# Color Digital Picture Recognition Based on Fuzzy Granulation Approach

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**Abstract.** The paper concerns specific problems of color digital picture recognition by use of the concept of fuzzy granulation, and in addition rough information granulation. This idea employs information granules that contain pieces of knowledge about digital pictures such as location of objects as well as their size and color. Each of those attributes is described by means of linguistic values of fuzzy sets, and the shape attribute is also considered with regard to the rough sets. The picture recognition approach is focused on retrieving a picture (or pictures) from a large collection of color digital pictures (images) - based on the linguistic description of a specific object included in the picture to be recognized.

## 1 Introduction

The main idea of the fuzzy granulation approach to color digital picture recognition - developed and employed in this paper - is based on the concept introduced by the authors in [15]. Some problems mentioned in the last section of [15], within the context of further research, are considered in this paper. In particular, the third dimension of the CIE chromaticity triangle (color model) - that is the luminance - is included in our approach. Besides, in addition to the size attribute, approximate shape of an object located in the picture to be recognized is taken into account, and - apart from the fuzzy granules - application of the rough granulation is proposed.

Nowadays we collect a lot of various color digital pictures, and the number of such pictures are still growing. Moreover, the picture resolution increases, so we need new methods for searching, recognition, and retrieving a particular picture from a large collection of them. Let us imagine a problem of searching for a picture based on a piece of knowlegde about a particular object that we remember as located in this picture. Let us assume that we may roughly describe the location as well as shape, size, and color of that object. In such a case, we can employ the approach proposed in this paper, and it seems to be very useful.

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Two main attributes, considered in the linguistic fuzzy description of the specific object included in the picture, are color and location. Other attributes, such as shape and size are strictly related with the location, and concern the same 2-dimensional space of pixels. The color attribute may refer to different space of the color spaces, e.g. the following color models: the CIE chromaticity triangle, RGB three-dimensional space, HSL (hue, saturation, lightness), and similar HSV. More information about various color models can be found in [15] and many other publications, including [3].

Color is a very important attribute of digital pictures. It carries significant information that helps to distinguish, recognize, compare, and classify different pictures or objects presented on various pictures. We can use only this attribute in the case when we do not have any information about the specific object except its color. Of course, with less knowledge, it is more difficult to find the proper picture (or pictures).

In this paper, and in [15], the concept of fuzzy granulation, originally introduced by Zadeh [17], is proposed to describe fuzzy location of pixels as well as fuzziness of their color. In consequence, we can consider a color digital picture as a fuzzy set of pixels or groups of pixels that we call macropixels, according to the fuzzy set theory [16]. The attributes of the shape and size may be considered within the framework of the rough sets (also called Pawlak sets) [4]. However, we can apply fuzzy sets represented by various types of membership functions; for details, see e.g. [10]. In particular, specific shapes of functions defined on two-dimensional space, expressed by proper mathematical formulas are very useful as the membership functions, with regard to both the location space and CIE color space.

It should be mentioned that some other authors combine fuzzy set and rough set theories [2], [9] in different applications, e.g. [8]. Granulation approaches have been developed within the framework of both fuzzy granulation [17] and rough granulation [5]. Information granules are applied in pattern recognition and image processing, e.g. [5] and [12], with fuzzy and rough granulation, respectively.

This paper developes the approach, introduced in [15], for solving the problems of picture recognition based on the vague knowlegde about a specific object (or any detail) included in the picture to be recognized and retrieved. The bigger is the knowledge the easier to find the proper solution, however the algorithm is more complicated because of processing more information. Thus, the special algorithm depends on the knowledge about the picture we want to recognize and retrieve.

The paper is organized as follows. The next section describes the concept of fuzzy granulation. In Section 3, a new algorithm is proposed and employed for granulating a color digital picture (pixel space) into the so-called "macropixels". Section 4 presents a new method for color space granulation. This is fuzzy granulation of the CIE chromaticity triangle with the third dimension, i.e. luminance. In Section 5, results from two previous sections are combined in order to obtain fuzzy information granules concerning the color digital pictures. Moreover, rough granules are also considered. Section 6 illustrates the problem of color digital picture recognition based on the information granules. Conclusions and final remarks are included in Section 7.

## 2 Fuzzy Granulation

As emphasized at the beginning of Section 1, the idea of fuzzy granulation in application to the color digital picture recognition, presented in [15], is considered in this paper.

Image processing is one of examples where information granulation may be applied and play an important role in pattern recognition [7]. In this case, the similarity of objects that are candidats for grouping into a granule usually refers to the closeness of pixels located spatially close to each other. In the concept of information granulation, the granules can take a form of sets, fuzzy sets, rough sets, etc., but most often are concentrated on the use of fuzzy sets. In this paper, we also apply fuzzy granulation to digital color pictures. This is presented in Sections 3 and 4. However, in Section 5, we propose to apply rough granulation.

The idea of macropixels, introduced in [15] and developed in Section 3, is strictly related to the fuzzy granulation approach. As a matter of fact, the algorithm proposed in Section 3, for creating the macropixels, realizes granulation of the pixel space. In this case, the granules refer to the closeness of pixels located spatially close to each other, and previously take a form of sets. Then, fuzzy membership functions are defined, so the macropixels are viewed as fuzzy sets.

The fuzzy color areas of the CIE diagram, discussed in [15], are combined with the luminance in Section 4, and considered as color space granulation. It is worth emphasizing that in this way we granulate the 3D color space (CIE chromaticity triangle with the luminance) that is the color model representing colors as perceived by humans, unlike the RGB. The fuzzy regions of the CIE chromaticity triangle are viewed as fuzzy sets, as well as the luminance intervals. In [15], the granules as groups of points with similar pure color (hue) are applied to the fuzzy granulation approach. In this paper, the luminance enriches the information granules.

Both the fuzzy location of pixels and fuzzy color, considered in Sections 3 and 4, respectively, are considered in the framework of the fuzzy granulation. When pixels of the same (or similar) color are located within a macropixel, we have a granule of the same color and location. In addition, as mentioned in [15], the third attribute, i.e. the size of the macropixels may be taken into account. Thus, we can see a digital color picture as a collection of macropixels associated with corresponding granules that carry information about color, location, and size. This concept is especially useful with regard to the problem of color digital picture recognition discribed in Section 6. In this paper, we also introduce another attribute, that is shape of an object to be recognized, and in this context we propose to employ the rough granulation (see Section 5).

## 3 Algorithm of Pixel Space Granulation

As explained in Section 1, the shape and size attributes are strictly related with the location that is considered in the 2-dimensional space of pixels. The color digital pictures are composed of pixels belonging to this pixel space. The location of an object in a picture can be pointed by means of the "macropixels" defined as groups of pixels. The idea of the macropixels is introduced in [15]. As a matter of fact, the macropixels can be treated as fuzzy granules, and the algorithm - proposed to determine the macropixels - granulates the pixel space, i.e. the digital picture area of pixels (smallest picture elements).

The algorithm that creates the macropixels - dividing the width and height of the picture into intervals (what is a proces of granulation) - is introduced in this section and illustrated in Fig. 1.

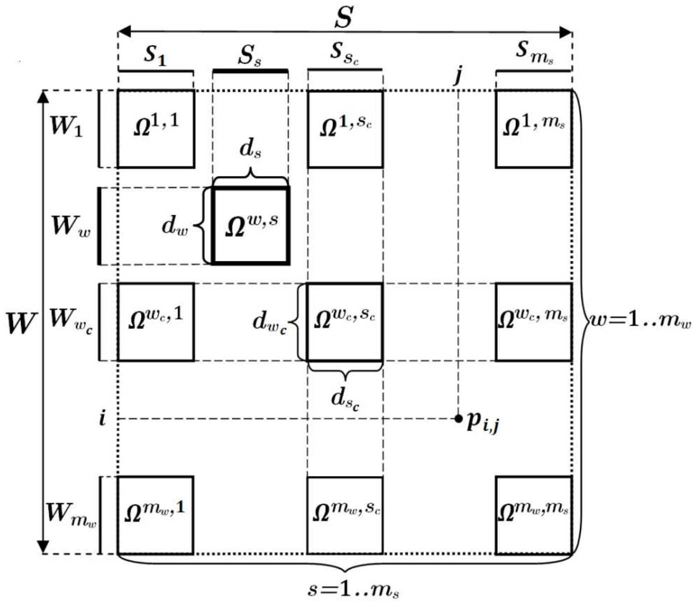


**Fig. 1.** Illustration of picture granulation into macropixels

Let $\Omega$ denotes a digital picture, composed of pixels, $p_{i,j}$, for $i = 1, ..., M_w$, and $j = 1, ..., M_s$. Thus, the number of pixels in the picture $\Omega$ equals $M = M_w M_s$ where $M_w$ and $M_s$ determine height $W$ and width $S$ of the picture, respectively.

Figure 1 shows the digital picture, $\Omega$, of size $WS$, with pixels $p_{i,j} \in \Omega$. In addition, the macropixels, denoted as $\Omega^{w,s}$, where $w = 1, ..., m_w$ and $s = 1, ..., m_s$ are depicted. This means that $m_w m_s$ is the number of macropixels $\Omega^{w,s}$ in the picture $\Omega$, and the following equation fullfils

$$\Omega = \bigcup_{\substack{w=1,\dots,m_w \\ s=1,\dots,m_s}} \Omega^{w,s} \tag{1}$$

To create the macropixels, the height $W$ and width $S$ of the picture $\Omega$ are divided into intervals, denoted as $W_w$ and $S_s$, for the macropixel's height and width, respectively:

$$W = \cup_{w=1}^{m_w} W_w \tag{2}$$

and

$$S = \cup_{s=1}^{m_s} S_s \tag{3}$$

The central intervals, denoted as $W_{w_c}$ and $S_{s_c}$, may be of different sizes than the rest ones that are intervals of the same height $W_w$ and width $S_s$. This is important and must be taken into account in the algorithm of creating the macropixels. The central macropixel is denoted as $\Omega^{w_c,s_c}$ in Fig. 1.

With regard to all the macropixels, for $w = 1, ..., w_{c-1}, w_c, w_{c+1}, ..., m_w$ and $s = 1, ..., s_{c-1}, s_c, s_{c+1}, ..., m_s$, where $m_w$ and $m_s$ are the number of intervals $W_w$ and $S_s$ in the height $W$ and width $S$ of the picture, respectively, we have:

$$p_{i,j} \in \Omega^{w,s} \Leftrightarrow i \in W_w,\ j \in S_s \tag{4}$$

Formally, we define the macropixels as Cartesian products of their heigth and width:

$$\Omega^{w,s} = W_w \times S_s \tag{5}$$

The intervals $W_w$ and $S_s$ can be expressed as follows:

$$W_w = [b_{W_w}, ..., e_{W_w}] \tag{6}$$

$$S_s = [b_{S_s}, ..., e_{S_s}] \tag{7}$$

where $b_{W_w}$, $e_{W_w}$, and $b_{S_s}$, $e_{S_s}$, denote the begin and end of the intervals, respectively.

Each macropixel $\Omega^{w,s}$ forms the granule (5), and may be viewed in the same way as the picture $\Omega$. The number of pixels (4) in the macropixel $\Omega^{w,s}$ equals:

$$M_{w,s} = M_W^{w,s} M_S^{w,s} \tag{8}$$

where $M_W^{w,s}$ and $M_S^{w,s}$ define the number o pixels corresponding to the height $W_w$ and width $S_s$ of the macropixel, given by (6) and (7), respectively, according to the following formulas:

$$M_W^{w,s} = e_{W_w} - b_{W_w} + 1 \tag{9}$$

$$M_S^{w,s} = e_{S_s} - b_{S_s} + 1 \tag{10}$$

As mentioned earlier, all makropixels in Fig.1, except the central ones, are of the same height and width that we denote $d_w$ and $d_s$, respectively, and determine as follows:

$$d_w = M_w\ div\ m_w \tag{11}$$

$$d_s = M_s\ div\ m_s \tag{12}$$

The width and height of central intervals, denoted as $d_{w_c}$ and $d_{s_c}$, respectively, are determined according to the following formulas:

$$d_{w_c} = M_w\ div\ m_w\ +\ M_w\ mod\ m_w \tag{13}$$

$$d_{s_c} = M_s\ div\ m_s\ +\ M_s\ mod\ m_s \tag{14}$$

and

$$w_c = m_w \ div \ 2 + 1, \quad s_c = m_s \ div \ 2 + 1 \tag{15}$$

Now, let us present the algorithm that allows to obtain all the intervals $W_w$ and $S_s$ that determine the height and width of the macropixels, respectively, for $w = 1, ..., w_{c-1}, w_c, w_{c+1}, ..., m_w$ and $s = 1, ..., s_{c-1}, s_c, s_{c+1}, ..., m_s$.

This algorithm is based on Equations (6) and (7), as well as (11) - (14); in order to get the begin and end values of the $W_w$ and $S_s$ intervals, denoted as $b_{W_w}$, $e_{W_w}$, and $b_{S_s}$, $e_{S_s}$, respectively.

For the first, $W_1$ and $S_1$, intervals, we have:

$$W_1 : \quad b_{W_1} = 1, \quad e_{W_1} = b_{W_1} + d_w - 1 \tag{16}$$

$$S_1 : \quad b_{S_1} = 1, \quad e_{S_1} = b_{S_1} + d_s - 1 \tag{17}$$

Then, because of the fact that the size of central macropixels may differ from others, we consider two cases:

I - Let us notice, from (11) and (13), that if $M_w \ mod \ m_w = 0$ then $d_w = d_{w_c}$. Analogously, from (12) and (14), if $M_s \ mod \ m_s = 0$ then $d_s = d_{s_c}$.

II - Otherwise, if $M_w \ mod \ m_w \neq 0$ then $d_w \neq d_{w_c}$, and if $M_s \ mod \ m_s \neq 0$ then $d_s \neq d_{s_c}$; from (11), (13), and (12), (14), respectively.

In case I,
for $w = 2, ..., m_w$

$$W_w : \quad b_{W_w} = e_{W_{w-1}} + 1, \quad e_{W_w} = b_{W_w} + d_w - 1 \tag{18}$$

for $s = 2, ..., m_s$

$$S_s : \quad b_{S_s} = e_{S_{s-1}} + 1, \quad e_{S_s} = b_{S_s} + d_s - 1 \tag{19}$$

In case II,
for $w = 2, ..., w_{c-1}, w_{c+1}, ..., m_w$ use formula (18)
and for $w = w_c$ determine the central interval as follows:

$$W_{w_c} : \quad b_{W_{w_c}} = e_{W_{w_{c-1}}} + 1, \quad e_{W_{w_c}} = b_{W_{w_c}} + d_{w_c} - 1 \tag{20}$$

Analogously, for $s = 2, ..., s_{c-1}, s_{c+1}, ..., m_s$ use formula (19)
and for $s = s_c$ determine the central interval as follows:

$$S_{s_c} : \quad b_{S_{s_c}} = e_{S_{s_{c-1}}} + 1, \quad e_{S_{s_c}} = b_{S_{s_c}} + d_{s_c} - 1 \tag{21}$$

In this way, we obtain $m_w m_s$ macropixels that may be viewed as the granules within the pixel space. These granules include information about location of pixels in a digital picture.

As mentioned earlier, we can treat each makropixel $\Omega^{w,s}$ like the picture $\Omega$, then implement the presented algorithm to the macropixels, and get more but smaller ones. The number of the makropixels at first level of recursion equals $m_s m_w$, and generally at level g, we get $(m_s m_w)^g$ macropixels. Of course, we can modify the algorithm to obtain different number of the macropixels.

The algorithm proposed in this section performs crisp granulation. However, the results may be viewed as the granulation with fuzzy boundaries between the macropixels, so appropriate membership functions can be introduced for the fuzzy granules (see [15] and Section 5).

## 4 Color Space Granulation

As mentioned in Section 1, we consider two main attributes: location and color. The location attribute concerns the pixel space that is granulated according to the algorithm proposed in Section 3. Now, we are interested in the color attribute and color space granulation.

Color digital pictures are composed of pixels. In computers, the color attribute associated with each pixel, is expressed as an RGB triplet $(r, g, b)$. Every component (RGB coordinate), in the RGB color model, can vary from zero to a defined maximum value (e.g. 1 or 255). An RGB triplet $(r, g, b)$ represents the 3-dimensional coordinate of the point of the given color within the cube created by 3 axes (red, blue, and green). The triplets $(r, g, b)$ are viewed as ordinary Cartesian coordinates in the Euclidean space. The $(r, g, b)$ coordinates can be transformed into the CIE chromaticity triangle, i.e. to the color areas located on the 2-dimensional space (of the CIE diagram) with $(x, y)$ coordinates.

Mathematical formulas describing transformations between different color spaces can be found in many publications, e.g. [3]. The transformation from the RGB to CIE is also explained and the mathematical equations are included in [14]. For considerations in this paper, it is sufficient to express the transformation in the following, general form:

$$x = f_1(r, g, b), \quad y = f_2(r, g, b), \quad Y = f_3(r, g, b) \tag{22}$$

where $(x, y)$ denotes 2-dimensional coordinates in the CIE triangle, and $Y$ is the additional coordinate corresponding the luminance. Knowing the functions (22), we can transform each $(r, g, b)$ triplet assigned to particular pixels of a digital color picture to the CIE chromaticity triangle, and also to the third dimension that is the luminance. In this way, we can determine the proper color area of the CIE diagram (that represents a pure color called hue) and the luminance to every pixel of the digital picture. The hue with the luminance constitutes the color that people perceive and recognize.

Now, let us denote, like in Section 3:
$\Omega$ – digital color picture,
$M$ – number of pixels in the picture $\Omega$,
but unlike in Section 3:
$p_j$ – $j$-th pixel in the picture $\Omega$, where $j = 1, ..., M$,
and additionally:
$h_j = (x_j, y_j)$ calculated from (22) for triplet $(r_j, g_j, b_j) = p_j$, where $j = 1, ..., M$

$l_j$ – luminance of the pixel $p_j$, for $j = 1, ..., M$.
$c_j = (h_j, l_j)$ – full color attribute of the pixel $p_j$, for $j = 1, ..., M$.

Let $\Delta_{CIE}$ denotes the CIE chromaticity triangle, and $\{H_1, H_2, ..., H_N\}$ - crisp color areas (regions with sharp boundaries) of the $\Delta_{CIE}$. Hence, we have the following equation:

$$\Delta_{CIE} = \bigcup_{n=1}^{N} H_n \tag{23}$$

The color areas (regions) of the CIE chromaticity triangle, $\{H_1, H_2, ..., H_N\}$, may be treated as fuzzy regions, with fuzzy boundaries between them. This means that the fuzzy color areas are fuzzy sets of points $(x, y)$ belonging to these regions with membership grades expressed by a value from the interval [0.1]. The membership functions of the fuzzy sets may be defined in different ways. An algorithm for creating such membership functions for the fuzzy color areas of the CIE triangle is proposed in [14]. Like in [15], let us denote the fuzzy regions of the CIE diagram as $\{\widetilde{H}_1, \widetilde{H}_2, ..., \widetilde{H}_N\}$. Other types of membership functions, for the fuzzy CIE areas, may also be employed.

Table TP in Table 1 contains values of the membership functions concerning the hue attribute of the pixels. This refer to the fuzzy sets $\{\widetilde{H}_1, \widetilde{H}_2, ..., \widetilde{H}_N\}$. Table TL in Table 1 includes values of membership functions that define fuzzy sets in the luminance space. The fuzzy sets $\{\widetilde{L}_1, \widetilde{L}_2, ..., \widetilde{L}_{m_L}\}$ can be represented by triangular or trapezoidal membership functions, with the meaning, e.g. "small", "medium", "large", with regard to the luminance. Both the hue and luminance attribute produce the color attribute of the pixels $p_j$, for $j = 1, ..., M$. Fuzzy granules are created, as the Cartesian product of corresponding fuzzy sets $\{\widetilde{H}_1, \widetilde{H}_2, ..., \widetilde{H}_N\}$ and $\{\widetilde{L}_1, \widetilde{L}_2, ..., \widetilde{L}_{m_L}\}$. These granules contain information about the color as the combination of the hue and luminance.

The luminance $l_j$, for $j = 1, ..., M$, and the fuzzy sets $\{\widetilde{L}_1, \widetilde{L}_2, ..., \widetilde{L}_{m_L}\}$, are defined in the luminance space $L = [0, ..., 255]$. By use of formulas (22), the $(r_j, g_j, b_j)$ values of every pixel, $p_j$, for $j = 1, ..., M$, in the digital color picture $\Omega$, can easily be transformed to the corresponding point $(x_j, y_j)$ in the $\Delta_{CIE}$ space defined by Equation (23). In addition, values of the luminance attribute can be obtained as $l_j = Y_j$, for $j = 1, ..., M$. Hence, we can determine membership values of $h_j$ to the fuzzy sets $\{\widetilde{H}_n\}$, for $n = 1, ..., N$, as $\mu_{\widetilde{H}_n}(h_j) = \mu_{\widetilde{H}_n}(x_j, y_j)$, and membership values of luminance $l_j$ to fuzzy sets $\widetilde{L}_t$, for $t = 1, ..., m_L$. In this paper, we employ trapezoidal membership functions, $\mu_{\widetilde{L}_t}(l_j)$, defined in the range of the luminance that is different for each CIE region $H_n$, for $n = 1, ..., N$. However, as mentioned earlier, other types of the membership functions $\mu_{\widetilde{L}_t}(l_j)$ can be applied. Assuming that $m_n$ denotes the number of the fuzzy sets (fuzzy luminance intervals) associated with $H_n$, we have $m_L = \sum_{n=1}^{N} m_n$ fuzzy sets $\widetilde{L}_t$ in Table TL. For example, if we consider three fuzzy sets, defining "small", "medium", and "big" luminance, respectively, for each CIE region $H_n$, then $m_L = 3N$.

**Table 1.** Membership table of color of pixels; hue (TP) and luminance (TL)

| **TP** | $h_1$ | $h_2$ | ... | | ... | $h_j$ | ... | | ... | $h_M$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $\widetilde{H}_1$ | $\mu_{\widetilde{H}_1}(h_1)$ | $\mu_{\widetilde{H}_1}(h_2)$ | ... | | ... | $\mu_{\widetilde{H}_1}(h_j)$ | ... | | ... | $\mu_{\widetilde{H}_1}(h_M)$ |
| $\widetilde{H}_2$ | $\mu_{\widetilde{H}_2}(h_1)$ | $\mu_{\widetilde{H}_2}(h_2)$ | ... | | ... | $\mu_{\widetilde{H}_2}(h_j)$ | ... | | ... | $\mu_{\widetilde{H}_2}(h_1)$ |
| ... | | | | | | | | | | |
| | | | | | | | | | | |
| $\widetilde{H}_n$ | $\mu_{\widetilde{H}_n}(h_1)$ | $\mu_{\widetilde{H}_n}(h_2)$ | ... | | ... | $\mu_{\widetilde{H}_n}(h_j)$ | ... | | ... | $\mu_{\widetilde{H}_n}(h_M)$ |
| ... | | | | | | | | | | |
| $\widetilde{H}_N$ | $\mu_{\widetilde{H}_1}(h_1)$ | $\mu_{\widetilde{H}_N}(h_2)$ | ... | | ... | $\mu_{\widetilde{H}_N}(h_j)$ | ... | | | $\mu_{\widetilde{H}_N}(h_M)$ |

| **TL** | $l_1$ | $l_2$ | | | | $l_j$ | | | | $l_M$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $\widetilde{L}_1$ | $\mu_{\widetilde{L}_1}(l_1)$ | $\mu_{\widetilde{L}_1}(l_2)$ | ... | | ... | $\mu_{\widetilde{L}_1}(l_j)$ | ... | | ... | $\mu_{\widetilde{L}_1}(l_M)$ |
| ... | | | | | | | | | | |
| $\widetilde{L}_t$ | $\mu_{\widetilde{L}_t}(l_1)$ | $\mu_{\widetilde{L}_t}(l_2)$ | ... | | ... | $\mu_{\widetilde{L}_t}(l_j)$ | ... | | ... | $\mu_{\widetilde{L}_t}(l_M)$ |
| ... | | | | | | | | | | |
| $\widetilde{L}_{m_L}$ | $\mu_{\widetilde{L}_{m_L}}(l_1)$ | $\mu_{\widetilde{L}_{m_L}}(l_2)$ | ... | | ... | $\mu_{\widetilde{L}_{m_L}}(l_j)$ | ... | | ... | $\mu_{L_{m_L}}(l_M)$ |

## 5 Fuzzy and Rough Information Granulation

In the previous sections, new methods for fuzzy granulation of both pixel and color spaces are presented. Now, we can use the granules produced by these methods in order to obtain information granules about the color digital picture. These granules may carry information concerning the location, size, shape, and color of a specific object located on the picture. We can consider the fuzzy information granules as well as rough granules.

The pixel and color space granulation, presented in Sections 3 and 4, respectively, allows to create fuzzy granules that carry information about location, size, and color of the granules.

With regard to the color, we obtain the following fuzzy granules:

- For the pure color (hue) — fuzzy CIE color areas, $\{\widetilde{H}_n\}$, for $n = 1, ..., N$, where e.g. $N = 23$, with the membership functions $\mu_{\widetilde{H}_n}(h_j)$, for $j = 1, ..., M$; see Table TP in Table 1
- For the luminance — $\{\widetilde{L}_t\}$, for $t = 1, ..., m_L$, where $m_L$ equals to the number of fuzzy luminance intervals (fuzzy sets) defined for every CIE region $H_n$, for $n = 1, ..., N$; see Table TL in Table 1

Thus, for both the hue and luminance, we constract granules as the Cartesian product of corresponding fuzzy sets $\{\widetilde{H}_n\}$, and $\{\widetilde{L}_t\}$, for $n = 1, ..., N$, and $t = 1, ..., m_L$, where $t$ is appropriate for $H_n$, what means that the number of the fuzzy luminance granules associated with $H_n$ equals to $m_n$. It should be explained that the crisp granules $H_n$, for $n = 1, ..., N$, are viewed as $\alpha - cuts$, i.e. crisp $\alpha - level$ sets (see e.g. [11]) of the fuzzy sets $\widetilde{H}_n$, respectively, where $\alpha - level$ equals 0.5. The color granules are fuzzy sets, defined according to the definition of the Cartesian product of fuzzy sets ((see e.g. [11]), as follows:

$$\mu_{\widetilde{C}_t}(c_j) = \mu_{\widetilde{H}_n \times \widetilde{L}_t}(h_j, l_j) = \min[\mu_{\widetilde{H}_n}(h_j), \mu_{\widetilde{L}_t}(l_j)] \tag{24}$$

for $j = 1, ..., M$, where $c_j = (h_j, l_j)$, and $\widetilde{C}_t = \widetilde{H}_n \times \widetilde{L}_t$ is the Cartesian product of fuzzy sets $\widetilde{H}_n$ and $\widetilde{L}_t$, represented by membership functions $\mu_{\widetilde{H}_n}(h_j)$ and $\mu_{\widetilde{L}_t}(l_j)$, respectively. Let us notice that the number of the color granules, $\widetilde{C}_t$, equals to the number of the luminance granules, $\widetilde{L}_t$, that is $m_L$; see Table 1. The color granules are defined in the 3D color space, where every $c_j = (h_j, l_j)$, for $j = 1, ..., M$, corresponds to a point given by formula (22).

Referring to the location, we consider granules as the fuzzy macropixels, taking into account both their size and location in the color digital picture:

- For the location — Fig.1 shows the macropixels $\Omega^{w,s}$, for $w = 1, ..., m_w$ and $s = 1, ..., m_s$, located in the picture $\Omega$; the location is determined by the intervals $W_w$ and $S_s$, defining the macropixel's height and width. As mentioned at the end of Section 3, fuzzy boundaries between the macropixels may be expressed by appropriate membership functions. Hence, the macropixels are viewed as fuzzy sets (fuzzy granules) defined e.g. by trapezoidal membership functions as presented in [15].
- For the size — fuzzy sets with membership functions that describe size of the macropixels $\Omega^{w,s}$, for $w = 1, ..., m_w$ and $s = 1, ..., m_s$ as e.g. "small", "medium", "big" (3 fuzzy sets) or "very small", "small", "medium", "big", "very big" (5 fuzzy sets), may be created as trapezoidal functions. The size of macropixels obtained by use of the algorithm proposed in Section 3 may be evaluated according to these membership functions.

In this way, we can construct fuzzy information granules, as the Cartesian products of fuzzy sets corresponding to the location and size of the macropixels, analogously to the color granules created according to formula (24). Thus, the location attribute can be viewed in a wider sense, including both the location and size - and information about this attribute is carried in the fuzzy granules.

Combining the location and color, in the same way, i.e. as the Cartesian product of fuzzy sets, we may obtain fuzzy information granules that contain information concerning fuzzy values of all the attributes considered with regard to the location and color.

Now, let us focus our attention on another, additional attribut that can be included into the information granules. With regard to color digital pictures, we may be interested in shape of an object located in a picture (see Section 6).

Figure 2 illustrates a hat shape object located in the picture $\Omega$. Its size and location can easily be described by use of the fuzzy information granules above discussed. This additional attribute - shape - may also be defined by a membership function in the 2-dimensional pixel space. The object's shape can be approximated by specific mathematical functions, similarly to the membership functions of the CIE pure color regions.

However, with regard to the shape attribute, we propose to apply rough granulation, based on the rough set theory introduced by Pawlak [4], in addition to the fuzzy approach. As we see in Fig.2, the shape of the object can easily be determined by the lower and upper approximations of the group of macropixels corresponding to the object in the picture. The rough granularity, developed e.g. in [5] and [8], may be employed in the problem of color digital picture recognition considered in this paper (Section 6). According to Zadeh [17], the rough set theory is one of the approaches that use crisp granulation. Thus, we apply the rough granulation to the crisp granules (macropixels) obtained by use of the algorithm presented in Section 3. Then, we create the information granules that contain both the fuzzy and rough information about the color, location (including size), and shape attributes.

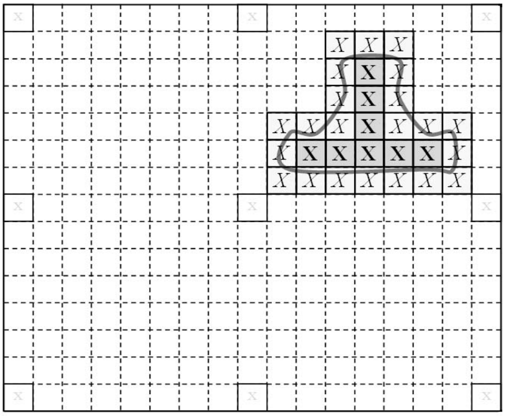

**Fig. 2.** Rough localisation of shape of an object in a digital picture

## 6 Picture Recognition

Now, let us consider two problems of color digital picture recognition based on the information granules described in Section 5.

Problem 1: Having a large collection of the pictures, we would like to find a picture (or pictures) presenting an object characterized by three attributes – size, color, and location – with fuzzy values (e.g. a big object of a color close to red, located somewhere in the center). In order to recognize such a picture (or a group of similar pictures), we can employ the idea of macropixels (created by use of the algoritm proposed in Section 3), considered with regard to the fuzzy location and size (Section 5), along with the fuzzy approach to the color granulation (Section 4). The fuzzy granulation, considered in Section 5, with the information granules that contain information about size, color, and location, is especially useful with regard to this problem. Macropixels of different sizes and the same (or similar) color and location may form the information granules.

Problem 2: Having a large collection of the pictures, we would like to find a picture (or pictures) presenting an object characterized by the same three attributes as in Problem 1 – size, color, location – but with the additional attribute i.e. shape. In this case, the rough aproach is proposed to be applied, as described in Section 5. The rough granulation with regard to the shape, combined with the fuzzy approach to the other attributes, may be employed to this problem.

The fuzzy granules discussed in Section 5 are multidimensional fuzzy sets that represent fuzzy relations between color, location, and size (attributes of the macropixels). In the fuzzy granulation approach, a digital color picture is viewed as a composition of the fuzzy granules that carry information about the color, location, and size, as well as interactions between them (expressed by the fuzziness that results in overlapping of the granules).

Both classification problems can be solved by use of a fuzzy system with the inference method based on fuzzy logic and fuzzy IF-THEN rules; for details see e.g. [11], and also [15] where the fuzzy granules are considered. In general, the rules of the following form may be employed:

$$\text{IF } G \text{ THEN class } D \tag{25}$$

where $G$ is the fuzzy granule, and $D$ is the class of pictures that suit to the description of the attributes concerning the object. In this way, we expect to obtain a group of pictures belonging to the specific class (e.g. with a big object of a color close to red, somewhere in the center of the picture). Then, having relatively small number of such a pictures (after the classification), it is much easier to find this one that we are searching for. Of course, it is possible to get just the only one picture from a large collection of others.

With regard to Problem 2, we may be interested, for example, in an object of hat shape located in the right upper corner of the picture (as presented in Fig.2), in addition to other attributes like size and color. Concerning the shape, a user of the system can indicate these macropixels that fully belong to the shape, and those that may belong (which means - belong partially); as Fig.2 shows. The former corresponds to the lower approximation while the latter to upper approximation, referring to the rough set terminology. However, the hat shape object can be viewed as a fuzzy set, and we can define the membership function of this set as follows: it equals 1 for the lower approximation area (inside the hat shape), and 0.5 for the area of macropixels included the boundary of the hat shape, and 0 for other area of the picture; see Fig.2. Thus, all the attributes may be considered within a fuzzy granule. Hence, the fuzzy system based on the rules of the form (25) can be employed.

## 7 Conclusions and Final Remarks

This paper concerns the concept of fuzzy granulation, and also combined with the rough granulation, in application to digital color pictures. In particulary, we consider a problem of picture recognition based on information granules that contain a piece of knowledge about the specifc picture to be recognized.

Fuzzy granulation approach, as mentioned in Section 1, has been introduced by Zadeh [17]. Some information one also can find in e.g. [11]. New ideas concerning the fuzzy granulation approach have been developed by Pedrycz, especially with regard to neural networks (see e.g. [6]) but also to pattern recognition [7]. Rough granulation is presented and developed e.g. in [5] and [8].

In this paper, as well as in [15], we consider problems of color digital pictures recognition that can be viewed as a special case of image processing and image recognition. However, we are interested in a large collection of the color digital pictures that are images, of course, but typical, taken by popular digital cameras, not e.g. medical images. Therefore, we use the name "picture" rather that "image", in order to focus our attention on the application to usual photos. Moreover, the important issue is that we are now not going to recognize the exact image presented in the picture but only its specific part described by an approximate color, location, and also shape.

It should be emphasized that the main idea concerning the problems considered in this paper is to describe a picture by linguistic terms that refer to color, location, and size, i.e. the attributes of macropixels. Then, our task is to recognize (and e.g. classify) pictures with specific features, expressed by the linguistic description, such as "a big object of a color close to red, located somewhere in the center of the picture". Thus, our aim is not to recognize details of the image but only selected features characterized by approximate (fuzzy) values.

Further research on this subject may concern very interesting problems of image understanding (see e.g.[13]), based on the fuzzy granulation approach, also combined with the rough information granules discussed in Section 5.

Color and shape attributes may also be considered with regard to the content-based image retrieval (see e.g. [1]), where shape representation techniques are usually boundary-based and region-based.

It is important to note that our approach to image recognition does not require to process every pixel in particular pictures but only the area of selected macropixels. Furthermore, we do not need to realize segmentation with crisp boundaries, so we do not have to employ any algorithm for edge detection.

Of course, more practical experiments will be realized to illustrate performance of the system proposed in this paper to solve the digital picture recognition problems. In this way, we will be able to compare results of the picture recognition depending on the information about attributes of the object presented in the picture to be recognized. It seems to be obvious, as mentioned in Section 1, that the more information we have the better recognition results.

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