Chaos Level Measurement in Logistic Map Used as the Chaotic Numbers Generator in Differential Evolution

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Abstract. In present time some researchers use chaotic numbers generators in evolutionary algorithms like differential evolution, SOMA or particle swarm optimization. These chaotic numbers generators are based on chaotic discrete systems which replace pseudorandom numbers generators like Mersenne Twister, Xorshift etc. In this paper we will investigate the influence of chaos level in logistic map which is used as chaotic numbers generator to the convergence's speed of differential evolution to the global minimum of testing functions.

1 Introduction

Differential evolution (DE) uses pseudorandom numbers generators in many steps of the algorithm. At first pseudorandom numbers generator (PSNG) is used when the first population is created – parameters of individual are generated randomly in lower and upper bounds. Then DE needs PSNG in random choosing of three different parents, then PSNG is needed in crossing, etc. In this paper PSNG is replaced by chaotic discrete system – Logistic map. We know that the level of chaos is given by Lyapunov exponents. If Lyapunov exponent is greater than zero, system's behavior can be described as chaotic. This paper will deal with influence of chaos level to convergence's speed of DE. In section 2 we explain motivation of this paper, in section 3 experiments design is described. In section 4 we can see results of experiments and section 5 summarize findings.

1.1 Differential Evolution

In DE we will see all principles of evolutionary algorithms – natural selection, crossing and mutation. The principle of DE can be described like this:

– First population is generated randomly. Number of individuals is given by the parameter NP. The values of parameters can be only from interval [lower

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bound, upper bound]. Individuals are evaluated – fitness value is computed according to the cost function. Fitness value says how the individual is good for population.
- Difference between the first chosen parent $x_{r1,j}$ and parent $x_{r2,j}$ is multiplied by mutation constant F and the third parent $X_{r3,j}$ is added to the result. The noise vector is created.
- New individual creation: Random number r from interval [0,1] is generated. If $r < CR$, where CR is crossing probability, parameter from the noise vector is taken to the new individual. If $r > CR$ parameter from the actual individual is taken. Fitness value of a new individual is computed. If the fitness value is better than fitness value of actual individual, the actual individual will be replaced by new individual in next generation. If not, actual individual will be taken to the next generation [1].

In this paper DE called DE/rand/1/bin is used.

In present time DE appears in many areas of research. In [3] authors deal with multi objective optimization by an adaptive DE. In [4] DE plays an essential role in identification time-delayed fractional order chaos. In [5] authors present a novel Particle Swarm Optimization (PSO) based on a non-homogenous Markov chain and DE and in [6] authors analyze the behavior of DE algorithm applied to the objective function, which are transformed by means of local searches. Authors of [7] use distributed DE in detecting moving objects from a video sequence. In [8] authors describe repairing the crossover rate in adaptive DE.

DE in connection with chaos and chaotic systems is mentioned for example in [9] – [13].

1.2 Chaos Level

Chaos level is given by Lyapunov exponent's value. Lyapunov exponent is computed for an orbit. We know that the Lyapunov exponent can be undefined for some orbits. In [2] authors says: "*In particular, an orbit containing a point x_i with $f'(x) = 0$ causes the Lyapunov exponent to be undefined.*"

Definition 1. *Let f be a smooth map of the real line $\Re$. The Lyapunov number $L(x_1)$ of the orbit $x_1, x_2, x_3 \ldots$ is defined as*

$$L(x_1) = \lim_{x\to\infty} |f'(x_1)| \ldots |f'(x_n)|)^{\frac{1}{n}} \tag{1}$$

if this limit exists. The Lyapunov exponent $h(x_1)$ is defined as

$$h(x_1) = \lim_{x\to\infty} (\frac{1}{n})[\ln|f'(x_1)| + \ldots + \ln|f'(x_n)|] \tag{2}$$

if this limit exists. Notice that h exists if and only if L exists and is nonzero, and $\ln(L) = h$ *[2].*

Definition 2. *Let f be a map of the real line $\Re$, and let $x_1, x_2, \ldots$ be a bounded orbit of f. The orbit is chaotic if:*

– $\{x_1, x_2, \ldots\}$ *is not asymptotically periodic*
– *the Lyapunov exponent is greater than zero [2].*

1.3 Chaos Level in Logistic Map

Logistic map is defined by Eg.3.

$$x_{n+1} = ax(1 - x_n) \tag{3}$$

In our research we will change value of parameter a and for each value of a the Lyapunov exponent will be computed. Then we will observe influence of the computed Lyapunov exponent value to the DE convergence's speed to the global minimum.

Logistic map appears for example in [14], where period 3 and chaos for unimodal maps are studied. In [15] Logistic map is mentioned in connection with chaos optimization algorithms based on chaotic maps with different probability distribution. In [16] authors describe logistic neural networks and their chaotic pattern recognition properties and in [17] discrete fractional Logistic map and its chaos is investigated.

2 Motivation

As it was mentioned above, the main goal of this research was to investigate differential evolution convergence's speed reliance on Lyapunov exponent's values. Chaos is defined by a Lyapunov exponent greater than zero [2]. In this paper we observe differential evolution convergence's speed when Lyapunov exponent acquires different values.

3 Experiment Design

Precise setting of DE parameters is mentioned in the Table 1, where NP means number of individuals in population, D dimension (number of parameters of the individual), $Generations$ means number of generation cycles, F mutation constant and CR crossing probability. In our research Schwefel's (see Eq.4), Griewangk's (see Eq.5), Rastrigin's (see Eq.6), Egg Holder's (see Eq.7) and Rana's (see Eq.8) functions have been used as cost functions. Schwefel's global minimum is $f(x) = -415.9829D$ where D denotes dimension, for Rastrigin's and Griewangk's the global minimum is $f(x) = 0$. For Egg Holder's and Rana's functions there is not common formula for easy calculation of global minimum value. For experiments HP Pavilion dv7-6050 with processor Intel Core i7 with frequency 2 GHz, 4 GB RAM and graphic card AMD Radeon HD 6770M and Microsoft Visual Studio 2010 have been used. The experiments have been processed by Mathematica 8.

$$\sum_{i=1}^{D} -x_i \sin\left(\sqrt{|x_i|}\right) \tag{4}$$

$$1 + \sum_{i=1}^{D} \frac{x_i^2}{4000} - \prod_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}}\right) \quad (5)$$

$$10D + \sum_{i=1}^{D} x_i^2 - 10cos\left(2\pi x_i\right) \quad (6)$$

$$\begin{array}{l}\sum_{i=1}^{D-1} \left(-x_i \sin(\sqrt{|x_i - x_{i+1} - 47|}) - \right. \\ \left. -(x_{i+1} + 47)\sin(\sqrt{|x_{i+1} + 47 + \frac{x_i}{2}|})\right)\end{array} \quad (7)$$

$$\begin{array}{l}\sum_{i=1}^{n-1} \left[(x_{i+1} + 1)\cos\left(\sqrt{|x_{i+1} - x_i + 1|}\right) + \right. \\ + \sin\left(\sqrt{|x_{i+1} + x_i + 1|}\right) + \\ + x_i \cos\left(\sqrt{|x_{i+1} + x_i + 1|}\right) \\ \left. \sin\left(\sqrt{|x_{i+1} - x_i + 1|}\right)\right]\end{array} \quad (8)$$

Table 1. DE setting

Parameter	Value
NP	50
D	20
$Generations$	1800
F	0.9
CR	0.4

At first parameter a of Logistic map had been set to the beginning value $a = 3.5$ and then it was increased by 0.01. For each value of parameter a one hundred experiments have been generated. For each cost function 5100 experiments have been generated. Initial value of x has been set to $x = 0.02$. This value has been chosen randomly.

4 Results

In Table 2 we can see resultant values of parameter a, Lyapunov exponent, average fitness and median fitness for all cost functions. These results are mentioned in connection with the biggest convergence's speed of DE. In Tables 3 and 4 we can find cost functions median fitness values intervals for $a \in [3.50, 3.60]$ and $a \in [3.61, 4.00]$. Dependence of Lyapunov exponents on parameter a is shown in Fig.1. Minimum, maximum and median fitness values reached during experiments are shown in Fig.2 for Schwefel's function, in Fig.3 for Griewangk's function, in Fig.4 for Rastrigin's function, in Fig.5 Egg Holder's function and in Fig.6 for Rana's function.

Table 2. The fastest convergence of DE for Schwefel's, Griewangk's, Rastrigin's, Egg Holder's and Rana's functions

	Schwefel	**Griewangk**	**Rastrigin**	**Egg Holder**	**Rana**
Parameter a	3.94	3.77	3.76	3.98	3.6
Lyapunov exp.	0.540	0.398	0.383	0.596	0.178
Average fitness	-6554.91	0.843	68.53	-6477.80	-4177.43
Median fitness	-6541.40	0.848	61.25	-8920.75	-4198.46

Table 3. Cost functions (Schwefel's, Griewangk's and Rastrigin's) median fitness values interval

Interval of a	**Schwefel**	**Griewangk**	**Rastrigin**
$a \in [3.50, 3.60]$	[-6399.96, -6307.58]	[0.849,0.862]	[68.58, 70.66]
$a \in [3.61, 4.00]$	[-6541.40,-6186.59]	[0.843, 0.864]	[61.25, 71.37]

Table 4. Cost functions (Egg Holder's, Rana's) median fitness values interval

Interval of a	**Egg Holder**	**Rana**
$a \in [3.50, 3.60]$	[-6507.70, -6412.98]	[-4211.54, -4142.86]
$a \in [3.61, 4.00]$	[-6587.57, -6390.79]	[-4223.99,-4105.34]

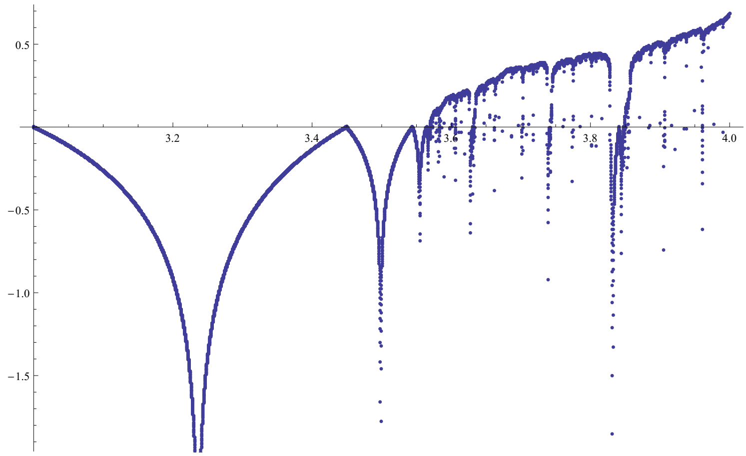

Fig. 1. Lyapunov exponents for Logistic map. X-axis is values of parameter a of Logistic map, y-axes is Lyapunov exponents.

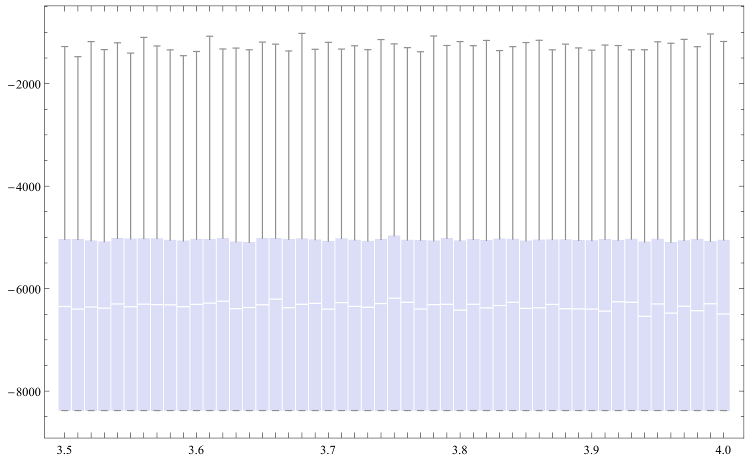

Fig. 2. Experiments results for Schwefel's function

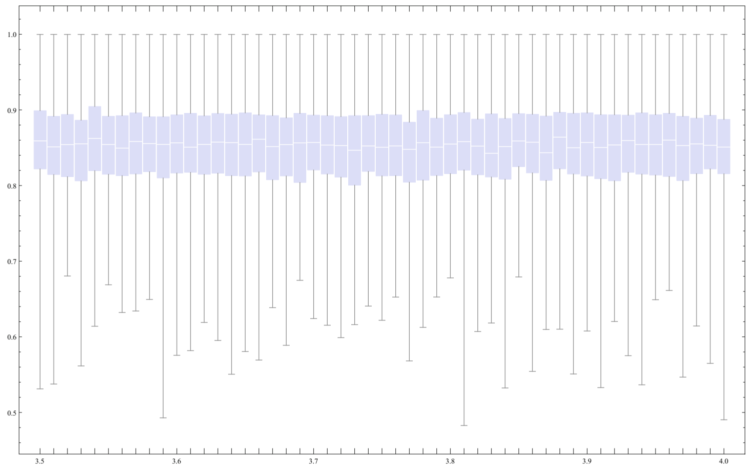

Fig. 3. Experiments results for Griewangk's function

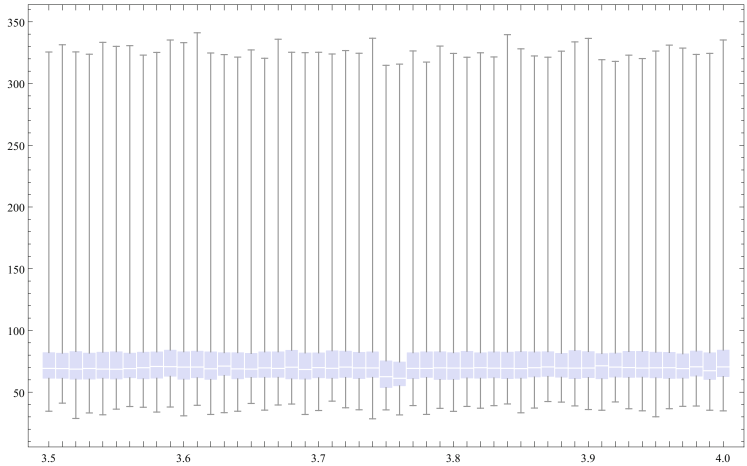

Fig. 4. Experiments results for Rastrigin's function

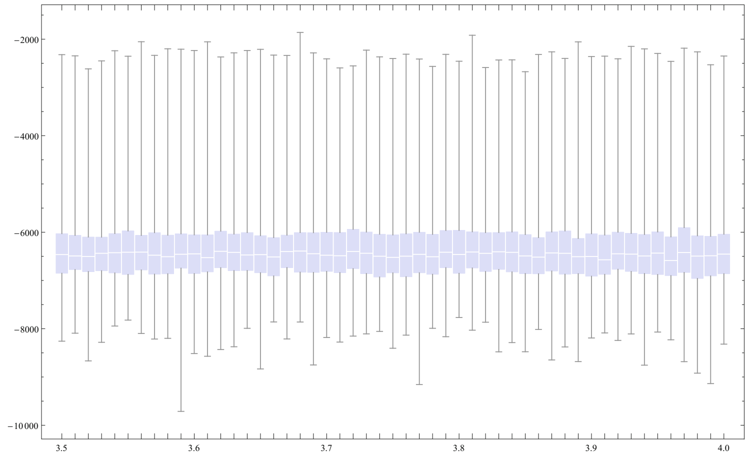

Fig. 5. Experiments results for Egg Holder's function

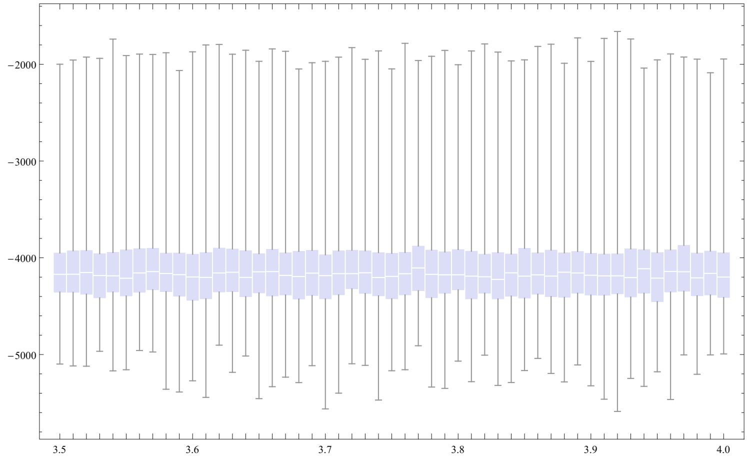

Fig. 6. Experiments results for Rana's function

5 Conclusion

From results mentioned in section 4 we can make some conclusions:

- **Schwefel's Function:** When we look at the Fig.2 we can see that DE has reached global minimum in all cases. When parameter a had been set to $a = 3.94$ DE convergence's speed was the biggest. In this case Lyapunov exponent gained the value 0.540 and average fitness value gained value -6554.91, median fitness value was then -6541.40. When $a \in [3.50, 3.60]$ median fitness values moved in interval [-6399.96, -6307.58]. When $a \in [3.61, 4.00]$ median fitness values moved in interval [-6541.40,-6186.59], see Table 3.
- **Griewangk's Function:** In Fig.3 there are results for Griewangk's function. It is clear that DE has not reached the global minimum in any case. It is probably caused by DE's setting. The biggest convergence's speed was observed when $a = 3.77$ and Lyapunov exponent gained the value 0.398, average fitness value was 0.843 and median fitness value was 0.848. When $a \in [3.50, 3.60]$ median fitness values moved in interval [0.849,0.862]. When $a \in [3.61, 4.00]$ fitness values moved in interval [0.843, 0.864], see Table 3. The smallest fitness value was reached when $a = 3.81$, fitness value gained the value 0.483.
- **Rastrigin's Function:** In Fig.4 we can see that DE has not reached global minimum. It is probably caused by DE's setting. The smallest fitness value was reached when $a = 3.74$, its value was 28.44. When $a = 3.76$ Lyapunov exponent gained value 0.383 and DE's convergence's speed was the biggest, the average fitness value was 68.53 and median fitness value 61.25. When $a \in [3.50, 3.60]$ median fitness values moved in interval [68.58, 70.66]. On the other hand when $a \in [3.61, 4.00]$ median fitness values moved in interval [61.25, 71.37], see Table 3.

- **Egg Holder's Function:** The results for Egg Holder's function are showed in Fig.5. We know that for Egg Holder's function there is not described global minimum in the literature. The smallest fitness value was reached when $a = 3.59$. Lyapunov exponent for this value of parameter a has the value 0.138. When $a = 3.98$ DE's convergence's speed was the biggest, average fitness value gained the value -6477.80 and median fitness value -8920.75. Lyapunov exponent for $a = 3.98$ is 0.596. When $a \in [3.50, 3.60]$ median fitness values moved in interval [-6507.70, -6412.98]. When $a \in [3.61, 4.00]$ median fitness values moved in interval [-6587.57, -6390.79], see Table 4.
- **Rana's Function:** As well as Egg Holder's function for Rana's function there is not described global minimum in the literature. The results for Rana's function are in Fig.6. The smallest fitness value -5588.24 was reached when $a = 3.92$. In this case Lyapunov exponent gained the value 0.517 for $a = 3.92$. When $a = 3.6$, Lyapunov exponent gained the value 0.178 and the convergence's speed of DE was the biggest. Average fitness value was -4177.43 and median fitness value -4198.46. When $a \in [3.50, 3.60]$ median fitness values moved in interval [-4211.54, -4142.86]. When $a \in [3.61, 4.00]$ median fitness values moved in interval [-4223.99,-4105.34], see Table 4.
- When we look at the results mentioned above, we can make conclusion that DE convergence's speed was the biggest when Lyapunov exponent had gained values greater than zero for all testing functions. For Schwefel's function its value was 0.540 ($a = 3.94$), for Griewangk's 0.398 ($a = 3.77$), for Rastrigin's 0.383 ($a = 3.76$), for Egg Holder's 0.138 ($a = 3.98$) and for Rana's 0.517 ($a = 3.6$). For all functions DE convergence's speed was the biggest when $a \geq 3.6$.

In the future, we would like to extend our research by adding other cost functions as Michalwicz's, Rosenbrock's, Patological's etc. We would like to try these experiments to other evolutionary algorithms like PSO, Self-organizing migrating algorithm etc.

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