.9\beta^5 \right)$$
(A2)

Table 3 Numerical values of Y' , present results compared with those of Tutluoglu and Keles (2011)	$\beta = a/R$	s/2R	Present results	Tutluoglu and Keles (2011)
	0.3	0.5	2.495	2.538
	0.5	0.5	3.679	3.550
	0.67	0.5	5.835	6.209

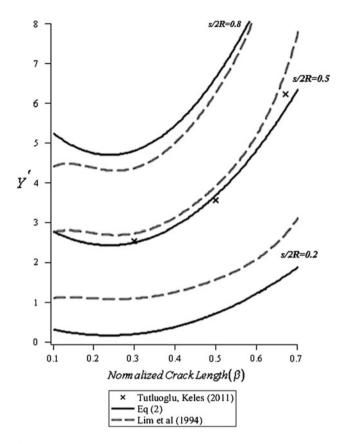


Fig. 7 The *curves* plotted based on Eqs. (2) and (A2)

Figure 7 shows a comparison between the curves plotted based on Eqs. (2) and (A2) for different values of β and $\frac{s}{2R}$. Significant discrepancies can be seen between these two sets of results.

Having checked our finite element results by different mesh designs and element numbers, we concluded that the observed discrepancy can be due to less accurate method used by Lim et al. (1994) for determining the stress intensity factors of the SCB specimen. The displacement/stress extrapolation method employed by Lim et al. was a common technique in the 1990s for deriving stress intensity factors from finite element results. But, later more accurate methods were proposed like the contour integral techniques (e.g. *J*-integral method). It is now well established that the numerical errors in the region of high stress gradient around the crack tip affects the J-integral method much less than the displacement/stress extrapolation technique.

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