Performance Testing of Multi-Chaotic Differential Evolution Concept on Shifted Benchmark Functions

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Abstract. This research deals with the hybridization of the two softcomputing fields, which are chaos theory and evolutionary computation. This paper aims on the investigations on the multi-chaos-driven evolutionary algorithm Differential Evolution (DE) concept. This paper is aimed at the embedding and alternating of set of two discrete dissipative chaotic systems in the form of chaos pseudo random number generators for the DE. In this paper the novel initial concept of DE/rand/1/bin strategy driven alternately by two chaotic maps (systems) is introduced. From the previous research, it follows that very promising results were obtained through the utilization of different chaotic maps, which have unique properties with connection to DE. The idea is then to connect these two different influences to the performance of DE into the one multi-chaotic concept. Repeated simulations were performed on the selected set of shifted benchmark functions in higher dimensions. Finally, the obtained results are compared with canonical DE.

Keywords: Differential Evolution, Deterministic chaos, Dissipative systems, Optimization.

1 Introduction

These days the methods based on soft computing such as neural networks, evolutionary algorithms (EA's), fuzzy logic, and genetic programming are known as powerful tool for almost any difficult and complex optimization problem. Differential Evolution (DE) [1] is one of the most potent heuristics available.

This research deals with the hybridization of the two softcomputing fields, which are chaos theory and evolutionary computation. This paper is aimed at investigating the novel concept of multi-chaos driven DE. Although a number of DE variants have been recently developed, the focus of this paper is the embedding of chaotic systems in the form of Chaos Pseudo Random Number Generator (CPRNG) for the DE.

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Firstly, the motivation for this research is proposed. The next sections are focused on the description of evolutionary algorithm DE, the concept of chaos driven DE and the used test function. Results and conclusion follow afterwards.

2 Motivation

This research is an extension and continuation of the previous successful initial experiment with chaos driven DE (ChaosDE) [2], [3] with test functions in higher dimensions.

In this paper the novel initial concept of DE/rand/1/bin strategy driven alternately by two chaotic maps (systems) is introduced. From the previous research it follows, that very promising results were obtained through the utilization of Delayed Logistic, Lozi, Burgers and Tinkerbelt maps. The last two mentioned chaotic maps have unique properties with connection to DE: strong progress towards global extreme, but weak overall statistical results, like average cost function (CF) value and std. dev., and tendency to premature stagnation. While through the utilization of the Lozi and Delayed Logistic map the continuously stable and very satisfactory performance of ChaosDE was achieved. The idea is then to connect these two different influences to the performance of DE into the one multi-chaotic concept.

A chaotic approach generally uses the chaotic map in the place of a pseudo random number generator [4]. This causes the heuristic to map unique regions, since the chaotic map iterates to new regions. The task is then to select a very good chaotic map as the pseudo random number generator.

The focus of our research is the embedding of chaotic systems in the form of CPRNG for evolutionary algorithms. The initial concept of embedding chaotic dynamics into the evolutionary algorithms is given in [5]. Later, the initial study [6] was focused on the simple embedding of chaotic systems in the form of chaos pseudo random number generator (CPRNG) for DE and Self Organizing Migration Algorithm (SOMA) [7] in the task of optimal PID tuning. Also the PSO (Particle Swarm Optimization) algorithm with elements of chaos was introduced as CPSO [8]. The concept of ChaosDE proved itself to be a powerful heuristic also in combinatorial problems domain [9]. At the same time the chaos embedded PSO with inertia weigh strategy was closely investigated [10], followed by the introduction of a PSO strategy driven alternately by two chaotic systems [11].

The primary aim of this work is to use and test the implementation of natural chaotic dynamics into evolutionary algorithm as a multi-chaotic pseudo random number generator.

3 Differential Evolution

DE is a population-based optimization method that works on real-number-coded individuals [1]. For each individual $\vec{x}_{i,G}$ in the current generation G, DE generates a new trial individual $\vec{x}'_{i,G}$ by adding the weighted difference between two randomly selected individuals $\vec{x}_{r1,G}$ and $\vec{x}_{r2,G}$ to a randomly selected third individual $\vec{x}_{r3,G}$. The resulting individual $\vec{x}'_{i,G}$ is crossed-over with the original individual $\vec{x}_{i,G}$. The fitness of the resulting individual, referred to as a perturbed vector $\vec{u}_{i,G+1}$, is then compared with the fitness of $\vec{x}_{i,G}$. If the fitness of $\vec{u}_{i,G+1}$ is greater than the fitness of $\vec{x}_{i,G}$, then $\vec{x}_{i,G}$ is replaced with $\vec{u}_{i,G+1}$; otherwise, $\vec{x}_{i,G}$ remains in the population as $\vec{x}_{i,G+1}$. DE is quite robust, fast, and effective, with global optimization ability. It does not require the objective function to be differentiable, and it works well even with noisy and time-dependent objective functions. Please refer to [1], [12] for the detailed description of the used DERand1Bin strategy (1) (both for ChaosDE and Canonical DE) as well as for the complete description of all other strategies.

$$u_{i,G+1} = x_{r1,G} + F \cdot \left(x_{r2,G} - x_{r3,G}\right) \tag{1}$$

4 The Concept of ChaosDE

The general idea of ChaosDE and CPRNG is to replace the default pseudorandom number generator (PRNG) with the discrete chaotic map. As the discrete chaotic map is a set of equations with a static start position, we created a random start position of the map, in order to have different start position for different experiments (runs of EA's). This random position is initialized with the default PRNG, as a one-off randomizer. Once the start position of the chaotic map has been obtained, the map generates the next sequence using its current position.

The first possible way is to generate and store a long data sequence (approx. 50-500 thousands numbers) during the evolutionary process initialization and keep the pointer to the actual used value in the memory. In case of the using up of the whole sequence, the new one will be generated with the last known value as the new initial one.

The second approach is that the chaotic map is not re-initialized during the experiment and any long data series is not stored, thus it is imperative to keep the current state of the map in memory to obtain the new output values.

As two different types of numbers are required in ChaosDE; real and integers, the use of modulo operators is used to obtain values between the specified ranges, as given in the following equations (2) and (3):

$$rndreal = \text{mod}\ (\text{abs}\ (rndChaos)\ ,\ 1.0) \tag{2}$$

$$rndint = \text{mod}\ (\text{abs}\ (rndChaos)\ ,\ 1.0) \times Range + 1 \tag{3}$$

Where *abs* refers to the absolute portion of the chaotic map generated number *rndChaos*, and *mod* is the modulo operator. *Range* specifies the value (inclusive) till where the number is to be scaled.

5 Chaotic Maps

This section contains the description of discrete dissipative chaotic maps used as the chaotic pseudo random generators for DE. In this research, direct output iterations of the chaotic maps were used for the generation of real numbers in the process of crossover based on the user defined CR value and for the generation of the integer values used for selection of individuals. Following chaotic maps were used: Burgers (4), and Lozi map (5).

The Burgers mapping is a discretization of a pair of coupled differential equations which were used by Burgers [13] to illustrate the relevance of the concept of bifurcation to the study of hydrodynamics flows. The map equations are given in (4) with control parameters a = 0.75 and b = 1.75 as suggested in [14].

$$\begin{aligned} X_{n+1} &= aX_n - Y_n^2 \\ Y_{n+1} &= bY_n + X_n Y_n \end{aligned} \tag{4}$$

The Lozi map is a discrete two-dimensional chaotic map. The map equations are given in (5). The parameters used in this work are: a = 1.7 and b = 0.5 as suggested in [14]. For these values, the system exhibits typical chaotic behavior and with this parameter setting it is used in the most research papers and other literature sources.

$$\begin{aligned} X_{n+1} &= 1 - a|X_n| + bY_n \\ Y_{n+1} &= X_n \end{aligned} \tag{5}$$

5.1 Graphical Example – Lozi Map and Burgers Map

The illustrative histograms of the distribution of real numbers transferred into the range <0 - 1> generated by means of studied chaotic maps are in Figures 1 and 2.

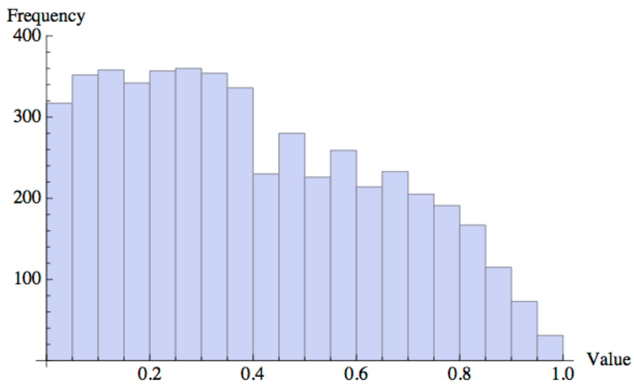

Fig. 1. Histogram of the distribution of real numbers transferred into the range <0 - 1> generated by means of the chaotic Lozi map – 5000 samples

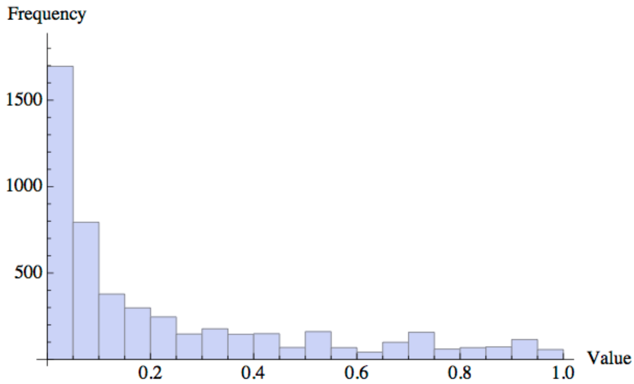

Fig. 2. Histogram of the distribution of real numbers transferred into the range <0 - 1> generated by means of the chaotic Burgers map – 5000 samples

6 Benchmark Functions

For the purpose of evolutionary algorithms performance comparison within this initial research, the shifted 1st De Jong's function (6), shifted Ackley's original function in the form (7) and shifted Rastrigin`s function (8) were utilized.

$$f(x) = \sum_{i=1}^{\dim} (x_i - s_i)^2 \tag{6}$$

Function minimum: Position for E_n: $(x_1, x_2 \ldots x_n) = \boldsymbol{s}$; Value for E_n: $y = 0$
Function interval: <-5.12, 5.12>.

$$f(x) = -20\exp\left(-0.02\sqrt{\frac{1}{D}\sum_{i=1}^{D}(x_i - s_i)^2}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos 2\pi(x_i - s_i)\right) + \\ + 20 + \exp(1) \tag{7}$$

Function minimum: Position for E_n: $(x_1, x_2 \ldots x_n) = \boldsymbol{s}$; Value for E_n: $y = 0$
Function interval: <-30, 30>.

$$f(x) = 10\dim + \sum_{i=1}^{\dim}(x_i - s_i)^2 - 10\cos(2\pi x_i - s_i) \tag{8}$$

Function minimum: Position for E_n: $(x_1, x_2 \ldots x_n) = \boldsymbol{s}$, Value for E_n: $y = 0$

Where s_i is a random number from the 50% range of function interval; $\boldsymbol{s}$ vector is randomly generated before each run of the optimization process.

7 Results

The novelty of this approach represents the utilization of discrete chaotic maps as the multi-chaotic pseudo random number generator for the DE. In this paper, the canonical DE strategy DERand1Bin and the Multi-Chaos DERand1Bin strategy driven alternately by two different chaotic maps (ChaosDE) were used.

The previous research [2], [3] showed that through utilization of Burgers and Tinkerbelt map the unique properties with connection to DE were achieved: strong progress towards global extreme, but weak overall statistical results, like average (benchmark function) Cost Function (CF) value and std. dev. Whereas through the utilization of the Lozi and Delayed Logistic map the continuously stable and very satisfactory performance of ChaosDE was achieved. The idea is then to connect these two different influences to the performance of DE into the one novel multi-chaotic concept. The moment of manual switching over between two chaotic maps as well as the parameter settings for both canonical DE and ChaosDE were obtained analytically based on numerous experiments and simulations (see Table 1)

Table 1. Parameter set up for canonical DE and ChaosDE

DE Parameter	Value
Popsize	75
F	0.8
Cr	0.8
Dimensions	30
Generations	100•D = 3000
Max Cost Function Evaluations (CFE)	225000

Experiments were performed in the combined environments of *Wolfram Mathematica* and *C language*, canonical DE therefore used the built-in *C language* pseudo random number generator *Mersenne Twister C* representing traditional pseudorandom number generators in comparisons. All experiments used different initialization, i.e. different initial population was generated in each run of Canonical or Chaos driven DE.

Within this initial research, one type of experiment was performed. It utilizes the maximum number of generations fixed at 3000 generations. This allowed the possibility to analyze the progress of DE within a limited number of generations and cost function evaluations.

The statistical results of the experiments are shown in Tables 2, 4, 6, which represent the simple statistics for cost function values, e.g. average, median, maximum values, standard deviations and minimum values representing the best individual solution for all 50 repeated runs of canonical DE and several versions of ChaosDE and Multi-ChaosDE.

Tables 3, 5 and 7 compare the progress of several versions of ChaosDE, Multi-ChaosDE and Canonical DE. These tables contain the average CF values for the generation No. 750, 1500, 2250 and 3000 from all 50 runs. The bold values within the all Tables 2 - 7 depict the best obtained result. The graphical comparison of the time evolution of average CF values for all 50 runs of ChaosDE/Multi-ChaosDE and canonical DERand1Bin strategy is depicted in Fig. 3 - 5. Following versions of Multi-ChaosDE were studied:

- *Burgers-Lozi-Switch-500*: Start with Burgers map CPRNG, switch to the Lozi map CPRNG after 500 generations.
- *Lozi-Burgers-Switch-1500*: Start with Lozi map CPRNG, switch to the Burgers map CPRNG after 1500 generations.

Table 2. Simple results statistics for the shifted 1st De Jong's function – 30D

DE Version	Avg CF	Median CF	Max CF	Min CF	StdDev
Canonical DE	5.929778	5.435726	11.69084	2.53501	2.432546
Lozi-No-Switch	3.73E-05	2.27E-05	0.000222	1.54E-06	4.17E-05
Burger-No-Switch	**1.02E-14**	**2.88E-15**	**5.73E-14**	**5.98E-17**	**1.49E-14**
Burger-Lozi-Switch-500	1.78E-06	4.2E-07	2.95E-05	1.59E-08	4.61E-06
Lozi-Burger-Switch-1500	8.34E-10	2.75E-10	1.2E-08	2.81E-11	1.76E-09

Table 3. Comparison of progress towards the minimum for the shifted 1st De Jong's function

DE Version	Generation No. 750	Generation No. 1500	Generation No. 2250	Generation No. 3000
Canonical DE	482.4017	114.3075	26.34619	5.929778
Lozi-No-Switch	90.40304	0.74516	0.004854	3.73E-05
Burger-No-Switch	**0.531726**	**1.33E-05**	**4.32E-10**	**1.02E-14**
Burger-Lozi-Switch-500	2.764289	0.022319	0.000201	1.78E-06
Lozi-Burger-Switch-1500	87.49406	0.709014	3.15E-05	8.34E-10

Table 4. Simple results statistics for the shifted Ackley's original function – 30D

DE Version	Avg CF	Median CF	Max CF	Min CF	StdDev
Canonical DE	3.791676	3.841045	4.518592	2.95934	0.341008
Lozi-No-Switch	0.005533	0.00452	0.014929	0.00154	0.003334
Burger-No-Switch	0.067287	**6.34E-08**	1.501747	**5.47E-09**	0.27714
Burger-Lozi-Switch-500	8.04E-04	7.24E-04	0.002667	1.85 E-04	4.84E-04
Lozi-Burger-Switch-1500	**1.77E-05**	1.03E-05	**7.6E-05**	1.95E-06	**1.46E-05**

Table 5. Comparison of progress towards the min. for the shifted Ackley's original function

DE Version	Generation No. 750	Generation No. 1500	Generation No. 2250	Generation No. 3000
Canonical DE	13.16276	8.511778	5.506989	3.791676
Lozi-No-Switch	8.199525	1.79389	0.081797	0.005533
Burger-No-Switch	**1.548046**	**0.071167**	0.067307	0.067287
Burger-Lozi-Switch-500	2.797948	0.168713	0.009855	8.04E-04
Lozi-Burger-Switch-1500	7.852258	1.621723	**0.003654**	**1.77E-05**

Table 6. Simple results statistics for the shifted Rastrigin`s function – 30D

DE Version	Avg CF	Median CF	Max CF	Min CF	StdDev
Canonical DE	270.9612	273.2324	305.4176	234.2218	15.99992
Lozi-No-Switch	50.68194	45.70853	110.4599	21.52906	21.71585
Burger-No-Switch	44.36785	43.06218	80.76961	16.9143	15.17985
Burger-Lozi-Switch-500	**38.67436**	**36.68143**	82.46749	**16.18175**	**11.82373**
Lozi-Burger-Switch-1500	42.94927	43.09553	**72.74598**	20.8219	13.53718

Table 7. Comparison of progress towards the minimum for the shifted Rastrigin`s function

DE Version	Generation No. 750	Generation No. 1500	Generation No. 2250	Generation No. 3000
Canonical DE	790.1378	404.8734	308.3072	270.9612
Lozi-No-Switch	370.954	177.9286	93.68944	50.68194
Burger-No-Switch	**189.6604**	**55.04461**	**44.56468**	44.36785
Burger-Lozi-Switch-500	221.8914	116.8081	60.55444	**38.67436**
Lozi-Burger-Switch-1500	365.1778	171.6624	57.50722	42.94927

Obtained numerical results given in Tables 2 - 7 and graphical comparisons in Figures 3 - 5 support the claim that all Multi-Chaos/ChaosDE versions have given better overall results in comparison with the canonical DE version. Although the shifted benchmark functions were utilized, from the presented data for the unimodal 1st De Jong's function it follows, that Multi-Chaos DE versions driven by Lozi/Burgers Map have given very satisfactory results, nevertheless the single-chaos concept of original ChaosDE has given the best overall results. High sensitivity of the differential evolution on the selection, settings and internal dynamics of the chaotic PRNG is fully manifested in the case of multi-modal functions. The influence of different internal chaotic dynamics and the exact moment of switching over of two different CPRNGs are clearly visible from Fig. 4 and Fig. 5. Multi-ChaosDE concept has reached the best results from all 50 runs.

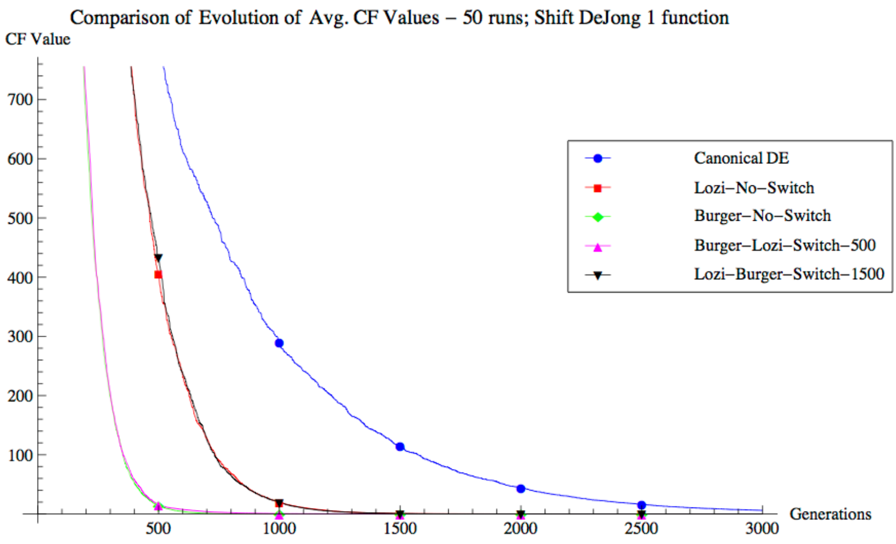

Fig. 3. Comparison of the time evolution of avg. CF values for the all 50 runs of Canonical DE, ChaosDE and Multi-ChaosDE. shifted 1st De Jong's function, $D = 30$.

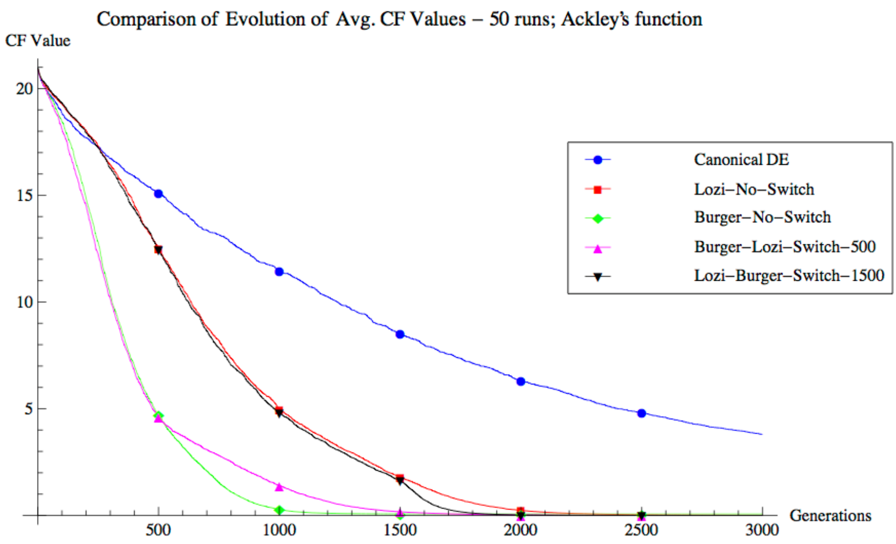

Fig. 4. Comparison of the time evolution of avg. CF values for the all 50 runs of Canonical DE, ChaosDE and Multi-ChaosDE. shifted Ackley's original function, $D = 30$.

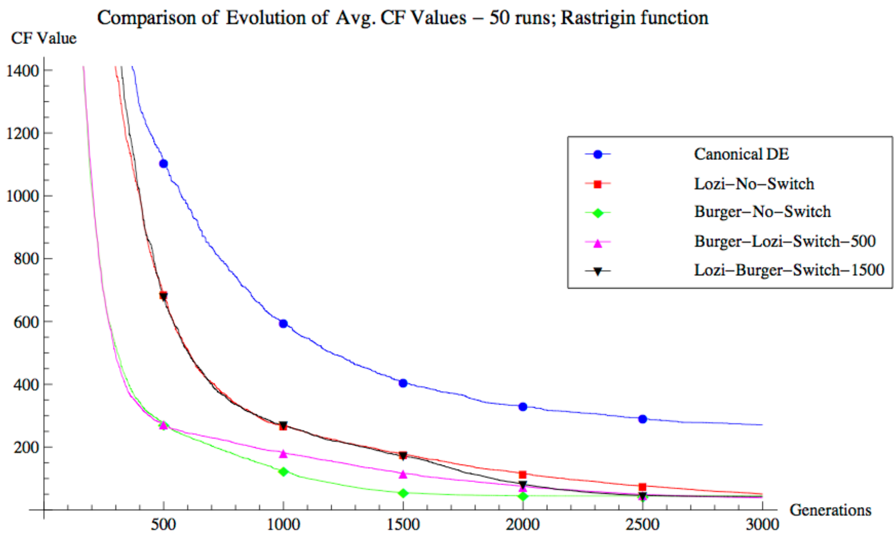

Fig. 5. Comparison of the time evolution of avg. CF values for the all 50 runs of Canonical DE, ChaosDE and Multi-ChaosDE. shifted Rastrigin`s function, $D = 30$.

For the *Burgers-Lozi-Switch-500* version the progressive Burgers map CPRNG secured the faster approaching towards the global extreme from the very beginning of evolutionary process. The very fast switch over to the Lozi map based CPRNG helped to avoid the Burgers map based CPRNG weak spots, which are the weak overall statistical results, like average CF value and std. dev.; and tendency to stagnation. The initial faster convergence (starting of evolutionary process) and subsequent continuously stable searching process without premature stagnation issues is visible from Figures 3 - 5 (magenta lines).

Through the utilization of *Lozi-Burgers-Switch-1500* version, the strong progress towards global extreme given by Burgers map CPRNG helped to the evolutionary process driven from the start by mans of Lozi map CPRNG to achieve almost the best avg. CF and median CF values. The moment of switch (at 1500 generations) is clearly visible from Figures 3 - 5 (black lines).

8 Conclusion

In this paper, the novel concept of multi-chaos driven DERand1Bin strategy was tested and compared with the canonical DERand1Bin strategy on the selected benchmark function in higher dimension. Based on obtained results, it may be claimed, that the developed Multi-ChaosDE gives considerably better results than other compared heuristics in cases of complex multimodal benchmark functions.

This research represents the example of hybridizing of two discrete chaotic systems as a multi-chaotic pseudo-random number generator with evolutionary algorithms. Presented data shows the high sensitivity of DE to the selection and internal dynamics of used CPRNG.

Since this was a preliminary study of the novel presented concept, only small set of shifted benchmark functions in higher dimensions was utilized to test the influence of alternating several CPRNGs to the performance of original previous ChaosDE concept. Nevertheless the original concept of ChaosDE itself was tested on huge set of both simple and complex benchmark functions based mostly on the IEEE CEC 2005 benchmark set and with nine different discrete dissipative chaotic systems. Thus based on the deeper analysis of results from the previous research the composition of the presented experiment was prepared.

Future plans include testing of combination of different chaotic systems as well as the adaptive switching and obtaining a large number of results to perform statistical tests.

Furthermore chaotic systems have additional parameters, which can by tuned. This issue opens up the possibility of examining the impact of these parameters to generation of random numbers, and thus influence on the results obtained using differential evolution.

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