Chapter 8
Model and Modelling

8.1 Introduction

The concept of model has deep historical roots and definitely it is not unambiguous. Especially it is used in both common life and science. In this book, we think of a model as a simplified representation of the reality. Depending on the goal of investigation, a human makes some simplifications intentionally leaving the real system without many details and many features, which are unnecessary from the awaited goal viewpoint.

The concept of model is very broad. In sociology, culture is treated as a collection of behaviour models and human activities. In linguistics, in the case of colloquial language, grammatical rules and writing are models.

The ideal scientific theory should include a collection of minimal axioms (principles and concepts taken without proof). One can obtain, basing oneself on the axioms, a solution of any problem with the use of formal logic, i.e. mathematically. It turns out that complexity of phenomena around us, as well as limitations originating from creative abilities of Man makes a scientific theory impossible to apply.

The examining processes proceeding in the reality is performed by means of suitable mathematical apparatus, based on the earlier chosen mathematical models. The structure of mathematical model is the key to describing phenomena and processes. It should be emphasized that any mathematical model is approximated and is not adequate to the process it describes.

When one constructs a mathematical model, one tends to catch the most characteristic features of the analysed process. On the other hand, the mathematical model should be quite simple and it should provide necessary information about the process under consideration. Consequently, some singularities of a process are completely taken into account, others at a certain degree and the rest is ignored. We say about the so-called idealization procedure and the success of the investigation strongly depends on this procedure.

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In order to make use of mathematical apparatus in practice, any problem should be simplified. Only experiments can verify the assumptions, the ones that should be verified not only physically but also mathematically.

One does not require experiments related to problems, which do not differ from each other too much, while qualitatively new problems require new experiments since the introduced simplifications can lead to physically unrealizable conclusions. During the idealization procedure it is necessary to introduce a kind of order of its elements by comparing them.

For example, if one of the system elements is 1 cm long then a natural question arises, namely: is this length small or large? Only initial formulation of the problem can give an answer to this question. It is obvious that when we examine the motion of an orbiting satellite, we can assume that 1 cm is negligible. On the other hand, if we consider distances between molecules then 1 cm is huge.

Let us take another example. It is well known that air is compressible. Does one always need to take into account the compressibility of air? Again, it depends on initially formulated problem. If an object moves in air at small velocity V, then we can neglect the compressibility. However, if the velocity is high, even close to the speed of sound or higher, then we must take into account the compressibility. In this case, it is convenient to introduce a dimensionless quantity $M = \frac{V}{a}$ called Mach number. It plays an important role in aerodynamics.

For example, as $M \ll 1$ one can use the idealized model of incompressible fluid, however at larger values of Mach number the compressibility of air should be taken into account. We have the similar situation during the building of a mathematical model and in other branches of science or technology, where other characteristic dimensionless numbers play an important role. These numbers are formed as combinations of three dimensionless quantities, such as: length L, time T and mass M. For the sake of convenience, one assumes that the dimension of the combination FT^2/ML equals 1 (where F is force). In other words, one of the quantities F, T, L and M can be chosen freely.

The concept of *model* can be defined in regard to a language L. In this case, the model consists of the universal set U containing all the objects under examination (symbols), which are significant from the goal of examinations viewpoint, and the map from L to U, called an interpretation function.

From the viewpoint of logic, a theory is defined as a consistent collection of sentences. The Austrian scientist Kurt Gödel stated the theorem of consistency, which says that a theory possesses a model if and only if it is consistent. This means that theory can lead to the contradiction. Gödel's theorem, proved in 1931, allows to verify correctness of a theory through the examining of its models. It follows from the Gödel's theorem that the theory T proves the sentence X if and only if each model of T fulfils X. Validity of a statement X is proved on the basis of statements belonging to the theory T, axioms and proving rules of classical logic.

Besides Gödel, the Polish mathematician and logician Alfred Tarski had an impact on the theory of models, called *logical semantics*. Tarski claimed that in a theory there must exist true statements, which are not proved by the theory. He emphasized the importance of meaning (*semantics*) of the examined statements and not only their *syntactical* relations. He pointed the attention that equivalence of syntactic and semantic consequences exhibits only internal equivalence of the theory but it does not allow to conclude about the validity of a sentence as a result of the analysis of syntactic relations.

In the 1960s of twentieth century, mathematicians and logicians obtained many significant results so that the theory of models isolated from logic as an individual science.

The reality around us is so rich that sciences which so far have not undergone absolute formalization and their results are based on deductive reasoning and intellectual evaluations dependent on subjective media. With regard to rich development of mathematics, a natural tendency of a researcher is to introduce a *mathematical model*, i.e. applying mathematical apparatus for description of the examined object or process. Most of the sciences undergo such an approach. One needs to mention such sciences as: mechanics, electrical engineering, physics, biophysics and biomechanics, biology, economy, sociology, medicine or even politics sciences.

Most of the examples given in this book and concerning the modelling will be related to applied sciences. Nowadays, it is difficult to imagine life without engineering achievements based on mathematical fundamentals. This concerns civil engineering, construction of airplanes and spaceships, transport of people and resources, production of machines and mechanisms, control of complex technological processes, novel solutions in bioengineering, or various solutions in electronics and high technology.

With regard to rapid changes and achievements in science, particular scientific disciplines underwent the transformations, which originate from the expectations imposed by the industry, as well as tendency to being interdisciplinary. The field of interests of mechanics ranges from atomic to astronomic phenomena. The tools of mechanics are used by physicists, biologists, chemists, ecologists, physicians, economists and even cryptologists which proves its interdisciplinary.

There exist indissoluble relations between mathematics and mechanics in historical course and one pointed out how the development of one of the mentioned scientific disciplines positively influenced the development of the second one. For example, problems appearing in a field of interests of mechanics stimulated the development of such branches of science as theory of dynamical systems or theory of optimization. Mechanics was a source of rise of individual scientific disciplines such as theory of control or theory of bifurcations and chaos.

Mechanics, in the sense of Gödel's approach, is a theory represented by the system hierarchically ordered. The highest level of the hierarchy is usually connected with a description of the whole "macro-body" with the use of time-varying parameters. On technical purposes, from mechanics one isolates mechanics of materials, solids mechanics and fluids mechanics.

Mechanics makes use of classical tools of fundamental physical laws, when relatively small number of material points is taken. While the number of these objects increases (in the case of molecules and atoms) then mechanics makes use of phenomenological theories. It turns out that there is no model, which describes transition on the basis of the knowledge concerning particular basic elements of structures such as atoms and molecules. Mechanics allows for explaining many biological or medical problems such as biological structures or hemostaza processes. The attention should be paid to biomechanics—a new and dynamically developing branch of mechanics.

It should be emphasized that examining any continuum medium is connected to the analyses of fields of displacements, deformations, temperature, velocity, acceleration and stress, but these fields vary in time randomly.

The outlined portrait of contemporary mechanics points the difficulties related to the theory of modelling. Moreover, modelling in this domain of science, supported by computational methods, gives hopes connected with solving of many courageous problems in the future such as: predictions based on computer simulations of properties of natural objects, working out new and complex models of continuum media, working out transition techniques from nano to micro scales, modelling of biological systems or yielding new materials of the desirable properties.

8.2 Mathematical Modelling

Let us start from the definition given by Gutenbaum [107]. A mathematical model is a collection of symbols and mathematical relations, and the rules how to operate them. The mentioned symbols and relations refer to specific elements of the modeled part of the reality. Interpretation of symbols and mathematical relations made a model come into being. Mathematical modelling is characterized by two essential features:

(i) *interdisciplinarity* (introducing the language of formal logic enables to examine many various scientific disciplines);
(ii) *universalism* (the same mathematical model can be applied in different branches of science).

Great scientists of eighteenth and nineteenth centuries pointed that applying the rules of mathematics and logic awaited empirical verification. Since mathematics and logic is a product of the human mind, and the latter undergoes evolutionary process according to the laws of the nature, then one can expect that the product of the mind is also consistent with the nature. Consequently, the product possesses a corresponding real element of the being.

There are many examples of application of outwardly pure abstract products, e.g. non-Euclidean geometry or Boole algebra. The aim of mathematical modelling is to describe our *reality*. An *experiment* is an ultimate verification of modelling correctness.

In general case, a sequence of events leading to appearance of a mathematical model is depicted in Fig. 8.1.

Mathematical modelling is successful and reaches sciences, which have been regarded as being resistant to mathematizing as e.g. history. Geometry

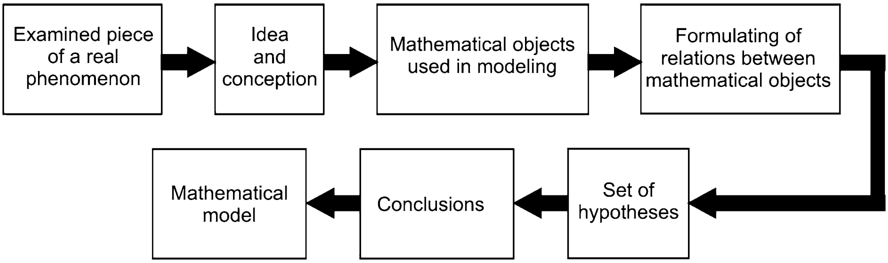

Fig. 8.1 Thinking process leading to the appearance of a mathematical model

undergoes unbelievable mathematical regime, as Spinoza (1632–1677) emphasized. He presented his philosophy in "a geometric manner" using definitions and axioms.

It was Galileo (1564–1642), who introduced mathematics into physics and Lavoisier (1743–1794) introduced mathematics into chemistry. In order to familiarize with the modelling procedure we will focus on mechanics.

8.3 Modelling in Mechanics

In order to examine an object or a phenomenon we are interested in, first we must build its physical model. A physical model is understood as a conception of physical phenomenon description and determination of parameters having the influence on its course.

The model should accurately represent processes proceeding in the real object. On the other hand, the model should not be too large. The next stage of modelling is the building of a mathematical model that is understood as mathematical formulation of physical laws. A mathematical model can be represented by algebraic equations, ordinary or partial differential equations, differential–integral equations, difference equations or difference equations with a delayed argument.

Although a mathematical model of a phenomenon should describe real conditions as accurately as possible (modern computational methods enable to analyse very complicated models with nonlinearity, stochasticity and various types of external interactions involved), it can happen that the costs connected with the analysis of so complicated problems are very high. Moreover, sometimes very simple models are sufficient to describe a phenomenon.

In the case of analysis of construction, one can treat the analysis as a system of coordinates, which are decision variables (constructional) and parameters from mathematical viewpoint. Moreover in the case of modelling of construction, one often makes use of a simplified model at the preliminary stage, while in final stages the model is more complicated.

The goal of mathematical description of construction is mathematical formulation of all constructional features, i.e. geometrical, material and dynamical ones. On the other hand, one needs to remember that constructional conditions make functional limitations. A complete mathematical model of construction covers determination of decision variables, determination of a set of good constructions and creating an optimization criterion—an objective function.

Mathematical models of construction can be classified as follows:

a) deterministic—all parameters are determined and each decision corresponds to one value of the objective function;
b) probabilistic—one or a few parameters are random variable with known probability distribution;
c) statistic—one or a few parameters are random variable with unknown probability distribution;
d) strategic—one or a few parameters can take on one of the determined values.

It is worth noticing that dynamical processes proceeding in a physical model are not accurate reflection of a real phenomenon but they rather present the actual knowledge about the examined phenomenon.

The differences between a real process and a process proceeding in a physical model are called disturbances. By the notion of disturbances we understand unknown or known influence of small changes of the system parameters on the process. This influence is intentionally ignored.

In order to understand the concept of disturbances better, let us consider an example of mechanism with ideally stiff links. Assumption of such a model is a kind of approximation, since in a real mechanism there are many (omitted) phenomena described by equations of theory of elasticity, rheology, fluid mechanics and others.

A mechanism with susceptible links can make the same movements as a mechanism with stiff links (classical), but additionally these movements can interfere with vibrations, which are connected with deformability of the constraints. The motion of a classical mechanism, called the base motion, is perturbed by additional motion, namely vibrations in this case. This additional motion will be called a disturbance of the base motion.

This observation also led to an original method applied in a new branch of mathematics and physics, namely the asymptotology. Due to the introduction of a naturally occurring quantity, namely a small parameter (perturbative), there exists a possibility of suitable separation of equations describing a modeled phenomenon into several systems. The systems describing processes, which differ in terms of only magnitude of some characteristics can be solved separately, that significantly simplifies the computing procedure. Next, the obtained particular solutions are combined into asymptotic series.

The next stage of modelling is the analysis of equations describing dynamics of machines. The analysis is performed in various ways, depending upon a type of the equations. Generally, one can isolate some essential steps applied at this stage. First, one seeks analytical solutions of such equations. One can do it if one deals with a linear mathematical model. It turns out that even very simple nonlinear systems do not possess unique solutions or it is very difficult to find solutions. Then, one uses analytical methods for finding approximated solutions based on elementary functions.

Such solutions allow for at least initial to familiarize with qualitative and often with sufficient accuracy quantitative dynamics of the system. However, so far numerical methods have been widely used in the analysis of the equations. On a present level of development of calculating machines (computers) and numerical methods, very many problems of dynamics of machines can be successfully solved.

Finally, the last stage of modelling covers the choice of system parameters so that one achieves the required dynamical properties. The choice is made within a framework of *synthesis* and *optimization*.

The next step, after synthesis and optimization, is a construction of a machine or a mechanism, which should be optimal with respect to the chosen optimization criteria. The detailed principles of constructing follows:

(a) functionality;
(b) reliability and durability;
(c) efficiency;
(d) lightness;
(e) cheapness and availability of materials;
(f) producibility;
(g) easiness of usage;
(h) ergonomicity;
(i) agreement with current norms and regulations;
(j) corrosion-proof and resistant to changes of temperature;
(k) aesthetics.

The optimization procedure can be defined as the choice of an element from a given set with regard to an order in this set [94]. Different variants of construction or control of particular processes can be included in the set. The set of solutions is usually bounded to the set of admissible solutions. Numbers (Euclidean space) or functions (Hilbert space) are components of a vector of decision variables.

The operator transforming an admissible set ϕ (or space of solutions E_x) into another set ϕ_q (or quality space E_q), which is called a set of attainable objectives is said to be an optimization criterion Q. The optimization criterion can be treated as a hypersurface located over the admissible set.

If $E_q = R^m(m = 2, 3, \ldots)$, then the operator $Q(q_1, \ldots, Q_m)$ associates each element belonging to the admissible set (ϕ) with m numbers characterizing the quality of the element. This is a job of poly-optimization, since m components of a vector of the function Q correspond m different optimization criteria. An element of a subset of m-dimensional Euclidean space $Q(x) = (q_1(x), \ldots, Q_m(x))$, called an *objective vector* for a fixed vector of decision variables, is a set of m numbers determining the value of particular quality coefficients. It is worth noticing that dimension n of a vector of decision variables does not depend on dimension of a vector of quality coefficients m.

From mathematical point of view, by optimization we understand finding such an element $x^* \in \phi$, that for $x \neq x^*$ the values of $Q(x)$ are not less than the values of a quality functional at the optimal point x^*, i.e. $(x^* \in \phi) : \wedge_{x\in\phi} Q(x) \geq Q(x^*)$.

The point $x^* = (x_1^*, \ldots, X_n^*)$ is said to be optimal, if the components $x_1^*, \ldots, x_n^*$ are optimal values of particular decision variables.

Depending on ϕ and Q we have to do with various optimization tasks. If an admissible set has a finite number of elements, then a problem of optimization is *discrete*. A problem of optimization is continuous if we have to do with continuous variation of decision variables. When we have to do with both mentioned tasks of optimization, then problems of optimization will be *mixed*.

Problems of optimization can be classified as problems with and without limitations, and static or dynamic problems. The case of static analysis in E^n belongs to a problem of linear or mathematical programming. If the dimension of Euclidean space is 1, then we have to do with minimization of a function of single or multi variables in one direction.

In the case of *dynamical* optimization, admissible solutions are elements of infinite-dimensional space, i.e. they are functions of independent variables. In practice, we mostly have to do with optimal choice of controlling, while an admissible set is given by equational and inequational constraints, and differential equations of a state.

Optimization problems can be classified with respect to the way they are solved. Generally, optimization problems can be classified as partial problems, and then we say about the *decomposition* of a problem. *Iterative* and *statistical* methods belong to the computational methods of optimization.

Besides the mentioned theoretical analysis, one should perform experimental research. The goal of such an examination is identification of elements of a physical model, identification of a mathematical model and verification of theoretical considerations. The experimental research is often performed in parallel with the theoretical research.

By the occasion, it is worth paying attention to significance of modern research techniques supported by reliable mathematical methods. As an example one can mention constructions consisting of non-uniform shells of various thickness. It turns out that the most reliable results can be obtained by using modern experimental methods.

There is a scheme of elements of modelling in dynamics of machines depicted in Fig. 8.2.

One distinguishes two classes of physical models. *Structural models*, belonging to the first class, whose organization is similar to organization of the analysed object and there exists correspondence between the elements of the object and model.

In the case of mechanical models, one distinguishes continuous and lumped models. The first ones are systems with continuously distributed parameters and they are usually described by means of partial differential equations, integral or differential-integral equations.

Lumped mechanical systems are described by ordinary differential equations and are simpler than continuous ones during the analysis. That is why continuous systems are often approximated by discrete systems. Moreover, one distinguishes linear and nonlinear models which are described by linear and nonlinear equations respectively. Usually, linear systems are approximations of nonlinear ones because the world outside is nonlinear.

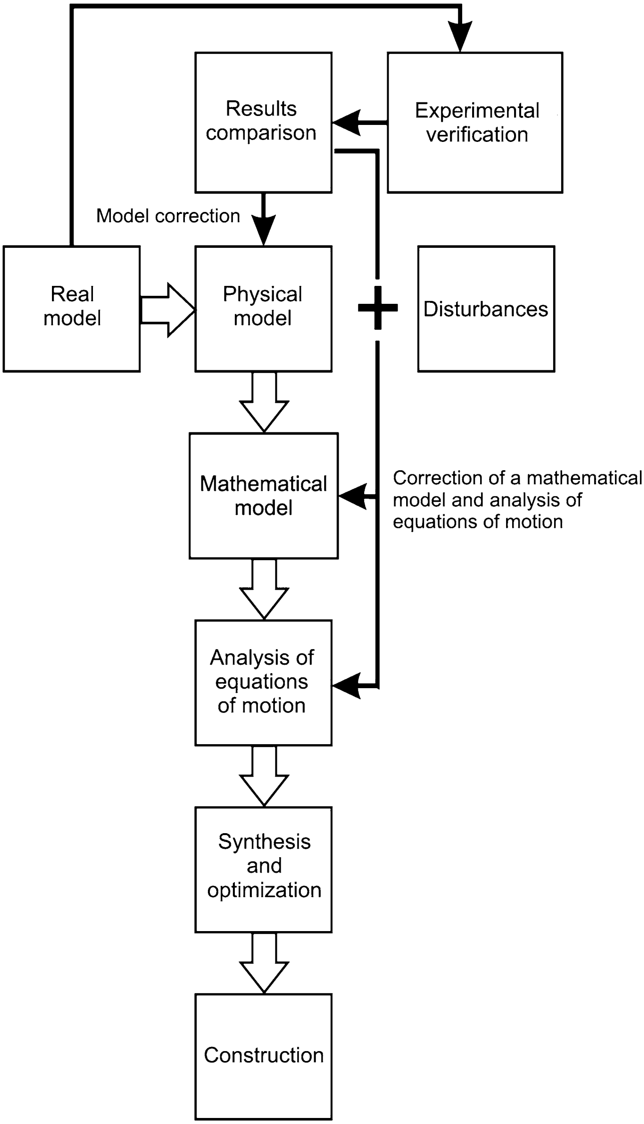

Fig. 8.2 Scheme of modelling stages in dynamics of machines

It is worth emphasizing that we will consider rather simplified mathematical models generated by problems related to nonlinear dynamics of machines. Moreover, we will show, even in regard to complicated systems of nonlinear equations, how to obtain much information, desired by engineers concerning the behaviour of the systems with the use of an associated simplified model, which is very often a system of linear differential equations.

Further considerations will be carried out on the basis of a mathematical model described by the equations

$$\frac{dy_s}{dt} = F_s(t, y_1, \ldots, y_n), \quad s = 1, 2, \ldots, n, \quad n \in N. \tag{8.1}$$

This is a very general formulation of equations. The processes proceeding between ideally stiff bodies, ones connected by a massless system of stiffness and damping can be brought to these equations. The system (8.1) needs more discussion. First of all, it is in normal form i.e. derivatives of the quantities are on the left-hand side of the equation. It can happen that the system $F_s(t, y_1, \ldots, y_n, \dot{y}_1, \ldots, \dot{y}_n) = 0$ is implicit, however such problems will not be discussed here. Most of methods and textbooks on numerical solutions of ordinary differential equations concern a system of the form (8.1). Moreover, the right-hand side of the system (8.1) is continuous, thus some of the processes proceeding in dynamical systems are not modelled by the system (8.1).

The techniques of transforming mechanics problems into a system of differential equations (8.1) are described in many textbooks on mechanics and will not be discussed here. With regard to a problem or needs of a user, on the medial stage leading to the system (8.1) one uses Lagrange's equations of the first or second kind, Lagrange's equation with multipliers, Boltzmann–Hamel or Maggi's equations (see [12, 13]).

The state of a dynamical system is described by $y_1, \ldots, y_n, t$, where t designates time, while $y_1, \ldots, y_n$ for each fixed $t \geq t_0 \geq 0$ take on real values. The Cauchy problem connected with the system (8.1) requires a "starting point", i.e. imposing initial conditions of the form $y(t_0) = y_0$, thus for the initial instant a state of the system is known, i.e. $y_{10}, y_{20}, \ldots, y_{n0}$.

Moreover, assume the function F is defined, continuous and its first derivative with respect to y is continuous as well for all $t \in I = t : t \geq t_0 \geq 0$ and $||y|| < \infty$, where $|| \cdot ||$ denotes the norm.

The mentioned assumptions ensure for each $t_0 \geq 0$ and $||y|| < \infty$ local existence and uniqueness of solutions as well as their continuous dependence on the initial instance y_0 in the finite time interval.

Moreover, we will deal with only such a class of equations for which solutions can be extended to $+\infty$, and this covers most of mechanical systems. Assume we know a particular solution to the system (8.1)

$$y_s = \varphi_s(t), \; s = 1, \ldots, n, \tag{8.2}$$

while $\varphi_s(t_0) = \varphi_{s0}$. This solution for $t > t_0$ means that for each value of t the instantaneous state of the system is generated by the initial state.

This raises a question: can one, on the basis of proximity of two states (proximity of two sets of initial conditions), come to conclusion about the distance between these two states as $t \to \infty$. Theory of *stability* of motion deals with such problems.

8.4 General Characteristics of Mathematical Modelling of Systems

One can distinguish the following types of models of systems connected with the goals of modelling:

(i) phenomenological model—describes and explains functioning of a system;
(ii) prognostic model—allows for predicting the behaviour of a model in future even when there are different conditions of environment interactions;
(iii) decision model—allows for a proper choice of input interactions satisfying required conditions;
(iv) normative model—allows for a proper choice of parameters and structure of a model for realization of particular tasks.

As one already mentioned, mathematical models should be compatible with the modeled system and easy in usage on the basis of the modelling procedure in mechanics. *Verification* and *validation* of a model plays a crucial role during the modelling procedure. Comparison of results obtained during the modelling procedure with a modeled object conserved (piece of the reality or more complex medial model) with respect of theory and experiment is called verification of a model.

The following criteria decide about consistency between an original and model:

(i) *Internal*, based on *formal* consistency (no logic and mathematical contradictions) and *algorithmic* (correctness of functioning of computational algorithms);
(ii) *External*, based on *heuristic* consistency (ability of interpretation of phenomena or verification of hypotheses and formulation new research task) and *pragmatic* (evaluating whether a built model is good).

In 1976 Zeigler [251] formulated the following types of consistency:

(i) *Replicative*—relies on evaluation of consistency with the same data as one used during identification of models;
(ii) *Predicative*—relies on evaluation of consistency with the modeled system under another conditions than during the performance of identification of a model;
(iii) *Structural*—relies on preservation of consistency also with respect to the structure of a model.

8.5 Modelling Control Theory

Now, we deal with modelling in automation and control theory which can be successfully applied in scientific disciplines such as: mechanics, electronics, physics, civil engineering, chemistry, chemical and process engineering, biochemistry and biomechanics, materials science and others. Moreover, many conceptions of the mentioned sciences can be easily transferred to another system, which is an object of considerations of cybernetics, psychology, linguistics or other arts. This chapter is based on the textbook [37].

Classification of control systems is performed on the basis of assumed criteria. One of the most important criterions is the one of required initial information (a priori) about a controlled system. According to this criterion, automatic control system can be divided into:

(i) ordinary control systems;
(ii) adaptive control systems;
(iii) distributed control systems.

Ordinary control systems require some initial information about a controlled process, i.e. detailed information about properties, equations, characteristics and parameters of the process before the start of controlling. Such systems are characterized by a constant structure and time-invariant values of parameters of particular elements during the system operation.

Ordinary control systems are widely discussed in literature and applied in practice. Since conditions variation of many physical processes is slow in time enough, it is sufficient to apply ordinary control systems (instead of more complicated control systems, namely adaptive ones).

In Fig. 8.3 one presented classification of ordinary control systems.

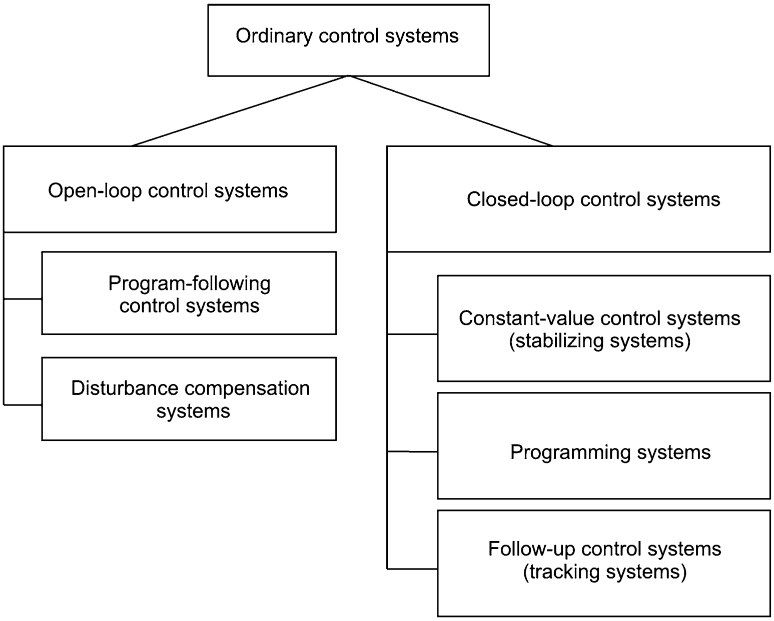

Fig. 8.3 Classification of ordinary control systems

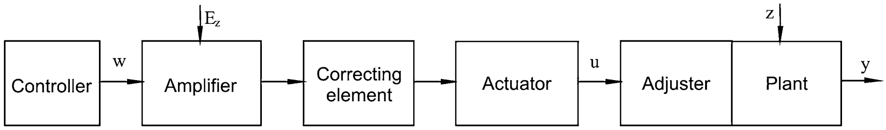

Fig. 8.4 Block diagram of an open-loop control

8.5.1 Ordinary Automatic Control Systems

Principles of operation and basic structure of control systems of different types will be presented in block diagrams.

8.5.1.1 Open-Loop Control Systems

Open-loop control systems constitute a group of systems, in which there is no feedback, which makes the input dependent on a selected output quantity. This type of systems requires complete initial information about a controlled system (process) with regard to the lack of feedback. The operating information is contained mainly in controlling and disturbing quantities. Open-loop control systems can be divided into open program-following control systems and open systems with disturbance compensation.

Principle of operation of an open-loop control system is depicted in Fig. 8.4.

Such systems perform jobs in order, preset by a controller, independently of the state of a controllable quantity y and disturbances acting on a plant (an object to be controlled). A correcting element in the system is necessary, when the response y_z of the object to the disturbance z differs from the response y_u of the object on the control signal u, and this can occur in both stationary and transient states. An amplifier is an element of the system which amplifies its output by means of the energy E_z taken from outside.

Open program-following control systems very often occur in processing industry in machines such as: numerically controlled machine, cyclic automata, etc. The information (program) is stored in controllers (memory devices), e.g. magnetic drums, perforated tapes or diskettes. The information in digital form is delivered to actuators and ensures the preset order and parameters of the procedure. In the similar way mechanical copy devices operate, in which a program is contained in a suitable type of cam mechanisms. Another example of an open-loop control system is control of the traffic lights.

A block diagram of an open system with disturbances compensation is depicted in Fig. 8.5.

Open systems with disturbance compensation are used to reduce undesirable influence of the environment (disturbances) on a process or object to be controlled by means of measurement of these interactions and compensation, which is performed by the additional inverse interaction on the object. The operating information is

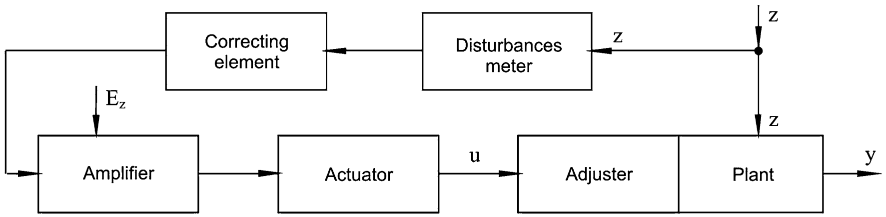

Fig. 8.5 Block diagram of open-loop control system with disturbance compensation

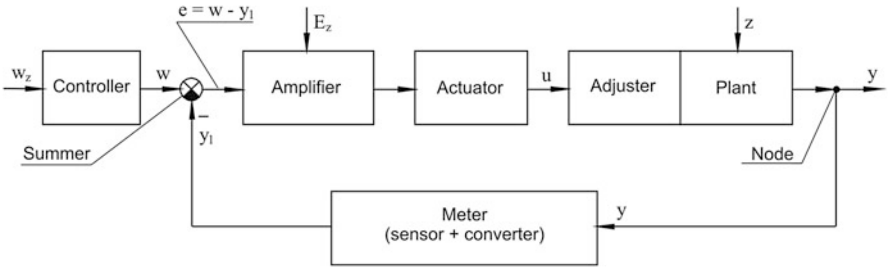

Fig. 8.6 Block diagram of a constant-value control system with a regulator of indirect operation

contained in disturbing quantities, but one needs to choose only those disturbances which influence the process most, since it is impossible to cover all disturbing interactions. As an example of this kind of systems, one can mention thermo-compensation systems in sensitive measurement devices.

8.5.1.2 Closed-Loop Control Systems (Automatic Control Systems)

Closed-loop control systems operate on the basis of the negative feedback principle, in other words measurement of the output quantity of an object (controllable quantity) and comparison of this result with the preset value of this quantity. Hence, a signal of control deviation arises. So the operating information is contained in the controlled signal itself. The initial information about a process to be controlled must be moderately complete, however less so than in open systems.

A control system with single controlled quantity and single feedback loop is called a one-dimensional system, while systems with many controlled quantities are called multi-dimensional.

Closed-loop control systems can be classified as follows: constant-value automatic control systems (stabilizing), program-following automatic control systems and follow-up control systems (tracking systems in other words).

A block diagram of a simple constant-value control system is depicted in Fig. 8.6.

As one can see in Fig. 8.6, the system possesses a backward feedback loop running through the meter of the controlled quantity y. Thus, it is a closed system.

In the main loop of the system there is an amplifier supplied with the energy E_z besides an actuator, adjuster and plant.

In consequence, the signal power from the controller is amplified and such systems are called systems with a controller of indirect operation. Otherwise, if a regulator does not amplify signals (there is no amplifier), then we have to do with simple systems with a regulator of direct action. A characteristic feature of constant-value control systems is constant value of the presetting quantity ($w(t) = \text{const.}$), which can be varied (if necessary) by means of various inputs w_z adjusted in the controller by means of handwheels. On the preset level w of the controlled value y the system will stabilize its course despite the undesirable disturbance z interaction on the object.

Control systems containing one loop of control are called one-loop systems. Such a system is depicted in Fig. 8.6. However, both one-dimensional and multi-dimensional systems can be multi-loop.

A set of elements of a simple regulator of indirect operation (see Fig. 8.6) is not always sufficient in practice. One introduces correcting serial and parallel elements into the system in order to ensure the necessary stability of operation, suitable quality and accuracy of control.

A block diagram of such a stabilizing one-dimensional system (which is then a one-loop system) is depicted in Fig. 8.7.

Constant-value control systems are the largest group of control systems, which are encountered in practice.

The second group of closed-loop control systems is different from stabilizing systems by that the presetting quantities $w(t)$ (preset values of controlled quantities) vary in time according to the program introduced by the service of the controller. Thus, these quantities are known, programmed functions of time ($w(t) = f(t)$).

A block diagram of the program-following control is similar to that in Fig. 8.7 but instead of an ordinary controller a program controller occurs. The controllable quantity y varies according to the program stored in a controller. Follow-up control systems do not possess a controller. The presetting quantity $w(t)$ is a random, previously unknown function of time ($w(t) = f_l(t)$). The controlled quantity y follows or tracks the course $w(t)$.

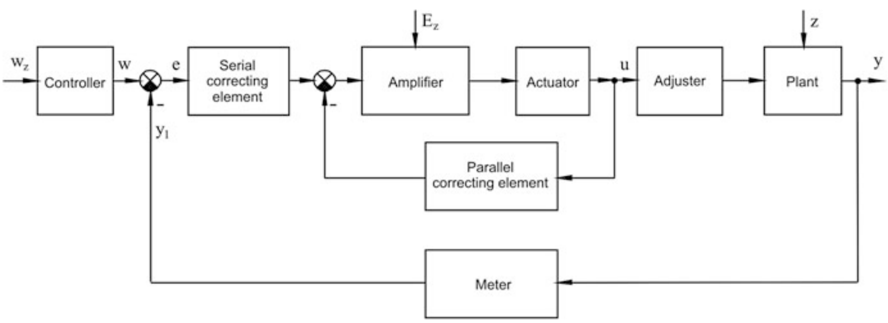

Fig. 8.7 Block diagram of a stabilizing system with a serial and parallel correcting element

In practice, one can also meet combined systems (closed-open), where feedback loop occurs with section of disturbances measurement.

8.5.1.3 Adaptive Automatic Control Systems

Adaptive control systems are such systems in which required initial information about a controlled process or object does not have to be complete. This means that in order to ensure the required accuracy and quality control, these systems require smaller range of the initial information than ordinary systems do. It follows from the fact that adaptive systems have an ability to adapt to such changes of the object operating conditions, which result in a change of its parameters or characteristics.

Activities connected with an object control in these systems rely on continuous or periodic examination of the object, and this results in completing the initial information.

Cars (changes of the tractive adhesion coefficient, changes of intensity of wind) and airplanes (changes of flight parameters according to altitude or weather conditions) are examples of objects, whose characteristics change as the operating conditions change.

Another type of objects of variable properties are objects with characteristics possessing extrema. The objective of an adaptive system is to keep an object operating point at maximal point of this characteristic. An example of such an object can be a radio receiver, whose circuit must be aligned to the frequency of a received electromagnetic wave in order to obtain a maximal value of intensity of the received signals. Another example is a water turbine, whose efficiency coefficient η under different loads Z_i varies according to curves possessing maxima according to the value of angle α of blades configuration (Fig. 8.8). Keeping a turbine operating point at maximal efficiency requires an adaptive system.

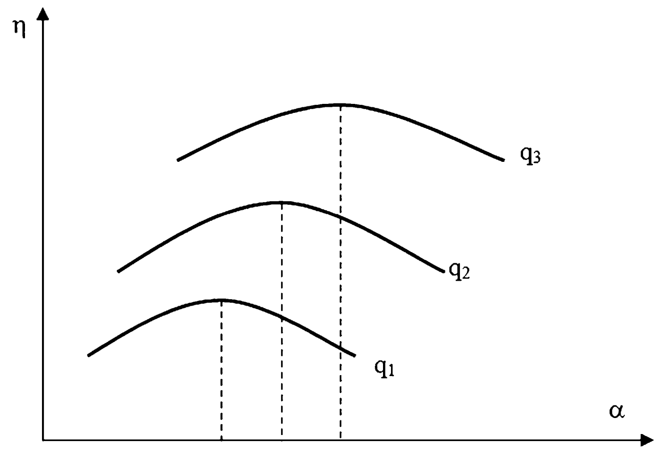

Fig. 8.8 Extremal characteristics of a water turbine

Adaptive systems can be divided into extremal control systems, control systems with self-adjustable correcting devices and self-optimizing systems. Extremal adaptive control systems refer to objects of characteristics, which possess extrema. The operating information in these systems is deviations from an extremum of a particular function of a single or multiple variable and it is not necessary to know precise position of the extremum at the beginning. In fact, it is sufficient to know a type of the extremal function and occurrence of the very extremum. The system gains the lacking information about position of the extremum during the seeking procedure.

Adaptive systems with self-adjustable devices allow to ensure the stability and required quality of control under incomplete knowledge of parameters and characteristics of a plant.

These systems are classified as follows: systems with open adjustment loops, closed adjustment loops and automatic control of object characteristics, and systems with extremal adjustment of correcting devices. Additional operating information in these systems is: information about disturbances influencing the parameters of a plant, information about deviation of transient processes in the main loop and information about deviation of quality control from the extremum of this quality.

Self-optimizing systems are a kind of control systems, which improves their characteristics during the control. The operating information, which concerns the existing interactions, and conclusions about necessary changes of the characteristics are gained by the system during the operation. Such processes as: prediction of occurrence of a random event, classification of complex situations, etc. are the basis of control algorithm of such systems. These systems are used in e.g. remote control and optimization of flight trajectories of rockets.

8.5.2 Distributed Automatic Control Systems

Distributed systems constitute a group of systems, which are meant to control motion of a large number of elements, means of transport or elements of communication network, etc.

Control in such systems is like playing game between two sides, from which one side is controlled by the system (computer) but the second side is not. The control is to ensure the most useful kind of motion of particular elements according to a criterion called a success function. This function can be e.g. required amount of transported goods in determined terms at the smallest costs of the transport. A system which is controlled is a set of means of transport, and the other side, the uncontrolled one, is a set of goods requirements, which pour in from various regions of a territory. In such systems, the required initial information about the uncontrolled side is poor. Only in successive game stages (control) the system gains the operating information about the other side.

A block diagram explaining the distributed system functioning is depicted in Fig. 8.9.

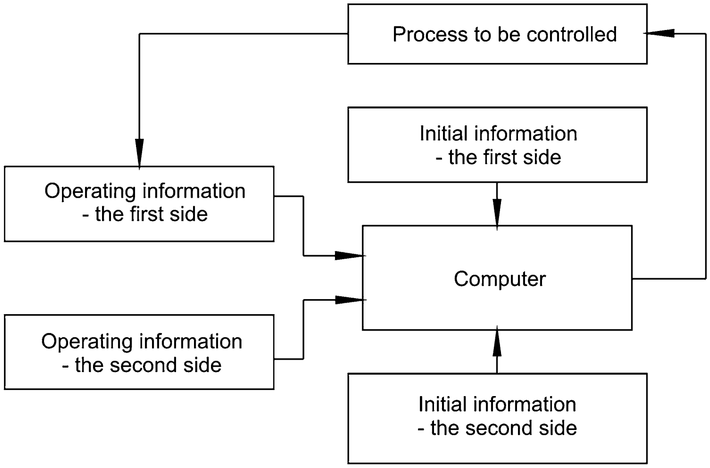

Fig. 8.9 Scheme of functioning of a distributed control system

8.5.3 Classification of Control Systems with Respect to Another Criteria

Besides the main criterion of classification of control systems with respect to initial information about a process to be controlled, it is advisable to discuss another classification criteria, which are used in theory and practice.

8.5.3.1 Classification with Respect to Purpose

It covers control systems or automatic regulation systems of: level, intensity, flow, power, voltage, angular velocity, pressure, temperature, humidity, etc.

8.5.3.2 Classification with Respect to Type of Energy of the Control Factor

It covers control systems such as: mechanical, electrical, electronical, hydraulic, pneumatic, electro-hydraulic, electro-pneumatic, chemical systems, etc.

8.5.3.3 Classification with Respect to Types of Signals Occurring in a System

We distinguish here: continuous systems, in which all occurring signals are continuous functions of time and discontinuous systems (discrete), in which at least one element generates a discrete quantized signal in time.

The following systems belong to the discrete systems:

(i) impulse systems, in which quantized in time signals occur, and this means that output quantities (coming from impulsators) appears only in pulsing instants (e.g. each 1 second), while the width of the pulse can be constant and the height can be variable or otherwise. One encounters impulse systems of variable period of pulsing;
(ii) relay systems, in which output signals of relays attain only two (e.g. zero–maximum) or three (minimum–zero–maximum) values, while transition from one to another level is done, when the input signal of a relay exceeds so-called switch point.

So-called finite automata belong to the class of discontinuous systems. These are systems, which take on (under influence of external signals and internal couplings) finite number of specific states, whose change obeys deterministic or stochastic rules (algorithms). Thus, a finite automaton will be a simple relay as well as digital mathematical machine. Complex finite automata can serve for modelling of neural networks of alive organisms.

8.5.3.4 Classification with Respect to Linearity of Elements

Linear systems, in which all elements have linear static characteristics (dependencies between inputs and outputs in stationary states) and are governed by linear differential, integral or algebraic equations. These are systems, for which one can apply the superposition principle, i.e. a complete output signal of the system with simultaneous action of several input signals is equal to the sum of output signals of the system.

Nonlinear systems are such systems in which there is at least one nonlinear element, and this makes that the whole system can be described by means of nonlinear equations, whose analysis is more difficult than the analysis of linear equations.

In the reality, one does not meet ideal linear elements. However with good approximation, one can often linearize for technical purposes nonlinear characteristic in neighbourhoods of operating points of an element.

8.5.3.5 Classification with Respect to Character of System Parameters

In this case we distinguish the following systems:

- **stationary**, whose parameters do not vary in time. In the case of stationary linear systems, they are described by linear equations of constant coefficients;
- **non-stationary (parametric)**, whose parameters clearly vary in time, mostly according to known functions of time. Equations describing these systems are so-called parametric equations, whose coefficients are known functions of time, e.g. harmonic functions.

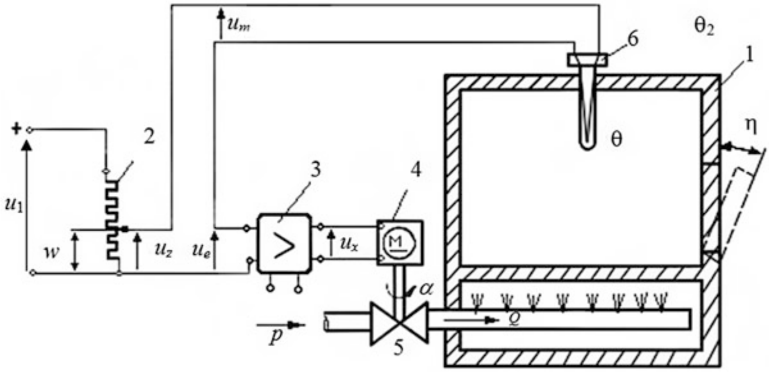

Fig. 8.10 Scheme of a temperature control system in a gas furnace

8.5.4 Examples of Control Systems and Their Block Diagrams

Classify elements occurring in control systems and make block diagrams (structural) of control systems, which are presented in schematic block diagrams and described with respect to their destination and operation. Moreover, determine the type of the considered control systems.

Example 8.1. Description of the system functioning.

The presented system (Fig. 8.10) serves for controlling the internal temperature of a furnace (1). It is achieved through the change of intensity Q of the heating medium (gas) delivered to a valve (5) under constant pressure p. The degree of valve opening is controlled by the electric engine (4), whose angle of rotation depends on the delivered voltage u_x, which is amplified in the amplifier (3) by the voltage u_e. Voltage u_e is a difference between the voltage u_z on the resistor (2) and the one u_m on the temperature sensor (6). So, the value of temperature θ can be changed by the service through adjusting position of the slider of the resistor (2), which is under constant voltage u_1.

After the analysis of the system functioning one can say that temperature control is carried on in a closed loop of signals, since the signal u_m from the measurement of the temperature θ backward-influences the value of this temperature, which in turn depends on the voltage u_z set on the resistor (2) by the service. The considered system is a constant-value automatic control system, in other words a stabilizing system.

Essential Elements in the System

1. Plant—a furnace (1), in which a process of temperature θ control proceeds by means of variation of inflow intensity Q of the heating medium.

2. Controller—a potentiometer (2); with its help the service can preset the required value of the temperature θ in the furnace (1) adjusting the potentiometer to the proper voltage value u_z.
3. Summer—an electric circuit of thermoelement connections (6), a potentiometer (2) and an amplifier (3).
4. Amplifier—an electrical amplifier (3), serves for amplifying a difference u_e voltage u_z and u_m to the value, which allows to start the engine (4).
5. Actuator—an electric engine (4), serves for changing the degree of valve opening (5). Elements mentioned in points 2–5 constitute a regulator.
6. Adjuster—a valve (5), serves for variation of inflow intensity Q of the heating medium.
7. Sensor—a thermoelement (6), serves for measurement of the temperature, which is transformed into voltage u_m.

Essential Quantities Occurring in the Considered Temperature Control System

1. Controllable quantity—temperature θ inside the furnace.
2. Input—position "w" of the potentiometer slider.
3. Presetting quantity—voltage u_z on the potentiometer.
4. Processed quantity (measuring)—voltage u_m on the thermo-element.
5. Control error—voltage u_e, which is a difference between the voltages u_z and u_m.
6. Controlling quantity (adjusting)—angle α of rotation of an electrical engine shaft (4) adjusting a valve (5).
7. Adjustable quantity—inflow intensity Q of the heating medium through the valve (5).
8. Disturbances—opening of the furnace (2), the external temperature θ_z, eventual changes of the supply pressure p.

Basing oneself on the analysis of signal courses in the considered system, one made its block diagram (Fig. 8.11).

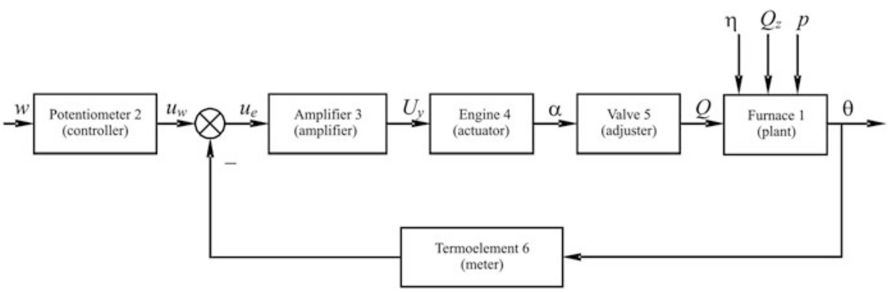

Fig. 8.11 Block diagram of the system from Fig. 8.10

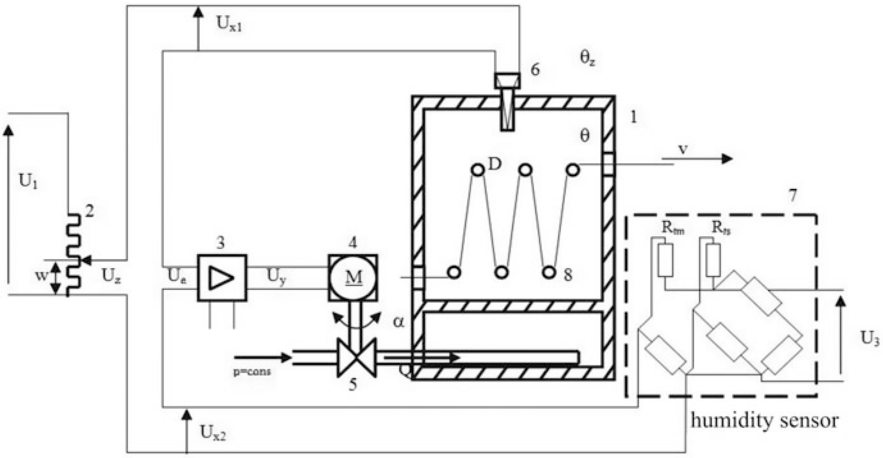

Fig. 8.12 Scheme of a system of humidity control of a material band

As one can see, the considered control system is a one-loop constant-value automatic control system (stabilizing), since there is a negative feedback and the presetting quantity U_z does not vary during the system operation.

Example 8.2. Description of the system functioning.

The presented system in Fig. 8.12 is meant for regulation of tapes humidity of material moving by means of system of rolls at velocity v. Functioning of this system is as follows: we insert the material (8) containing some amount of humidity into a drying chamber (1). The material containing some amount of humidity is subject to drying air flow of temperature θ measured by a sensor (6). The humidity D of the material is measured at the dryer output (1) by means of the humidity sensor (7). Signals from sensors (6) and (7) in a form of respective voltages U_{x1} and U_{x2} are subtracted from the adjusted voltage U_w on the potentiometer (2). The difference U_e amplified in the amplifier (3) arises, then it starts the engine (4), which change the degree of valve opening (5), and consequently the flow intensity Q. The value of temperature θ of the heating medium, which influences the temperature inside the dryer depends on the flow intensity Q. The temperature inside the dryer has an influence on the vaporization intensity of humidity in the material.

After the analysis of the system one can say that stabilization of humidity of the tape (8), coming out the dryer chamber (1) is made through the variation of the temperature θ inside the chamber. The signal coming from the temperature sensor (6) and the signal from the humidity sensor (7) are summed up with the input signal in the summer. Thus, we have to do with a double-loop system of humidity control.

Essential Elements in the System

1. Plant—a furnace (1) together with a material (8).
2. Controller—a potentiometer (2).
3. Summer—electric circuit of the thermoelement connections (6), a humidity sensor (7), a potentiometer (2) and an amplifier (3).
4. Amplifier—an electric amplifier (3).
5. Actuator—an electric engine (4).
6. Adjuster—a valve (5).
7. Sensor—a thermoelement (6) and a humidity sensor (7).

Essential Quantities Occurring in the Considered Temperature Control System

1. Controllable quantity—humidity D of a tape (8).
2. Input—position “w” of a slider on the potentiometer.
3. Presetting quantity—voltage U_z on a potentiometer (2).
4. Processed quantity—voltage U_{x1} and U_{x2} from the sensors.
5. Control error—input voltage U_e on an amplifier (3).
6. Adjusting quantity—angle α of rotation of a shaft of an engine (4).
7. Adjusted quantity—intensity Q of inflow of the heating medium.
8. Disturbances—opening (2) of the drying chamber, the outer temperature θ_z, variations of the velocity v of the tape displacement, variations of the tape humidity D_{wej} on the entry of the furnace chamber.

A block diagram of the considered system is depicted in Fig. 8.13

We can see from the block diagram that we have to do with a two-loop system. The first, internal loop concerns a temperature control and the second, external loop concerns the very humidity control. This is a constant-value control system of the humidity, whose value can be adjusted on the potentiometer (2). There are two negative feedbacks in the system, and the total control error U_e is a difference between the presetting quantity U_w and two measured quantities U_{x1} and U_{x2}.

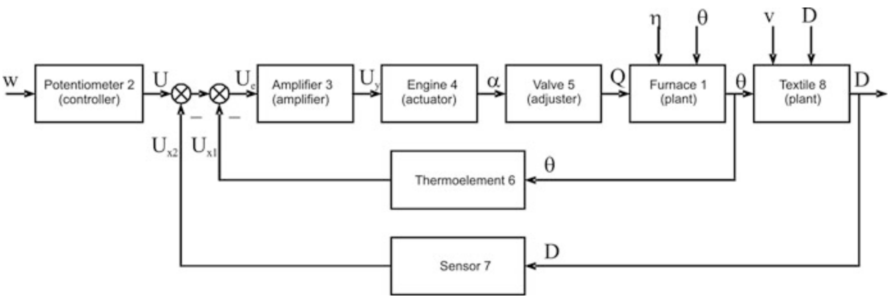

Fig. 8.13 Block diagram of the system from Fig. 8.12

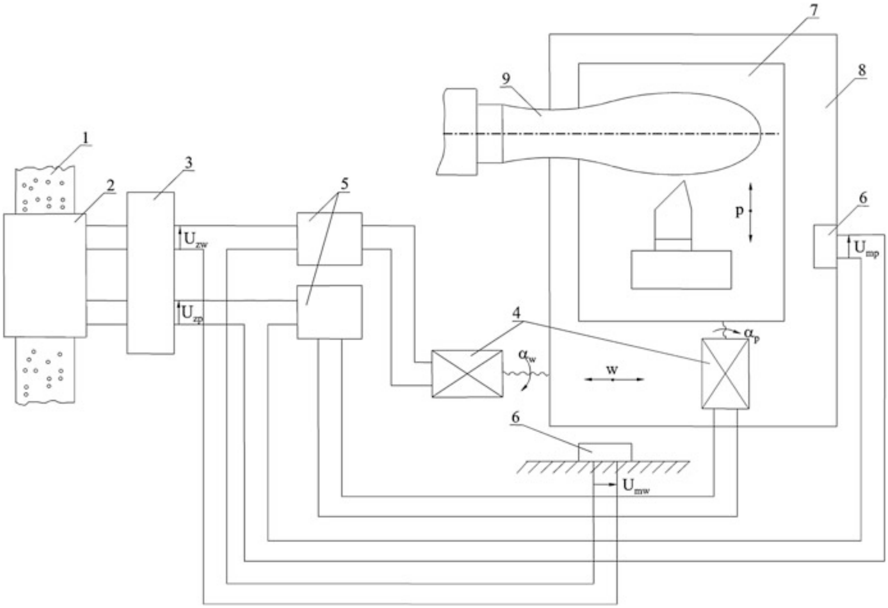

Fig. 8.14 Schematic diagram of the numerically controlled lathe

Example 8.3. Description of the system functioning.

Figure 8.14 presents a schematic block diagram of a numerically controlled lathe. The machining program of the item is saved on a perforated tape (1). Data read from the tape in the form of pulses sequence are changed into another kinds of pulses, from which each pulse corresponds to the precisely determined increment of the coordinates w and p. It is performed by an interpolator (3). Position of the board in arbitrary instant of time is measured by sensors (6). The information about this position is transmitted from the sensors in a form of signals, which come from the interpolator. Thus, the signals determine the required position of the board in a given instant of time. When there is a difference between these signals, the electric engines rotate and move the board by means of a gear, and this makes the difference vanish.

It follows from the above description that quantities presetting the position of the board in the form of signals U_{zw} and U_{zp} from the interpolator (3) are not constant, but they vary according to the treat program saved on a perforated tape (1). There are also two feedback loops reaching from the sensors (6) to the amplifiers (5). The considered system is a two-loop program-following control system.

Essential Elements in the System

1. Plant—a treated object (9) together with a cross slide (7) and carriage (8).
2. Controller—a perforated tape (1), a reader (2) and an interpolator (3).
3. Summer—an electric circuit of interpolator connections (3), sensors (6) and amplifiers (5).
4. Amplifier—an electric amplifiers (5).
5. Actuator—an electric engines (4).
6. Adjuster—gears moving the slide cross (7) and carriage (8).
7. Sensor—position sensors (6).

Essential Quantities Occurring in the System

1. Controllable quantity—coordinates y_w and y_p of positions of the slides.
2. Input—configuration of perforation on the tape (1) (w_z).
3. Presetting quantity—voltage U_{zw} and U_{zp} from an interpolator (3).
4. Processed quantity—voltage U_{mw} and U_{mp} from sensors (6).
5. Control error—input voltage of the amplifiers (5)
6. Adjusting quantity—angles α_w and α_p of rotation of shafts of electric engines (4).
7. Adjusted quantity—displacements of the slides on the gears
8. Disturbances—movements resistance of the slides, technological inaccuracy.

A block diagram of the considered system (Fig. 8.15) is as follows: the considered system is an example of a program-following control of two loops. The first loop serves for program-following control of the carriage position and the second loop serves for program-following control of the cross slide position; presetting

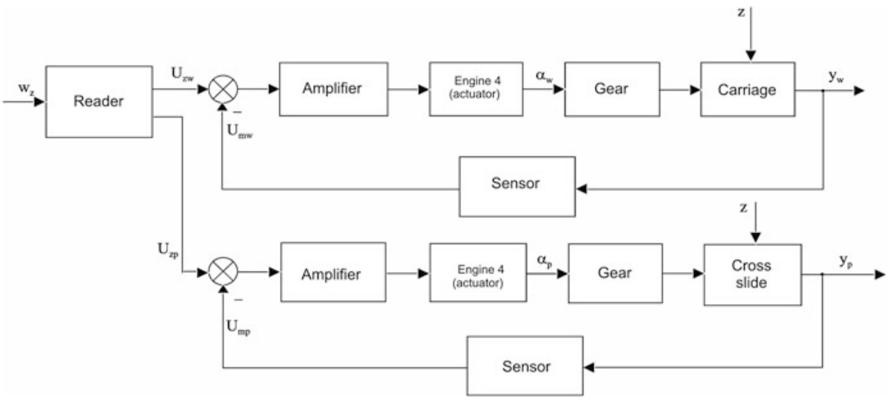

Fig. 8.15 Block diagram of the system in Fig. 8.14

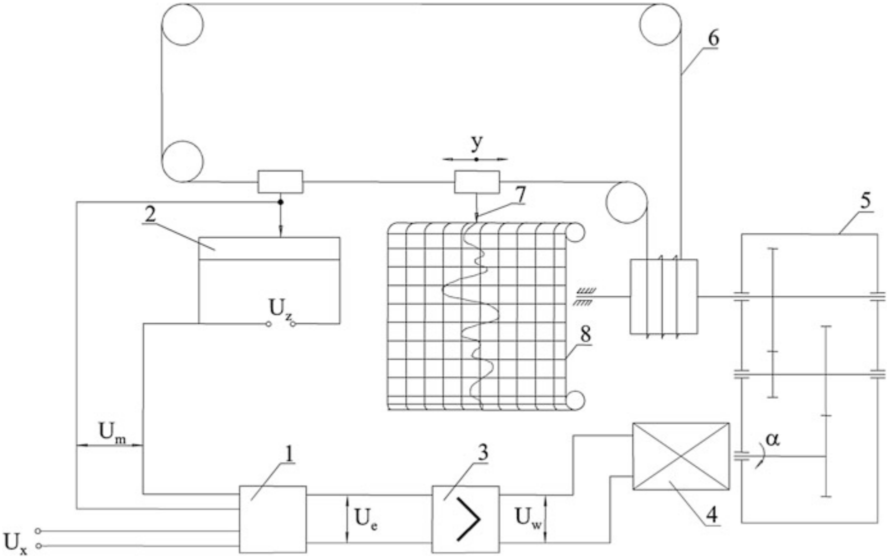

Fig. 8.16 Schematic block diagram of a register of voltage signals

quantities of these position are obtained from the controller consisting of a reader of the perforated tape (2) and interpolator (3).

Example 8.4. Description of the system functioning.

Figure 8.16 illustrates the principle of operation of a voltage signals register. A signal U_x, whose course is to be registered on a tape (8), is delivered to the input of a correcting element (1), where it is compared with the voltage U_m obtained from a slider of a potentiometer (2) supplied with a constant voltage U_z. Voltage $U_e = U_x - U_m$ is obtained on the output of the correcting element The voltage is amplified in an amplifier (3) and then it supplies the electric engine (4). The engine via the gear (5) and the strand (6) moves the slider of the potentiometer (2) in such a direction that the difference of voltage U_x and U_m decreases. The engine stops when the voltage U_e is zero. The strand (6) moves the pen (7). If the input signal U_x varies, then the slider of the potentiometer will be moved so that the voltage U_m follows the voltage U_x. The pen, which is displaced together with the slider saves on the tape (8) a graph of variation of the input signal U_x vs time.

It follows from the description of the system functioning that the input voltage U_x varies arbitrarily and independently of the service. This voltage is followed by the voltage U_m, whose value influences the position of the slider. The positions of the pen and the slider depend on the value of U_m. To conclude, the system depicted in Fig. 8.16 can be classified as a follow-up control system, tracking in other words, since the position y of a pen follows changes of the input voltage U_x and there is a closed cycle of signals in the system.

Essential Elements in the System

1. Plant—a pen (7) of the register.
2. Controller—it can be an electric device, whose voltage characteristics $U_x(t)$ is to be registered on the tape (8).
3. Summer—an electric comparison unit (1).
4. Amplifier—an electric amplifier (3).
5. Actuator—an electric engine (4).
6. Adjuster—a gear (5) with a strand (6).
7. Sensor—a potentiometer (2).

Essential Quantities Occurring in the System

1. Controllable quantity—position y of a pen of the register.
2. Presetting quantity—input voltage U_x.
3. Processed quantity—voltage $_m$ on the potentiometer.
4. Control error—voltage U_e on the correcting element (1).
5. Adjusting quantity—angle α of rotation of an engine shaft (4).
6. Adjusted quantity—displacement of the strand on the shaft.
7. Disturbances—resistance of the pen, elongation of the strand, variation of voltage U_z applied to the potentiometer, slides of the strand on a shaft.

A block diagram of the considered system has the following form.

In Fig. 8.17, one can see that this is a tracking system. Its structure is similar to the structure of a stabilizing system in Fig. 8.11, but the presetting quantity U_x varies arbitrarily and in a completely independent way, i.e. it can be neither adjusted nor programmed by the service.

Example 8.5. Description of the system functioning.

In Fig. 8.18 a schematic block diagram of a system is presented, which serves for keeping the flow intensity Q constant or in other words a constant discharge of the liquid in a common part of a pipeline. A discharge Q is, as one can see in Fig. 8.18, a sum of two discharges Q_1 and Q_2. The discharge Q_1 undergoes indeterminate changes, while the discharge Q_2 depends on the angular velocity of a

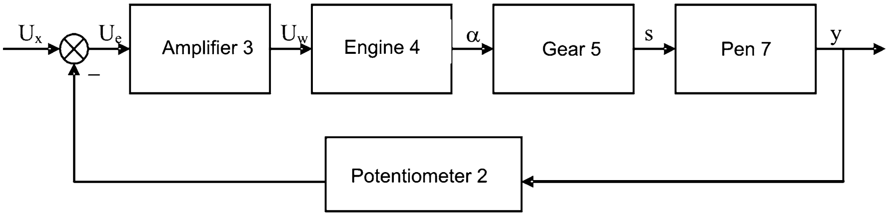

Fig. 8.17 Block diagram of a system in Fig. 8.17

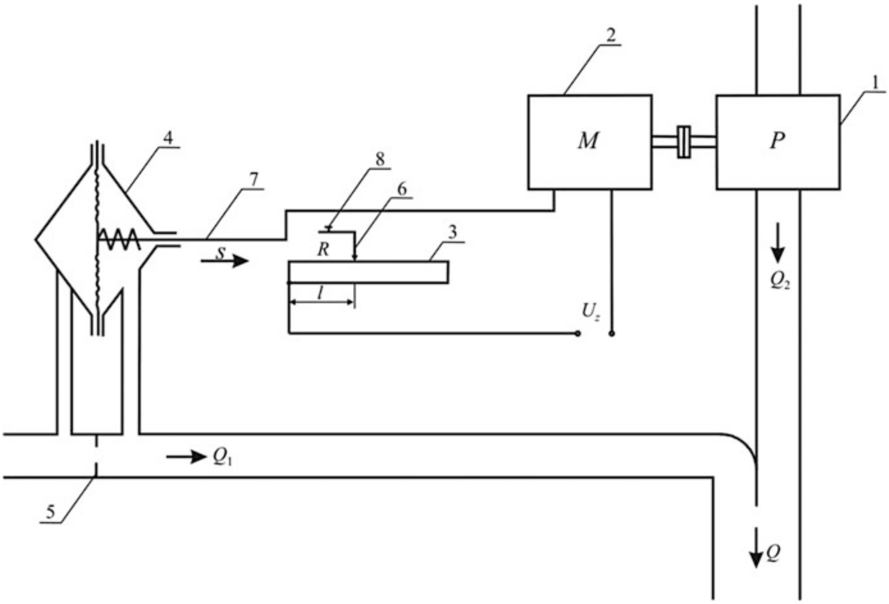

Fig. 8.18 Schematic block diagram of a system of keeping the flow Q constant

pump (1), which is driven by a direct-current motor (2). The motor is supplied with the constant voltage U_z, and its revolutions depend on the resistance R of a resistor (3), which in turn depends on a slider (6) position. The slider (6) position at fixed length of a mandrel (7) by means of a screw (8) depends on displacement of a sensor (4) membrane, which in turn depends on the pressure drop on a measuring orifice plate (5). This pressure drop depends on actual value of the flow intensity Q_1. As the flow intensity Q_1 grows, the resistance R grows as well, and in turn angular velocity of the engine (2) decreases. Hence, the discharge Q_2 of the pump (1) decreases. Thus, the total discharge Q is kept on a constant level. The service is able to change the value of the discharge Q by changing the mandrel (7) length.

It follows from the description of the system functioning that the system serves for keeping the discharge Q on a constant level. As follows from the schematic block diagram, the quantity Q is not measured. The quantity Q_1 is measured by means of the orifice (5) and the sensor (4). One needs to say that there is no classical feedback in the system. Thus, it is an open-loop automatic control system and not a closed-loop one. By further analysis we can conclude that it is an open-loop system with disturbance compensation. The disturbances are certain indeterminate changes of the flow intensity Q_1.

Essential Elements of the System

1. Plant—common part of the pipeline.
2. Controller—a mandrel (7) with a device (8) serving for adjusting of its initial position on a resistor (3).
3. Summer—a resistor (3).
4. Amplifier—there is none in this system.
5. Actuator—an electric engine (4).
6. Adjuster—a pump (1).
7. Disturbance sensor—an orifice (5) together with a membrane sensor (4).

Essential Quantities in the System

1. Controllable quantity—flow intensity Q.
2. Presetting quantity—initial position l of the slider on the resistor.
3. Processed quantity—displacement s of a sensor mandrel (4) corresponding to the flow intensity Q_1.
4. Control error—increment of the resistance se resulting from the displacement of the slider of the resistor with respect to the initial position l corresponding to the resistance R.
5. Adjusting quantity—angular velocity of an engine (2).
6. Adjusted quantity—flow intensity Q_2 from a pump (1).
7. Disturbances—flow intensity Q_1.

A block diagram of a system in Fig. 3.18 has the following form.

In Fig. 8.19 one can clearly see that it is an open-loop control system without feedback. In this system, there is a slotted line of disturbance influencing the controllable quantity.

Example 8.6. Description of the system functioning.

It follows from the schematic block diagram (see Fig. 8.20) that the system serves for controlling of a piston motion (3). It is performed with the use of a programmed motion of the slider (2) of a hydraulic divider, which guides the oil (which is

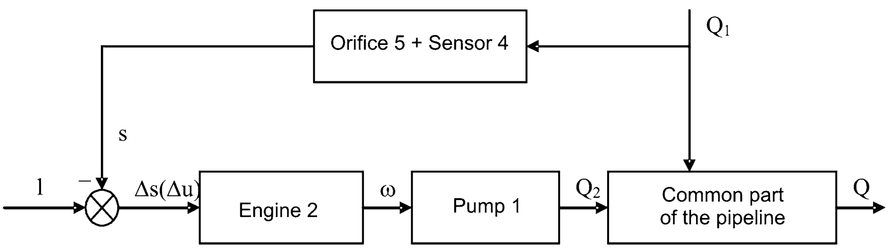

Fig. 8.19 Block diagram of the system in Fig. 8.18

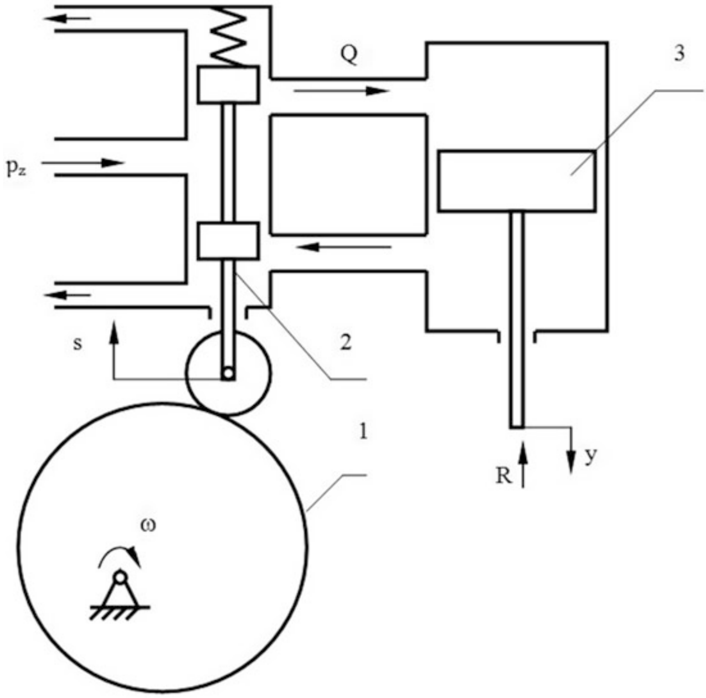

Fig. 8.20 Scheme of a piston motion control

delivered under constant pressure p_z) over or under the piston. The device presented in Fig. 8.20 consisting of a slider divider (2) and a piston, which moves in a cylinder is called a servo-mechanism. In this servo-mechanism slight movements of the slider make the piston move. Small movements of the slider requiring small forces to be applied result in large displacements of the piston (3), which can overcome significant resistance.

It follows from Fig. 8.20 that it is an open program-following control system since there is no feedback. This means that there is no connection between movement y of a piston (3) and movement s of a slider (2) of the hydraulic divider. The motion s, as can be seen, follows from the cam (1), which is a program of this motion, and motion of the piston (3) in consequence.

Essential Elements of the Considered System

1. Controller—a cam (1) of a particular angular velocity ω.
2. Amplifier—a hydraulic amplifier, which is a slider divider (2).
3. Actuator, adjuster and plant—a cylinder together with the piston (3).

A block diagram of a system in Fig. 8.20 has the following form (Fig. 8.21).

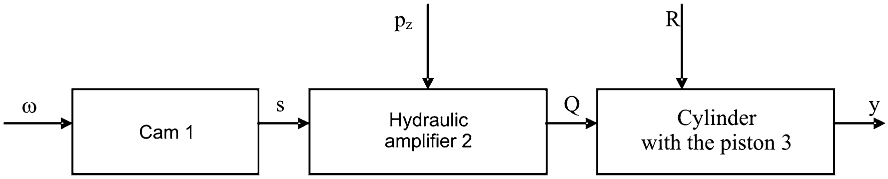

Fig. 8.21 Block diagram of the system in 8.20

Essential Quantities of the System

1. Controllable quantity—movements y of the piston (3).
2. Input—curvature of the cam (1).
3. Presetting quantity—position s of a divider slider.
4. Controlling quantity—degree of gaps opening s_1 delivering the oil to the engine.
5. Adjusted quantity—flow intensity Q of oil in the system.
6. Disturbances—resistance R of the piston movements, changes of the pressure p_z of oil supply.