Heterogeneous Households: Monopolistic Capitalists, Entrepreneurs and Employees

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Abstract. What are the implications of an uneven distribution of welfare on optimal stabilisation policy? I build a dynamic stochastic general equilibrium model with household heterogeneity in income and consumption with which to solve optimal fiscal and monetary policy over the business cycle. I include three types of household; capitalists, entrepreneurs and workers, and endogenise the selection process between the latter two.

Keywords: limited asset market participation, stabilisation policy, income inequality, business cycles.

1 Introduction

What are the implications of an uneven distribution of welfare on optimal stabilisation policy? This paper presents a framework with which to solve optimal monetary and fiscal policy in the presence of uneven distributions of wealth, income and consumption. To achieve this, I introduce household heterogeneity into the model with idiosyncratic productivity and the possibility of default. There are *ex post* three types of households in the economy including a fixed proportion of capitalists, that is, households whom own all the productive capital, with the remaining households either behaving as entrepreneurs or workers. The entrepreneurs borrow from the capitalists and combine labour and capital to produce output for consumption and investment. The entrepreneurs cannot be forced to commit to the loan contract and with idiosyncratic risk some will renege on the loan. If they do so they can no longer deal with the capitalists and must provide labour as workers. Monopolistic capitalists formulate a contract given that it is possible for the entrepreneurs to default.

We choose to differentiate between capitalists that are able to hold wealth and smooth consumption, and other households in the economy that are not able to do so. The motivation for this set-up begins with Mankiw in [8] who highlights that many households live hand-to-mouth holding virtually zero net worth. This

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has a significant effect on aggregate consumption smoothing and the transmission mechanism of stabilisation policy. Indeed, further research (e.g. Amato & Laubach in [1], Bilbiie in [2], and Bilbiie & Straub in [3]) has emphasised this, showing that the presence of this behaviour leads to endogenous persistence in output and inflation, and alters both the transmission of monetary policy and the welfare objective. We then endogenise the selection of households to act as either entrepreneurs or workers. This follows a literature beginning with Quadrini in [9], followed by Cagetti & De Nardi in [5]. Doing so enriches the analysis of distributional variables, and by capturing firm exit and employment, models labour market dynamics in a novel way. The model then distinguishes between the optimising behaviour of three types of household; the capitalist, the entrepreneur, and the worker.

2 The Model

The economy is comprised of a large number of identical families, each of which contains *ex post* three types of household; capital owners, entrepreneurs and workers.

2.1 Firms and Production

Firms use capital and labour to produce output according to a standard Cobb-Douglas production function

$$Y_{f,i,t} = z_t \hat{H}^{\alpha}_{f,i,t} K^{1-\alpha}_{f,t-1}$$

for firm i where subscript f identifies a firm-level variable. Capital is invariant across firms as the contract is agreed prior to realisation of the idiosyncratic productivity. The variable z_t represents total factor productivity experienced by the whole economy. In addition to the economy-wide productivity, each firm experiences an independent across time and population productivity shock such that at time t, firm i is defined by idiosyncratic labour productivity status $\varepsilon_{i,t}$. We assume that ε is uniformly distributed across time and space with mean 1 and with $\varepsilon_l \leq \varepsilon \leq \varepsilon_h$, and enters the production function by introducing efficiency hours $\hat{H}_{f,i,t}$ where

$$\hat{H}_{f,i,t} = \varepsilon_{i,t} H_{f,i,t}$$

Within each firm there is an employer and some workers. Indexing the total population to 1, the proportion of households that are the capitalists is exogenously determined and given by $1 - \phi$. The remaining ϕ of households are then made up of the entrepreneurs and workers. The proportion of households that have defaulted and supply labour as workers is given by $\phi\theta_{t-1}$. It follows then that the proportion of households in the economy that borrow capital and hire workers is given by $\phi(1 - \theta_{t-1})$. The firm's labour input is given by

$$H_{f,i,t} = H_{e,i,t} + H_{w,i,t}.$$

which is simply the sum of the labour input by the employer $H_{e,i,t}$ and that by all workers at the firm $H_{w,i,t}$.

2.2 Households

All households seek to maximise expected, discounted lifetime utility under the standard assumptions of rational expectations. We assume all households derive utility according the standard log utility

$$U_t = \ln(C_t) + \chi \ln(1 - H_t).$$

where C_t is household consumption, and H_t household labour supply. Other than access to the asset market, the capitalists differ from the other households in the economy only in that they do not provide labour. Just below we will describe the optimal default decision faced by the entrepreneur. We will then look at the labour market before solving the capitalist's problem.

Idiosyncratic Productivity and Default. If each firm is identical other than the idiosyncratic labour productivity ε then we can attempt to find some critical values of productivity at which point an entrepreneur will be indifferent between defaulting and not. The following equation describes the evolution of θ (or the firm entry and exit in the economy).

$$\theta_t = (1 - \zeta)\theta_{t-1} + \xi_t(1 - \theta_{t-1})$$

A worker faces a fixed probability ζ or being forgiven and so ξ_t is then the proportion of employers that default, or in other words, the proportion of firms that exit. Recalling that $\varepsilon \sim \mathcal{U}(a, b)$, if all employers that experience $\varepsilon > \varepsilon_c$ will decide to default, then we can say

$$\xi_t = (b - \varepsilon_{c,t})(b - a)^{-1}$$

Finding the critical productivity status, $\varepsilon_{c,t}$ will imply ξ_t and given θ_{t-1} we can solve θ_t. This, with the capital stock K_t, total factor productivity z_t, and any new exogenous shocks, determine the state of the economy in period $t + 1$. The value of $\varepsilon_{c,t}$ is solved by finding the firm that is indifferent between defaulting and not.

We let $V_{e,i,t}$ be the value to entrepreneur i of keeping to the contract at time t. This is given as the utility gained from doing so in period t plus the discounted expected future utilities of defaulting or not multiplied by their probabilities. Similarly, $V_{d,i,t}$ is the value to the household if they have defaulted. These are given by

$$V_{e,i,t} = U_{e,i,t} + \beta \mathbb{E}_t \left[\xi_{t+1} \left(V_{d,i,t+1} + \omega_{i,t+1}\right) + \left(1 - \xi_{t+1}\right) V_{e,i,t+1}\right] \quad (1)$$

$$V_{d,i,t} = U_{d,i,t} + \beta \mathbb{E}_t \left[\zeta V_{e,i,t+1} + (1 - \zeta) V_{d,i,t+1}\right] \quad (2)$$

where $\omega_{i,t}$ represents the additional utility from stealing the output, and $U_{e,i,t}$ and $U_{d,i,t}$ the period utility gained by an employer sticking to a contract, and by a worker respectively. For a firm, j, that realises the critical ε_c and is indifferent between defaulting or not, the condition $V_{e,j,t} = V_{d,j,t} + \omega_{j,t}$ must hold. Solving this leads to the indifference condition for the marginal entrepreneur

$$U_{e,j,t} = U_{s,j,t} + \beta \mathbb{E}_t \begin{bmatrix} (\zeta + \xi_{t+1} - 1)\left(V_{e,j,t+1} - V_{d,j,t+1}\right) \\ -\xi_{t+1}\left(U_{s,j,t+1} - U_{d,j,t+1}\right) \end{bmatrix}, \tag{3}$$

where $V_{e,t}$ and $V_{d,t}$ are expressed recursively as in equations 1 and 2 and $U_{s,j,t}$ is the utility gained in default. This condition determines the critical value of ε_t and so the proportion of employees that will default in period t, ξ_t. This then determines the value of θ_t which is the proportion of households that are workers in the following period $t+1$.

Capital Investment and the Contract Form. Before looking at the labour market, we state our assumption of the evolution of capital and the functional form of the contract. Recent research (such as Bose, Pal & Sappington in [4]) has shown linear contracts to be close approximations of fully optimal contracts and so we assume that contract offered by the capitalist will be of linear form. The contract is agreed between capitalist and entrepreneur and offers the payout

$$\mathcal{C}_{f,i,t} = \gamma_{1,t} + \gamma_{2,t} Y_{f,i,t}$$

where $\mathcal{C}_{f,i,t}$ is the payout to a firm based on the firm output and will be consumed by the entrepreneurs and workers. The capitalists own the capital and are responsible for the investment decisions given the capital accumulation process

$$K_t = I_t + (1-\delta)K_{t-1}.$$

with K_t and δ as the end of period capital stock and the depreciation rate respectively. The contracts are agreed prior to idiosyncratic productivity and so all firms are identical. We can then state, for all firms,

$$K_{f,t-1} = (\phi(1-\theta_{t-1}))^{-1} K_{t-1}.$$

Timing of Events. The timing of events in each period are important to clarify.

1. The proportion of workers that have defaulted θ_{t-1} and the capital stock K_{t-1} is inherited from the end of the previous period.
2. Any exogenous aggregate shocks occurring in the period are realised.
3. The monopolistic capitalists offer contracts to maximise their lifetime utility.
4. The firms realise idiosyncratic productivity status ε and choose labour demand, entrepreneurs and workers choose labour supply. Firms yield output Y_t.
5. The proportion of entrepreneurs ξ_t default and ζ workers are forgiven, realising θ_t.
6. The capitalists choose to consume or invest the income from the contracts which will realise the capital stock K_t to be used in the next period.

We can now derive the labour market conditions.

Labour Demand. The firm owner seeks to maximise their consumption and will choose labour demanded to do so. Let $C_{e,i,t}$ be the consumption of firm owner i with productivity status ε_i at time t.

$$C_{e,i,t} = \gamma_{1,t} + \gamma_{2,t} Y_{f,i,t} - w_t H_{w,i,t} \tag{4}$$

$H_{w,i,t}$ is the worker labour input and is allowed to take negative values in which case it represents the entrepreneur selling their own labour back to the labour market at the market wage rate. With many firms and many workers it is assumed that the labour market is perfect and so all agents are price takers. Maximising consumption and solving for firm i labour demand $H_{w,i,t}$ gives

$$H_{w,i,t} = \left(\alpha\gamma_{2,t} z_t w_t^{-1} \varepsilon_i^{\alpha}\right)^{\frac{1}{1-\alpha}} K_{f,t-1} - H_{e,i,t}, \tag{5}$$

Workers Labour Supply. Given that all workers receive the same wage rate, and with no wealth accumulation, every worker supplies the same labour. Whilst the labour demanded by each firm varies, the workers can be seen to provide labour to a central pool which is then distributed to the firms unevenly according to the firm's demand at the market wage. The labour supply condition for the representative worker is found by maximising their utility subject to the budget constraint

$$C_{d,t} = w_t H_{d,t}.$$

This leads to

$$H_d = (1 + \chi)^{-1}$$

for all workers. The time subscript is dropped as this is time-invariant.

Entrepreneur Labour Supply. The entrepreneurs have the same objective function as the workers but with the budget constraint as in equation 4. This leads to the labour supply condition

$$\chi \frac{1}{2} \frac{C_{e,i,t}}{1 - H_{e,i,t}} = w_t \tag{6}$$

Labour Market Equilibrium. We assume the labour market clears and so the total labour demanded must equate that supplied as in

$$\theta_{t-1} H_d = (1 - \theta_{t-1}) \frac{1}{\varepsilon_h - \varepsilon_l} \int_{\varepsilon_l}^{\varepsilon_h} H_{w,i,t} d\varepsilon$$

Substituting in the labour demand equation 5, we can solve for the market wage

$$w_t = A^{1-\alpha} \alpha\gamma_{2,t} z_t G_t^{1-\alpha}$$

where

$$A = \left({\varepsilon_h}^{\frac{1}{1-\alpha}} - {\varepsilon_l}^{\frac{1}{1-\alpha}} \right) (\varepsilon_h - \varepsilon_l)^{-1} (1-\alpha)$$

and $G_t = K_{t-1}/H_t$ is the capital-labour ratio over the whole economy with H_t as measure of aggregate labour given by

$$H_t = \phi(\theta_{t-1} H_d + (1-\theta_{t-1}) H_{e,t}) \tag{7}$$

with the representative entrepreneur labour supply as $H_{e,t} = \frac{1}{\varepsilon_h - \varepsilon_l} \int_{\varepsilon_l}^{\varepsilon_h} H_{e,i,t} d\varepsilon$. Taking the total labour demanded by firm i, $H_{f,i,t}$ and substituting in the expression for the market wage, we can find

$$H_{f,i,t} = \varepsilon_i^{\frac{\alpha}{1-\alpha}} A^{-1} H_t (\phi(1-\theta_{t-1}))^{-1}.$$

From here and 6 we can express the entrepreneur labour supply as a closed form expression

$$H_{e,i,t} = (2+\chi)^{-1} \left(2 - \chi \gamma_{1,t} w_t^{-1} - \chi A^{-1} \alpha H_t (\phi(1-\theta_{t-1}))^{-1} \varepsilon_i^{\frac{\alpha}{1-\alpha}} \right)$$

So we now conveniently have all firm-level variables as closed form functions of aggregate variables and the firm productivity status $\varepsilon_{i,t}$.

2.3 Aggregations

The aggregate variables can be expressed by integrating over all productivity status realisations. This leads to expressions for aggregate output, output from non-defaulting firms which will appear in the capitalist budget constraint and the representative employee labour supply

$$Y_t = A^{1-\alpha} z_t H_t^{\alpha} K_{t-1}^{1-\alpha}$$

$$\bar{Y}_t = \left(\varepsilon_{c,t}^{\frac{1}{1-\alpha}} - {\varepsilon_l}^{\frac{1}{1-\alpha}} \right) (\varepsilon_{c,t} - \varepsilon_l)^{-1} (1-\alpha) A^{-\alpha} z_t H_t^{\alpha} K_{t-1}^{1-\alpha}$$

$$H_{e,t} = (2+\chi)^{-1} \left(2 - \chi \gamma_{1,t} w_t^{-1} - \chi \alpha (1-\alpha) H_t (\phi(1-\theta_{t-1}))^{-1} \right)$$

2.4 Optimal Contract

To ensure saddle-path stability, we introduce a small adjustment cost to the contract. We can think of these as administrative costs that enter the capitalist's budget constraint as $X_{\gamma_1,t} + X_{\gamma_2,t}$. We want the adjustment costs to disappear in the long-run and so specify the functional forms of these costs as

$$X_{\gamma_j,t} = \varrho_1 \left(1 - \gamma_{j,t}/\gamma_{j,t-1}\right)^2$$

so that the costs are zero in steady state. This is then adequate to ensure saddle-path stability as required.

With the assumption of the availability of a commitment technology, the capitalists offer the contracts to the entrepreneur with full knowledge of the household behaviour and internalising their impact on aggregate conditions. The representative capitalist maximises expected lifetime utility subject to the budget constraint

$$(1-\phi)C_{c,t} + I_t = (1-\gamma_{2,t})\bar{Y}_t - (1-\xi_t)\,\phi\,(1-\theta_{t-1})\,\gamma_{1,t} - (X_{\gamma_1,t} + X_{\gamma_2,t})\,.$$

They achieve this through the appropriate choice of policy instruments $\gamma_{1,t}$ and $\gamma_{2,t}$, and investment I_t. The economy is described in full by the state variables K_{t-1} and θ_{t-1}, the policy choice, $\gamma_{1,t}$ and $\gamma_{2,t}$, and the exogenous shock z_t. The optimization problem for the capital owner is then to choose a path for capital stock, $\{K_{t+s}\}$, and the worker population, $\{\theta_{c,t+s}\}$, to maximise lifetime utility subject to the implementability constraints given by equations 1, 2, 3, and 7. All other variables can then be expressed recursively. We can express these four constraints as

$$\begin{aligned}
0 &= \Theta_t\begin{pmatrix} E_t\left[V_{e,t+1}\right], E_t\left[V_{d,t+1}\right], K_{t-1}, \theta_{t-1}, H_t, \gamma_{1,t}, \gamma_{2,t}, K_t, \theta_t, \\ E_t\left[\theta_{t+1}\right], E_t\left[\gamma_{1,t+1}\right], E_t\left[\gamma_{2,t+1}\right], E_t\left[H_{t+1}\right], z_t \end{pmatrix}\\
0 &= \mathcal{H}_t\left(K_{t-1}, \theta_{t-1}, H_t, \gamma_{1,t}, \gamma_{2,t}, z_t\right)\\
0 &= \mathcal{V}_{e,t}\begin{pmatrix} V_{e,t}, E_t\left[V_{e,t+1}\right], E_t\left[V_{d,t+1}\right], K_{t-1}, \theta_{t-1}, H_t, \gamma_{1,t}, \gamma_{2,t}, K_t, \theta_t, \\ E_t\left[\theta_{t+1}\right], E_t\left[\gamma_{1,t+1}\right], E_t\left[\gamma_{2,t+1}\right], E_t\left[H_{t+1}\right], z_t \end{pmatrix}\\
0 &= \mathcal{V}_{d,t}\left(V_{d,t}, E_t\left[V_{e,t+1}\right], E_t\left[V_{d,t+1}\right], K_{t-1}, H_t, \gamma_{2,t}, z_t\right)
\end{aligned}$$

The representative capitalist then maximises their objective by solving the Lagrangian leading to the *timeless perspective* optimal solution.

$$\mathcal{L} = E_0\sum_{t=0}^{\infty}\beta^t\left\{U_{c,t}(\cdot) + \lambda_{1,t}\Theta_t(\cdot) + \lambda_{2,t}\mathcal{H}_t(\cdot) + \lambda_{3,t}\mathcal{V}_{e,t}(\cdot) + \lambda_{4,t}\mathcal{V}_{d,t}(\cdot)\right\}$$

3 Results and Analysis

We solve the zero-growth steady-state around which we log-linearize to second order. Standard methods are employed to simulate, and find the moments and auto-correlations of the model. Impulse response functions to the productivity shock are also computed. This information is then compared to a benchmark real business cycle model and a set of stylized facts regarding business cycle dynamics.

Table 1 includes the correlations of key variables with output, and the mean of the variables and standard deviation relative to that of output. The last column contains stylized business cycle facts from Canova in [6]. In comparison to the data, the behaviour of the aggregate variables over the business cycle are similar to those in a benchmark real business cycle model. Investment is found to be more volatile and hours less so in comparison to the benchmark model and the data. The latter is partly consequence of the inelastic labour supply of the workers but it also finds that entrepreneur labour drops following a technology shock, a feature supported by the data (e.g. see Canova in [7]) and typically requiring nominal rigidities to replicate. We consider the proposed model a useful framework to begin to answer the question posed at the start.

Table 1. Correlation with output, mean and standard deviation relative to output, and some stylised facts

	Heterogeneous			RBC Baseline			Canova
	Corr w/ Y	Mean	rel. σ	Corr w/ Y	Mean	rel. σ	rel. σ
Y	1	0.894	1	1	1.168	1	1
I	0.91	0.106	7.3036	0.929	0.186	5.184	2.82
K	0.60	4.598	0.7538	0.577	8.07	0.764	0.61
w	0.99	0.155	0.7281	0.964	1.603	0.670	0.70
H	0.48	0.443	0.0937	0.892	0.510	0.403	1.06
$H_{e,a}$	0.01	0.467	0.1843	-	-	-	-
$H_{e,b}$	-0.21	0.396	0.2810	-	-	-	-
θ	0.81	0.219	1.7719	-	-	-	-
C	0.737	0.652	0.555	0.610	0.982	0.453	0.49
C_c	0.71	5.813	0.5076	-	-	-	-
$C_{e,a}$	0.99	0.207	0.7311	-	-	-	-
$C_{e,b}$	0.98	0.234	0.7734	-	-	-	-
C_d	0.99	0.086	0.7296	-	-	-	-

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