Chapter 78
A Linear Assignment Method of Simple Additive Weighting System in Linear Programming Approach Under Interval Type-2 Fuzzy Set Concepts for MCDM Problem

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Abstract The ranking phase is valuable to examines the final alternative rankings of decision making problems. Based on simple additive weighting (SAW) and linear programming (LP) within the context of interval type-2 fuzzy sets (IT2 FSs), we develop a linear assignment method to produce the final ranking order of all alternatives for interval type-2 fuzzy TOPSIS (IT2 FTOPSIS) method. A numerical example is used to check the efficiency and applicability of the proposed method. The results shows consistent outcomes of the decision making process. Thus, the proposed method offers an alternative, user-friendly method that is robust in the decision making framework.

78.1 Introduction

Ranking phase is the step to examines the results of decision making problems. The interpretation of multiple attribute decision making (MADM) results can show the differences in the rankings of the alternatives. It was extensively applied and strengthened the theoretical part of aggregating phase by many authors. A few of them were; Gao et al. [1] developed a fuzzy approach based on the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), where in the ranking phase, the distances of each alternative from the fuzzy positive ideal solutions (PIS) and the fuzzy negative ideal solutions (NIS) are computed respectively with

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a vertex method. Then, a closeness coefficient is obtained to rank order of all alternatives. Li [2] constructed nonlinear-programming models on the basis of the concepts of the relative-closeness coefficient and the weighted-Euclidean distance. Simpler auxiliary nonlinear-programming models were further deduced to calculate relative-closeness of intuitionistic fuzzy (IF) sets of alternatives to the interval-valued intuitionistic fuzzy-positive ideal solutions (IVIF-PIS), which can be used to generate the ranking order of alternatives. Jolai et al. [3], proposed the goal programming (GP) technique, and constructed a multi-objective mixed integer linear programming (MOMILP) model to determine the order quantities of each selected supplier for each product in each period.

Furthermore, for interval type-2 (IT2) fuzzy approach based on the TOPSIS, Chen and Lee [4] proposed a ranking value method to cumulative all the collective decisions and obtained the relative closeness through the traditional TOPSIS method computing process. However, the used of standard deviation in the ranking value method is believed influenced by extreme scores and the method is depended only on the dispersion's data. It is proved by Chen et al. [5], that the ranking value proposed by Chen and Lee [4] was difficult and higher in computational volume. Since that, various authors discussed on the ranking phase of IT2 FTOPSIS method. For example, Chen et al. [5] proposed a new method MADM based on the proposed ranking method of IT2FSs. Wang et al. [6] developed IT2 fuzzy weighted arithmetic averaging operator to aggregate all individual IT2 fuzzy decision matrices provided by the decision-makers (DMs) into the collective IT2 fuzzy decision matrix, then utilized the ranking-value measure to calculate the ranking value of each attribute value and constructed the ranking-value matrix of the collective IT2 fuzzy decision matrix. Chen and Wang [7] presented a new fuzzy ranking method based on the α-cuts of interval type-2 fuzzy sets (IT2 FSs). Chen [8] developed a new linear assignment method to produce an optimal preference ranking of the alternatives in accordance with a set of criterion-wise rankings and a set of criterion importance within the context of interval type-2 trapezoidal fuzzy numbers (IT2TrFNs) for MADM problems. However, little research has been conducted on the simple additive weighting (SAW) and linear programming (LP) for coping with IT2FSs. This linear assignment method (SAW and LP with IT2FSs method) is developed to handle the ranking phase of IT2 FTOPSIS method.

Thus, the purpose of this paper is to extend the SAW and LP methods in IT2FSs approach for ranking phase of IT2FTOPSIS. This paper proposes the linear assignment method with the identification of the SAW and LP methods to determine the final ranking orders respectively, for each pair of alternatives. The feasibility and the applicability of the proposed methods are illustrated using the MADM examples of Chen [9].

This paper is illustrated as follows. Section 78.2 discusses the concept of weighted average with linear programming. Section 78.3 proposes a linear assignment method based on SAW and LP methods in IT2 FSs concept. Section 78.4 illustrates a numerical example in order the check the efficiency of the proposed method. Finally, Sect. 78.5 presents the conclusions.

78.2 Weighted Average With Linear Programming

In the following, we recall basic notations and definitions of weighted average with linear programming.

Definition 78.1 [10, 11] The minimum and maximum for the fuzzy weighted average for each given α_j can be obtained by solving the following two fractional programming problems:

$$\begin{aligned} &\min f_L = \frac{w_1 a_1 + w_2 a_2 + \cdots + w_n a_n}{w_1 + w_2 + \cdots + w_n} \\ &\text{s.t. } c_i \le w_i \le d_i, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{78.1}$$

$$\begin{aligned} &\max f_U = \frac{w_1 b_1 + w_2 b_2 + \cdots + w_n b_n}{w_1 + w_2 + \cdots + w_n} \\ &\text{s.t. } c_i \le w_i \le d_i, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{78.2}$$

where c_i and d_i are the two end points of the interval w_i for a given α_j level cut.

The Charnes and Cooper's linear transformation is summarized in the following. Consider the following simple fractional programming problem:

$$\begin{aligned} &\min \frac{px}{qx} \\ &\text{s.t. } Ax \le b, \quad x \ge 0, \end{aligned} \tag{78.3}$$

where p and q are two n-dimensional constant vectors, x is the n-dimensional variable, A is an $m \times n$ matrix, and b is an m-dimensional constant vector.

To transform the above fractional programming problem into a linear problem, let

$$z = \frac{1}{qx} \quad \text{and} \quad zx = y, \tag{78.4}$$

where we assume that $qx \ne 0$. Multiplying both the objective function and the constraints by z and using the definitions given in Eq. 78.5, we obtain

$$\begin{aligned} &\min py \\ &\text{s.t. } Ay \le bz, \quad qy = 1, \quad y \ge 0, \quad z \ge 0, \end{aligned} \tag{78.5}$$

which is a linear programming problem.

This weighted average with linear programming is being used in defining a linear assignment method. Thus, the development of the proposed model is described in Sect. 78.3.

78.3 The Proposed Method

Original SAW method and LP are modified into an IT2 FS manner. Modifications are made to accommodate the objective of the research and also to simplify the computational procedure without losing the novelty of SAW and LP. The proposed method is then applied into a linear assignment method for IT2 FTOPSIS (MCDM method) to get the optimal preference ranking. This proposed method is believed to be more flexible rather than the existed model due to the fact that it is used the IT2 FS. On the other hand, this model is more suitable to represent uncertainties because it is involve end-users into the whole weighting process. Thus, suppose an IT2 FTOPSIS has n alternatives $(A_1, \ldots, A_n)$ and m decision criteria/attributes $(C_1, \ldots, C_m)$. Each alternative is evaluated with respect to the m criteria/attributes. All the values/ratings assigned to the alternatives with respect to each criterion from a decision matrix, denoted by $S = (y_{ij})_{n \times m}$, and the relative weight vector about the criteria, denoted by $W = (w_1, \ldots, w_m)$, that satisfying $\sum_{j=1}^{m} w_j = 1$. Therefore, the rest of the general process of this proposed method is listed as follows:

Rating State: In this state, all the matrices are transformed into the IT2 FS concept.

Step 1: **Establish a decision matrix and weight matrix**

Establish an IT2 decision matrix and IT2 fuzzy weight matrix.

Step 2: **Comparable Scale**

Construct a comparable scale for all elements in the decision matrix. The comparable scale is used to divide the outcome of a certain criterion by its maximum value, provided that the criteria are defined as benefit criteria. Therefore, the comparable scale is represented as follows:

$$\tilde{\tilde{r}}_{ij} = 1 \Big/ FOU\left(\tilde{\tilde{r}}_{ij}\right) = \left[\underline{\tilde{\tilde{r}}_{ij}}, \bar{\tilde{\tilde{r}}}_{ij}\right] \tag{78.6}$$

$$\text{For positive criteria} \quad \tilde{\tilde{r}}^{*}_{ij} = \left(\left[\frac{\underline{\tilde{\tilde{f}}}_{ij}}{\underline{\tilde{\tilde{f}}}^{*}_{ij}} \right], \left[\frac{\bar{\tilde{\tilde{f}}}_{ij}}{\bar{\tilde{\tilde{f}}}^{*}_{ij}} \right] \right) \tag{78.7}$$

$$\text{For negative criteria} \quad \tilde{\tilde{r}}_{ij} = \left(\left[\frac{\underline{\tilde{\tilde{f}}}^{\min}_{ij}}{\underline{\tilde{\tilde{f}}}_{ij}} \right], \left[\frac{\bar{\tilde{\tilde{f}}}^{\min}_{ij}}{\bar{\tilde{\tilde{f}}}_{ij}} \right] \right) \tag{78.8}$$

Then the decision matrix can be expressed as follows:

$$D = \begin{matrix} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} \begin{matrix} \begin{matrix} C_1 & C_2 & \cdots & C_n \end{matrix} \\ \begin{bmatrix} \tilde{\tilde{r}}_{11} & \tilde{\tilde{r}}_{12} & \cdots & \tilde{\tilde{r}}_{1n} \\ \tilde{\tilde{r}}_{21} & \tilde{\tilde{r}}_{22} & \cdots & \tilde{\tilde{r}}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{r}}_{m1} & \tilde{\tilde{r}}_{m2} & \cdots & \tilde{\tilde{r}}_{mn} \end{bmatrix} \end{matrix} \tag{78.9}$$

where $\tilde{\tilde{r}}_{mn}$ is the comparable scale value in the decision matrix.

Weighting State: Modification of the existed SAW method with "modified SAW in IT2 FS concepts".

Step 3: **Weight of attributes of SAW**

Construct the weighting matrix W_p using the SAW formulae of the attributes of the decision-maker and construct the pth average weighting matrix $\bar{W}$.

Step 4: **Weighted decision matrix**

Construct the weighted decision matrix.

Aggregation State: Upgrading the calculation of separations of each alternative with linear programming concepts.

Step 5: **Positive ideal solution and negative ideal solution**

Determine the matrices that include positive and negative ideal solutions.

Step 6: **Construct the separation of each alternative of SAW by linear programming approach**

Calculate the separation of each alternative from the positive ideal solution I^* and negative ideal solution I^- using the formulae as follows:

$$\begin{aligned} C_i\left(b_{ij}, d_{ij}; s\right) &= \frac{\sum_{j=1}^{n}\left[\tilde{\tilde{w}}_j\tilde{\tilde{b}}_{ij} + \tilde{\tilde{\rho}}_j\left(1-\tilde{\tilde{d}}_{ij}\right)\right]}{\sum_{j=1}^{n}\left[\tilde{\tilde{w}}_j\tilde{\tilde{b}}_{ij} + \tilde{\tilde{\rho}}_j\left(1-\tilde{\tilde{d}}_{ij}\right)\right] + \sum_{j=1}^{n}\left[\tilde{\tilde{w}}_j\left(1-\tilde{\tilde{b}}_{ij}\right) + \tilde{\tilde{\rho}}_j\left(\tilde{\tilde{d}}_{ij}\right)\right]} + s^l + s^u \\ &= \frac{\sum_{j=1}^{n}\left[\tilde{\tilde{w}}_j\tilde{\tilde{b}}_{ij} + \tilde{\tilde{\rho}}_j\left(1-\tilde{\tilde{d}}_{ij}\right)\right]}{\sum_{j=1}^{n}\left[\tilde{\tilde{w}}_j + \tilde{\tilde{\rho}}_j\right]} + s^l + s^u \end{aligned} \tag{78.10}$$

and let

$$z = \frac{1}{\sum_{j=1}^{n}\left[\tilde{\tilde{w}}_j + \tilde{\tilde{\rho}}_j\right]} \tag{78.11}$$

Assigned the value for

$$\tilde{\tilde{t}}_j = z\tilde{\tilde{w}}_j \tag{78.12}$$

and

$$\tilde{\tilde{y}}_j = z\tilde{\tilde{\rho}}_j \quad (j = 1, 2, \ldots, n) \tag{78.13}$$

Since

$$z = \frac{1}{\sum_{j=1}^{n}\left[\tilde{\tilde{w}}_j + \tilde{\tilde{\rho}}_j\right]} \therefore \quad \frac{1}{z} = \sum_{j=1}^{n}\left[\tilde{\tilde{w}}_j + \tilde{\tilde{\rho}}_j\right] \tag{78.14}$$

and

$$\tilde{\tilde{t}}_j = z\tilde{\tilde{w}}_j \quad \therefore \quad \tilde{\tilde{w}}_j = \frac{\tilde{\tilde{t}}_j}{z} \tag{78.15}$$

$$\tilde{\tilde{y}}_j = z\tilde{\tilde{\rho}}_j \quad \therefore \quad \tilde{\tilde{\rho}}_j = \frac{\tilde{\tilde{y}}_j}{z} \tag{78.16}$$

Thus, based on the above Charnes and Cooper's transformations [12], Eq. 78.10 can be transformed into the equivalent linear programming models as follows:

$$C_i^u\left(\tilde{\tilde{b}}_{ij}, \tilde{\tilde{d}}_{ij}; s\right) = \max\left\{\sum_{j=1}^{n}\tilde{\tilde{t}}_i b_{ij}^u + \tilde{\tilde{y}}_j\left(1 - d_{ij}^l\right) + s^l + s^u\right\}$$
$$\text{s.t.}\begin{cases} z\tilde{\tilde{w}}_j^l \le \tilde{\tilde{t}}_j \le z\tilde{\tilde{w}}_j^u \quad (j = 1, 2, \ldots, n) \\ z\tilde{\tilde{\rho}}_j^l \le \tilde{\tilde{y}}_j \le z\tilde{\tilde{\rho}}_j^u \quad (j = 1, 2, \ldots, n) \\ \sum_{j=1}^{n}\left(\tilde{\tilde{t}}_j + \tilde{\tilde{y}}_j\right) = 1 \\ z \ge 0 \\ s^l = n \quad (n = 0, \ldots, 1) \\ s^u = n \quad (n = 0, \ldots, 1) \end{cases} \tag{78.17}$$

and

$$C_i^l\left(\tilde{\tilde{b}}_{ij}, \tilde{\tilde{d}}_{ij}; s\right) = \max\left\{\sum\nolimits_{j=1}^{n} \tilde{\tilde{t}}_i b_{ij}^l + \tilde{\tilde{y}}_j\left(1 - d_{ij}^u\right) + s^l + s^u\right\}$$

$$\text{s.t.} \begin{cases} z\tilde{\tilde{w}}_j^l \le \tilde{\tilde{t}}_j \le z\tilde{\tilde{w}}_j^u \quad (j = 1, 2, \ldots, n) \\ z\tilde{\tilde{\rho}}_j^l \le \tilde{\tilde{y}}_j \le z\tilde{\tilde{\rho}}_j^u \quad (j = 1, 2, \ldots, n) \\ \sum\nolimits_{j=1}^{n}\left(\tilde{\tilde{t}}_j + \tilde{\tilde{y}}_j\right) = 1 \\ z \ge 0 \\ s^l = n \quad (n = 0, \ldots, 1) \\ s^u = n \quad (n = 0, \ldots, 1) \end{cases} \tag{78.18}$$

where $C_i\left(b_{ij}, d_{ij}; s\right)$ is an IT2 FS, denoted by $\left[C_i^l, C_i^u\right]$.

Step 7: **Define the closeness coefficient**

Calculate the relative degree of closeness to the ideal solution for each alternative.

Ranking State:

Step 8: **Rank all alternatives**

Sort the values of $(CC)_i$ in a descending sequence, where $1 \le j \le n$. The larger the value of $(CC)_i$, the higher the preference of the alternative for $(CC)_i$.

In this section, we have successfully introduced a new concept of linear assignment method. In order to check the efficiency of the proposed method, a numerical example is provided in Sect. 78.4 to illustrate the proposed method.

78.4 Illustrative Example

In this section, we used an example from Chen [9] to illustrate the proposed method. This numerical example is used to test the ability of the proposed method to handle the IT2 MCDM problems in many areas. All the relative importance weights in this numerical example are described using the linguistic variables which are defined in Table 78.1.

Table 78.1 Linguistic variables for the relative importance weights of criteria

Linguistic variable	Interval type-2 fuzzy number (IT2FN)
Very low (VL)	((0,0.1;1), (0,0.5;1))
Low (L)	((0,0.3;1), (0.05,0.2;1))
Medium low (ML)	((0.1,0.5;1), (0.2,0.4;1))
Medium (M)	((0.3,0.7;1), (0.4,0.6;1))
Medium high (MH)	((0.5,0.9;1), (0.6,0.8;1))
High (H)	((0.7,1.0;1), (0.8, 0.95;1))
Very high (VH)	((0.9,1.0;1), (0.95,1.0,1))

Table 78.2 Linguistic variables for the ratings of criteria

Linguistic variable	Interval type-2 fuzzy number (IT2FN)
Very poor (VP)	((0,1;1), (0,0.5;1))
Poor (P)	((0,3;1), (0.5,2;1))
Medium poor (MP)	((1,5;1), (2,4;1))
Medium (M)/fair (F)	((3,7;1), (4,6;1))
Medium good (MG)	((5,9;1), (6,8;1))
Good (G)	((7,10;1), (8,9.5;1))
Very good (VG)	((9,10;1), (9.5,10,1))

Table 78.3 Weights of the attributes evaluated by decision-makers

Attributes	Decision-makers		
	D_1	D_2	D_3
C_1	H	VH	MH
C_2	VH	VH	VH
C_3	VH	H	H
C_4	VH	VH	VH
C_5	M	MH	MH

Moreover, all the relative importance ratings (i.e. the criteria values) in this numerical example are described using the linguistic variables which are defined in Table 78.2.

Assume that there are three decision-makers, D_1, D_2, and D_3 of a software company to hire a system analysis engineer and assume that there are three alternatives x_1, x_2, x_3 and five attributes "Emotional Steadiness" (denoted by C_1), "Oral Communication Skill" (denoted by C_2), "Personality" (denoted by C_3), "Past Experience" (denoted by C_4), "Self-Confidence" (denoted by C_5). Let X be the set of alternatives, where $X = \{x_1, x_2, x_3\}$, and let F be the set of attributes, where F = {Emotional Steadiness, Oral Communication Skill, Personality, Past Experience, Self-Confidence}. Assume that there are three decision-makers D_1, D_2, and D_3 used the linguistic terms shown in Table 78.1 to represent the weights of the four attributes, respectively, as shown in Table 78.3.

Then theses three decision-makers D_1, D_2, and D_3 used the linguistic terms shown in Table 78.2 to represent the evaluating values of the alternatives with respect to different attributes, respectively, as shown in Table 78.4.

Using the linguistic scales from Tables 78.3 and 78.4, and the eight steps of the proposed method (in Sect. 78.3), results for Chen [9] example is shown in Table 78.5. Table 78.5 shows the min value and max value from Step 6 and calculates the closeness coefficient $(CC)_i$ for each of alternatives.

Table 78.4 Linguistic of decision matrix

Attributes	Alternatives	Decision-makers		
		D_1	D_2	D_3
C_1	x_1	MG	G	MG
	x_2	G	G	MG
	x_3	VG	G	F
C_2	x_1	G	MG	F
	x_2	VG	VG	VG
	x_3	MG	G	VG
C_3	x_1	F	G	G
	x_2	VG	VG	G
	x_3	G	MG	VG
C_4	x_1	VG	G	VG
	x_2	VG	VG	VG
	x_3	G	VG	MG
C_5	x_1	F	F	F
	x_2	VG	MG	G
	x_3	G	G	MG

Table 78.5 Final ranking order

	Min	Max	Closeness coefficient, $(CC)_i$
x_1	2.3575	2.6364	0.4721
x_2	2.3835	2.5944	0.4788
x_3	2.3668	2.6216	0.4745

As shown in Table 78.5, results for the relative closeness of Chen's method [9] for three alternatives are 0.4721 for x_1, 0.4788 for x_2 and 0.4745 for x_3; which lead to the ranking of $x_2 > x_3 > x_1$. Chen [9]'s result coincides with the proposed results.

78.5 Conclusion

This paper distributed a linear assignment method which consisted with the SAW and LP method for IT2 FTOPSIS. This proposed method is able to produce an optimal ranking order of the alternatives. Besides, we provided a numerical example to analyze the applicability of the proposed method. The proposed method can capture the imprecise and uncertain decision information instead of the optimal ranking orders. Furthermore, the proposed method offers an alternative ways of ranking phase for IT2 FTOPSIS method.

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