New Method for Dynamic Signature Verification Using Hybrid Partitioning

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Abstract. Dynamic signature is behavioural biometric attribute which is commonly used to identity verification. Methods based on the partitioning are one of the types of methods for identity verification using signature biometric attribute. These methods divide trajectories of the signature into parts and during verification phase compare created fragments of trajectories in each partition. Partitioning is performed on the basis of values of signals describing dynamics of signing process (e.g. pen velocity or pen pressure). In this paper we propose a new method for dynamic signature verification using hybrid partitioning. Partitions in the proposed method can be interpreted as, for example, high velocity in the first phase of the signing process or low pressure in the final phase of the signing process. Our method assumes use of all partitions during classification process and our classifier is based on the flexible neuro-fuzzy system of the Mamdani type. Simulations were performed using public SVC2004 dynamic signature database.

1 Introduction

Signature is a behavioural biometric attribute used to verify identity of the individual. This attribute is very interesting from the practical point of view because identity verification using the signature is commonly accepted in the society. However, verification based on the behavioural global features is more difficult than verification based on physiological ones, like fingerprint or iris.

Dynamic signature (called also on-line signature) is signature created in the real time using some kind of input digital device, e.g. graphic tablet. It contains also information about the dynamics of signing, like velocity and pressure signals changing over time. This information are very useful during verification process and increases its accuracy.

Approaches to identity verification based on dynamic signature may be categorized into few groups, one of them are methods based on signature partitioning (see [23]). In this paper we propose a new method for dynamic signature verification based on hybrid of horizontal and vertical partitioning (see [9], [65]).

L. Rutkowski et al. (Eds.): ICAISC 2014, Part II, LNAI 8468, pp. 216–230, 2014.

First, signature is divided into partitions on the basis of time indexes values, because we assume that some regions of the signature acquired in certain timeframe can be more characteristic for the user than other regions. Next, trajectory in each selected partition is divided into two parts on the basis of velocity and pressure signals average values. Previous researches have shown that combination of velocity and pressure with shape makes verification more effective than use of the separated dynamic features (see [17], [23]-[24], [63]-[65]). The partitioning allows selection of the most discriminative features of the signature which belong to the user. In the verification phase we propose flexible neuro-fuzzy system of the Mamdani type (see e.g. [6], [8], [14], [50]-[51]). Our method assumes partitioning signatures into few subspaces (number of subspaces results from the product of the number of horizontal and vertical partitions) which are weighted by weights of importance and used during classification process. In this process we use data from all partitions created during training phase.

This paper is organized into 4 sections. Section 2 contains detailed description of the algorithm. Simulation results are presented in Section 3. Conclusions are drawn in Section 4.

2 Detailed Description of the Algorithm

In our method we use four signals of the signature over time: x-trajectory, y-trajectory, pressure and velocity. First three of them are acquired directly from the graphic tablet and the velocity is first derivative of the signature trajectory. Before beginning of the main phase of the method, all training signatures of the signer i should be pre-processed by commonly used methods to remove some intra-class variations (see e.g. [17], [18], [33], [40]). Signatures are pre-processed with reference to one signature of the user (called base signature) which is the most similar to all training signatures. During pre-processing the length, rotation, scale and offset of the signatures are matched. After a pre-processing, main phase of training process is performed.

The individual steps of the algorithm are detailed below: **Step 1. Partitioning of signatures.** First, signatures are partitioned on the basis of time indices values into two parts. Next, fragment of the signature in each partition is divided into two parts on the basis of the average value of pressure and velocity signal. This second step is also performed in two phases: 1) velocity and pressure signals are divided into two parts, 2) partitioning of the whole signature is performed, signature elements which time points corresponding to the velocity and pressure signals are assigned to the appropriate partition. After this phase signatures are divided into eight parts (four partitions related to the velocity and four partitions related to the pressure). This step is performed during the training and the test phase. **Step 2. Templates generation.** In this step templates, which contains average values of training signatures signals, are generated for each partition. The templates are regarded as the reference signature of the user. This step is performed only during training phase. **Step 3. Determination of similarities between signatures and template in each partition.** In this step similarities between each signature of the user and template are calculated for each partition. In the training phase the similarities are used for determination of the classifier. In the test phase the similarities are created only for the test signature. They are used in the classification process. This step is performed during training and test phase. **Step 4. Determination of the partition importance in the classification process.** In this step weights of importance for each partition are created. They allow to evaluate which partition contains information characteristic for the user. The weights are used in the verification process. This step is performed only during training phase. **Step 5. Preliminary separation of the reference signatures in the partition.** During this step linear boundary of the inclusion of genuine signatures in each partition is created (see [64]). The boundary is used to determine fuzzy sets applied in the classification process. This step is performed only during training phase. **Step 6. Determination of the parameters of fuzzy classifier of genuineness of the signatures.** The parameters describe fuzzy sets of the classifier, which is used in the classification phase. Fuzzy rules describe a way of test signature classification. The fuzzy sets in the rules are based on decision boundaries determined in the step 5. Therefore they may be interpretable. This step is performed only during training phase. **Step 7. Classification of the genuineness of the signatures.** In this step signature is classified as genuine or forgery. In this process flexible neuro-fuzzy system of the Mamdani type is used. This step is performed only during test phase.

We can see that steps 1-6 are performed during training phase, while steps 1,3,7 are performed during test phase.

After training phase, velocity and pressure signals of the base signature, information about partitions and parameters of the classifier are stored into the database. These information will be used in the test phase.

2.1 Vertical Signature Partitioning

First, vertical partitioning based on selected time intervals of signing is performed. This is possible because lengths of the signals of all signatures are the same through the pre-processing. Alignment of the length is performed using Dynamic Time Warping algorithm (see e.g. [22]), which operates on the basis of matching velocity and pressure signals. Result of this matching is a map of corresponding points of the signatures signals, which is used to match trajectories of the signature (see [9]). Vertical partitions $partv^{\{s\}}_{i,j,k}$ of the sample k of the signature j of the signer i based on signal s (velocity v or pressure z) are created using the following equation:

$$partv^{\{s\}}_{i,j,k} = \begin{cases} 1 & \text{for} \quad 0 < k \leq \frac{L_i}{P^{\{s\}}} \\ 2 & \text{for} \quad \frac{L_i}{P^{\{s\}}} < k \leq \frac{2L_i}{P^{\{s\}}} \\ \vdots & \\ P^{\{s\}} & \text{for} \quad \frac{(P^{\{s\}}-1)L_i}{P^{\{s\}}} < k \leq L_i \end{cases}, \tag{1}$$

where s is a signal type (velocity or pressure) used during alignment phase, i is the user number ($i = 1, 2, \ldots, I$), j is the signature number ($j = 1, 2, \ldots, J$), L_i is a number of samples of the user i, k is the sample number ($k = 1, 2, \ldots, L_i$) and $P^{\{s\}}$ is a number of partitions ($P^{\{s\}} \ll L_i$). In this method we have assumed, that $P^{\{v\}} = P^{\{z\}} = 2$.

2.2 Horizontal Signature Partitioning

After vertical partitioning, horizontal partitioning of the signature is performed. In the first step of this process average values $avg_{i,p}^{\{s\}}$ of velocity and pressure signals of the base signature are computed for each vertical partition. This is described by the following formula:

$$avg_{i,p}^{\{s\}} = \frac{1}{K_{i,p}} \sum_{k=1}^{K_{i,p}} s_{i,j=jBase,p,k}, \tag{2}$$

where $K_{i,p}$ in number of samples in the vertical partition p ($p = 1, 2$) of the user i, $s_{i,j,p,k} \in \{v_{i,j,p,k}, z_{i,j,p,k}\}$ is signal (velocity v or pressure z) value of the sample k ($k = 1, 2, \ldots, K_{i,p}$), which belongs to the vertical partition p, of the base signature (for which $j = jBase$) of the signer i.

Next, division into horizontal partitions on the basis of values determined in (2) is performed. Horizontal partition $parth_{i,j,p,k}^{\{s\}}$ of the sample k, which belongs to the vertical partition p (of the index specified in the formula (1)), of the signature j of the signer i based on signal s (velocity v or pressure z) is determined as follows:

$$parth_{i,j,p,k}^{\{s\}} = \begin{cases} 1 \text{ for } s_{i,j,p,k} < \mathrm{avg}_{i,p}^{\{s\}} \\ 2 \text{ for } s_{i,j,p,k} \geq \mathrm{avg}_{i,p}^{\{s\}} \end{cases}. \tag{3}$$

We use two horizontal partitions, because our previous research have shown that method based on two partition achieves best performance.

In the next step templates of the signatures for each partition are generated.

2.3 Generation of the Templates

Template $\mathbf{ta}_{i,p,r}^{\{s\}}$ of the partition p, r (p denotes index of the vertical partition described by the formula (1), r denotes index of the horizontal partition described by the formula (2)) of the signer i for signatures aligned with use of signal s (velocity v or pressure z) and trajectory a (x or y) is described by the following equation:

$$\mathbf{ta}_{i,p,r}^{\{s\}} = \left[ta_{i,p,r,1}^{\{s\}}, ta_{i,p,r,2}^{\{s\}}, ..., ta_{i,p,r,K_{i,p,r}^{\{s\}}}^{\{s\}} \right], \tag{4}$$

where $K_{i,p,r}^{\{s\}}$ in number of samples in the partition p, r ($r = 1, 2$), determined for signal s, of the user i, $ta_{i,p,r,k}^{\{s\}}$ is template value for the time step k of the partition p, r of the signer i for signatures aligned with use of signal s (velocity v or pressure z) and trajectory a (x or y) which is calculated by the formula:

$$ta_{i,p,r,k}^{\{s\}} = \frac{1}{J}\sum_{j=1}^{J} a_{i,j,p,r,k}^{\{s\}}, \tag{5}$$

where $a_{i,j,p,r,k}^{\{s\}}$ is trajectory (x or y) value in the sample k of the partition p, r, determined for signal s (velocity v or pressure z), of the signature j of the signer i.

Next, distances between templates from all partitions and each signature trajectory are calculated.

2.4 Determination of Similarities between Signatures and Template in Each Partition

Distance $da_{i,j,p,r}^{\{s\}}$ between template of the partition p, r, determined for signal s (velocity v or pressure z) of the signer i and trajectory a (x or y), and the signature j of the signer i is described by the following equation:

$$da_{i,j,p,r}^{\{s\}} = \sqrt{\sum_{k=1}^{K_{i,p,r}^{\{s\}}} \left(ta_{i,p,r,k}^{\{s\}} - a_{i,j,p,r,k}^{\{s\}}\right)^2}. \tag{6}$$

The next phase of this step is calculation of distances between templates and signatures in two dimensional space. Distance $d_{i,j,p,r,}^{\{s\}}$ between the trajectory of signature j of the signer i and template of the signer i in the partition p, r, determined for signal s (velocity v or pressure z), is calculated by the formula:

$$d_{i,j,p,r}^{\{s\}} = \sqrt{\left(dx_{i,j,p,r}^{\{s\}}\right)^2 + \left(dy_{i,j,p,r}^{\{s\}}\right)^2}. \tag{7}$$

The values $d_{i,j,p,r}^{\{s\}}$ are used directly to determine the parameters of the fuzzy sets of the signature classifier.

In the next step, weights of importance for partitions are calculated.

2.5 Determination of the Partition Importance in the Classification Process

The weights are created on the basis of mean distances $\bar{d}_{i,p,r}^{\{s\}}$ between signatures and template in partitions and standard deviation of distances in each partition. The mean distance $\bar{d}_{i,p,r}^{\{s\}}$ between signatures of the signer i and the template of the signer i in the partition p, r, determined for signal s (velocity v or pressure z), is calculated by the formula:

$$\bar{d}_{i,p,r}^{\{s\}} = \frac{1}{J}\sum_{j=1}^{J} d_{i,j,p,r}^{\{s\}}. \tag{8}$$

Standard deviation of signatures $\sigma_{i,p,r}^{\{s\}}$ of the user i from the partition p, r, determined for signal s (velocity v or pressure z), is calculated using the following equation:

$$\sigma_{i,p,r}^{\{s\}} = \sqrt{\frac{1}{J}\sum_{j=1}^{J}\left(\bar{d}_{i,p,r}^{\{s\}} - d_{i,j,p,r}^{\{s\}}\right)^2}. \tag{9}$$

Next, weights of importance are calculated. Weight $w_{i,p,r}^{'\{s\}}$ of the partition p, r, determined for signal s (velocity v or pressure z), of the user i is calculated by the following formula:

$$w_{i,p,r}^{'\{s\}} = \bar{d}_{i,p,r}^{\{s\}} \cdot \sigma_{i,p,r}^{\{s\}}. \tag{10}$$

After that, weights should be normalized to simplify the classification phase. Weight $w_{i,p,r}^{\{s\}}$ of the partition p, r, determined for signal s (velocity v or pressure z), of the user i is normalized by the following equation:

$$w_{i,p,r}^{\{s\}} = 1 - \frac{c_w \cdot w_{i,p,r}^{'\{s\}}}{\max\left\{w_{i,p,r}^{'\{s\}}\right\}}, \tag{11}$$

where $c_w \in (0, 1]$ is the auxiliary constant of the normalization, which prevents elimination of the partitions associated with a small values of the weights from the classification process (in our simulations we assumed that $c_w = 0.9$).

In the next step, preliminary separation of the reference signatures in the partitions is realized.

2.6 Preliminary Separation of the Reference Signatures in the Partition

In the considered problem, immediate adaptation of the method for verification of new users' signature is required. This eliminates the possibility of machine learning in the classifier selection. Therefore, we developed a flexible neuro-fuzzy classifier which requires properly prepared descriptors, determined once on the basis of the reference signatures of the user.

The boundary of the inclusion of genuine signatures is determined by exploiting the consistency of dissimilarity measures in training signatures (see [23], [24]). Parameters of the boundary are computed using the means and standard deviations of the distances. The mean distance $\bar{da}_{i,p,r}^{\{s\}}$, between signatures of the signer i and template of the signer i in the partition p, r, determined for signal s (velocity v or pressure z) and trajectory a (x or y), is calculated by the formula:

$$\bar{da}_{i,p,r}^{\{s\}} = \frac{1}{J}\sum_{j=1}^{J} da_{i,j,p,r}^{\{s\}}. \tag{12}$$

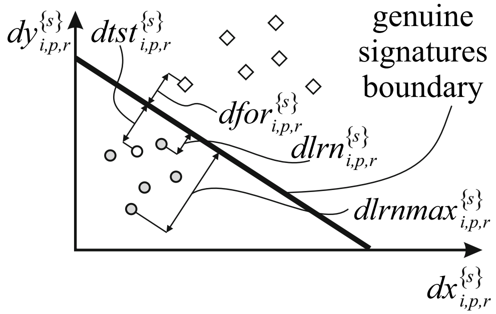

Fig. 1. Illustration of the genuine signature boundary. Genuine training signatures of the user are described as circles, genuine training signatures of other users are described as diamonds.

Standard deviation of signatures $\sigma a_{i,p,r}^{\{s\}}$ of the user i from the partition p, r, determined for signal s (velocity v or pressure z) and trajectory a (x or y), is calculated using the following equation:

$$\sigma a_{i,p,r}^{\{s\}} = \sqrt{\frac{1}{J} \sum_{j=1}^{J} \left(\bar{da}_{i,p,r}^{\{s\}} - da_{i,j,p,r}^{\{s\}} \right)^2}. \tag{13}$$

The linear boundary of the inclusion of genuine signatures in the slope-intercept form is presented as follows:

$$dy\left(dx\right) = -\frac{\sigma y_{i,p,r}^{\{s\}}}{\sigma x_{i,p,r}^{\{s\}}} dx + c_{i,p,r}^{\{s\}} \cdot \bar{dx}_{i,p,r}^{\{s\}} \cdot \left(\frac{\sigma y_{i,p,r}^{\{s\}}}{\sigma x_{i,p,r}^{\{s\}}} + 1 \right), \tag{14}$$

where $c_{i,p,r}^{\{s\}}$ is constant parameter used to adjust the position of the line, which is determined in such a way that $dlrn_{i,p,r}^{\{s\}}$ is equal to $dfor_{i,p,r}^{\{s\}}$, as depicted in Fig. 1.

Remarks on Fig. 1 can be summarized as follows: **(a)** Grey circles in Fig. 1 represent the distances between signatures and templates created individually for each user (see (6), (7)). Therefore, they represent the instability of the signature created by the individual user within each partition and they are not interpretable clusters of data. Theoretically, grey circles should lie exactly in the centre of the coordinate system. In practice, large distance between grey circles and the origin of the coordinate system means that quality of the acquired signatures is low and the reliability of the dynamic signature of the user is also low. In other words, in the context of considered user the method is not reliable, because the user is unable to create in a similar way a few signatures at the same time. **(b)** White circles in Fig. 1 theoretically should also be exactly in the centre of the coordinate system, because they represent signatures created by the user in the test phase. In practice, it is expected that the white circles will be placed at a certain distance from the origin of the coordinate system (e.g. due to changes of signature in time). **(c)** Diamonds in Fig. 1 represent the signatures of other users. Therefore, they should be significantly further from the origin of the coordinate system than white circles. **(d)** Fuzzy rules of the classifier define a way of signature classification which depends on the location of the descriptors ($dtst_{i,p,r}^{\{s\}}$) of the test signature in relation to the boundary of the inclusion of the reference signatures of the user. Please note that the sample (white circle) does not have to be classified as false, even if it is located over the boundary of the inclusion of the reference signatures of the user in the partition (within the inclusion area of false signatures). This happens when: (1) sample in the other partitions is more similar to the template, (2) the reliability of the partition is small (the partition is associated with the low value of the weight). It is a distinctive feature of our method against the methods presented in other works. **(e)** Values $dlrnmax_{i,p,r}^{\{s\}}$ (see Fig. 1) have an impact on spacing of fuzzy sets, which represent values $\{\text{low}, \text{high}\}$ assumed by the linguistic variables "the truth of the signature of user i from the partition p, r, determined for signal s".

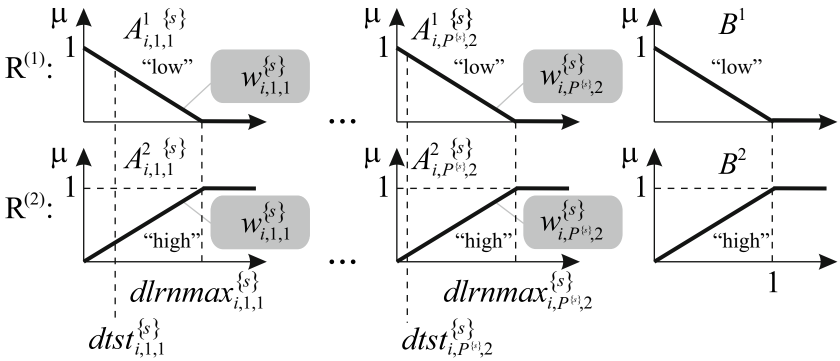

Fig. 2. Input and output fuzzy sets of the flexible neuro-fuzzy system of the Mamdani type for signature verification

Next, determination of the classifier is performed.

2.7 Determination of the Parameters of Fuzzy Classifier of Genuineness of the Signatures

In this step flexible Mamdani-type neuro-fuzzy system is used. Neuro-fuzzy systems combine the natural language description of fuzzy systems (see e.g. [1]-[5], [10]-[13], [16], [21], [25], [28], [34], [45]-[47], [60]-[61]) and the learning properties of neural networks (see e.g. [7], [26], [29]-[32], [35]-[39], [41]-[42], [48], [55]-[56], [58]-[59]). Alternative approaches to classification can be found in [15], [20], [43]-[44], [49], [52]-[54], [57]. Our system works on the basis of two fuzzy rules presented as follows:

$$
\left\{
\begin{array}{l}
R^{(1)}:\left[\begin{array}{c}
\text{IF}\left(dtst_{i,1,1}^{\{s\}}\,\text{is}\,A_{i,1,1}^{1}{}^{\{s\}}\right)\Big|w_{i,1,1}^{\{s\}}\ \text{OR}\\
\left(dtst_{i,1,2}^{\{s\}}\,\text{is}\,A_{i,1,2}^{1}{}^{\{s\}}\right)\Big|w_{i,1,2}^{\{s\}}\ \text{OR}\\
\vdots\\
\left(dtst_{i,P^{\{s\}},1}^{\{s\}}\,\text{is}\,A_{i,P^{\{s\}},1}^{1}{}^{\{s\}}\right)\Big|w_{i,P^{\{s\}},1}^{\{s\}}\ \text{OR}\\
\left(dtst_{i,P^{\{s\}},2}^{\{s\}}\,\text{is}\,A_{i,P^{\{s\}},2}^{1}{}^{\{s\}}\right)\Big|w_{i,P^{\{s\}},2}^{\{s\}}\ \text{THEN}\,y_i\,\text{is}\,B^{1}
\end{array}\right]\\
R^{(2)}:\left[\begin{array}{c}
\text{IF}\left(dtst_{i,1,1}^{\{s\}}\,\text{is}\,A_{i,1,1}^{2}{}^{\{s\}}\right)\Big|w_{i,1,1}^{\{s\}}\ \text{OR}\\
\left(dtst_{i,1,2}^{\{s\}}\,\text{is}\,A_{i,1,2}^{2}{}^{\{s\}}\right)\Big|w_{i,1,2}^{\{s\}}\ \text{OR}\\
\vdots\\
\left(dtst_{i,P^{\{s\}},1}^{\{s\}}\,\text{is}\,A_{i,P^{\{s\}},1}^{2}{}^{\{s\}}\right)\Big|w_{i,P^{\{s\}},1}^{\{s\}}\ \text{OR}\\
\left(dtst_{i,P^{\{s\}},2}^{\{s\}}\,\text{is}\,A_{i,P^{\{s\}},2}^{2}{}^{\{s\}}\right)\Big|w_{i,P^{\{s\}},2}^{\{s\}}\ \text{THEN}\,y_i\,\text{is}\,B^{2}
\end{array}\right]
\end{array}
\right., \tag{15}
$$

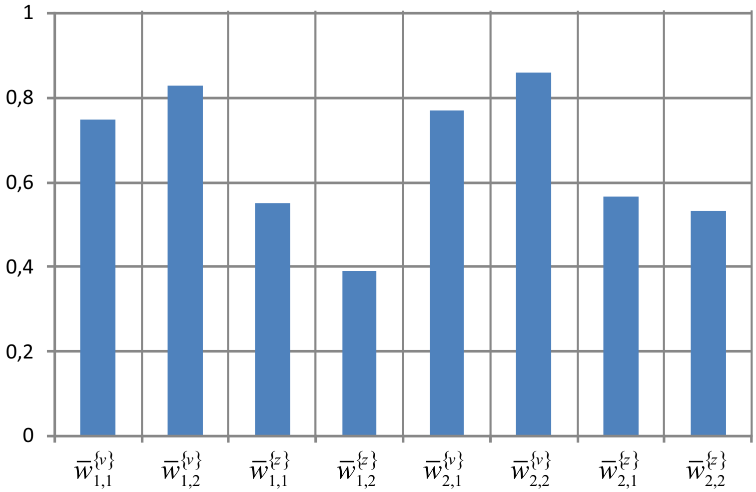

Fig. 3. Average values of weights of the users determined for each partition of dynamic signature

where **(a)** $dtst_{i,p,r}^{\{s\}}$ are input linguistic variables, whose numeric value is a distance between the test signature trajectory of the signer i and the linear boundary of the inclusion of genuine signatures in the partition p, r, determined for signal s. **(b)** $A_{i,p,r}^{1}{}^{\{s\}}$, $A_{i,p,r}^{2}{}^{\{s\}}$ are input fuzzy sets related to the signal $s \in \{v, z\}$ shown in Fig. 2. Fuzzy sets $A_{i,p,r}^{1}{}^{\{s\}}$ and $A_{i,p,r}^{2}{}^{\{s\}}$ represent values $\{\text{low}, \text{high}\}$ assumed by input linguistic variables $dtst_{i,p,r}^{\{s\}}$. **(c)** y_i is input linguistic variable interpreted as reliability of signature. **(d)** B^1, B^2 are output fuzzy sets shown in Fig. 2. Fuzzy sets B^1, B^2 represent values $\{\text{low}, \text{high}\}$ assumed by output linguistic variable determining the reliability of signature. **(e)** $w_{i,p,r}^{\{s\}}$ are weights of the partition p, r, determined for signal s, of the user i.

Signature is considered true if the following assumption is satisfied:

$$\bar{y}_i = \frac{S^* \left\{ \begin{array}{c} \mu_{A_{i,1,1}^2{}^{\{s\}}}\left(dtst_{i,1,1}^{\{s\}}\right), \mu_{A_{i,1,2}^2{}^{\{s\}}}\left(dtst_{i,1,2}^{\{s\}}\right), \ldots, \\ \mu_{A_{i,P\{s\},1}^2{}^{\{s\}}}\left(dtst_{i,P\{s\},1}^{\{s\}}\right), \mu_{A_{i,P\{s\},2}^2{}^{\{s\}}}\left(dtst_{i,P\{s\},2}^{\{s\}}\right); \\ w_{i,1,1}^{\{s\}}, w_{i,1,2}^{\{s\}} \ldots, w_{i,P\{s\},1}^{\{s\}}, w_{i,P\{s\},2}^{\{s\}} \end{array} \right\}}{\left(\begin{array}{c} S^* \left\{ \begin{array}{c} \mu_{A_{i,1,1}^2{}^{\{s\}}}\left(dtst_{i,1,1}^{\{s\}}\right), \mu_{A_{i,1,2}^2{}^{\{s\}}}\left(dtst_{i,1,2}^{\{s\}}\right), \ldots, \\ \mu_{A_{i,P\{s\},1}^2{}^{\{s\}}}\left(dtst_{i,P\{s\},1}^{\{s\}}\right), \mu_{A_{i,P\{s\},2}^2{}^{\{s\}}}\left(dtst_{i,P\{s\},2}^{\{s\}}\right); \\ w_{i,1,1}^{\{s\}}, w_{i,1,2}^{\{s\}} \ldots, w_{i,P\{s\},1}^{\{s\}}, w_{i,P\{s\},2}^{\{s\}} \end{array} \right\} + \\ S^* \left\{ \begin{array}{c} \mu_{A_{i,1,1}^1{}^{\{s\}}}\left(dtst_{i,1,1}^{\{s\}}\right), \mu_{A_{i,1,2}^1{}^{\{s\}}}\left(dtst_{i,1,2}^{\{s\}}\right), \ldots, \\ \mu_{A_{i,P\{s\},1}^1{}^{\{s\}}}\left(dtst_{i,P\{s\},1}^{\{s\}}\right), \mu_{A_{i,P\{s\},2}^1{}^{\{s\}}}\left(dtst_{i,P\{s\},2}^{\{s\}}\right); \\ w_{i,1,1}^{\{s\}}, w_{i,1,2}^{\{s\}} \ldots, w_{i,P\{s\},1}^{\{s\}}, w_{i,P\{s\},2}^{\{s\}} \end{array} \right\} \end{array} \right)} > cth_i, \tag{16}$$

where **(a)** $S^*\{\cdot\}$ is a weighted t-conorm (see [6]). **(b)** $\bar{y}_i$ is the value of the output signal of applied neuro-fuzzy system (see e.g [27]) described by rules (15). Detailed description of the system can be found in [51]. Formula (16) was created by taking into account in the description of system simplification resulting from the spacing of fuzzy sets shown in Fig. 2: $\mu_{A_{i,p,r}^1{}^{\{s\}}}(0) = 1$, $\mu_{A_{i,p,r}^1{}^{\{s\}}}\left(dlrnmax_{i,p,r}^{\{s\}}\right) = 0$, $\mu_{A_{i,p,r}^2{}^{\{s\}}}(0) = 0$, and $\mu_{A_{i,p,r}^2{}^{\{s\}}}\left(dlrnmax_{i,p,r}^{\{s\}}\right) = 1$. **(c)** $cth_i \in [0,1]$ - coefficient determined experimentally during training phase for each user to eliminate disproportion between FAR and FRR error (see [62]). The parameters $cth_i \in [0,1]$, computed individually for the user i, are used during verification process in the test phase.

3 Simulation Results

The simulation was performed using public SVC2004 signature database which contains signatures of 40 users. The signatures were acquired in two sessions using the digitizing tablet. In the first session each user created 10 genuine signatures. In the second session, each user came again to create another 10 genuine signatures. In this session he/she also created four skilled forgeries for five other users. The SVC2004 database contains 20 genuine signatures and 20 skilled forgeries for each user.

Test procedure proceeded as follows for signatures of each from 40 signers available in the database. During training phase we used 5 randomly selected (from 20) genuine signatures of each signer. During test phase we used 10 randomly selected (from the remaining 15) genuine signatures and all 20 skilled forgeries of each signer. The process was performed five times, and the results were averaged. The described method is commonly used in evaluating the effectiveness of methods for dynamic signature verification, which corresponds to the standard crossvalidation procedure. The test was performed using the authorial testing environment implemented in C# language.

We also implemented some other methods based on partitioning to compare the results of our simulations: 1) method presented in [17] which achieves very good results, 2) our previous method based on vertical partitioning proposed in [9], 3) our previous method based on horizontal partitioning proposed in [65].

Table 1 contains simulation results described as values of FAR (False Acceptance Rate) and FRR (False Rejection Rate), which are commonly used in biometrics (see e.g. [19]). As mentioned earlier, in the simulations we assumed that a number of vertical partitions is equal to 2 and a number of horizontal partition is also equal to 2. Moreover, we present average values of weights of importance for each partition ($\bar{w}_{p,r}^{\{s\}}$), averaged in the context of the users (see Fig. 3), which describe reliability of the signature in the partitions.

Table 1. Results of simulation performed by the system (16)

Method	**Average FAR**	**Average FRR**	**Average error**
Ibrahim et al. [17]	11.05 %	13.75 %	12.40 %
Zalasiński & Cpałka [65]	12.15 %	11.00 %	11.58 %
Cpałka & Zalasiński [9]	10.51 %	10.45 %	10.99 %
Our method	**11.73 %**	**9.95 %**	**10.84 %**

4 Conclusions

In this paper we presented a new method for dynamic signature verification using hybrid partitioning. In this method the signature is divided into few vertical parts, which are divided into two horizontal parts. All created partitions are used during classification process. The method assumes use of the classifier based on the Mamdani type neuro-fuzzy system which is characterized by very good accuracy and ease of interpretation of the collected knowledge. Accuracy achieved in our simulations performed using SVC2004 database proves the correctness of the proposed assumptions. Moreover, the simulations show that partitions created on the basis of the velocity signal are more reliable than partitions created on the basis of the pressure signal. This is due to the higher value of weights (11) associated with the partitions of the signal v (see 3). The most reliable partition is the one created in the final phase of the signing process ($p = 2$) and associated with the high value ($r = 2$) of the velocity signal v.

Acknowledgment. The project was financed by the National Science Centre (Poland) on the basis of the decision number DEC-2012/05/B/ST7/02138.

References

1. Bartczuk, Ł., Dziwiński, P., Starczewski, J.T.: A New Method for Dealing with Unbalanced Linguistic Term Set. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2012, Part I. LNCS (LNAI), vol. 7267, pp. 207–212. Springer, Heidelberg (2012)
2. Bartczuk, Ł., Dziwiński, P., Starczewski, J.T.: New Method for Generation Type-2 Fuzzy Partition for FDT. In: Rutkowski, L., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2010, Part I. LNCS, vol. 6113, pp. 275–280. Springer, Heidelberg (2010)
3. Bartczuk, Ł., Przybył, A., Dziwiński, P.: Hybrid state variables - fuzzy logic modelling of nonlinear objects. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2013, Part I. LNCS, vol. 7894, pp. 227–234. Springer, Heidelberg (2013)
4. Bartczuk, Ł., Rutkowska, D.: A New Version of the Fuzzy-ID3 Algorithm. In: Rutkowski, L., Tadeusiewicz, R., Zadeh, L.A., Żurada, J.M. (eds.) ICAISC 2006. LNCS (LNAI), vol. 4029, pp. 1060–1070. Springer, Heidelberg (2006)
5. Bartczuk, Ł., Rutkowska, D.: Medical Diagnosis with Type-2 Fuzzy Decision Trees. In: Kącki, E., Rudnicki, M., Stempczyńska, J. (eds.) Computers in Medical Activity. AISC, vol. 65, pp. 11–21. Springer, Heidelberg (2009)
6. Cpalka, K.: A Method for Designing Flexible Neuro-fuzzy Systems. In: Rutkowski, L., Tadeusiewicz, R., Zadeh, L.A., Żurada, J.M. (eds.) ICAISC 2006. LNCS (LNAI), vol. 4029, pp. 212–219. Springer, Heidelberg (2006)
7. Cpałka K., Łapa K., Przybył A., Zalasiński M.: A new method for designing neuro-fuzzy systems for nonlinear modelling with interpretability aspects, Neurocomputing (in print, 2014), `http://dx.doi.org/10.1016/j.neucom.2013.12.031`
8. Cpałka, K., Rutkowski, L.: Flexible Takagi Sugeno Neuro-fuzzy Structures for Nonlinear Approximation. WSEAS Transactions on Systems 9(4), 1450–1458 (2005)
9. Cpałka, K., Zalasiński, M.: On-line signature verification using vertical signature partitioning. Expert Systems with Applications 41, 4170–4180 (2014)
10. Dziwiński, P., Bartczuk, Ł., Starczewski, J.T.: Fully controllable ant colony system for text data clustering. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) EC 2012 and SIDE 2012. LNCS, vol. 7269, pp. 199–205. Springer, Heidelberg (2012)
11. Dziwiñski, P., Rutkowska, D.: Algorithm for generating fuzzy rules for WWW document classification. In: Rutkowski, L., Tadeusiewicz, R., Zadeh, L.A., Żurada, J.M. (eds.) ICAISC 2006. LNCS (LNAI), vol. 4029, pp. 1111–1119. Springer, Heidelberg (2006)
12. Dziwiński, P., Rutkowska, D.: Ant focused crawling algorithm. In: Rutkowski, L., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2008. LNCS (LNAI), vol. 5097, pp. 1018–1028. Springer, Heidelberg (2008)
13. Dziwiński, P., Starczewski, J.T., Bartczuk, Ł.: New linguistic hedges in construction of interval type-2 FLS. In: Rutkowski, L., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2010, Part II. LNCS, vol. 6114, pp. 445–450. Springer, Heidelberg (2010)
14. Gabryel, M., Cpałka, K., Rutkowski, L.: Evolutionary strategies for learning of neuro-fuzzy systems. In: Proceedings of the I Workshop on Genetic Fuzzy Systems, Granada, pp. 119–123 (2005)
15. Greblicki, W., Rutkowska, D., Rutkowski, L.: An orthogonal series estimate of time-varying regression. Annals of the Institute of Statistical Mathematics 35(2), 215–228 (1983)

16. Greenfield, S., Chiclana, F.: Type-reduction of the discretized interval type-2 fuzzy set: approaching the continuous case through progressively finer discretization. Journal of Artificial Intelligence and Soft Computing Research 1(3), 183–193 (2011)
17. Ibrahim, M.T., Khan, M.A., Alimgeer, K.S., Khan, M.K., Taj, I.A., Guan, L.: Velocity and pressure-based partitions of horizontal and vertical trajectories for on-line signature verification. Pattern Recognition 43 (2010)
18. Jain, A.K., Griess, F.D., Connell, S.D.: On-line signature verification. Pattern Recognition 35, 2963–2972 (2002)
19. Jain, A.K., Ross, A.: Introduction to Biometrics. In: Jain, A.K., Flynn, P., Ross, A.A. (eds.) Handbook of Biometrics. Springer (2008)
20. Jaworski, M., Duda, P., Pietruczuk, L.: On fuzzy clustering of data streams with concept drift. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2012, Part II. LNCS (LNAI), vol. 7268, pp. 82–91. Springer, Heidelberg (2012)
21. Jelonkiewicz, J., Przybył, A.: Accuracy improvement of neural network state variable estimator in induction motor drive. In: Rutkowski, L., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2008. LNCS (LNAI), vol. 5097, pp. 71–77. Springer, Heidelberg (2008)
22. Jeong, Y.S., Jeong, M.K., Omitaomu, O.A.: Weighted dynamic time warping for time series classification. Pattern Recognition 44, 2231–2240 (2011)
23. Khan, M.A.U., Khan, M.K., Khan, M.A.: Velocity-image model for online signature verification. IEEE Trans. Image Process 15 (2006)
24. Khan, M.K., Khan, M.A., Khan, M.A.U., Lee, S.: Signature verification using velocity-based directional filter bank. In: IEEE Asia Pacific Conference on Circuitsand Systems, APCCAS, pp. 231–234 (2006)
25. Korytkowski, M., Nowicki, R., Rutkowski, L., Scherer, R.: AdaBoost ensemble of DCOG rough–neuro–fuzzy systems. In: Jędrzejowicz, P., Nguyen, N.T., Hoang, K. (eds.) ICCCI 2011, Part I. LNCS, vol. 6922, pp. 62–71. Springer, Heidelberg (2011)
26. Korytkowski, M., Rutkowski, L., Scherer, R.: On combining backpropagation with boosting. In: Proceedings of the IEEE International Joint Conference on Neural Network (IJCNN), vol. 1-10, pp. 1274–1277 (2006)
27. Korytkowski, M., Rutkowski, L., Scherer, R.: From Ensemble of Fuzzy Classifiers to Single Fuzzy Rule Base Classifier. In: Rutkowski, L., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2008. LNCS (LNAI), vol. 5097, pp. 265–272. Springer, Heidelberg (2008)
28. Kroll, A.: On choosing the fuzziness parameter for identifying TS models with multidimensional membership functions. Journal of Artificial Intelligence and Soft Computing Research 1(4), 283–300 (2011)
29. Laskowski, L.: A Novel Continuous Dual Mode Neural Network in Stereo-Matching Process. In: Diamantaras, K., Duch, W., Iliadis, L.S. (eds.) ICANN 2010, Part III. LNCS, vol. 6354, pp. 294–297. Springer, Heidelberg (2010)
30. Laskowski, Ł.: A novel hybrid-maximum neural network in stereo-matching process. Neural Comput & Applic. 23, 2435–2450 (2013)
31. Laskowski, Ł.: Hybrid-Maximum Neural Network for Depth Analysis from Stereo-Image. In: Rutkowski, L., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2010, Part II. LNCS, vol. 6114, pp. 47–55. Springer, Heidelberg (2010)
32. Laskowski, Ł.: Objects Auto-selection from Stereo-Images Realised by Self-Correcting Neural Network. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2012, Part I. LNCS, vol. 7267, pp. 119–125. Springer, Heidelberg (2012)
33. Lei, H., Govindaraju, V.: A comparative study on the consistency of features in on-line signature verification. Pattern Recognition Letters 26, 2483–2489 (2005)

34. Li, X., Er, M.J., Lim, B.S., Zhou, J.H., Gan, O.P., Rutkowski, L.: Fuzzy Regression Modeling for Tool Performance Prediction and Degradation Detection. International Journal of Neural Systems 20(5), 405–419 (2010)
35. Łapa, K., Przybył, A., Cpałka, K.: A new approach to designing interpretable models of dynamic systems. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2013, Part II. LNCS (LNAI), vol. 7895, pp. 523–534. Springer, Heidelberg (2013)
36. Łapa, K., Zalasiński, M., Cpałka, K.: A New Method for Designing and Complexity Reduction of Neuro-fuzzy Systems for Nonlinear Modelling. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2013, Part I. LNCS (LNAI), vol. 7894, pp. 329–344. Springer, Heidelberg (2013)
37. Nowicki, R.: Rough-neuro-fuzzy structures for classification with missing data. IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics 39(6), 1334–1347 (2009)
38. Nowicki, R., Scherer, R., Rutkowski, L.: A method for learning of hierarchical fuzzy systems. In: Sincak, P., Vascak, J., Kvasnicka, V., Pospichal, J. (eds.) Intelligent Technologies - Theory and Applications, pp. 124–129. IOS Press (2002)
39. Nowicki, R., Scherer, R., Rutkowski, L.: A hierarchical neuro-fuzzy system based on simplication. In: Proceedings of International Joint Conference on Neural Networks, IJCNN 2003, Portland, Oregon, pp. 20–24 (2003)
40. O'Reilly, C., Plamondon, R.: Development of a Sigma-Lognormal representation for on-line signatures. Pattern Recognition 42, 3324–3337 (2009)
41. Patan, K., Patan, M.: Optimal Training strategies for locally recurrent neural networks. Journal of Artificial Intelligence and Soft Computing Research 1(2), 103–114 (2011)
42. Peteiro-Barral, D., Bardinas, B.G., Perez-Sanchez, B.: Learning from heterogeneously distributed data sets using artificial neural networks and genetic algorithms. Journal of Artificial Intelligence and Soft Computing Research 2(1), 5–20 (2012)
43. Pietruczuk, L., Duda, P., Jaworski, M.: A new fuzzy classifier for data streams. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2012, Part I. LNCS, vol. 7267, pp. 318–324. Springer, Heidelberg (2012)
44. Pietruczuk, L., Duda, P., Jaworski, M.: Adaptation of decision trees for handling concept drift. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2013, Part I. LNCS, vol. 7894, pp. 459–473. Springer, Heidelberg (2013)
45. Pławiak P., Tadeusiewicz R, Approximation of phenol concentration using novel hybrid computational intelligence methods. Applied Mathematics and Computer Science 24(1) (in print, 2014)
46. Przybył, A., Jelonkiewicz, J.: Genetic algorithm for observer parameters tuning in sensorless induction motor drive. In: Rutkowski, L., Kacprzyk, J. (eds.) Neural Networks and Soft Computing (6th International Conference on Neural Networks and Soft Computing 2002), Zakopane, Poland, pp. 376–381 (2002)
47. Przybył, A., Smoląg, J., Kimla, P.: Distributed Control System Based on Real Time Ethernet for Computer Numerical Controlled Machine Tool (in Polish). Przeglad Elektrotechniczny 86(2), 342–346 (2010)
48. Rutkowski, L.: Computational intelligence. Springer (2008)
49. Rutkowski, L.: On Bayes risk consistent pattern recognition procedures in a quasi-stationary environment. IEEE Transactions on Pattern Analysis and Machine Intelligence PAMI-4(1), 84–87 (1982)
50. Rutkowski, L., Cpałka, K.: Flexible structures of neuro-fuzzy systems. In: Quo Vadis Computational Intelligence. STUDFUZZ, vol. 54, pp. 479–484. Springer, Heidelberg (2000)

51. Rutkowski, L., Cpałka, K.: Flexible weighted neuro-fuzzy systems. In: Proceedings of the 9th International Conference on Neural Information Processing (ICONIP 2002), Orchid Country Club, Singapore, November 18-22 (2002)
52. Rutkowski, L., Jaworski, M., Pietruczuk, L., Duda, P.: Decision trees for mining data streams based on the gaussian approximation. IEEE Transactions on Knowledge and Data Engineering 26(1), 108–119 (2014)
53. Rutkowski, L., Jaworski, M., Pietruczuk, L., Duda, P.: The CART decision tree for mining data streams. Information Sciences 266, 1–15 (2014)
54. Rutkowski, L., Pietruczuk, L., Duda, P., Jaworski, M.: Decision trees for mining data streams based on the McDiarmid's bound. IEEE Transactions on Knowledge and Data Engineering 25(6), 1272–1279 (2013)
55. Rutkowski, L., Przybył, A., Cpałka, K.: Novel on-line speed profile generation for industrial machine tool based on flexible neuro-fuzzy approximation. IEEE Transactions on Industrial Electronics 59, 1238–1247 (2012)
56. Rutkowski, L., Przybył, A., Cpałka, K., Er, M.J.: Online speed profile generation for industrial machine tool based on neuro-fuzzy approach. In: Rutkowski, L., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2010, Part II. LNCS, vol. 6114, pp. 645–650. Springer, Heidelberg (2010)
57. Rutkowski, L., Rafajlowicz, E.: On optimal global rate of convergence of some nonparametric identification procedures. IEEE Transaction on Automatic Control, AC-34(10), 1089–1091 (1989)
58. Scherer, R.: Neuro-fuzzy relational systems for nonlinear approximation and prediction. Nonlinear Analysis Series A: Theory, Methods and Applications 71(12), e1420–e1425 (2009)
59. Scherer, R., Rutkowski, L.: Connectionist fuzzy relational systems. In: 9th International Conference on Neural Information and Processing; 4th Asia-Pacific Conference on Simulated Evolution and Learning; 1st International Conference on Fuzzy Systems and Knowledge Discovery, Singapore. Computational Intelligence for Modelling and Prediction. SCI, vol. 2, pp. 35–47. Springer, Heidelberg (2005)
60. Starczewski, J.T., Scherer, R., Korytkowski, M., Nowicki, R.: Modular Type-2 Neuro-fuzzy Systems. In: Wyrzykowski, R., Dongarra, J., Karczewski, K., Wasniewski, J. (eds.) PPAM 2007. LNCS, vol. 4967, pp. 570–578. Springer, Heidelberg (2008)
61. Szaleniec, M., Goclon, J., Witko, M., Tadeusiewicz, R.: Application of artificial neural networks and DFT-based parameters for prediction of reaction kinetics of ethylbenzene dehydrogenase. Journal of Computer-Aided Molecular Design 20(3), 145–157 (2006)
62. Yeung, D.-Y., Chang, H., Xiong, Y., George, S.E., Kashi, R.S., Matsumoto, T., Rigoll, G.: SVC2004: First International Signature Verification Competition. In: Zhang, D., Jain, A.K. (eds.) ICBA 2004. LNCS, vol. 3072, pp. 16–22. Springer, Heidelberg (2004)
63. Zalasiński, M., Cpałka, K.: A new method of on-line signature verification using a flexible fuzzy one-class classifier. In: Selected Topics in Computer Science Applications, pp. 38–53. EXIT (2011)
64. Zalasiński, M., Cpałka, K.: Novel algorithm for the on-line signature verification. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2012, Part II. LNCS (LNAI), vol. 7268, pp. 362–367. Springer, Heidelberg (2012)
65. Zalasiński, M., Cpałka, K.: New approach for the on-line signature verification based on method of horizontal partitioning. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) ICAISC 2013, Part II. LNCS (LNAI), vol. 7895, pp. 342–350. Springer, Heidelberg (2013)