Chapter 6
Systems

In the free encyclopedia "Wikipedia", one can find that the entry "system" is ambiguous and old. The word "system" comes from Greek and means a "compound object" (σύτημα), and this explains why it occurs in many different branches of science such as: *social science* (social system, law system), *astronomy* (solar system, system of planets, heliocentric system), *philosophy* (philosophical system), *geology* (geological system), *anatomy* (nervous system), *mathematics* (number system, decimal system, binary system), *computer science* (operating system), *logic* (deductive system) and many others. It is difficult to give a single definition of a system suitable for any branch of science. That is why we give only a few definitions and choose the most suitable one for our purposes (see [45, 48, 107, 132, 150, 168, 235]). There is a tendency to use a concept of system as a primary one, i.e. indefinable. However, in this case it is a composed object which contains interacting elements. We give our own definition within this background.

(i) *Definition (generalizing, asymptotical).* System is a collection of material elements or marks, which undergo slow evolutionary processes in space–time. The interchange of interactions among the systems, ones extracted from the environment (one being a paradigm determined by a particular branch of science), is of order ε, where $\varepsilon \ll 1$. Below, we give some features of such defined system.

   (a) elements of the system can vanish or appear in time of order $(1/\varepsilon)^n$, $n \in N$;
   (b) in the system one can distinguish subsystems, which interact in a region of space and time. The way they interact obeys the rules related to a particular branch of science. The role of a subsystem can be understood as "system residence" in a particular region of space and time of order $(1/\varepsilon)^n$ as *opened* or *closed* subsystem;
   (c) the function and purpose of the system is determined by a generating entity (human, laws of the nature);

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(d) there exists a border between the system extracted from the environment and the rest of the environment. Matter and energy (the order of magnitude ε) or information (depending on a given paradigm represented by a particular branch of science) are interchanged through the border. Note that sometimes in particular cases the interchange may lead to qualitative changes of the system (or another one, quantitative changes can transfer into qualitative ones);
(e) we will assume that the mentioned borders undergo slow changes in parallel with the mentioned changes of the system state;
(f) one can use the definitions from Wikipedia, i.e. a system can (but in a defined time interval of order $(1/\varepsilon)^n$) characterize with morphostasis (keeping form and shape the same) or morphogenesis (tendency to changes);
(g) equilibrium state (dynamical or static state, understood as a particular case of the dynamical one between the system and the rest of the environment is homeostatic). Note that from Greek *homoios* means similar (new), and *stasis* means residing in time interval $(1/\varepsilon)^n$;
(h) one admits the possibility of various results (after exceeding time $(1/\varepsilon)^n$) or preservation of the system with the same causes (perturbations of order ε), i.e. so-called equifinality holds (it is just a result of slow evolution of the considered system);
(i) similarly, one can explain the *equipotentiality* principle. The causes (perturbations) originating from the same source can cause various effects (responses or behaviour) to the system. It depends on places in the border region where the causes arrive to the analysed system.

Now, we give three definitions of a system used in mathematics, cybernetics and natural sciences.

(ii) Definition (mathematical)
System (*S*) is a subset of *N*-elements relation, which is the Cartesian product ($\times$) of properties set (objects of a system or a process), namely

$$S \subset (U_1 \times U_2 \times \cdots \times U_N), \tag{6.1}$$

where U_i denotes *i*th set of properties.

In many branches of science, an object is regarded as a "black box", i.e. we do not know a mathematical model of an examined object or a process but we know completely a set of the inputs X and the outputs Y (responses) of the system. There also exists a concept of a "white box", when the knowledge of our object is complete. Such defined system has the form

$$S \subset (X \times Y) \tag{6.2}$$

The aim of every examination is to determine the black box. The examination is equivalent to identification of the system. The identification can be performed basing itself on the knowledge, investigations, development of science, leading to a kind of mathematical model, e.g. based on algebraic, differential, differential-integral or integral equations. Some parameters of the system are determined by the *identification*, i.e. based on the measured or known inputs X and outputs Y (this problem will be considered in more detail). It can happen that behaviour of the black box is so complicated that it is impossible to assume a priori a model.

Then, it is useful to apply the concept of neural networks. One needs to "teach" the network, usually basing oneself on a large amount of input signals, so that the response of the system is generated by the taught network. In other words, when we teach our network we make it change its dye from black to white, i.e. a white box.

Note that this method is often used in the behaviour analysis in psychology, medicine, chemistry and even in mechanics (though mechanics is highly saturated by mathematics). This way is often more economic with regard to the time duration of solving the problem or it is not possible to perform the identification of the whole object due to the costs (e.g. bridge support are immersed deep in water or some of them are not accessible for examining due to detrimental conditions such as chemical or nuclear). There is a concept of relation used in the given definition. Below, we give the definition of relation

Definition of relation (set theory).

Relation ρ between elements of the sets X and Y is any subset of the Cartesian product of X and Y, i.e. $\rho \subset X \times Y$. The relation defines any set of the ordered pairs *(x,y)*, where $x \in X$ and $y \in Y$. Notation $(x, y) \in \rho$ is equivalent to $x \rho y$.

As an example, consider relation in a set of integer numbers C such that $y = kx$. Such defined relation is called the *relation of divisibility*. For example $(2, -6)$ is the element of the relation of divisibility ρ, whereas $(-5, 3) \notin \rho$.

(iii) *Definition (natural).* System is a collection of interacting material elements, whose mutual interactions make a common goal (function), which cannot be reduced to a function of a single or a few selected elements.

It is easy to give an example of a natural object, namely a plant or an animal. For example, the brain consists of cells that are connected by cells of the nervous system and it receives stimulus's. The brain would not be able to function without cells of the nervous system.

(iv) *Definition (cybernetic).* System is a functional entirety that is extracted from environment, which influences the system by means of stimulus's (actions, signals, input quantities), and the system influences the environment through "feedback loop" by means of the outputs (reactions).

The notion of cybernetics comes from Greek (kybernetes—steersman, manager; kyberan—to control) and is a science of control systems, transmitting and processing information.

Cybernetics is one of the mainstreams of the so-called systematic research and is an interdisciplinary science. It is assumed that it was Andre Marie Ampere (1775–1836), who first used this concept in "Essay on philosophy of science".

The Polish philosopher Bronislaw F. Trentowski used this concept for the first time in 1843, in "The relation of philosophy to cybernetics".

The American mathematician Norbert Wiener is regarded as a creator of cybernetics as a sole branch of science. Now, one can distinguish *theoretical cybernetics* (emerged from biology and mathematics) and *applied cybernetics* (technical, economical or biocybernetics).

To sum up, it seems that the definition (i) has the broadest range and can be applied in many different branches of science. Many of the derived properties (a)–(i) will be explained further with the help of many examples.

At the end of this chapter, we refer to the genesis of the earlier given definition (i). The genesis is related to asymptotological understanding of the nature laws and to the conception of quantitative changes and their transformations into qualitative changes. In classical textbooks, one usually contrasts traditional Aristotle's mechanics with Newtonian mechanics. However, both conceptions can be seen by asymptotology. One can show that rough Aristotle's approach could have been a stimulus that made modern mechanics arise after asymptotological pattern. The way the science develops is not like pulling down old theories and building new ones.

It is not unusual for an old theory to be a source of new ideas and concepts. Although there are many various opinions on development of science today's, one can see asymptotological relations between them. Aristotle thought that force is a cause of motion and a state of rest is when no force is applied. According to Aristotle, a dropped ball from a horizontally moving object falls downward. Despite the beliefs outwardly distant from present conceptions, it turns out that one can point some asymptotological relations between Aristotle's and Newtonian theory. In order to make a thorough study of such relations, we consider the motion of a body experiencing the force F and the viscous resistance with the coefficient c. Aristotle's idea can be expressed through the relationship

$$F = c\dot{x} = cv. \tag{6.3}$$

The relationship confirms the earlier described Aristotle's ideas. When there is no force ($F = 0$), there is no motion (since $\dot{x} = 0$). If the force is constant, then it makes the motion velocity grow. To Aristotle, the resistance force was an attribute of motion and was not external. Newton introduced inertial force and resistance force which are external. The equation of motion of a massive body reads

$$m\frac{dv}{dt} = F - cv, \tag{6.4}$$

and after integrating it leads to the relationship

$$v = \frac{F}{c}\left(1 - e^{-\frac{c}{m}t}\right). \tag{6.5}$$

One can see as $t \to \infty$ we get Aristotle's law. Aristotle did not know the law of inertia and it was Galileo, who made this breakthrough discovery. Thus, Aristotle's approximation is a kind of asymptotics of motion for sufficiently long time intervals. The larger resistance and lower mass, the greater difference is between Aristotle's and Galileo's principle. For small values $\frac{c}{m}t$ we obtain

$$v \cong \frac{F}{c}\left(1 - 1 + \frac{c}{m}t\right) \cong \frac{F}{m}t. \tag{6.6}$$

The above equation describes additional asymptotics. This equation holds for sufficiently small time t intervals. Moreover, the lower resisting coefficient c is, the longer time will pass before Aristotle's asymptotics comes into play. This example shows the described asymptotological transition between two theories, namely Aristotle's and Galileo–Newton's. Aristotle's theory "generated" the asymptotics, and after differentiating we get

$$m\frac{dv}{dt} = F, \tag{6.7}$$

that is definitely Newton's second law. Did Aristotle discover Newton's law without noticing it?

The way through the asymptotics led us to a completely different world, alien to Aristotle, to the world of Hamiltonian mechanics. Theory of conservative systems is one of the asymptotics of Newtonian mechanics. In this new world, completely new notions emerged such as vibrations, periodic and quasiperiodic dynamics, chaotic orbits. This new, idealized world is a reflection of the real one and approximates it.

Is Aristotle's theory already dead? Does not it have a reflection in the reality? On the contrary, it does. Movements of polymer molecules in solutions and relaxation movements can be well approximated by Aristotle's principle.

It was Einstein, who made the next asymptotological transformation arise. This led to the Special Relativity. Let us discuss the asymptotic transition between Newton's and Einstein's theory.

Consider a particle of mass m, moving at the velocity v and experiencing the force F. According to the Special Relativity, the velocity is described by the formula

$$v = \frac{v_0}{\sqrt{1 + \left(\frac{v_0}{c}\right)^2}}, \tag{6.8}$$

where $v_0 = \frac{Ft}{m_0}$ and c is the speed of light. For small values of the velocity we stay in the world of Newton's theory and then $v \cong v_0$.

The equation allows to discover additional asymptotics. It is easy to see that

$$\lim_{t\to\infty} v = \lim \frac{1}{\sqrt{\varepsilon + \left(\frac{1}{c}\right)^2}} = c, \tag{6.9}$$

where $\varepsilon = v_0^{-2}$.

We seek the solution in the form

$$v(\varepsilon) = v(0) + \frac{dv}{d\varepsilon}(0)\varepsilon + 0(\varepsilon)^2. \tag{6.10}$$

Since

$$\frac{dv}{d\varepsilon}(\varepsilon) = -\frac{1}{2}\frac{1}{\left(\sqrt{\varepsilon + \left(\frac{1}{c}\right)^2}\right)^3}, \tag{6.11}$$

then

$$\frac{dv}{d\varepsilon}(0) = -\frac{1}{2}c^3. \tag{6.12}$$

By (6.10) we get

$$v = c\left(1 - \frac{1}{2}\frac{c^2}{v_0^2}\right). \tag{6.13}$$

As one can see, as $t \to \infty$ we get completely new effects. With the help of the asymptotics, one succeeded to either discover or explain a few new physical phenomena, namely deviation of light rays in gravitational fields, retardation of electromagnetic signals propagating in gravitational fields and many others.

Another example of asymptotical relations is classical mechanics and quantum mechanics. First, let us investigate analogous relations between wave optics and geometrical one. It turns out that there exists a justified, asymptotical transition between these theories. The transition from wave optics to geometrical optics is related to neglecting of the wave of the length $\varepsilon(\varepsilon \approx 10^{-7}m)$ in comparison with the size of the object. Let a light wave be described by the following formula:

$$u = A(x, y, z, \varepsilon)e^{\frac{i\varphi(x,y,z)}{\varepsilon}}, \tag{6.14}$$

where A is the amplitude and it can be put into the form of series

$$A = A_0 + A_1\varepsilon + \cdots \tag{6.15}$$

and $i^2 = -1$ and $\varphi(x, y, z)$ is a phase-shift at the point (x, y, z). One puts the sought solution into the wave equation. Next, one equates the terms occurring by the same powers of ε. Thus we obtain a nonlinear differential equation to determine $\varphi(x, y, z)$. This is the equation which corresponds to the approximation introduced by theory of geometrical optics. It was E. Schrödinger, one of the first, who transferred asymptotical similarities between the mentioned theories to asymptotical relation between quantum mechanics and classical mechanics. Similarly, one can find asymptotological relations between both mechanics. The point is that the initially given probability distribution of a particle position varies according to the classical mechanics laws.

At the end, it is worth mentioning that the methodology of asymptotical methods enables to comprehend the real world deeply. There are no eternally accurate rules and laws—they are less or more approximated as Einstein noticed. Such fundamental laws as follows: Ohm's law, Hooke's law, Coulomb's friction law $T = \mu N$ are the approximated laws and the given relations are obtained through linearization of nonlinear laws.

Even one of the fundamental models of fluid mechanics, namely the Navier–Stokes model, is the asymptotics of gas flow in the framework of Boltzmann's theory as $\varepsilon \to 0$, where ε is the mean free path of molecules. Such asymptotical transition allows many completely new concepts to come into being; the concepts which could not appear in the framework of old theories.