Modelling Travel Routes in Transport Systems by Means of Timed and Hybrid Petri Nets

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Abstract. Timed Petri nets (TPN) and first order hybrid Petri nets (FOHPN) are tested here in order to model transport systems and to find the suitable travel routes in different non-standard situations during the increased traffic density (i.e. at the bounded traffic or congestion). This work extends our previous work where the flexible routes in transport systems were found by means of the place/transition Petri nets (P/T PN). While at usage of the TPN only the time parameters are assigned to the P/T PN model transitions, the FOHPN model is different and it yields the continuous flows of vehicles.

Keywords: Agent, cooperation, first order hybrid Petri nets, hybrid Petri nets, place/transition Petri nets, timed Petri nets, transport systems.

1 Introduction and Preliminaries

Place/transition Petri nets (P/T PN) [11, 12] are the effective tool for modelling discrete event systems (DES). However, their extended (modified) version - timed Petri nets (TPN) - are suitable for DES behaviour in time. Hybrid Petri nets (HPN) [6, 8] are suitable for modelling hybrid systems (HS) containing both the continuous part and the discrete one. Simplified version of HPN - the first order HPN (FOHPN) [2–5, 7, 13] - are especially suitable for modelling HS because of the existence of the handy simulation tool HYPENS for Matlab [13, 14]. P/T PN are (as to their structure) bipartite directed graphs with two kinds of nodes - places $p_i \in P,\ i = 1, \ldots, n$, and transitions $t_j \in T,\ j = 1, \ldots, m$, and two kinds of edges - the set $F \subseteq P \times T$ of edges from places to transitions and the set $G \subseteq T \times P$ of edges from transitions to places. But moreover, P/T PN have also dynamics - the evolution of marking of their places given as $\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{B}.\mathbf{u}_k$, where $\mathbf{x}_k = (\sigma_{p_1}, \ldots, \sigma_{p_n})^T$ with $\sigma_{p_i} \in \{0, 1, \ldots, c\}$ (here c, being the capacity of the places, may be either infinite or finite) is the marking vector expressing the state of the marking of the particular places, $\mathbf{u}_k = (\gamma_{t_1}, \ldots, \gamma_{t_m})^T$ with $\gamma_{t_j} \in \{0, 1\}$ is the vector of the states of transitions (disabled or enabled). The matrix $\mathbf{B} = \mathbf{G}^T - \mathbf{F}$ expresses the structure. $\mathbf{F}$ (**Pre**) and $\mathbf{G}^T$ (**Post**) are the incidence matrices of the arcs corresponding, respectively, to the sets F and G.

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P/T PN do not depend on time. Their transitions, places, arcs and tokens do not contain any time specifications. In TPN [15] time specifications are defined. In general, the time specification can be assigned to TPN places, transitions, directed arcs, even to tokens. However, in this paper the time specifications will be assigned exclusively to the TPN transitions (more precisely to the P/T PN transitions). Namely, in the deterministic case they will represent time delays of the transitions while in the non-deterministic cases they will express a kind of probability distribution of timing (e.g. exponential, discrete uniformed, Poisson's, Rayleigh's, Weitbull's). HPN in general are [6] an extension of Petri nets (PN). HPN model the hybrid systems where discrete and continuous variables coexist. HPN have two groups of places and transitions - discrete and continuous. Consequently, three kinds of directed arcs exist in HPN: (i) the arcs between discrete places and discrete transitions; (ii) the arcs between continuous places and continuous transitions; (iii) the arcs between discrete places and continuous transitions as well as the arcs between the continuous places and discrete transitions. While HPN discrete places and transitions handle discrete tokens, the HPN continuous places and transitions handle continuous variables (different kinds of flows). FOHPN are a simplified kind of HPN. FOHPN are defined in details in [2–4, 7, 13]. The comprehensive definition will not be introduced here, of course. But to give the basic idea about FOHPN it is necessary to introduce that the set of places $P = P_d \cup P_c$, where P_d is a set of discrete places (figured by circles) and P_c is a set of continuous places (figured by double concentric circles). Analogically, the set of transitions $T = T_d \cup T_c$, where T_d is a set of discrete transitions (figured by rectangles) and T_c is a set of continuous transitions (figured by double rectangles). T_d contains a subset of immediate (no-timed) transitions like those used in P/T PN and/or a subset of timed transitions (deterministic and/or non-deterministic like in TPN). Consequently, the FOHPN marking consists of two parts: (i) discrete - integer number of tokens in discrete places; (ii) continuous - an amount of a fluid in continuous places. The instantaneous firing speed (IFS) [2–4], determining an amount of fluid per a time units in a time instance τ, is assigned to each of the continuous transition T_j. For all time instances τ holds $V_j^{min} \leq v_j(\tau) \leq V_j^{max}$, where min and max denote the minimal and maximal values of the speed $v_j(\tau)$. IFS is piecewise constant. The rules for enabling the continuous transitions are as follows. An empty continuous place P_i is filled through its enabled input transition. So, the fluid can flow to its output transition. The continuous transition T_j is enabled in the time τ [2–4, 13] if and only if its input discrete places $p_k \in P_d$ have marking $m_k(\tau)$ at least equal to the element $Pre_{dc}(p_k, T_j)$ of the incidence matrix $\mathbf{Pre}_{dc}$ of arcs from discrete places to continuous transitions and all of its input continuous places $P_i \in P_c$ satisfy the condition that their markings $M_i(\tau) \geq 0$ - i.e. the places P_i are filled. If all of the input continuous places of the transition T_j have non-zero marking then T_j is strongly enabled, otherwise T_j is weakly enabled. The continuous transition T_j is disabled if some of its input places is not filled. Namely, T_j cannot take more fluid from any empty input continuous place than the amount entering the place from other transitions. This corresponds to the principle of mass conservation.

Segments of a transport system (e.g. whole city or a part of country) with an increased traffic density are understood to be agents. The agents are mutually connected by the roads. The cooperation among the adjacent agents is realized by means of passing the vehicles in/out each other. Here, we will be interested only in the interior of one of such agents (segments) and test its internal dynamic behaviour. Firstly, the segment will be modelled by the TPN and tested by simulation in Matlab by means of the HYPENS tool. Secondly, the modelling will be performed by FOHPN and tested by means of the HYPENS tool too.

2 Case Study

Let us start from the P/T PN model of the segment of a transport system given in Fig. 1 studied in [1]. The P/T PN places (denoted by circles) represent the intersections of roads while the arcs between the places containing the P/T PN transitions (denoted by rectangles) represent the roads between the intersections. In order to find the route from the crossroad modelled by the place p_1 at respecting the bounded throughput of the particular roads at the increased traffic density (i.e. when in some roads either the traffic is bounded or a congestion occurs) to the crossroad modelled by the place p_6, usage of TPN and FOHPN will be tried. Consider that the distance of each road section is given by d_{12} = 40m, d_{24} = 30m, d_{13} = 30m, d_{34} = 20m, d_{45} = 50m, d_{65} = 30m, d_{76} = 30m, d_{73} = 40m, d_{37} = 40m, d_{87} = 40m, d_{18} = 30m, where d_{ij} is the distance from intersection point i to j. Assume that the average vehicle speed of each road section is estimated by an Adaptive Gray Threshold Traffic Parameters Measurement (AGTTPM) [9, 10] system as v_{12} =10m/s, v_{24} = 5m/s, v_{13} = 5m/s, v_{34} = 5m/s, v_{45} = 10m/s, v_{65} = 10m/s, v_{76} = 3.33m/s, v_{73} = 10m/s, v_{37} = 5m/s, v_{87} = 10m/s, v_{18} = 10m/s (where m/s stands for meters per second), the estimated travel time τ_{ij} from the place p_i to p_j can be computed as the ratio of the distance to the average vehicle speed and assigned to the corresponding transition as (τ_{12} = 4s) $\rightarrow t_1$, (τ_{24} = 6s) $\rightarrow t_2$, (τ_{13} = 6s) $\rightarrow t_3$, (τ_{34} = 4s) $\rightarrow t_4$, (τ_{45} = 5s) $\rightarrow t_5$, (τ_{56} = 3s) $\rightarrow t_6$, (τ_{76} = 9s) $\rightarrow t_7$, (τ_{73} = 4s) $\rightarrow t_8$, (τ_{37} = 8s) $\rightarrow t_9$,

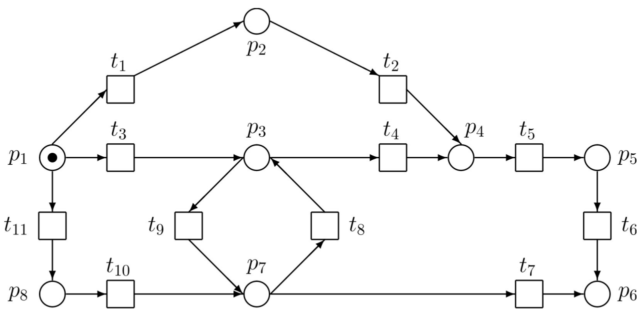

Fig. 1. The P/T PN-based model of the segment of the transport system

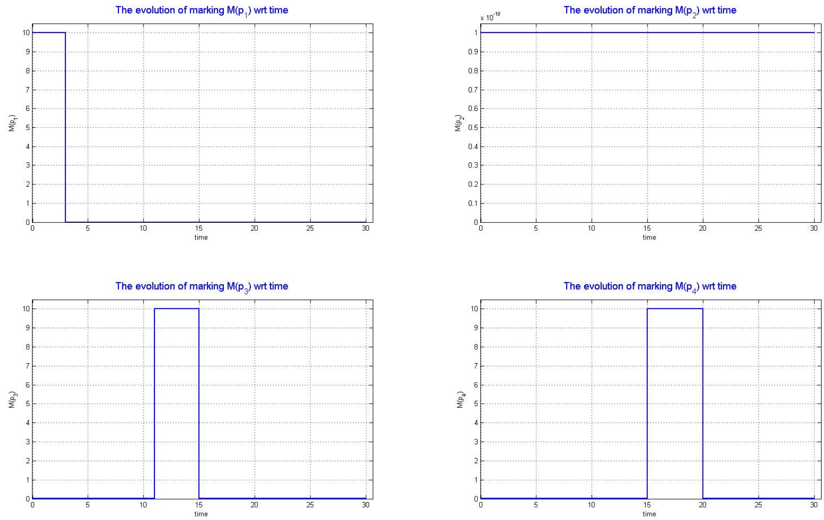

Fig. 2. TPN marking of the places $p_1 - p_4$ with respect to time in the deterministic case. The particular courses give us information about the occupation of the corresponding crossroads. The crossroad modelled by p_2 (its marking is 1.10^{-10} is passed not a bit.

$(\tau_{87} = 4\text{s}) \rightarrow t_{10}$, $(\tau_{18} = 3\text{s}) \rightarrow t_{11}$ (where s stands for seconds). These estimated travel times can be incorporated into the transitions of the TPN model, namely, in the deterministic case as their time delays, while in the non-deterministic case as a kind of probability distributions of their timing. The graphical results obtained by means of the HYPENS tool (it is able to model not only FOHPN but also TPN) are given in Fig. 2, Fig. 3. While in Fig. 2 the marking of TPN places $p_1 - p_4$ are displayed, in Fig. 3 the marking of TPN places $p_5 - p_8$ are shown. The results correspond to the input parameters $\mathbf{m}_0 \equiv \mathbf{x}_0^T = (10, 0, 0, 0, 0, 0, 0, 0)$ and the vector of the TPN transitions weights (it is the internal HYPENS parameter) $\alpha = (5, 3, 3, 5, 4, 6, 1, 5, 2, 5, 6)$ depending on the time delays (the maximal priority is assigned to the transition with the shortest time delay). It can be seen that the order of the sequence of activating the TPN places in time is: $p_1 \rightarrow p_8 \rightarrow p_7 \rightarrow p_4 \rightarrow p_5 \rightarrow p_6$. Just this sequence represents the route upon which the passing from p_1 to p_6 happens in the shortest time.

Now, let us use the FOHPN model given in Fig. 4. The sense of the discrete blocks can be explained on the block $\{p_1, p_2, p_3, p_4, t_1, t_2, T_i\}$ displayed in Fig. 5 as follows. The discrete place p_1 has to be active (i.e. to have the token) in order that the continuous transition T_i might be open. When p_2 is active, T_1 is closed. The active place p_3 makes possible to open the closed T_i, while the active place p_4 makes possible to close the opened T_i. The transitions t_1, t_2 may be either deterministic (with deterministic time delays only) or non-deterministic (with a kind of the probability distribution of its timing). The flow through the T_i is

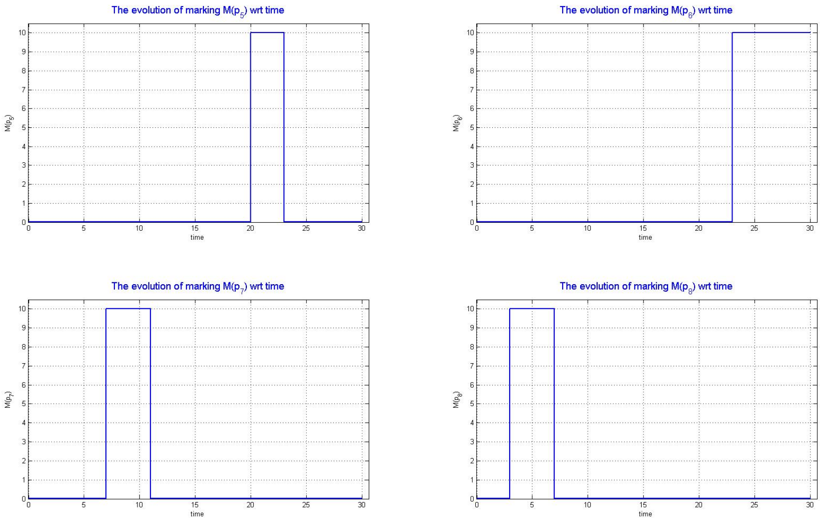

Fig. 3. TPN marking of the places $p_5 - p_8$ with respect to time in the deterministic case show the occupation of the corresponding crossroads. All of them are passed during the corresponding time interval.

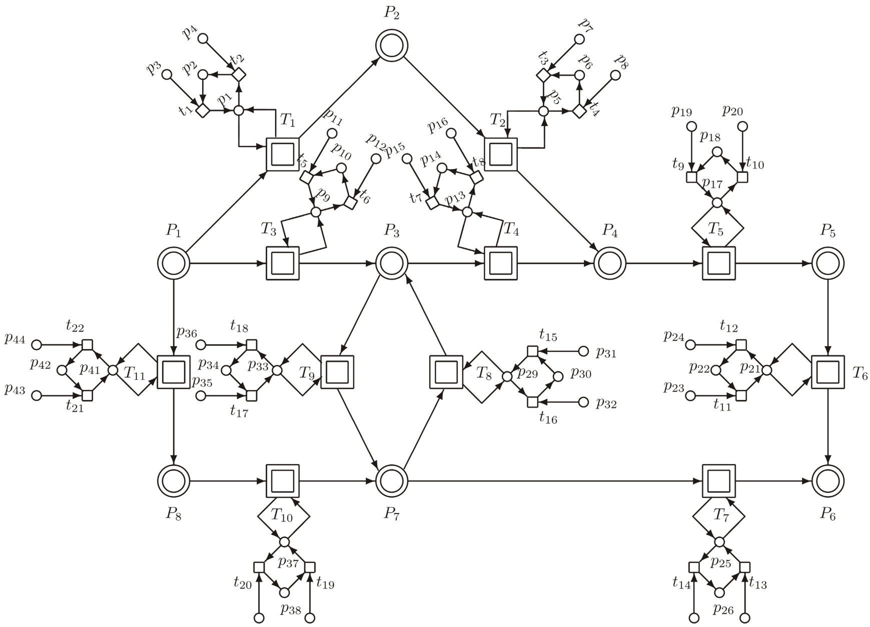

Fig. 4. The FOHPN model of the situation in the transport segment

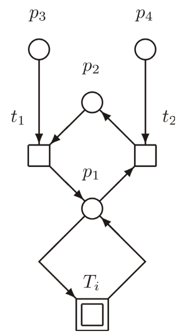

Fig. 5. The P/T PN-based discrete block in FOHPN

liable to the rules (mentioned above) explained in details in [2–4, 13] concerning the evolution of FOHPN continuous marking. Using the model in HYPENS tool we can obtain the graphical simulation results in deterministic case as the courses given in Fig. 6, Fig. 7, where the following input parameters were used: the initial continuous marking $M_0 = (100, 0, 0, 0, 0, 0, 0, 0)$, the initial discrete marking $m_0 = (1\,0\,1\,1\,1\,0\,1\,1\,1\,0\,1\,1\,1\,0\,1\,1\,1\,0\,1\,1\,1\,0\,1\,1\,1\,0\,1\,1\,1\,0\,1\,1\,1\,0\,1\,1\,1\,0\,1\,1\,1\,0\,1\,1)$, the limits of IFS for continuous transitions $V^{min} = (0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0)$, $V^{max} = (11, 6, 6, 6, 11, 11, 3.7, 11, 6, 11, 11)$. Simultaneously, the parameters of the discrete uniform probability distribution - $f_x = 1/(b-a)$ when $x \in (a, b)$ and $f_x = 0$ otherwise - for the discrete transitions timing are $\delta = 15*([2\ 6], [2\ 6], [4\ 8], [4\ 8], [4\ 8], [4\ 8], [2\ 6], [2\ 6], [3\ 7], [3\ 7], [1\ 5], [1\ 5], [7\ 11], [7\ 11], [2\ 6], [2\ 6], [6\ 10], [6\ 10], [2\ 6], [2\ 6], [1\ 5], [1\ 5])$, where the pairs $[a_i\, b_i]$, $i = 1, \ldots, 22$, create the parameters for particular discrete transitions $t_1 - t_{22}$. The weights of the discrete transition were not predefined while the weights of the continuous transitions are: (5, 3, 3, 5, 4, 6, 1, 5, 2, 5, 6). It can be seen from the results that at passing the routes the crossroads modelled by the continuous places P_5 and P_8 are attended less than the other ones. More or less it is confirmed also by the graphical simulation results obtained at using the exponential probability distribution of timing the discrete transitions: $f_x = \lambda.e^{-\lambda.x}$ for $x \geq 0$ and $f_x = 0$ otherwise. These results are given in Fig. 8, Fig. 9. In this non-deterministic case the parameters were the same, except the parameters characterizing the probability distribution. Here, the parameters of the exponential probability distribution are: $\lambda = (4, 40, 6, 60, 6, 60, 4, 40, 5, 50, 3, 30, 9, 90, 4, 40, 8, 80, 4, 40, 3, 30)$. It means that the transport is largely realized throughout the other places (out of the P_5 and P_8). The results obtained by using the exponential probability distribution are more smoothed than those gained at the discrete uniform one.

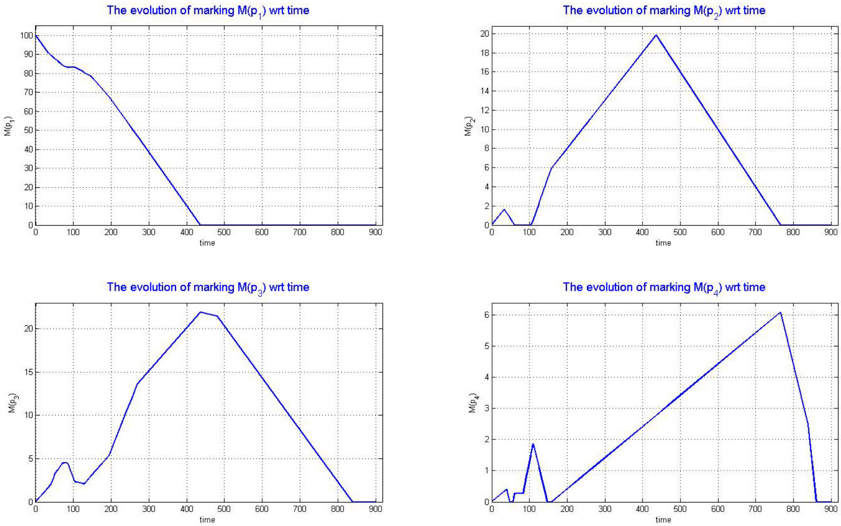

Fig. 6. FOHPN continuous marking of $P_1 - P_4$ with respect to time in non-deterministic case with the discrete uniform probability distribution of timing the discrete transitions

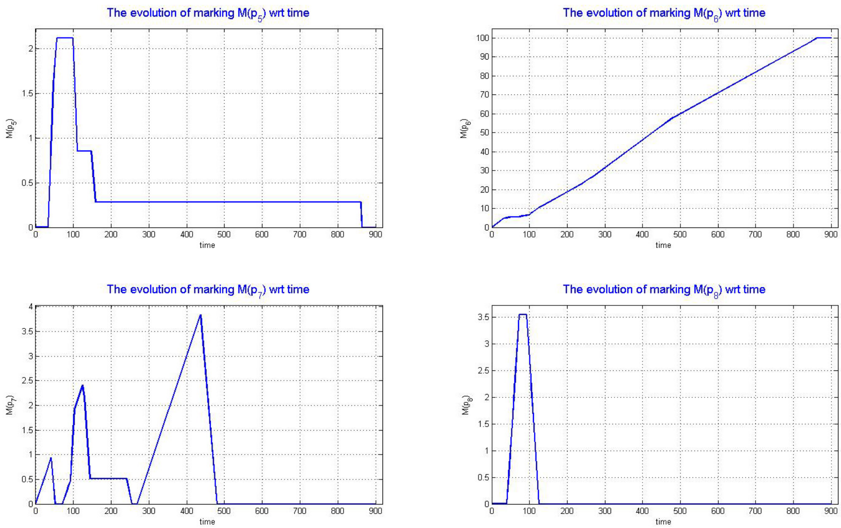

Fig. 7. FOHPN continuous marking of $P_5 - P_8$ with respect to time in non-deterministic case with the discrete uniform probability distribution of timing the discrete transitions

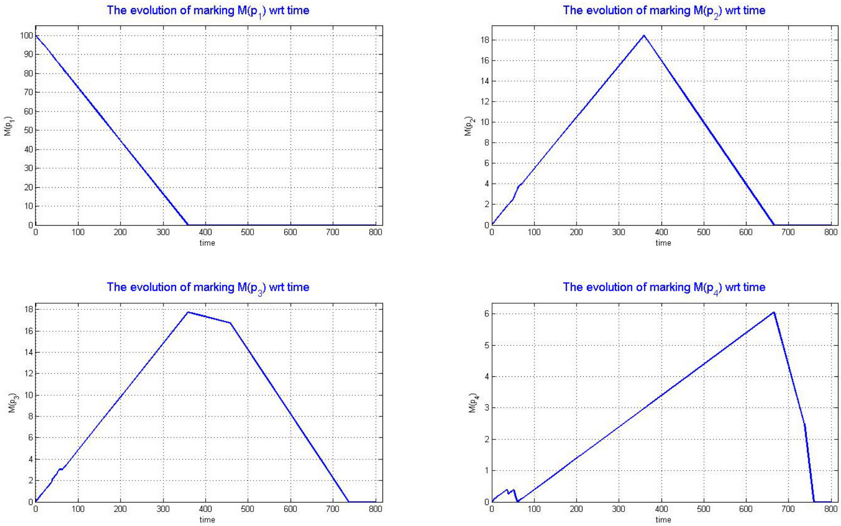

Fig. 8. FOHPN continuous marking of $P_1 - P_4$ with respect to time in non-deterministic case with the exponential probability distribution of timing the discrete transitions

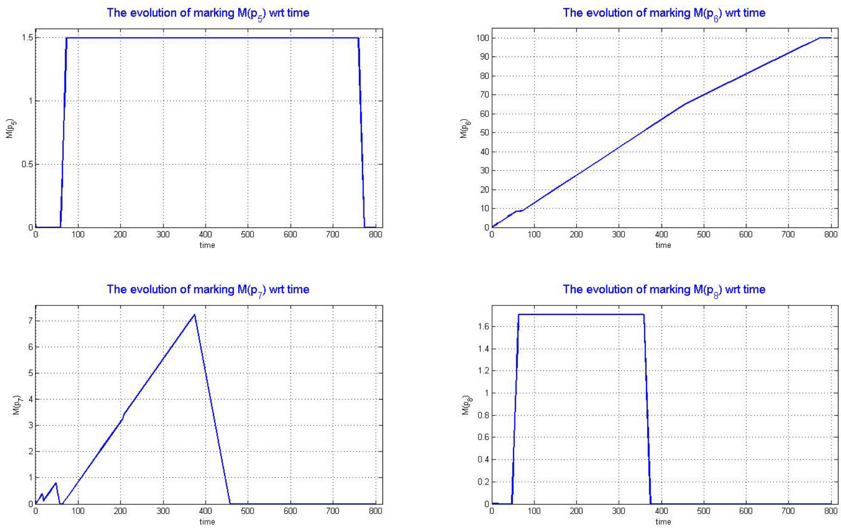

Fig. 9. FOHPN continuous marking of $P_5 - P_8$ with respect to time in non-deterministic case with the exponential probability distribution of timing the discrete transitions

3 Conclusion

The main idea of this paper is to point out the further possibilities of modelling the transport systems throughput at the increased traffic density. The information from the AGTTPM system makes possible to find the suitable routes. This work extends our previous work [1] where the P/T PN-based approach was presented. While there the time specifications were missing, here, at the application of TPN and FOHPN, the time specifications can be applied in a wide range. Consequently, the simulation of the time behaviour of the transport segment (agent) of a global transport system is possible. Such a procedure is very useful, because it yields the flows of the vehicles in time. Then, the cooperation among the adjacent segments (agents) can be realized by means of passing the vehicles in/out each other - i.e. by means of the mutual exchange of the vehicles. Of course, here (on the prescribed limited space) only one segment is dealt with. The TPN model presented here was built directly from the P/T PN model presented in [1]. Namely, the time specifications - i.e. either simple time delays in deterministic case of timing or a kind of the probability distributions of timing - were assigned to the P/T PN transitions. In such a way the TPN model of the segment arose. Using the model for simulation in Matlab by means of the HYPENS tool the graphical simulation results can be obtained in the form of stepped time functions. To illustrate the soundness of such an approach the graphical results for the deterministic case (where exclusively the deterministic time delays were used) were introduced and verbally described in order to document the abilities of the approach. The approach yields the most suitable passing the roads with respect to prescribed conditions. Then, the FOHPN model was proposed and used for the simulation in Matlab by means of the HYPENS tool. In such a way the continual courses of the vehicle flows passing the intersections of the roads can be found in the form of the continuous piecewise-linear real time functions. To illustrate the soundness of such and approach two graphical simulation results were presented. They correspond to non-deterministic case of timing the discrete transitions of the FOHPN. While in the former illustration the discrete uniform probability distribution was used at timing the discrete transitions of FOHPN model, in the latter illustration the exponential probability distribution at timing the discrete transitions was used. The results were corroborated by parameters used at the simulations and verbally interpreted.

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