A New Method for Group Decision Making Using Group Recommendations Based on Interval Fuzzy Preference Relations and Consistency Matrices

Shyi-Ming Chen and Tsung-En Lin

Department of Computer Science and Information Engineering,
National Taiwan University of Science and Technology,
Taipei, Taiwan, R. O. C.

Abstract. This paper presents a new method for group decision making using group recommendations based on interval fuzzy preference relations and consistency matrices. First, it constructs consistency matrices from interval fuzzy preference relations. Then, it constructs a collective consistency matrix, constructs a weighted collective preference relation, and constructs a group collective preference relation. Then, it constructs a consensus relation for each expert and calculates the group consensus degree for the experts based on the constructed consensus relations. If the group consensus degree is smaller than a predefined threshold value, then it modifies the interval fuzzy preference values in the interval fuzzy preference relations. The above process is performed repeatedly, until the group consensus degree is larger than or equal to the predefined threshold value. Finally, based on the group collective preference relation, it calculates the score of each alternative. The larger the score of the alternative, the better the preference order of the alternative. The proposed method can overcome the drawbacks of the existing methods for group decision making using group recommendations.

Keywords: Consistency Matrices, Group Consensus Degree, Group Decision Making, Group Recommendations, Interval Fuzzy Preference Relations.

1 Introduction

Some group decision making methods have been presented [2]-[19]. In [18], Xu presented a method for group decision making based on the consistency of interval fuzzy preference relations. In [19], Xu and Liu presented a group decision making method based on interval multiplicative preference relations and interval fuzzy preference relations by using the projection with a consensus process. However, in [19], Xu and Liu pointed out that Xu's method [18] has the drawbacks that the weights of experts are not considered, which is not reasonable. Furthermore, it does not consider the consensus level which is necessary in group decision making. Moreover, in this paper, we also found that Xu and Liu's method [19] has the following drawbacks: 1) It has the "divided by zero" problem when the interval preference

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relation of an expert and the group collective preference relation of all experts are the same and 2) It is unreasonable that their method which calculates the consensus degree in the consensus relation for each expert does not hold the commutative law. Therefore, we must develop a new method for group decision making using group recommendations based on interval fuzzy preference relations and consistency matrices to overcome the drawbacks of Xu's method [18] and Xu and Liu's method [19].

In this paper, we present a new method for group decision making using group recommendations based on interval fuzzy preference relations and consistency matrices. The proposed method can overcome the drawbacks of Xu's method [18] and Xu and Liu's method [19] for group decision making using group recommendations.

2 Preliminaries

In this section, we briefly review the concept of interval fuzzy preference relations from [18], briefly review the concept of consistency matrices from [11], and briefly review the concept of consistency degrees from [6].

Definition 2.1 **[18]:** Let P be an interval fuzzy preference relation for the set X of alternatives, where $X = \{x_1, x_2, \ldots, x_n\}$, shown as follows:

$$P = (p_{ij})_{n\times n} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix}, \tag{1}$$

where $p_{ij} = [p_{ij}^-, p_{ij}^+]$ denotes an interval preference value for alternative x_i over x_j. Then, $0 \le p_{ij}^- \le p_{ij}^+ \le 1$, $p_{ji} = 1 - p_{ij} = [1 - p_{ij}^+, 1 - p_{ij}^-]$, $p_{ii}^+ = p_{ii}^- = 0.5$, $1 \le i \le n$, and $1 \le j \le n$.

Definition 2.2 **[11]:** Given a complete fuzzy preference relation $P = (p_{ij})_{n\times n}$, where p_{ij} denotes preference value for alternative x_i over alternative x_j, $p_{ij} + p_{ji} = 1$, $p_{ii} = 0.5$, $1 \le i \le n$, and $1 \le j \le n$. The consistency matrix $\overline{P} = (\overline{p}_{ik})_{n\times n}$ is constructed based on the complete fuzzy preference relation P, shown as follows:

$$\overline{p}_{ik} = \frac{1}{n}\sum_{j=1}^{n}(p_{ij} + p_{jk}) - 0.5. \tag{2}$$

The consistency matrix $\overline{P} = (\overline{p}_{ik})_{n\times n}$ has the following properties:

(1) $\overline{p}_{ik} + \overline{p}_{ki} = 1$,

(2) $\overline{p}_{ii} = 0.5$,

(3) $\overline{p}_{ik} = \overline{p}_{ij} + \overline{p}_{jk} - 0.5$,

(4) $p_{ik} \le p_{is}$ for all $i \in \{1, 2, \ldots, n\}$, where $k \in \{1, 2, \ldots, n\}$ and $s \in \{1, 2, \ldots, n\}$.

***Definition 2.3* [6]:** Let $\overline{P} = (\overline{p}_{ik})_{n \times n}$ be a consistency matrix constructed by a fuzzy preference relation $P = (p_{ij})_{n \times n}$ given by an expert. The consistency degree d between P and $\overline{P}$ is defined as follows:

$$d = 1 - \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \left| p_{ij} - \overline{p}_{ij} \right|, \tag{3}$$

where $d \in [0, 1]$, p_{ij} denotes the preference value in the fuzzy preference relation P for alternative x_i over alternative x_j, $\overline{p}_{ij}$ denotes a preference value in the consistency matrix $\overline{P}$ for alternative x_i over alternative x_j, $1 \leq i \leq n$, and $1 \leq j \leq n$. The larger the value of d, the more consistent the fuzzy preference relation given by the expert. If the value of d is close to one, then the information of the fuzzy preference relation given by the expert is more consistent.

3 A New Method for Group Decision Making Using Group Recommendations Based on Interval Fuzzy Preference Relations and Consistency Matrices

In this section, we present a new method for group decision making using group recommendations based on interval fuzzy preference relations and consistency matrices. Assume that there are m interval fuzzy preference relations P^1, P^2, …, and P^m given by m experts E_1, E_2, …, and E_m, respectively, and assume that there are n alternatives x_1, x_2, …, and x_n. Assume that the interval fuzzy preference relation P^k given by expert E_k for alternative x_i over x_j is shown as follows:

$$P^k = (p_{ij}^k)_{n \times n} = \begin{bmatrix} p_{11}^k & p_{12}^k & \cdots & p_{1n}^k \\ p_{21}^k & p_{22}^k & \cdots & p_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1}^k & p_{n2}^k & \cdots & p_{nn}^k \end{bmatrix}, \tag{4}$$

where p_{ij}^k is an interval-valued preference value, $p_{ij}^k = [p_{ij}^{-k}, p_{ij}^{+k}]$, $0 \leq p_{ij}^{-k} \leq p_{ij}^{+k} \leq 1$, $p_{ji}^k = 1 - p_{ij}^k = [1 - p_{ij}^{+k}, 1 - p_{ij}^{-k}]$, $p_{ii}^{+k} = p_{ii}^{-k} = 0.5$, $1 \leq i \leq n$, $1 \leq j \leq n$, and $1 \leq k \leq m$. The proposed method is now presented as follows:

Step 1: Initially, let $r = 0$. Construct the fuzzy preference relation $B^k = (b_{ij}^k)_{n \times n}$ for expert E_k, construct the consistency matrix $\overline{B}^k = (\overline{b}_{ij}^k)_{n \times n}$ for expert E_k, construct the collective consistency matrix $\overline{B}^* = (\overline{b}_{ij}^*)_{n \times n}$ for all experts, and calculate the consistency degree d_k of expert E_k, shown as follows:

$$b_{ij}^k = \frac{1}{2}(p_{ij}^{-k} + p_{ij}^{+k}), \tag{5}$$

$$\bar{b}_{ij}^{k} = \frac{1}{n}\sum_{t=1}^{n}(b_{it}^{k} + b_{tj}^{k}) - 0.5, \tag{6}$$

$$d_k = 1 - \frac{2}{n(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|b_{ij}^{k} - \bar{b}_{ij}^{k}\right|, \tag{7}$$

$$\bar{b}_{ij}^{*} = \frac{1}{m}\sum_{k=1}^{m}\bar{b}_{ij}^{k}, \tag{8}$$

where $1 \le i \le n$, $1 \le j \le n$ and $1 \le k \le m$.

Step 2: Calculate the weight λ_k of expert E_k using the consistency degree d_k, shown as follows:

$$\lambda_k = \frac{d_k}{\sum_{t=1}^{m} d_t}, \tag{9}$$

where $1 \le k \le m$. Construct the weighted collective preference relation $P^* = \left(p_{ij}^{*}\right)_{n\times n}$ for all experts and construct the group collective preference relation $U = \left(u_{ij}^{*}\right)_{n\times n}$ for all experts, shown as follows:

$$p_{ij}^{*} = \sum_{k=1}^{m}\lambda_k (p_{ij}^{k}) = \left[\sum_{k=1}^{m}\lambda_k\left(p_{ij}^{-k}\right), \sum_{k=1}^{m}\lambda_k\left(p_{ij}^{+k}\right)\right] = \left[p_{ij}^{-*}, p_{ij}^{+*}\right], \tag{10}$$

$$u_{ij}^{*} = \left[\frac{p_{ij}^{-*} + \bar{b}_{ij}^{*}}{2}, \frac{p_{ij}^{+*} + \bar{b}_{ij}^{*}}{2}\right] = \left[u_{ij}^{-*}, u_{ij}^{+*}\right], \tag{11}$$

where $1 \le i \le n$, $1 \le j \le n$, and $1 \le k \le m$.

Step 3: Construct the consensus relation $C^k = \left(c_{ij}^{k}\right)_{n\times n}$ for expert E_k and calculate the group consensus degree CD for all experts, shown as follows:

$$c_{ij}^{k} = 1 - \frac{1}{2}\left|u_{ij}^{*} - p_{ij}^{k}\right| = 1 - \frac{1}{2}\left(\left|u_{ij}^{-*} - p_{ij}^{-k}\right| + \left|u_{ij}^{+*} - p_{ij}^{+k}\right|\right), \tag{12}$$

$$CD = \frac{\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n}\sum_{k=1}^{m} c_{ij}^{k}}{m\times(n^2 - n)}, \tag{13}$$

where $c_{ii}^{k} = 1$, $1 \le i \le n$, $1 \le j \le n$, and $1 \le k \le m$. If the group consensus degree CD is smaller than the predefined threshold value γ, where $\gamma \in [0, 1]$, then let $r = r + 1$ and go to Step 4. Otherwise, Step 9.

Step 4: Construct the proximity relation $F^k = \left(f_{ij}^{k}\right)_{n\times n}$ for expert E_k, shown as follows:

$$f_{ij}^{k} = \left[u_{ij}^{-} - p_{ij}^{-k}, u_{ij}^{+} - p_{ij}^{+k} \right] = \left[f_{ij}^{+k}, f_{ij}^{-k} \right], \tag{14}$$

where $\gamma \in [0, 1]$, $1 \leq i \leq n$, $1 \leq j \leq n$, and $1 \leq k \leq m$. If the consensus value c_{ab}^{k} in the consensus relation C^{k} is smaller than the group consensus degree CD, then get the set H^k of pairs (a, b) of alternatives x_a and x_b which satisfy "$c_{ab}^{k} < CD$", shown as follows:

$$H^{k} = \left\{ (a,b) \middle| c_{ab}^{k} < CD \right\}, \tag{15}$$

where the corresponding preference value of c_{ab}^{k} in the interval fuzzy preference relation P^k given by expert E^k is p_{ab}^{k}, $1 \leq a \leq n$, $1 \leq b \leq n$, and $1 \leq k \leq m$. Construct the modified interval fuzzy preference relation $P^{k(r)} = (p_{ij}^{k(r)})_{n\times n}$ for expert E_k using the proximity relations F^k and the modified constant δ, where $\delta \in (0, 1]$ and $1 \leq k \leq m$, shown as follows:

$$p_{ij}^{k(r)} = \begin{cases} p_{ij}^{k(r-1)} - \delta \times f_{ij}^{k}, & \text{if } (i, j) \in H^{k} \\ p_{ij}^{k(r-1)}, & \text{otherwise} \end{cases} \tag{16}$$

where $\delta \in (0, 1]$, (r) denotes the rth round, $1 \leq i \leq n$, $1 \leq j \leq n$, and $1 \leq k \leq m$. Go to Step 5.

Step 5: Based on Eqs. (5)-(8), update the fuzzy preference relation $B^{k} = (b_{ij}^{k})_{n\times n}$ for expert E_k, update the consistency matrix $\overline{B}^{k} = (\overline{b}_{ij}^{k})_{n\times n}$ for expert E_k, calculate the consistency degree d_k of expert E_k, and update the collective consistency matrix $\overline{B}^{*} = (\overline{b}_{ij}^{*})_{n\times n}$ for all experts, respectively.

Step 6: Based on Eq. (9), calculate the weight λ_k of expert E_k using the updated consistency degree d_k, where $1 \leq k \leq m$. Based on Eqs. (10) and (28), update the weighted collective preference relation $P^{*} = \left(p_{ij}^{*} \right)_{n\times n}$ for all experts, and update the group collective preference relation $U = \left(u_{ij}^{*} \right)_{n\times n}$ for all experts, respectively.

Step 7: Based on Eqs. (12) and (13), update the consensus relation $C^{k} = \left(c_{ij}^{k} \right)_{n\times n}$ for expert E_k and calculate the group consensus degree CD for all experts, respectively, where $1 \leq k \leq m$. If the consensus degree CD is smaller than the predefined threshold value γ, where $\gamma \in [0, 1]$, then let $r = r + 1$ and go to Step 8. Otherwise, go to Step 9.

Step 8: Based on Eq. (14)-(16), update the proximity relations $F^{k} = \left(f_{ij}^{k} \right)_{n\times n}$ for expert E_k, where $1 \leq k \leq m$, get the set H^k of pair (a, b) of alternatives x_a and x_b, and update the modified interval fuzzy preference relation $P^{k(r)} = (p_{ij}^{k(r)})_{n\times n}$ for expert E_k using the proximity relations F^k and the modified constant δ, respectively, where $\delta \in (0, 1]$ and $1 \leq k \leq m$. Go to Step 5.

Step 9: Based on the group collective preference relation U for all experts, calculate the score $R(x_i)$ of each alternative x_i, shown as follows:

$$R(x_i) = \frac{1}{n^2} \sum_{j=1}^{n} (u_{ij}^{+} + u_{ij}^{-}), \tag{17}$$

where $1 \le i \le n$ and $1 \le j \le n$. The larger the value of $R(x_i)$, the better the preference order of alternative x_i, where $1 \le i \le n$.

In the following, we use an example to illustrate the group decision making process of the proposed method.

Example 3.1 **[18]:** Assume that there are five alternatives x_1, x_2, x_3, x_4 and x_5 and assume that the interval fuzzy preference relations P^1, P^2 and P^3 given by the experts E_1, E_2 and E_3, respectively, are shown as follows:

$$P^1 = \begin{pmatrix} [0.5, 0.5] & [0.6, 0.8] & [0.7, 1] & [0.2, 0.3] & [0.4, 0.5] \\ [0.2, 0.4] & [0.5, 0.5] & [0.4, 0.6] & [0.7, 0.8] & [0.3, 0.5] \\ [0, 0.3] & [0.4, 0.6] & [0.5, 0.5] & [0.6, 0.9] & [0.4, 0.7] \\ [0.7, 0.8] & [0.2, 0.3] & [0.1, 0.4] & [0.5, 0.5] & [0.3, 0.4] \\ [0.5, 0.6] & [0.5, 0.7] & [0.3, 0.6] & [0.6, 0.7] & [0.5, 0.5] \end{pmatrix},$$

$$P^2 = \begin{pmatrix} [0.5, 0.5] & [0.5, 0.7] & [0.8, 0.9] & [0.3, 0.5] & [0.3, 0.6] \\ [0.3, 0.5] & [0.5, 0.5] & [0.6, 0.7] & [0.5, 0.6] & [0.4, 0.5] \\ [0.1, 0.2] & [0.3, 0.4] & [0.5, 0.5] & [0.7, 0.9] & [0.6, 0.7] \\ [0.5, 0.7] & [0.4, 0.5] & [0.1, 0.3] & [0.5, 0.5] & [0.5, 0.6] \\ [0.4, 0.7] & [0.5, 0.6] & [0.3, 0.4] & [0.4, 0.5] & [0.5, 0.5] \end{pmatrix},$$

$$P^3 = \begin{pmatrix} [0.5, 0.5] & [0.7, 0.9] & [0.8, 1] & [0.4, 0.5] & [0.3, 0.4] \\ [0.1, 0.3] & [0.5, 0.5] & [0.6, 0.7] & [0.4, 0.7] & [0.4, 0.6] \\ [0, 0.2] & [0.3, 0.4] & [0.5, 0.5] & [0.7, 0.8] & [0.5, 0.8] \\ [0.5, 0.6] & [0.3, 0.6] & [0.2, 0.3] & [0.5, 0.5] & [0.4, 0.7] \\ [0.6, 0.7] & [0.4, 0.6] & [0.2, 0.5] & [0.3, 0.6] & [0.5, 0.5] \end{pmatrix},$$

Assume that the predefined threshold value $\gamma = 0.94$ and assume that the modified constant $\delta = 2/3$. Table 1 shows the scores of the alternatives and the group consensus degree for each round by applying the proposed method. Fig. 1 shows the scores of the alternatives for different rounds when the predefined threshold value $\gamma = 1$ by applying the proposed method. Fig. 2 shows the group consensus degrees for different rounds when the predefined threshold value $\gamma = 1$ by applying the proposed method.

Table 2 makes a comparison of the experimental results of the proposed method with Xu and Liu's method [19] and Xu's method [18]. From the Table 2, we can see that the preference order of the alternatives x_1, x_2, x_3, x_4 and x_5 obtained by the proposed method and Xu and Liu's method [19] are the same, i.e., $x_1 > x_5 > x_3 > x_2 > x_4$. However, the preference order of the alternatives x_1, x_2, x_3, x_4 and x_5 obtained by Xu's method [18] is: $x_1 > x_2 > x_5 > x_3 > x_4$. In [19], Xu and Liu [19] have pointed out that Xu's method [18] has the drawbacks that 1) the weights of experts are not considered, which is not reasonable and 2) it does not consider the consensus level which is necessary in group decision making. Therefore, Xu's method [18] gets an unreasonable result of the preference order of the alternatives in this situation.

Table 1. The scores of the alternatives and the group consensus degree at the *r*th round by the proposed method for Example 3.1

Round Number r	Scores of the Alternatives					Consensus Degrees CD
	$R(x_1)$	$R(x_2)$	$R(x_3)$	$R(x_4)$	$R(x_5)$	
0	0.2279	0.1974	0.1934	0.1786	0.2027	0.8989
1	0.2252	0.1978	0.1945	0.1798	0.2027	0.9268
2	0.2211	0.1972	0.1986	0.1784	0.2047	0.9436
3	0.2196	0.1950	0.2008	0.1806	0.2041	0.9571
⋮	⋮	⋮	⋮	⋮	⋮	⋮
33	0.2169	0.1958	0.2026	0.1794	0.2053	0.9999
34	0.2169	0.1958	0.2026	0.1794	0.2053	1.0000

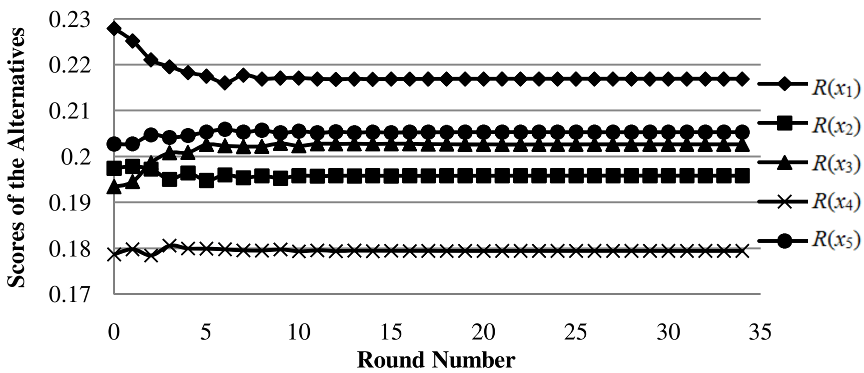

Fig. 1. The scores of the alternatives for different rounds when the predefined threshold value $\gamma = 1$ by the proposed method.

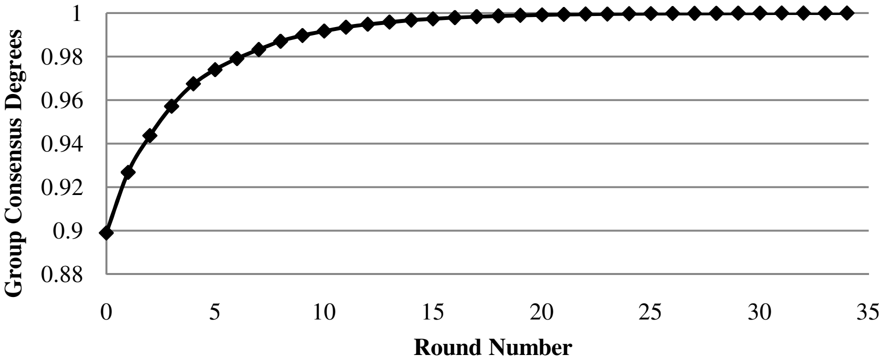

Fig. 2. The group consensus degrees for different rounds when the predefined threshold value $\gamma = 1$ by the proposed method

Table 2. A comparison of the experimental results for different methods for Example 3.1

Methods	Preference Order
Xu's Method [18]	$x_1 > x_2 > x_5 > x_3 > x_4$
Xu and Liu's Method [19]	$x_1 > x_5 > x_3 > x_2 > x_4$
The Proposed Method	$x_1 > x_5 > x_3 > x_2 > x_4$

4 Conclusions

We have presented a new method for group decision making using group recommendations based on interval fuzzy preference relations and consistency matrices. It can overcome the drawbacks of Xu's method [18] and Xu and Liu's method [19], where Xu's method [18] has the drawbacks that the weights of experts are not considered, which is not reasonable, and it does not consider the consensus level which is necessary in group decision making; Xu and Liu's method [19] has the following drawbacks: 1) It has the "divided by zero" problem when the interval preference relation of an expert and the group collective preference relation of all experts are the same and 2) It is unreasonable that their method which calculates the consensus degree in the consensus relation for each expert does not hold the commutative law. The proposed method provides us with a useful way for group decision making using group recommendations based on interval fuzzy preference relations and consistency matrices.

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