Probabilistic Considerations in the Damage Analysis of Ship Collisions

Abayomi Obisesan, Srinivas Sriramula, and John Harrigan

Abstract Ship collision events are often analyzed by following the approach of internal mechanics and external dynamics. The uncertainties in collision scenario parameters, which are used in the calculation of external dynamics, are usually quantified during ship collision analysis. However, uncertainties in the material and geometric properties are often overlooked during the analysis of internal mechanics. Consequently, it may lead to overestimation or underestimation of ship structural design capacity, which could impact on system performance.

This study aims to show a framework for assessing the reliability of ship hull structures during collision events. Finite element analysis using ABAQUS software and simplified analytical methods have been utilized to model the resistance of ship hull plates against the impact from the bulbous bow of a striking ship. The particulars of a general cargo vessel involved in a real-life collision have been used in the study to determine the external force required for the considered hull plate to resist. Based on Monte Carlo simulation, reliability analysis has been carried out to model the uncertainties of hull plate displacement. Two thousand design sets of the geometric and material properties were propagated through the simplified analytical model to obtain the resulting displacement data. These data were subsequently analyzed to obtain a probabilistic model for hull plate displacement, along with the variable sensitivity.

A. Obisesan • S. Sriramula (✉) • J. Harrigan
Lloyd's Register Foundation (LRF) Centre for Safety and Reliability Engineering,
School of Engineering, University of Aberdeen, Fraser Noble Building,
Aberdeen AB24 3UE, UK
e-mail: s.sriramula@abdn.ac.uk

1   Introduction

Due to the increase in energy and commodity demand worldwide, there have been an increase in sea transportation by Cargo and tanker vessels. The greatest risk associated with sea transportation is loss of containment, which then poses a threat to marine biodiversity, human lives, structural assets, and the reputation of the companies involved. According to IHS Fairplay world casualty statistics for 2011 (IHS Fairplay 2012), 126 ships with a total of approximately 770,000 gross tonnage were lost at sea in 2011. Approximately 19 % of the total ship losses were due to ship collisions and contact. Despite the advancement in proactive measures such as the ship navigation systems, ship collisions are still being recorded every year. This reiterates the importance of analyzing the structural performance of ships during collisions.

The two most common methods for analyzing ship collisions are external dynamics and internal mechanics. External dynamics is concerned with the rigid body motion of the striking ship and the struck ship as well as the energy dissipated by the ship structures due to collision. External dynamics are calculated either by numerical or analytical methods (Zhang 1999). Using an analytical approach, closed-form expressions can be derived for the impact impulses and the kinetic energy loss during ship collisions. With the application of the principles of conservation of momentum and energy, the analysis is achieved by calculating the deformation due to the body motion of the struck ship and the striking ship, based on the assumption that the structural response comes from the local contact only. Detailed discussion on the internal mechanics is provided in Sect. 2 of this study.

There are existing models which incorporate the processes of internal mechanics and external dynamics for ship collision analysis. The most common approaches in the literature are ALPS/SCOL (Paik and Pedersen 1996), DAMAGE (Simonsen 1999), SIMCOL (Chen 2000), and DTU (Lutzen 2001) models. The common denominator for these models is the assigned stopping criteria which compare the energy dissipation calculated from the external dynamics and the energy absorption calculated from the internal mechanics. Once an equilibrium condition is met, these models evaluate output parameters such as the maximum penetration and damage length. External dynamics and internal mechanics are estimated either separately or simultaneously in the models.

The outcome of the ship collision analysis provides input to decisions on design specifications, for example, on the quality of steel or the appropriate plate thickness to be used in shipbuilding. Improved understanding that leads to design improvements can then reduce the risk to ship structures and the marine environment. To improve our understanding, uncertainties that are inherent in the analysis variables need to be considered. Models such as SIMCOL (Chen 2000) and DTU (Lutzen 2001) quantified the uncertainties in collision scenarios during the estimation of output characteristics of the external dynamics. The variables of the collision scenario were identified as collision angle, speed, length, type, draught of the striking ship and struck ship.

However, the uncertainties associated with the internal mechanics of ship collisions also need to be analyzed. The design strength of ship structures, which is a major factor in the analysis of internal mechanics, possesses random behavior. Such behavior could be attributed to the basic strength and geometry variables of structural members. Unfortunately, these variables tend to be represented by their deterministic or design values only, with possible variances often ignored. However, these values are applied with the inclusion of a generalized safety margin between the applied load and the strength of structural members, called safety factor. Due to the nonflexible nature of the safety factor, safety margin of structural members may not be able to accommodate possible changes to certain factors, such as strength and load variability, possibility of correlation among load and strength parameters, and uncertainties in structural analysis. It is important to understand how such changes may affect the characteristics and reliability of structural members in order to achieve optimal design.

Uncertainty quantification of basic random variables for reliability assessment can contribute to the advancement and improvement of decisions made in ship structural design procedures by providing new design measures and a reliability framework that is applicable to both conventional and advanced ship structural members. It is worth noting that the application of reliability assessment to ship structures is not a new phenomenon in ship structural design. The study by Nikolaidis et al. (1993) proposed a method for assessing the reliability of ship deck panels to identify and rank the most important uncertainties and the most effective design improvement. However, the assessment is limited to the type of ship assessed as a result of the deterministic values assigned to the main geometric variables of the deck panel. The study by Fang and Das (2005) assessed and compared the reliability of damaged ships during collision and grounding scenarios by fitting suitable distributions to model the uncertainties from load and strength variables. It was concluded that the risk caused by collision is far greater than that of grounding when a damaged ship continued en route.

Chowdhury (2007) assessed the failure probability of the mid-ship section due to yielding by quantifying uncertainties from the load and strength bending moments. The reliability assessment study by Khan and Das (2008) followed a similar approach and identified that particular types of double hull tanker and bulk carriers in two different service conditions (intact and damage) would have a higher failure probability in sagging than in hogging during side collision and grounding scenarios. They quantified the model uncertainties that are attributed to the variables of the ultimate/residual strength of the mid-ship section and also to the load variables such as still water, wave-induced vertical and horizontal bending moments.

The literature discussed earlier justified the necessity to consider various uncertainties of ship collision analysis in a reliability framework. Failure of the ship structure, such as buckling of hull girder members, can be defined and modeled using the corresponding limit state functions. The functional relations can be solved using structural reliability techniques such as First and Second Order Methods (FORM/SORM) and Monte Carlo Simulation (MCS).

In this study, it is intended to apply reliability assessment for the purpose of improving existing knowledge on the performance of ship hull structures during collisions. The uncertainties from the strength and geometry variables of ship hull plating are quantified using data from existing literature. The probabilistic characteristics of the strength and geometry variables are then propagated through both the finite element and simplified analytical models using a suitable sampling technique. Reliability and sensitivity assessment are then performed to model the outcome: MCSs to analyze the structural performance and Sobol's first-order indices to determine the contribution of input variables on the model output variation.

2 Internal Mechanics

The internal mechanics approach is concerned with the local deformation of the structural members at the bow of the striking ship and at the side of the struck ship and how they would respond during loading, in terms of energy absorption, displacement, and failure load. The bow of the striking ship is usually modeled to be perfectly rigid, hence internal mechanics mostly consider the deformation of the structural members of the stuck ship.

The first step in the internal mechanics analysis is the identification of ship structural members affected by the impact and the classification of their energy absorbing deformation mechanisms. This is followed by the derivation of a closed-form expression of their failure load and energy absorbed. Ship structural members may be classified into basic elements as follows:

- The side shell panels, which are the outer and the inner shell panels.
- The web girders, such as longitudinal stringers, transverse frames, transverse bulkheads, decks, and floors.
- The intersection elements, created at the junction between transverse and longitudinal members.

Failure of these members is generally defined beyond material yield points, hence failure and energy absorbing mechanisms are governed by a complex mixture of buckling, folding, tearing, rupture, and crushing of the members.

In recent years, the finite element method and simplified method have been the preferred technique employed to predict ship damage due to a collision. Due to its costly nature, the experimental method is mostly used to have a better understanding of the deformation process and to validate results from other methods. Computer advancement has made finite element method a useful tool for ship collision analysis. For example, Amdahl and Kavlie (1992), Kitamura (1997), and Paik and Thayamballi (2003) have investigated the behavior of ship structures using nonlinear finite element methods. However, due to the huge computational effort required for FE analysis of ship collision problems, simplified methods are commonly employed for the verification of ship impact resistance.

Minorsky (1959) made the first attempt to derive a simplified expression for collision resistance. An empirical equation was derived which relates the collision energy absorbed with the material volume of the damaged ship structures based on past history of collision cases. Good agreement was achieved for high energy collision cases. Following this pioneering work, several authors have developed improved analytical solutions. These solutions have produced closed-form expressions that give a good prediction of the basic features of structural deformations.

Simplified analytical models assess the resistance of ship hull structures when subjected to collision. The structural members that make up the hull structure are outer shell plating, transverse frames, longitudinal stringers, transverse bulkhead, and the inner shell plating. Several authors have developed simplified analytical formulae for the resistance force during deformation of these members. The resistance force is calculated from the consideration of the internal energy dissipated due to a kinematically admissible deformation of a structural member. Common assumptions in this method include:

- Structural members are independent of each other, but they can be combined to derive the total collision resistance of the ship structure.
- Loading by the bow of the striking ship is normal to the hull structure; the material of the structural member is rigid-perfectly plastic and rate independent.
- Most of the simplified analytical formulae from the literature consider the loading regime for which ship structures deform in a quasi-static mode.

3   Simplified Analytical Formulae for Impact Resistance

The evaluation of the impact resistance of ship structures using simplified analytical formulae is usually computed using energy conservation techniques. This involves the application of idealized assumptions and empirical data.

The shape of the striking ship bow, which collides with the hull structure of the struck ship is considered first in the evaluation of impact resistance. This is particularly important when establishing the resistance and displacement relationship for the hull shell plating. In the literature, the contact between the bow and side panel has been idealized as a point load as well as a spherical or elliptical paraboloid. Zhang (1999) and Buldgen et al. (2012) established a resistance formula for a rectangular plate subjected to transverse and oblique impact, respectively, by a point load. Simonsen and Lauridsen (2000) and Wang (2002) developed a resistance formula for a circular plate impacted by a spherical-shaped indenter. Zhang (1999) and Haris and Amdahl (2012) idealized the striking ship bow as elliptical paraboloid and evaluated the resistance of a rectangular plate to the bow impact.

Zhang (1999) further simplified the bow shape into a circular paraboloid using the assumption that vertical and horizontal radii are equal, but this idealization may not be appropriate for all ship bulbous bows. Haris and Amdahl (2012) introduced two parameters to their resistance formulae in order to consider the possibility of

Fig. 1 An idealized model of
bulbous bow impact on shell
plate
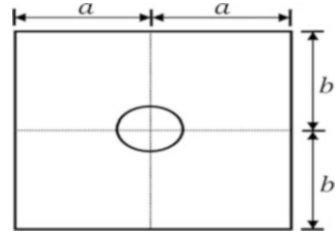
unequal curvatures of the bow in the *x*- and *y*-directions. The resistance of a square
shell plate (see Fig. 1) to impact by an elliptical paraboloid is given as follows
(Zhang 1999):

$$F_p = 1.5396\sigma_0 t_{\mathrm{p}}\delta \left[1 + 3\frac{R^2\delta}{R_{\mathrm{L}}a^2} + 6\left(\frac{R^2\delta}{R_{\mathrm{L}}a^2}\right)^2 \right], \tag{1}$$

where F_p is the external force, σ_0 is the plate material flow stress usually defined as
the average of the material yield stress and ultimate stress, t_{p} is the plate thickness,
δ is the displacement, *a* represents the half-length and half-breadth of the square
plate, R_{L} is the bulb length, and *R* is the bulb radius.

4 Numerical Analysis of Damage to Hull Structure

4.1 Finite Element Models

In this study, a square shell plate and the bulb section of a ship bow were modeled.
The dimensions applied to the two models are given in Table 1. The bulb was
idealized as a circular paraboloid (see Fig. 2) and modeled using the following
parametric equation:

$$\frac{x}{R_{\mathrm{L}}} = \frac{y^2 + z^2}{R^2}, \tag{2}$$

where *x*, *y*, and *z* are the coordinates of the bulb. The shell plate and the bulb were
modeled using Abaqus/CAE (ABAQUS 2011). Numerical analysis was carried out
using the nonlinear FE software, Abaqus/Standard. The shell plate was modeled by
a general-purpose shell element in order to incorporate changes in plate thickness
as the plate elements deform. Therefore, the shell plate was modeled using an S4R
element, which is a quadrilateral shell element with linear interpolation and reduced
integration. Frictionless contact was assumed between the bulb and plate.

Table 1 Geometrical variables for square plate impact
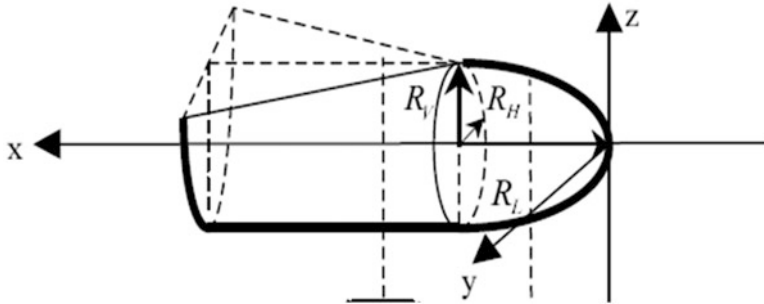
R_L	R	a	t_p
8 m	2.83 m	10 m	0.04 m

Fig. 2 Geometry of the idealized bulb (Zhang 1999)

Table 2 Material properties of the considered square plate

E	ν	σ_y	σ_u	k	ε_{σ_u}
210 GPa	0.3	400 MPa	600 MPa	400 MPa	0.5

4.2 Material Properties

Shipbuilding steel, suchas ordinary strength steel (NVA, ABS A, ASTM A36), was considered as the material of construction for the shell plate.

The shell plate material was assumed to have linear isotropic hardening, which is a simple and common form (Yaw 2012). The plastic stress was modeled using the expression:

$$\sigma\left(\varepsilon_p\right) = \sigma_y + k\varepsilon_p, \tag{3}$$

where σ_y is the yield stress, ε_p is the equivalent plastic strain, and k is the strength index, which for linear hardening, can be defined as plastic modulus. Other key materials are the ultimate stress (σ_u) and its equivalent plastic strain (ε_{σ_u}) . The elastic properties are the Young modulus (E) and Poisson ratio (ν). The values of the material properties used in the analysis are provided in Table 2.

4.3 Numerical Simulation

The plate deformation due to transverse loading by the bulb section of a bow is largely governed by membrane action. This was observed on the plating when the equivalent plastic strain distribution was analyzed, as shown in Fig. 3. Itwas also observed that the largest strain occurs in a localized area of the plate, close to the contact area. Figure 4 shows the simulation result of the finite element
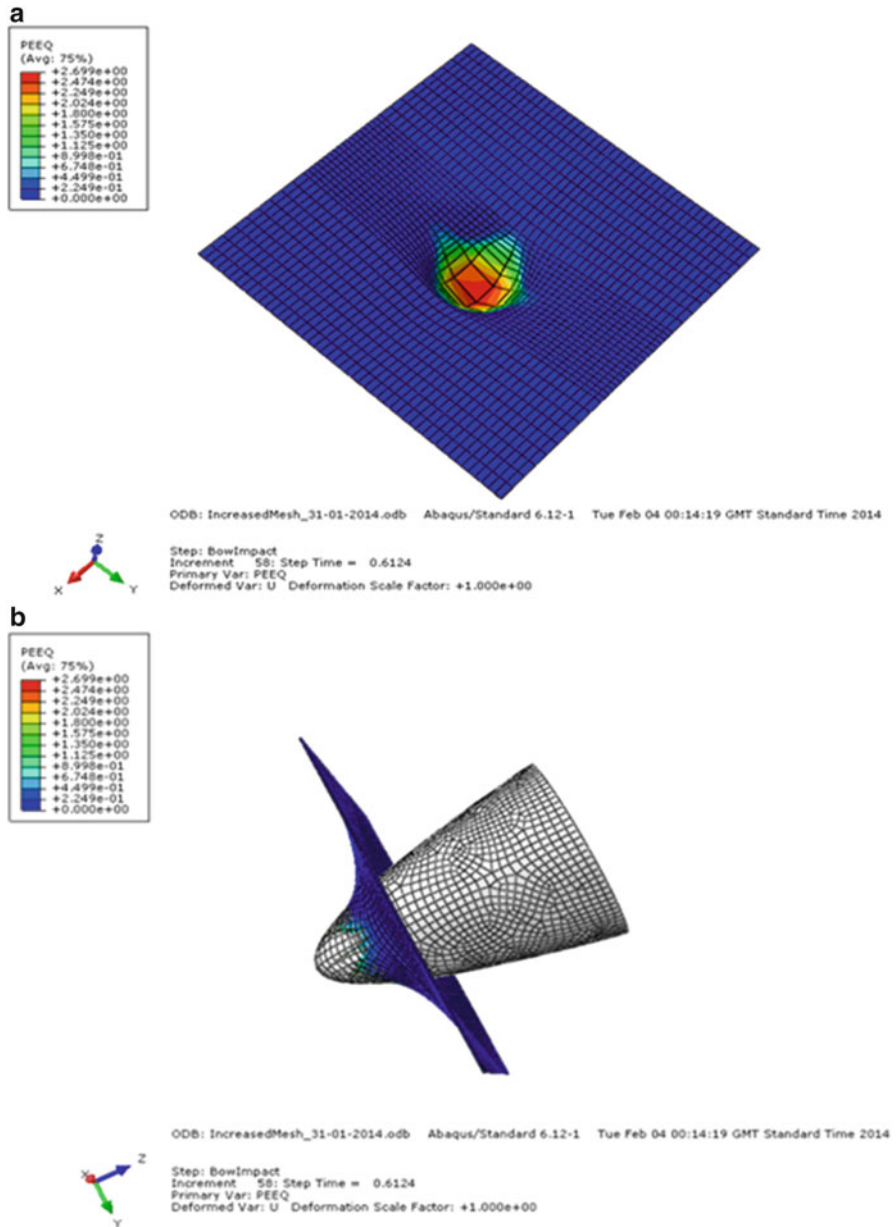
Fig. 3 Indentation of the square plate showing (**a**) plastic strain distribution and (**b**) the deformation of the plate
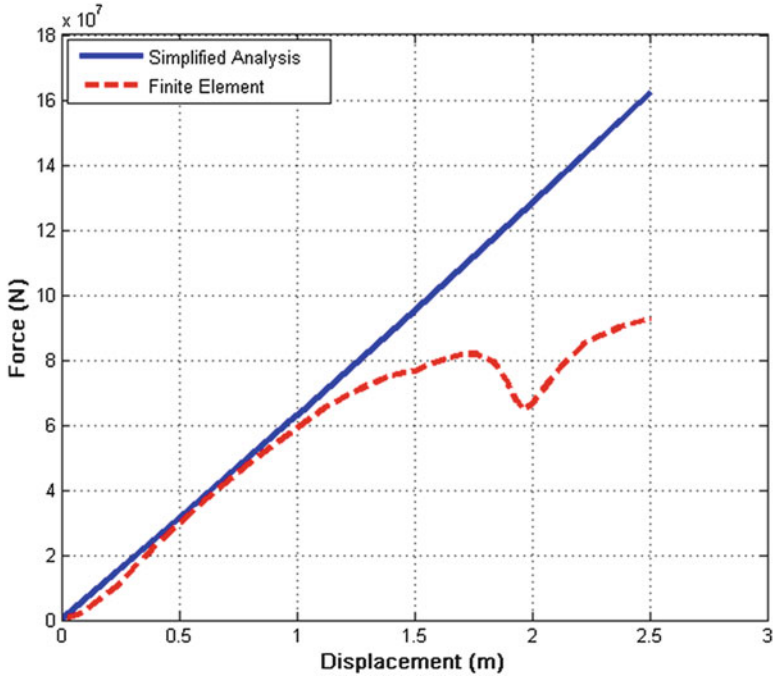
Fig. 4 Force–displacement plot of square plate impact

method plotted alongside the result of the force–displacement relationship of the simplified analytical method (Eq. (1)). The two methods show large differences after approximately 1.0 m displacement.

The linear trend of the simplified method predicts the early load–displacement behavior well. The simplified method diverges from the FE prediction at a displacement of approximately 1.0 m. This is because, simplified models assume that plate thickness is constant as the plate deforms, whereas the FE model captures the large local strains in the contact area of the plate. This is illustrated in a deformed mesh plot shown in Fig. 3a. This is accompanied by reductions in thickness in the contact region. At this stage, damage (ductile failure) is not incorporated into either of these models. Future investigations will incorporate a damage model in the FE analysis. At present, the change in slope at 1.0 m displacement is assumed to correspond to failure of the square plate.

5 Uncertainty Quantification

The outcome of most structural designs and their performances are largely governed by expert judgments. These judgments are made with the qualitative and/or quantitative consideration of uncertainties that could be inherent in the design from the

conceptual stage right through to the decommissioning stage. In the end, judgments are made on the most advantageous option for the design. However, without a complete understanding of the variability of performance, expert judgment could lead to overestimation or underestimation of design capacity, which could impact on the system reliability.

According to Lindley (1971), the choice of a course of action is largely controlled by human activities which depend on the personality of the decision makers. However, the decision taken can be positively influenced through quantitative approach. Uncertainty is mostly introduced into a system through parameters which take values within a number of ranges. Parameters with varying values, also called variables, could be either from the load or strength factors and could also be from both factors. The variables would therefore require single representations when analyzing system performance and safety. These representations are currently achievable by quantifying the variable probabilistic characteristics.

The structural strength and geometry variables of a ship shell plate are considered in the study. The strength variables are Young's modulus, yield strength, ultimate strength, and the strength index. The geometric variables are the plate length, breadth, and thickness. The quantification process of the variables from the two classes usually involves data collection and analysis for the determination of probabilistic characteristics such as probability distribution function (PDF), mean, standard deviation, and coefficient of variation (C.O.V). Fortunately, this process has already been performed for different kinds of shipbuilding steel and is available in the open literature.

Hess et al. (2002) and Guo et al. (2012) compiled statistical information and data for several shipbuilding steels. Collectively, the literature provided a data bank with more than 10,500 tests carried out on 11 different steel grades used for marine applications. However, the common steel types considered are ordinary, high tensile, high-strength low-alloy (c), and high yield steels. Hess et al. (2002) achieved the generalization of the compiled probabilistic characteristics using a normalization technique called bias. The bias was calculated as the ratio of the variable mean value to its nominal (or design) value. The recommended probabilistic characteristics were derived by taking an average of the weighted values. The goodness-of-fit tests were applied to the output of the 10,500 tests to identify suitable PDFs for the strength and geometric variables. It was observed that normal and lognormal distributions are valid choices for describing the considered variables.

The study by Guo et al. (2012) used the statistical information to develop probabilistic models of geometric and material properties for the purpose of determining the failure probability level for the corroding deck plate of aging tankers. Their study provided the most up-to-date list of statistical information for the following strength variables: Young's modulus, yield strength, and plate thickness. For the purpose of this study, the statistical information for mild steel variables are used. The bias factors and ranges for probabilistic characteristics of basic strength variables recommended by Hess et al. (2002) as well as the nominal values specified in Tables 1 and 2 are applied to the finite element and simplified models to carry out a reliability analysis on a square plate impacted by the bulb

Table 3 Probabilistic characteristics of strength random variables (Hess et al. 2002)

Variable	Distribution	Mean	C.O.V
E (GPa)	Normal	$0.987E$	0.076
σ_y (MPa)	Normal	$1.3\sigma_y$	0.124
σ_u (MPa)	Normal	$1.05\sigma_u$	0.075
k (MPa)	Deterministic	400	–
t_p (m)	Lognormal	$1.05t_p$	0.044
a (m)	Lognormal	$0.988a$	0.046

section of a striking ship bow. These characteristics are presented in Table 3 and their application is discussed in the following sections. It is worth noting that the mean of the strength random variables is from quoted values of the design steel material, but multiplied by a bias factor, so that the probabilistic characteristics would be a representative of a wide spectrum of steel used for shipbuilding.

6 External Force

The striking ship dissipates a considerable amount of kinetic energy during a collision. The energy dissipated is calculated by taking the time integral of the force exerted by the striking ship. However, the force value varies in terms of the striking ship characteristics and sea condition at the moment of impact. The combination of physical laws of motion and the energy absorbed by the deformation modes in the structural components are used to predict the extent of the structural damage.

The external force is usually calculated for the *xyz*-coordinates of the striking ship and in terms of the ship motion for each coordinate which are surge, sway, and yaw motions. For simplicity, the surge motion is only considered in this study, and the expression for the external force is given as:

$$F = M (1 + m_x) a_x, \tag{4}$$

where M is the mass of the striking ship, m_x is the added mass coefficient, and a_x is the acceleration of the striking ship. The added mass coefficient represents the effects of ship interaction with its surrounding waters. For the current study, a real-life collision case is used to estimate the contact force between the two ships.

The collision event selected for this study is a 2013 collision event where the general cargo vessel, Sulpicio Express Siete, impacted on a ferry with about 870 passengers on board. The accident, which claimed the lives of 55 people with 65 more passengers reported missing, happened outside a major port in Talisay. The general cargo vessel which had approximately 160,000 L of hydrocarbon spilled a considerable amount of oil into the sea (The Philippine Star 2013).

The main particulars of the striking vessel required for the study are provided in Table 4. It should be noted that due to the unavailability of sufficient data for the studied ship in the literature, assumptions were made for two ship parameters.

Table 4 Main particulars for Sulpicio Express Siete (Marine Connector 2013; Marine Traffic 2013)

Ship particular	Value
Deadweight tonnage (DWT)	11,464 t
Length (L)	146 m
Breadth (B)	21 m
Draught (D)	6.8 m
Block coefficient (C_B)	0.7
Lightship displacement (LD)	14,714.06 t
Ship mass (M)	26,178.06 t
Maximum speed recorded (v)	7.3 knots

These parameters are the ship acceleration and its block coefficient. However, the assumptions were made from both ship guidelines and data from a similar vessel. A block coefficient value of 0.7 was assigned to the vessel and this was used to calculate the lightship displacement using the following expression:

$$LD = 1.008 \times L \times B \times D \times C_B. \tag{5}$$

The definition of the parameters in Eq. (5) is provided in Table 4. The total mass of the ship is calculated by doing the sum of the DWT and LD. The added mass coefficient usually takes a value ranging from 0.02 to 0.07. A value of 0.05 was assigned to the striking ship, based on the study by Zhang (1999). The ship acceleration was calculated by taking an average of the acceleration at different units of a cargo vessel, following the guideline for cargo stowage and securing (see DNV 2004). For ships with lengths greater than 100 m, a correction factor should be applied to the derived acceleration value. The correction factor (i) is calculated from the expression (DNV 2004):

$$i = \left(\frac{0.345v}{\sqrt{L}}\right) + \left(\frac{58.62L - 1034.5}{L^2}\right). \tag{6}$$

With the 146 m length of Sulpicio Express Siete general cargo vessel, a correction factor of 0.56 was applied to the ship acceleration to give a value of 3.2 m/s^2. The external force was then calculated to be 87.96 MN.

7 Random Variable Generation and Propagation

Abaqus/CAE used in this study can be interfaced with the programming language tool—Python (ABAQUS 2011). Python can be applied to an existing model through one of two different methods, the Python scripting interface method and the scripting parametric method. The use of scripting parametric method for the current study would require the use of the minimum and maximum values of the random variables to generate, execute, and derive results of multiple analyses. The method

requires the development of a python script file (*.psf) and the creation of an input file (*.inp) from Abaqus/CAE model. The method first generates multiple design points for the variables by using one of the available sampling techniques. These techniques are interval, number, reference, and values. The random values generated for each variable are combined to generate design points that can be propagated one by one through the model input file.

The second method is through the use of the python development environment (PDE) which is accessible from the menu bar of Abaqus/CAE. Python scripting command provides basic functions—*random( )*—for generating pseudo-random numbers for various PDFs. The parameters required by the function are the distribution type, mean, and standard deviation of the basic random variable. The developed basic function for the random variables can be applied to the journal file (*.jnl) of an existing model by using them to replace their corresponding deterministic values to generate outputs of multiple analyses. This method was adopted for propagating the strength and geometry variables through the finite element model in this study.

The FE model was run with 2,000 sampled strength and geometry variable design sets by using the Python script developed for the model. The reactive force across the plate and the displacement of the reference nodal point were derived for each design set. MCSs were performed by propagating the generated design sets for the strength and geometry variables through the simplified model using MATLAB. First, the model (Eq. (1)) was rewritten to make displacement the subject of the cubic polynomial:

$$\left(6\frac{R^2}{R_{\rm L}}\right)\delta^3 + \left(3a^2\frac{R^2}{R_{\rm L}}\right)\delta^2 + a^4\delta - \frac{0.406a^4 F_p}{\sigma_0 t_{\rm p}} = 0. \tag{7}$$

This means that the resulting displacement value would be a vector with three roots, which contains both real and imaginary values. Second, a MATLAB code was written to propagate the design sets through the cubic polynomial. Complex roots of Eq. (7) were rejected, leaving only the real numbers as possible solutions. The results of the analysis are discussed in the following section. It is worth noting that the displacement term used in this section and subsequent sections relate to the local hull plate indentation and not the global ship displacement.

8 Stochastic Analysis

8.1 Determination of Displacement Probability Model

In order to understand the variation in the displacement values for the sampled strength and geometry variable design sets, the values were represented using a histogram plot, as shown in Fig. 5. It was observed that the displacement data from
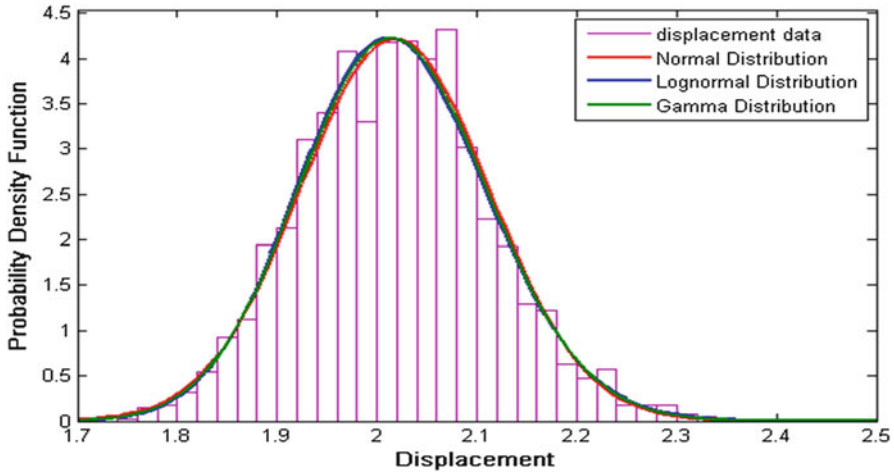
Fig. 5 Probability distributions fitted to the displacement frequency distribution

the MCS varied within the range of 1.7234 and 2.3386 m. Approximately 66 % of the displacement data lied within the range of 1.9 and 2.1 m. The mean and standard deviation of the displacement data were also calculated to be 2.018 and 0.0946 m, respectively.

A statistical approach was implemented by investigating the distribution of the displacement data. Several PDFs were fitted to the frequency distribution of the displacement data. By visual observation, three distributions—Normal, Lognormal, and Gamma distributions—provided good fits for the displacement data with minimum parameter standard error, as shown in Fig. 5. The estimated mean and standard deviation of the three distributions was similar to that derived from the displacement data. Further verification was required to discern the probability distribution model that would be plausible for the displacement data.

To this end, goodness-of-fit tests were used to statistically identify the validity and superiority among Normal, Lognormal, and Gamma distributions for the displacement data. There are several types of goodness-of-fit tests available in the literature. Prominent among them are the Kolmogorov–Smirnov (KS) and the Anderson–Darling (AD) tests, due to their applicability to both small and large data sets. KS and AD tests both use the cumulative distribution function approach to assess the suitable distribution of a data set (Romeu 2003). The AD test is particularly useful because it is more sensitive to the tails of a distribution. However, both KS and AD tests were applied to the displacement data using MATLAB. A significance level of 5 % was considered for both tests. The results of KS and AD tests showed that the null hypothesis that the displacement data could be represented by either Gamma or Lognormal distributions should be rejected, while that of Normal distribution should not be rejected at a significance level of 0.05.

8.2 Sensitivity Analysis

Due to the presence of various uncertainties, uncertainty exists in the calculated ship hull plate displacement. To characterize this, sensitivity analysis was carried out to investigate the contribution or the importance of the input variables on the variance in displacement values. The importance measure, Sobol's first-order indices were used to quantify the expected amount of variance reduction in the output data if a deterministic value of an input factor was known (Wagner 2007). The approach was implemented by defining a simplified model of displacement (δ) as a function of the random variables:

$$\delta = f\left(a, t_p, \sigma_y, \sigma_u\right). \tag{8}$$

To measure how much the variance of δ would change by fixing a variable, the variance of δ given that a variable is fixed was calculated. For example, the contribution of a fixed value of t_p on output variance would be quantified using the sensitivity index:

$$S_{t_p} = \frac{V\left(\delta \middle| t_p\right)}{V\left(\delta\right)}, \tag{9}$$

where $V(\delta|t_p)$ is the variance of δ for a known value of t_p and $V(\delta)$ is the variance of δ. However, assigning a fixed value for a random variable in the model can have a strong influence on the model outcome. In the context of the present example, the study by Saltelli et al. (2006) avoided the dependency of fixing a variable on the model outcome by replacing the numerator of Eq. (9) with the variance of the expected values of δ, obtained for each of the randomly generated values of thickness, t_{p_i}. The new expression for the sensitivity index of t_p was defined as:

$$S_{t_p} = \frac{V\left[E\left(\delta \middle| t_{p_i}\right)\right]}{V\left(\delta\right)}. \tag{10}$$

The numerator of Eq. (10) was calculated by applying the law of total variance:

$$V\left(\delta\right) = V\left[E\left(\delta \middle| t_{p_i}\right)\right] + E\left[V\left(\delta \middle| t_{p_i}\right)\right], \tag{11}$$

where $E\left[V\left(\delta \middle| t_{p_i}\right)\right]$ is the average of the variance of δ which is calculated by fixing t_p with all its possible values. A graphical representation of the percentage influence of all the variables on the displacement data is shown in Fig. 6. It was observed that t_p is the most sensitive of the variables. The variable a is the least sensitive to the
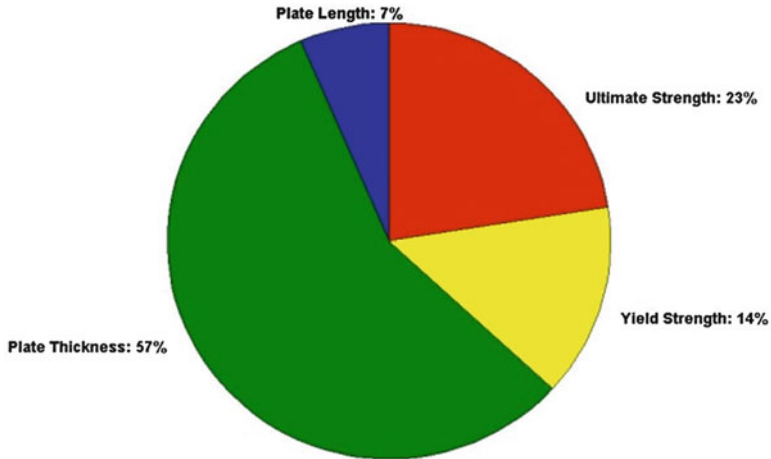
Fig. 6 Sensitivity indices of random variables on displacement data

displacement data with a sensitivity index of approximately 7 %. This result means that the variation in *a* is not significant and could be represented by a deterministic value in the simplified model.

The results of the stochastic analysis can be used to develop a probabilistic framework to predict the reliability index and failure probability of a ship hull plate under different collision scenarios. For this purpose, a limit state function can be developed to estimate the likelihood of a ship hull plate displacement exceeding a chosen design displacement capacity.

9   Discussion and Conclusion

In this study, finite element analysis of a ship hull plate impacted by a bulbous bow was combined with statistical and reliability techniques, in order to assess the effect of random strength and geometry variables on the extent of the plate deformation. The availability of Python scripting interface and sampling from pseudo-random numbers in Abaqus/CAE makes it valuable to generate multiple simulations. MATLAB was used to perform reliability assessments using MCS.

The analysis results provided knowledge and quantification of the variation in the displacement of shell plates during collisions. Normal distribution was found to be the best fit distribution to the displacement data. Parameters such as the mean value and standard deviation were identified for the displacement variable. Normal distribution and its identified parameters can then be used to represent the displacement of shell plates in any probabilistic framework.

The following are the conclusions from the reliability analysis of ship shell plate:

- The uncertainties of strength and geometry variables need to be considered during ship collision analyses and structural reliability analyses.
- The most probable displacement value of a ship shell plate would lie within the range of 1.9 and 2.1 m for the considered case study.
- Normal distribution is the best fit distribution to model the variation in displacement of a shell plate.
- Variations in the thickness of the shell plate are observed to significantly effect the hull plate displacement.

The presented study is being improved further by validating the probabilistic characteristics of displacement through the application of other sampling and reliability techniques.

References

ABAQUS (2011) ABAQUS 6.11 analysis user's manual. *Online documentation help*. Dassault Systèmes

Amdahl J, Kavlie D (1992) Experimental and numerical simulation of double hull stranding. DNV-MIT workshop on mechanics of ship collision and grounding

Buldgen L, Le Sourne H, Besnard N, Rigo P (2012) Extension of the super-elements method to the analysis of oblique collision between two ships. Mar Struct 29(1):22–57

Chen D (2000) Simplified collision model (SIMCOL). Master's thesis, Department of Ocean Engineering, Virginia Tech

Chowdhury M (2007) On the probability of failure by yielding of hull girder midship section. Ships Offshore Struct 2(3):241–260

DNV (2004) Cargo securing model manual (no. 97-0161)

Fang C, Das PK (2005) Survivability and reliability of damaged ships after collision and grounding. Ocean Eng 32(3–4):293–307

Guo JJ, Wang GG, Perakis AN, Ivanov L (2012) A study on reliability-based inspection planning—application to deck plate thickness measurement of aging tankers. Mar Struct 25(1):85–106

Haris S, Amdahl J (2012) An analytical model to assess a ship side during a collision. Ships Offshore Struct 7(4):431–448

Hess PE, Bruchman D, Assakkaf IA, Ayyub BM (2002) Uncertainties in material and geometric strength and load variables. Nav Eng J 114(2):139–165, +209

IHS Fairplay (2012) World casualty statistics 2011

Khan IA, Das PK (2008) Reliability analysis of intact and damaged ships considering combined vertical and horizontal bending moments. Ships Offshore Struct 3(4):371–384

Kitamura O (1997) Comparative study on collision resistance of side structure. Mar Technol SNAME News 34(4):293

Lindley DV (1971) Making decisions. Wiley, London

Lutzen M (2001) Ship collision damage. Technical university of Denmark

Marine Connector (2013) Sulpicio Express Siete—7724344—general cargo. http://maritime-connector.com/ship/sulpicio-express-siete-7724344/. Accessed 3 Jan 2014

Marine Traffic (2013) Sulpicio Express Siete. https://www.marinetraffic.com/en/ais/details/ships/7724344/vessel:SPAN_ASIA_17. Accessed 3 Jan 2014

Minorsky VU (1959) An analysis of ship collision with reference to protection of nuclear power ships. J Ship Res 3(2):1–4

Nikolaidis E, Hughes O, Ayyub BM, White GJ (1993) A methodology for reliability assessment of ship structures. In: Ship structure symposium, SSC/SNAME, Arlington, VA, 17–17 Nov 1993, pp H1–H10

Paik JK, Pedersen PT (1996) Modelling of the internal mechanics in ship collisions. Ocean Eng 23(2):107–142

Paik JK, Thayamballi AK (2003) A concise introduction to the idealized structural unit method for nonlinear analysis of large plated structures and its application. Thin Wall Struct 41(4):329–355

Romeu JL (2003) Anderson-Darling: a goodness of fit test for small samples assumptions. RAC START 10(5). http://src.alionscience.com/pdf/A_DTest.pdf

Saltelli A, Ratto M, Tarantola S, Campolongo F (2006) Sensitivity analysis practices: strategies for model-based inference. Reliab Eng Syst Saf 91(10–11):1109–1125

Simonsen BC (1999) Theory and validation for the DAMAGE collision module. Technical report 67, Joint MIT-Industry Program on Tanker Safety

Simonsen BC, Lauridsen LP (2000) Energy absorption and ductile failure in metal sheets under lateral indentation by a sphere. Int J Impact Eng 24(10):1017–1039

The Philippine Star (2013) 55 confirmed dead in Cebu ship collision. http://www.philstar.com/nation/2013/08/19/1108751/updated-55-confirmed-dead-cebu-ship-collision. Accessed 3 Jan 2014

Wagner S (2007) Global sensitivity analysis of predictor models in software engineering. Paper presented at the proceedings—ICSE 2007 workshops: third international workshop on predictor models in software engineering, PROMISE'07

Wang G (2002) Some recent studies on plastic behaviour of plates subjected to large impact loads. J Offshore Mech Arct Eng 124(3):125–131

Yaw LL (2012) Nonlinear static—1D plasticity—various forms of isotropic hardening. Walla Walla University, College Place, WA

Zhang S (1999) The mechanics of ship collisions. Technical University of Denmark, Kongens Lyngby