Time in Economic Theory

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Abstract

Much of spatial economic theory is based on general economic equilibrium theory, although there are problems in a direct addition of a spatial dimension. The most striking is the lack of an analysis of the role of non-material and material public capital (or infrastructure) in the deduction of a static equilibrium structure or patterns of growth and development of economies. In this paper I demonstrate how different aspects of time can improve our understanding of dynamics of economies.

In this paper I furthermore show that a proper refocusing on the time dimension can also shed light on the structure of economies in space. Four approaches are necessary for such a synthesis.

1. Subdivision of products and systems of production according to their different and always positive durability, implying that everything produced is capital.
2. Subdivision of products according to the time used in their production.
3. Subdivision into private and public goods, allowing for non-linearity.
4. Allowing for differences in time scales of economic processes.

With these distinctions it can be shown that the economic development in time **and** space is determined by the impact of economies of scale, duration of the production process, durability of products and the—relative to most other kinds of capital—much slower growth of public capital (i.e. material and non-material infrastructure).

This technical paper is closely related to my popular and thus non-technical paper published as Andersson Å.E. (2013) Time, space and capital. In: Andersson DE (ed) Advances in Austrian economics—the spatial market process. Emerald, Bingley

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5.1 Time in Economics

Walter Isard was concerned with the importance of correctly analyzing the role of the dimensions of time and space in economics as witnessed by his research together with Liossatos (Isard and Liossatos 1979). My intention with this paper is to shed some more light on how the many aspect of time ought to be represented in spatial and non-spatial economic theory.

First, the theoretically most convenient way is to represent time as a continuous variable, as is common in the modeling of many dynamic economic processes and especially in growth analysis. This implies that the processes are modeled as ordinary differential equations or in two dimensional space as partial differential equations, as shown by Puu (2003).

Second, a procedure that is popular in applied economic models, is to represent the dynamic economic processes as a discrete set of periods (e.g. weeks, months, quarters or years).

Third, a quite novel approach in dynamic economic theory is to subdivide the dynamic analysis into substantially different interactive **time scales** of the economic processes.

A quite different and economically realistic aspects of time is the representation of *each good in terms of durability (or alternatively the rate of depreciation) and each production process in terms of its' duration.*

5.2 Time as the Essential Element of Capital

More than a century ago in a critique of the Marxian definition of capital as accumulated labour Knut Wicksell (1966, 1967) proved that the time use or *duration of a production process* determines the value of capital.

This had earlier been demonstrated in Böhm Bawerk's numerical tables describing roundabout processes [Böhm Bawerk (1959–1921); Burmeister (1974); Morgenstern (1935); Marschak (1934); Dorfman (1959); Hicks (1970); Hicks (1973)].

The mathematician Wicksell realized that the numerical tables used by Böhm Bawerk could be densely represented as a mathematical maximization problem. This became the famous wine storing problem. See also Jevons (1871–1970). He assumed that the value of the wine would be growing with the time of storage. During the storage time a natural biological process using solar energy and the activity of yeast would contribute to the growing value of the wine, finally to be determined by the willingness to pay for the matured wine. The limiting factor on the time of storage is the opportunity cost of storage, including the loanable funds rate of interest.

In his model V(T) is the net value of the wine, if it is brought to the market at time T.

The present value (PV) of stopping the maturing by storage at time T is determined as:

$$\text{Maximize PV} = \mathrm{V(T)}e^{-rT}; \tag{5.1}$$

The necessary condition of optimality of storing time is:

$$\mathrm{dV/V = r;} \tag{5.2}$$

Optimal economic duration of the process thus implies that the storing should be stopped when:

The rate of growth of value equals the rate of interest.

This rule of thumb is not a special dynamic case, only relevant for point input, point output decision problems. The following dynamic optimization model shows that the value maximization condition also holds for harvesting sequences over time of some renewable biological resource (e.g. wine in some plantation, trees in a natural forest, or fish in the sea) as shown by the following model.

Maximize harvesting income = Maximize harvesting income = $\int_0^{\infty} p\mathrm{u(t)x(t)}\, e^{-rt}\mathrm{dt}$;

Subject to the growth condition:

$$\mathrm{dx/dt = ax(t) - bx(t)^2 - u(t)x(t)}$$

u(t) = the rate of harvesting at time t

x(t) = the stock of the biological resource at time t

We assume that the price is kept at a constant level.

Maximizing the Hamiltonian

$$\mathrm{H = pu(t)x(t)e^{-rt} - \lambda\big(ax(t) - bx(t)^2 - u(t)x(t)\big);} \tag{5.3}$$

leads to an optimal rate of harvesting at each instance of time.

One of the conditions of optimality requires that the rate of harvesting is determined when

$$\mathrm{d\lambda/\lambda = r} \tag{5.4}$$

Again we get the rule of thumb of optimal harvesting when:

The rate of growth of value should equal the rate of interest at the value maximizing rate of harvesting.

The Marxian labor theory of value was finally proved to be wrong. Not only is land (i.e. natural resources needed) an indispensable fundamental factor of production beside labor, as had already been shown by Johann Heinrich von Thünen (1826, 1960). Also the flow of time itself had now been shown to be a crucial variable in the determination of the dynamics of capital value.

5.3 Durability, Depreciation of Capital and the Rate of Economic Growth

The average durability of capital goods (i.e. all goods) of the economy is determined as the ratio between the value of capital as a stock and the value of production as a flow and has the *dimension of time* (Hawkins 1948; Hawkins and Simon 1949; Bródy 1970).

This implies that the average durability of the all goods (i.e. total capital) is determined as the ratio of the aggregate value of capital to the aggregate value of production. Under certainty, the optimal depreciation δ per year is 1/T. With an uncertain durability with known mean value the optimal rate of depreciation (and the implicit durability, T) is a constant fraction of the net asset value (Lev and Theil 1978).

Assume that the outputs of production processes in the economy are subdivided into currently used production and production for future use (i.e. investment). Current production requires inputs in fixed proportions, represented by input-output coefficients. Investment (I) is determined by an accelerator relation. I = Bgx, where I and x are vectors, g is the rate of growth and the matrix B gives the fixed capital requirements per unit of increase of production. b_{ij} divided by a_{ij} equals the durability of the good, $T_{i,}$ where T_i is the durability of good i. B = T′A where T′ is a diagonal matrix of durability of goods (i = 1,. . .,n):

$$x = Ax + gBx = (I + gT)Ax; \tag{5.5}$$

The primal condition of a general equilibrium production structure and rate of growth.

$$p = pA + rpB = pA(I + rT); \tag{5.6}$$

The dual price structure and rate of interest condition of a general growth equilibrium.

where x = production vector
p = price vector
A = n × n semi-positive input/output matrix
B = n × n semi-positive capital/output matrix = T'A.
g = the maximal rate of growth at the general growth equilibrium
r = the minimal rate of interest at the general growth equilibrium

A unique equilibrium growth and interest rate with an associated pair of equilibrium quantity and price vectors can be proved to exist [with the use of optimization procedures or the use of Perron's or Nikaido's theorems (Andersson and Beckmann p. 26, pp. 204–206 and p.235 and Debreu and Herstein 1953)].

Thus: The rate of interest equals the rate of growth in an economically sustainable equilibrium of this deterministic growing economy.

The larger the durability of any one product, ceteris paribus, the lower would be the rate of interest and growth. A compensating reduction of the use of current inputs is the only way of maintaining equilibrium rates of interest and growth, if any goods durability is increased.

5.4 von Neumann and the Birth of Mathematical General Equilibrium Economics

The models of growth shown in Eqs. (5.5) and (5.6) are special cases of the general equilibrium theory formulated by the mathematicians John von Neumann (1937) and Abraham Wald (1936, 1951). They developed the theory, when collaborating in a Vienna colloquium in the 1930s on the mathematics of general equilibrium theory as formulated by Walras (1874) and Cassel (1918, 1932). Wald proved the existence of a static general equilibrium and von Neumann proved the existence of a dynamic general equilibrium of a growth model based on a simpler model, formulated in Cassel's textbook. In Cassel's model the equilibrium rate of growth is determined by the ratio of the savings ratio and the capital-output ratio. Von Neumann proceeded to generalize this model into a theory of an economically sustainable dynamic general equilibrium, based on his saddle point theorem, proved in the 1920s.

He introduced time into his equilibrium growth theory in two ways:

First, he formulated the basic model in terms of discrete period dynamics.

Second, the durability of all products were introduced in an inverse form as constant rates of depreciation between periods, which is consistent with the assumption of a deterministic economic system, as shown by Lev and Theil (ibid).

Von Neumann assumed joint production in order to treat depreciation and durability efficiently in his model, as for example in the process of making paper in which wood, energy and machines are used as inputs at the start of the process. At the end of the paper making process a **joint product** vector of outputs consisting of paper, store of energy and of machines, which have depreciated and thus have become smaller in capacity. Formally the model is given by Eq. (5.7).

$$\begin{gathered}\mathbf{q}^{\mathbf{T}}\mathbf{B} \geq \alpha\mathbf{q}^{\mathbf{T}}\mathbf{A} \\ \mathbf{B}\mathbf{p} \leq \beta\mathbf{A}\mathbf{p} \\ \mathbf{q}^{\mathbf{T}}(\mathbf{B} - \alpha\mathbf{A})\mathbf{p} = 0 \\ \mathbf{q}^{\mathbf{T}}(\mathbf{B} - \beta\mathbf{A})\mathbf{p} = 0 \\ \mathbf{q} \geq 0; \mathbf{p} \geq 0\end{gathered} \tag{5.7}$$

Where $\mathbf{q}$ = vector of outputs
$\mathbf{p}$ = vector of prices
$\alpha = 1 +$ rate of growth
$\beta = 1 +$ rate of interest
$\mathbf{A}$ = mn matrix of inputs
$\mathbf{B}$ = mn matrix of outputs

The model allows for joint production and substitution of inputs.

Von Neumann proved that for the economic system (5.7) a sustainable equilibrium exists and is a saddle-point solution determining the equilibrium price and quantity vector. At the equilibrium point the minimum rate of interest β and the maximum sustainable rate of growth α are equalized.

von Neumann's theory does not presume utility maximization by the decision makers. This assumption probably stems from Gustav Cassel, who dismissed individual utility functions as necessary for the existence of demand functions.

Von Neumann and Wald had by then initiated modern mathematical economics with the use of saddle point and fixed point theorems. These ideas were later to be used in game theory as created by John von Neumann and Oscar Morgenstern (1944) and much later in general equilibrium theory as reformulated by Debreu (1959). von Neumann's introduction of inequalities in the formulation of saddle point theory also became one of the main preconditions for the development of linear and non-linear programming theory. The other important set of mathematical theorems to be used as a basis of programming theory was the book *Inequalities* by Hardy, Littlewood and Polya (HLP) (1933). Hardy, Littlewood and Polya proved all theorems behind the Constant Elasticity of Substitution (CES) function, which was much later to be used in the formulations of neoclassical economic growth theory (mostly without any references to HLP). Spatial analysis based on these theories later became important in theoretical and applied Regional Science.

5.5 Growth, Institutions, Uncertainty and Risk in the Determination of the Rate of Interest and the Value of Capital

It has been shown in the former section that a dynamic equilibrium in a deterministic economy requires equality of the growth rate and the rate of interest.

However, Wicksell (1914) and later Keynes showed that institutional factors may make the rate of interest deviate from this rule. The interest rate, as charged for loanable funds is a macroeconomic variable, determined by central banks, often using the interest rate as an instrument of monetary policies, sometimes in an international game between different governments. This monetary policy determined rate of interest can thus easily deviate from the "natural rate of interest" as determined by general equilibrium requirements. Such a deviation would then lead to inflation, deflation or unemployment, depending on the sign and size of the deviation and the institutional conditions ruling in the region.

Beside disregarding monetary institutions, most of the early analysis of the relation between the rates of growth and interest was based on deterministic equilibrium modelling. However, it is quite obvious that there could be uncertainty about the future among decision makers, for example as a consequence of variations in the conditions influencing production or demand. There would then be a required risk compensation in the form of a higher rate of interest in order to bridge the gap between lenders and borrowers. In the real world there will always be some uncertainty about the future (Knight 1934). This implies that a general dynamic economic equilibrium with uncertainty among decision makers requires that *the rate of interest is kept above the rate of economic growth*. How much above depends on the level and character of uncertainty.

5.6 Uncertainty and the Value of the Aggregate Stock of Capital

Early capital theorists had in vain tried to develop a consistent method of aggregation of the smallest units of durable goods into a consistent aggregate capital stock (Hayek 1941). Very often the starting point was the assumption that each little unit of durable good would have to be valued at some given unit price. The unsolved (and unsolvable) problem was how to determine the right micro level durable goods prices to be used in the aggregation procedure.

Paradoxically, the problem of capital aggregation can be resolved, as soon as we accept the necessary risk of all capital investments, organized into production units. These risks are revealed in the pricing of firms in financial markets and especially in the stock market. A firm, traded in the stock market, is essentially an already aggregated value of all the different capital goods of the firm, including information and knowledge capital in disembodied and embodied forms.

The theory of the stock market as a capital value determining machinery was initially formulated by Markowitz (1952) and further developed by Modigliani and Miller (M-M) (1958), Sharpe (1964), Lintner (1965), and Mossin (1966).

This modern financial market theory claims that the total equilibrium values of capital of all traded firms is determined (as an average over some period of observation) in the markets for securities and bonds, taking expected returns, perceived risk (as a measure of uncertainty) and the real rate of interest into consideration.

The generic claim is that the capital market is M-M-efficient, implying that the total value of all capital allocation opportunities can be captured by the expected return r(m) and the risk or standard deviation of returns (β(m)) for the market portfolio of all traded instruments. The value of a firm *as an aggregate of material and non-material capital* is determined in a similar way as a combination of expected returns and risk.

The risk-free or deterministic capital value would give $\beta(0) = 0$ with $r(0) = g$. Any other portfolio would imply a rate of returns (natural interest rate) higher than the natural rate of growth.

From this follows the conclusion that the heterogeneous capital value, aggregated by the firm and valued in the stock market, when divided by the scale of production of the firm would generate the average durability of the capital, invested in the firm.

5.7 The Importance of Organization of Capital

The theory of the firm as formulated by Oliver Williamson (1981) and others, being based on Ronald Coase's transaction cost assumptions (Coase 1937), is a useful starting point for an analysis of the formation of firms. However, it says nothing about the best spatial and other allocation of material and human capital in the organization of the firm.

A haphazard arrangement of the carriers of human capital and the machinery and other material capital will not give the same high level of output as a profit maximizing organization. However, it could be hard or sometimes impossible to find such an optimal organization. It can be shown that maximizing the global profitability between a large number of such discrete interdependent human and material capital objects can often not be found, even with the aid of powerful computers.[1] For a firm with only 50 groups of employees with their machinery to be allocated to 50 different tasks there are in fact more than one trillion possible patterns of assignment employees to tasks. With quadratically represented interaction advantages, there are usually a large number of local profit maxima in this class of problems and the search for the global maximum is thus very hard.

The quadratic optimal assignment problem of Koopmans and Beckmann can be approached as in the following integer programming model, proposed by Andersson and Kallio (1982).

$$\begin{aligned} &\text{Maximize } \mathrm{x}' \, \mathrm{S} \, \mathrm{x} + \mathrm{Rx} \\ &\text{Subject to} \sum (j) x_{ij} \leq 1 \text{ Specialist groups available} \\ &\qquad\qquad \sum (i) x_{ij} \geq 1; \text{Tasks to be fulfilled} \\ &\qquad\qquad \mathrm{x} = (0 \text{ or } 1) \end{aligned} \tag{5.8}$$

S is typically a non-definite matrix giving the positive or negative advantages of collaborating (possibly at a distance) between each pair of employees and R gives the revenue effects of each individual if operating a task on her own.

Andersson and Kallio (ibid.) developed a computer algorithm that would efficiently search for a local optimum, when started from randomly selected starting points. The numerical procedures found a number of local optima, with quite different organization patterns. For problems with many tasks and groups of specialists the number of such local optima could be extremely large. In such a situation there is no guarantee that a global optimum would be found in finite computer time.

[1] If we assume indivisible units of machines and humans and that the productivity of a machine or a human (x(i)) depends on interaction with (x(j)) and if these interaction net benefits can be captured by the quadratic form x'Cx, then there is no simple incentive mechanism or computerized search algorithm that would provide the route to a global maximum for most interaction matrices C (Koopmans and Beckmann 1959).

However, the probability of finding solutions close to the global maximum is vastly increased if different decision agents are simulated to be experimenting in different ways with their organization of production. Competitive search will then (if started at some ridge to be defined below) in the long run reveal the agent with superior organization in terms of profitability.

An evolutionary procedure mimicking a competitive, evolutionary search in problems like (5.8) has been developed by Stuart Kaufmann and his associates (1996). They called it the *Patch Procedure,* where a patch can be a predetermined team of employees with some given equipment. The computation experiences were summarized as follows:

The results hint at something deep and simple about why flatter, decentralized organizations may function well: contrary to intuition, breaking an organization into "patches" where each patch attempts to optimize for its own selfish benefit, even if that is harmful to the whole, can lead, as if by an invisible hand, to the welfare of the whole organization. The trick, as we shall see, lies in how the patches are chosen. We will find an ordered regime where poor compromises for the entire organization are found, a chaotic regime where no solution is ever agreed on, and a phase transition between order and chaos where excellent solutions are found rapidly (Kauffmann, p. 147)…. He concludes: *Therefore, as a general summary, it appears that the invisible hand finds the best solution if the coevolving system of patches is in the ordered regime rather near to the transition to chaos* (Kauffman, p.264).

It is clear that the evolutionary search must start in a rather special position on a ridge between order and chaos that might be hard to find. However, if it would be found, at the end of such an evolutionary process the superior firms with their structure of teams will have a capital value far above what would be indicated by their book-value of purchased machines and human capital.

The part of the capital value that cannot be easily accounted for as book value is often in accounting practice called Good Will Value. That value is always included in the valuation of the competing firms in the stock market.

5.8 Durability of Products and Patterns of Location of Production

The problem of the spatial structure of production is determined by the sustainable scale of firms and the total and spatial distribution of demand for their products. The sustainable scale of a firm is determined by the minimum of long term total average cost, including capital, transaction and transport (or logistics) costs. A long term equilibrium of the firm requires the price that can be charged to correspond to this minimal long run average cost.

Durability of the goods and the duration of production processes will have an important impact on the spatial structure of production. In order to determine the impact of durability of duration on the spatial structure of production we need to specify the long run average production cost function (C) and dependence on the scale of production, C(x). A common assumption is to disaggregate total production cost into fixed cost (F), and variable cost (V(x)). Fixed cost is the cost of all material and non-material capital of the production unit and is thus independent of the scale of operation as soon as the production unit has been established. For simplicity we assume that the firm is the production unit.

The duration (τ) of the production process increases the amount of capital needed and thus of the fixed cost of production, i.e. $F(\tau)$ with $F'(\tau) > 0$. This is especially pronounced in knowledge intensive production, needing a long period of research and development before actual production can occur. Typical examples are firms of the airplane producing industry, the movie industry, and the large pharmaceutical firms. Such firms regularly invest more than a fifth of the sales value in creation and innovation of new products and associated production equipment.

The variable cost is normally monotonously increasing with the scale of operation up to the capacity limit of the capital of the firm. In the sequel we assume that the optimal scale of operation is smaller than or equal to that upper limit. The simplest variable cost function is the linear case $V(x) = vx$. The total production cost function would be $P = F + vx$; and the average production cost function would thus be $C = F(\tau)/x + v$, where $\tau =$ duration of the production process.

Transport and transactions or logistics costs depend on the deliveries and other contacts between firm and customers. With a scattered distribution of customers in space around the firm, total transport and transaction cost would increase progressively with the increase in the scale of production and sales. Thus average cost of logistics (L), i.e. transactions and transport, would be increasing with the scale of operations. Assume $L = kx$. The term k can be decomposed into cost per unit of shipments, a, and the frequency of contacts, which is inversely depending on the durability, T, of the product. The average logistics cost is thus $L = (a/T)\,x$. Thus, the larger the durability, the lower is the average logistics cost.

The total average cost A equals the sum of average production cost C and average logistics cost L:

$$A = F(\tau)/x + c + (a/T)\,x; \tag{5.9}$$

Minimization of A implies that the optimal scale of production of the representative firm is:

$$\boldsymbol{x}(\boldsymbol{opt}) = \sqrt{\frac{\boldsymbol{F}(\tau)\boldsymbol{T}}{\boldsymbol{a}}}; \tag{5.10}$$

The optimal scale of production of the firm is thus increasing with increasing **duration** of the production process and also with increasing **durability** of the product.

The optimal number of firms is determined by the total scale of the market. The maximal total market scale is today the world market, to the extent that it is integrated by information and transport networks. The existence of an integrated world market is a precondition for perfect—or at least free—competition for many tradable goods.

The total number of firms in the world market, *N*, for a good is then determined as:

$$\mathrm{N} = \text{Total demand}/\mathrm{x(opt)}; \qquad (5.11)$$

Total demand is determined by the distribution of consumers in space and the minimal A, facing each consumer. However, with any spatial consumer distribution, the total number of firms would be decreasing with the **duration of production process and with the durability of the good** being analyzed.

An increased fixed cost, as influenced by an increased duration of the production process and durability of the good produced reinforce each other in decreasing the number of firms if the demand of the world market is given. In some cases the number of firms could be so severely constrained, that the assumption of perfect or free competition cannot be upheld even if the product is globally traded. Examples are trains, ships, airplanes and nuclear reactors, which are produced only in a few locations, serving a global market. These firms have an extremely long duration of production from the initial research stage through many stages of laboratory experiments.

The number of firms is thus determined by the procedure given above, but not the geographical locations of firms For that a connection with the theory of location and trade is needed. A theory of location and trade summarizing the contributions by Ricardo, von Thünen, Heckscher and Ohlin, Isard and Beckmann is the variational inequality model as formulated by Anna Nagurney (1999). In her model demand at each location and supply in each location is represented dually by the prices announced in the locations. An increased flow of a good from a location to another requires the price difference to be larger than the sum of logistics (including interest) cost, associated with a unit trade flow between the two locations, possibly at different instances of time.

The pattern of location and trade flows comes to an equilibrium when each good price difference is equal to (or smaller than) the sum of transaction and transport costs. As we have seen above the durability of each good determines their logistics cost. The larger the durability of the good the smaller is this cost. Trade will increase until there are no price differences between different locations for the limiting case of extremely large durability of a good. For goods of extreme durability and low logistical cost the law of one price must rule. An example is the pricing and trading in currencies.

5.9 Infrastructure: Capital That Is Durable and Public

Equilibrium theory and associated models have provided the fundaments for modern theoretical and applied economics. But they are inadequate in at least one important respect. These theories and models are not compatible with the dynamics of durable public goods (i.e. tangible and intangible infrastructure). Examples of infrastructure are fundamental values, constitutions, scientific knowledge, transportation and communication networks. In general equilibrium theory infrastructure is a mainly implicit, but always exogenously determined and stable stage on which the economic games are played. A dynamic theory of the interdependent evolution of the infrastructure and general economic equilibrium theory has been lacking.

The reason for the omission of a link between GET and infrastructure theory is quite clear. The necessary mathematical foundation for such an interdependency analysis did not exist before the 1970s. The first attempt to analyze catalytic and other collective/public phenomena was by Reneé Thom in his *Structural stability and morphogenesis: an outline of a general theory of models* (1989). In this book, originally published in French in 1972, he showed how collective phenomena could be modelled with singularity theory and applied to biological phenomena, such as the simultaneous blooming of a certain species by the influence of the slowly rising temperature, acting as a collective/public good.

Related to Thom's bifurcation theory is Synergetics, formulated by Hermann Haken as a way of solving some hard dynamic, non-linear problems in physics (Haken 1977). Haken showed that system predictability can often be achieved by subdividing dynamic processes according to their widely separated time scales. A general equilibrium of the combined dynamic system then becomes a possibility, if a few slowly changing variables are causally impacting a large collective of rapidly changing variables.

The institutional and material infrastructure can be defined to be such a collectively impacting (or public) variable moving on a qualitatively slower time scale than the private goods allocated in the markets. Thus any economic entity is defined to be an infrastructure if it is:

- simultaneously used by many firms or households and
- very durable, compared with other goods.

The following dynamic model of a market economy illustrates the power of subdividing the variables of the economic system into widely different time scales.

The dynamics of the markets for ordinary goods is determined as in general economic equilibrium theory by excess demand differential equations determining the price trajectories:

$$dp/dt = f(p, A); \tag{5.12}$$

where

p = a vector of prices of ordinary market goods including factor services (possibly in different regions),

A = a vector of infrastructure accessible in different regions

The development of infrastructure (as e.g. represented by accessibility values) can be represented by the equation:

$$s(A)dA/dt = m(p, A); \tag{5.13}$$

where s(A) represents the very large durability of infrastructure, indicating that s (A) is a very small, positive number, possibly in the order of 0.01 or lower.

This implies that in the time frame of the other variables of this system dA/dt can be set approximately equal to zero, **most of the time** (but not always). The fast and slow processes will rarely be synchronized and the whole system will then go into a period of creative destruction, eventually to come into rest at a new economic structure.

We thus have a dynamic system:

$$dp/dt = f(p, A^*), \tag{5.14a}$$

to be solved for an equilibrium, i.e. with f = 0, subject to the **temporary constraint**:

$$m(p, A^*) \leq 0, \tag{5.14b}$$

where A* indicates a given level of infrastructure in all parts of the economy.

For systems of this kind we can apply Tikhonov's theorem (Sugakov 1998):

Assume a dynamic system of N ordinary differential equations, which can be divided into two groups of equations. The first group consists of m fast equations, the second group consists of m + 1, . . ., *N* slow equations.

Tikhonov's theorem states that such a system has an equilibrium solution under certain economically reasonable conditions:

For each position of the slow subsystem, representing the dynamics of infrastructure, the fast general equilibrium market price subsystem has plenty of time to stabilize. Such an approximation is called adiabatic. (For a proof see Sugakov 1998)

In the very long run dA/dt cannot be assumed to be approximately equal to zero and thus the infrastructure would have substantially changed. The structure of prices and quantities of goods, as determined by f(p,A) could then cease to be as well behaved as in the short term dynamics, given by (Eq. 5.12).

The system would in the very long term have all the bifurcation properties, typical of non-linear, interactive dynamic systems. However, between periods of change of the economic structure, there could be periods of stable General Economic Equilibrium.

Most neoclassical economists have become skeptical about the possibility to mathematically model the dynamics of economic systems. Modern mathematical theory of dynamic systems supports this view. Chaos is the generic outcome of a non-linear economic system if all interactive economic variables are moving on the same time scale. General Equilibrium Theory, as formulated by e.g. Arrow and Hurwisz (1957), Debreu (1959) and others, is thus in fact not general enough to be expandable into a well behaved dynamic economic systems theory (and even less into combined spatial and dynamic systems).

However, I have shown above that this impossibility can be resolved if the dynamic models of the economy contain proper distinctions between the time scales of markets for goods, and the slow changes of the infrastructural stage on which the markets operate.

Conclusions

The theories and observations of the role of different time dimensions are fundamental to our understanding of economic processes in time and space. The most important time dimensions in dynamic and spatial economic theory are:

Duration of production.

Durability of goods and the inverse—the rate of depreciation.

Differences in time scales between infrastructure and market goods.

The choice of duration of a production process is important both for the temporal and spatial structure of production. With the increasing importance of large costs of scientific and industrial research and technological development this issue has become increasingly important.

But duration of the production process must be complemented by the durability of the goods produced. All goods are durable although to different degree—and all goods are consequently capital—and durability of capital is thus an irreducable determinant of many aspects of the economy as a growing spatial system.

The durability of goods and duration of production processes contribute in determining capital-output ratios, optimum rates of interest and growth and the spatial extent and pattern of the competitive markets.

The extreme durability and public nature of institutions, knowledge, networks and other infrastructure provides the basis of a new theory of complex dynamic spatial economic system. Within such an evolving system a general equilibrium of prices can exist and be stable.

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