Preventive Maintenance and Replacement Scheduling in Multi-component Systems

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Abstract Maintenance and replacement schedule is one of the most important issues in industrial-production systems to ensure that the system is sufficient. This chapter presents a multi-objective model to schedule preventive maintenance activities for a series system of several standby subsystems where each component has an increasing rate of occurrence of failure (ROCOF). The planning horizon divided into the same length and discrete intervals that in each period three different maintenance actions such as maintenance, replacement, and do nothing can be performed. The objectives of this model are maximizing the system reliability and minimizing the total system cost. Because of nonlinear and complex structure of the mathematical model, non-dominated sorting genetic algorithm (NSGA-II) is used to solve this model. Finally, a numerical example is illustrated to show the model's effectiveness.

Notation

N	Number of subsystems
T	Length of planning horizon
J	Number of intervals
K	Number of maintenance levels
C	Number of components in each subsystem
λ	Characteristic life (scale) parameter of component c of subsystem i
β_i^c	Shape parameter of component c of subsystem i
α_i^k	Improvement factor of subsystem i in maintenance level k

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$\vartheta_i^c(t)$	ROCOF of component c of subsystem i
F_i	Unexpected failure of subsystem i
$M_i^{c,k}$	Level kth maintenance cost of component c of subsystem i
R_i^c	Replacement cost of component c of subsystem i
S_i	Switching cost in subsystem i
Z	System outage cost
Cost	Total system cost
$E(N_{i,j})$	Number of expected failures in subsystem i in period j
$\text{Reliability}_{i,j}^c$	Reliability of component c of subsystem i in period j
$\text{Reliability}_{i,j}^{SS}$	Reliability of subsystem i in period j
Reliability	Total system reliability
$Re_{i,j}^{SS,c}$	Reliability of subsystem i if component c be loaded at the start of period j
$Re_{i,j}^c(t)$	Operation probability of component c of subsystem i in interval $[0,t]$ of period j
$Q_{i,j}^c(t)$	Failure probability of component c of subsystem i in interval $[0,t]$ of period j

1 Introduction

Preventive maintenance and replacement is a schedule of planned maintenance and replacement activities in order to prevent system failures. The main objective of preventive maintenance and replacement is to prevent failure occurrences earlier than in reality. This concept says that by replacing old components, the reliability could be kept or improved.

Several misconceptions exist about preventive maintenance and replacement. One of them is that preventive maintenance and replacement is very expensive. This logic shows that planned maintenances are more expensive than the time a component works till its failure and by a corrective maintenance become repaired. It may be true for some components but just costs shouldn't be compared, also long-term benefits and savings should be considered. For example, without preventive maintenance and replacement, by unplanned failure occurrences, some costs like lost production cost will impose to the system. Also, by increasing in system service effective age, some savings will be brought (http reliawiki com Preventive_Maintenance).

Long-term benefits of preventive maintenance include:

1. Improved system reliability.
2. Decreased cost of replacement.
3. Decreased system downtime.
4. Better spares inventory management.

Long-term costs and benefits comparison usually shows preventive maintenance and replacement superiority.

One of the fundamental questions is that, when preventive maintenance and replacement is effective? The answer is preventive maintenance is a logical choice if, and only if, the following two conditions are met (http reliawiki com Preventive_Maintenance):

1. The component in question has an increasing failure rate. In other words, the failure rate of the component increases with time, implying wear-out. Preventive maintenance of a component that is assumed to have an exponential distribution (which implies a constant failure rate) does not make sense
2. The overall cost of the preventive maintenance action must be less than the overall cost of a corrective action

Modern world has realized the importance of preventive maintenance. So all system types, including conveyers, vehicles, and overhead cranes, have predefined maintenance and replacement schedules to reduce system failure risk. Preventive maintenance and replacement activities usually include inspection, cleaning, lubrication, adjustment, worn components replacement, etc. In addition, in preventive maintenance and replacement, labors can record equipment failures and maintain or replace old components before their failure. An ideal maintenance plan prevents all equipment failures earlier than their occurrence in reality. Regardless of specific systems, preventive maintenance and replacement activities could be divided into two categories: component maintenance and its replacement (Usher et al. 1998).

A simple example of component maintenance is air pressure controlling in a car tires in desirable limits. It's notable that this task changes the tire age characteristics and if occurs correctly will reduce the failure rate. Replacing a tire with new one is a simple replacement example.

Standby systems are widely used in different industries. For example considering a spare tire for cars, in fact is using a standby system to start up the car after a tire failure. Using standby systems reduce system total costs and cause improvement in reliability level. Outage in steel factories imposes enormous costs to the system for its restart. So, for different parts of this factory, standby systems will be considered. As mentioned before this policy causes earlier system restart after its outage and prevents imposed costs.

In real world, factories consider multi-level maintenances for each system. It's obvious that different maintenances cause different changes in age characteristics. For example, primary maintenances can be performed on equipment that don't change its effective age so much. But by equipment overhaul, it will improve deeply and will change to a state like a new equipment.

It is known that preventive maintenance and replacement includes a trade off between costs of conducting maintenance activities and saving costs resulted from the reduction in the overall rate of system failure. Preventive maintenance and replacement scheduling designers must measure these costs separately to minimize total system operating costs. They may like to improve system reliability to highest level and maximize it according to budget constraint.

In this chapter the problem is finding the best sequence of maintenance and replacement activities for each component of the considered system in each period of a specified planning horizon, in order to minimize total cost and maximize system reliability.

Optimization problems, in terms of objective functions and optimization criteria, are dividable into two categories: single objective function problems and multi-objective optimization problems. In single objective optimization problems, a unique performance index improves, so that its minimum or maximum value shows the obtained responses quality completely. But in multi-objective models more than one objective should be optimized. In other words, in this type of problems, several objective functions or operating indexes must be defined and optimized simultaneously.

Multi-objective optimization is one of the known research fields among optimization concepts. Usually, multi-objective optimization known as multicriteria optimization and vector optimization. So far, several methods have been introduced for solving multi-objective optimization problems.

NSGA-II algorithm is one of the most popular and powerful algorithms in solving multi-objective optimization problems and its performance is proofed in solving different problems.

2 Problem Definition

Consider a system consists of N subsystems that are connected to each other in series. Each subsystem is a standby system with two or more components. It means that if the loaded component fails, one of the other standby components become active and the system continues its working. Figure 1 shows the assumed system.

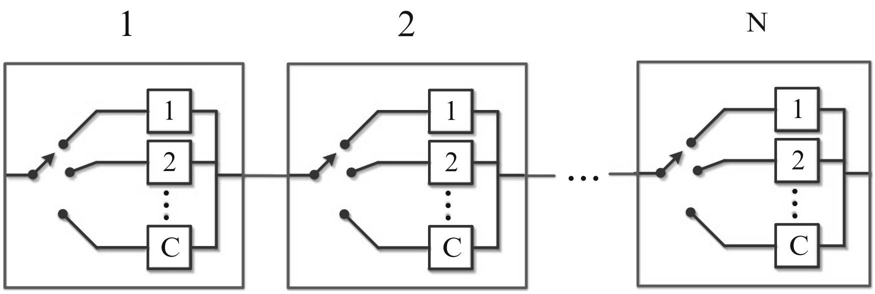


Fig. 1 Schematic view of the assumed system

The main purpose of this system modeling is finding a set of maintenance and replacement schedules for the components of each subsystem during the planning horizon to minimize the total system cost and maximize its total reliability.

For this end, it is assumed that component c of subsystem i has an increasing rate of occurrence of failure (ROCOF) $\vartheta_i^c(t)$, in which $t \cdot (t > 0)$ shows actual time. It is assumed that component failures follow well-known Non-Homogenous Poisson Process (NHPP) with ROCOF given as:

$$\vartheta_i^c(t) = \lambda_i^c \cdot \beta_i^c \cdot t^{\beta_i^c - 1} \text{ for } i = 1, \ldots, N \text{ and } c = 1, \ldots, C \tag{1}$$

Where λ_i^c and β_i^c are the characteristic life (scale) and the shape parameters of component i, respectively. It's notable that NHPP is similar to HPP with this difference that NHPP is a function of time.

As mentioned above, we seek to find a schedule for future maintenance and replacement activities on each component in interval $[0, T]$. For this purpose, the planning horizon is divided into J separated periods that each period length is equal to T/J. Similar to John Usher assumption, at the end of period j, one of these activities can be performed on each component: do nothing, maintenance, and replacement (Usher et al. 1998). These actions will review deeply as follows. One of the main assumptions of this model is that performing maintenance or replacement activities reduce the age of the components effectively. So that the ROCOF of the component will decrease. For simplicity, it is assumed that these activities perform instantaneously. In other words, the required time for maintenance or replacement is negligible compared to the planning horizon. Although this time is zero, but a cost associated to the maintenance and replacement have been considered. As mentioned above, one of these activities can be performed on each component at the end of a period:

1. Do nothing: In this policy, nothing performs on the component and the component age stays on the state "bad as old." So the component continues its working without any changes.
2. Maintenance: By maintaining a component, its age changes into a state between "bad as old" and "good as new." In this model, maintenance action reduces the effective age as a percentage of total lifetime. It's obvious that by a reduction in a component effective age, the component ROCOF decreases. An important point is that, the percentage of age reduction depends on the maintenance performance. So, multi-level maintenance with different improvement factors can be considered.
3. Replacement: In this case, the component replaces with a new one. So the component effective age drops to zero and the component state will be "good as new."

3 Effective Age of a Component at the Start of Each Period

3.1 Maintenance

For simplicity, an assumption is considered in which maintenance and replacement actions perform at the end of period j instantaneously. As mentioned above, according to the maintenance type, the age reduction differs. For example, three maintenance levels are defined: primary, intermediate, and moderate maintenance. When a primary maintenance performs, less age reduction occurs with less cost. But by overhauling a component, that component improves effectively with more cost. So:

$$\begin{aligned}&X^{c}_{i,j+1} = a^{k}_{i} \cdot X'^{c}_{i,j}\\&\text{for } i = 1,\ldots,N;\quad j = 1,\ldots,T-1; k = 1,\ldots,K;\\&c = 1,\ldots,C \text{ and } \left(0 \le a^{k}_{i} \le 1\right)\end{aligned} \tag{2}$$

To consider instantaneous changes in system age and its failure rate, $X^{c}_{i,j}$ is defined as the effective age of component c of subsystem i at the start of period j and $X'^{c}_{i,j}$ as the effective age of component c of subsystem i at the end of period j. α^{k}_{i} displays an improvement factor. Actually, this factor shows the effectiveness of the maintenance action. When $\alpha^{k}_{i} = 0$, the effect of the maintenance action is similar to replacement and the effective age becomes zero. But in the state $\alpha^{k}_{i} = 1$, the maintenance effect is similar to do nothing policy with no changes in the component age. It means that, increasing the improvement factor shows the maintenance weaknesses.

As is evident in Fig. 2, maintenance action at the end of period j, creates an instantaneous drop in ROCOF of component c. So by performing a maintenance action on component c of subsystem i at the end of period j, its ROCOF will change from $\vartheta^{c}_{i}(X'^{c}_{i,j})$ to $\vartheta^{c}_{i}(X^{c}_{i,j+1})$.

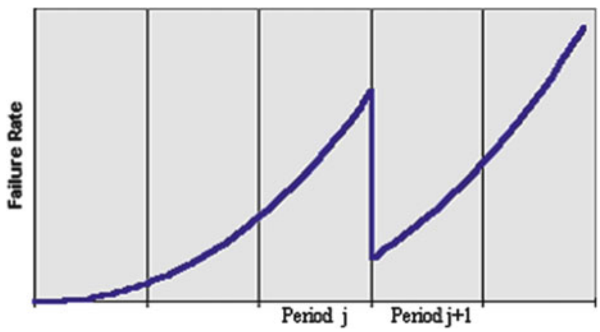


Fig. 2 Effect of period-j maintenance on component ROCOF (Usher et al. 1998)

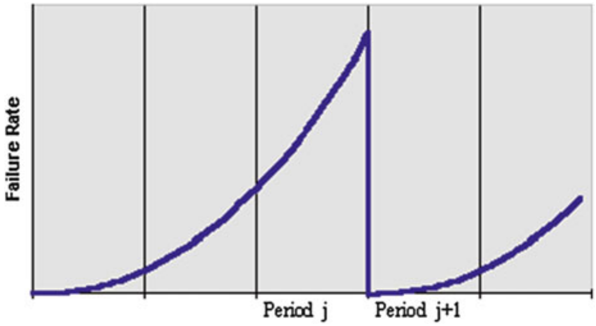


Fig. 3 Effect of period-j replacement on component ROCOF (Usher et al. 1998)

3.2 Replacement

If component c of subsystem i is replaced with a new one at the end of period j, its effective age at the start of period $j+1$ will be:

$$X^c_{i,j+1} = 0 \\ \text{for } i = 1, \ldots, N; j = 1, \ldots, T-1; c = 1, \ldots, C \tag{3}$$

In other words, system will return to a state "good as new," in which the component effective age becomes zero like a new component. So the ROCOF of this component drops from $\vartheta^c_i(X'^c_{i,j})$ to $\vartheta^c_i(0)$. Figure 3 shows the replacement effect on the component failure rate.

3.3 Do Nothing

If no action performs on a component in period j, that component continues its working without any changes in its effective age and ROCOF. So:

$$X^c_{i,j+1} = X'^c_{ij} \\ \text{for } i = 1, \ldots, N; j = 1, \ldots, T-1; c = 1, \ldots, C \tag{4}$$

$$\vartheta^c_i\left(X^c_{i,j+1}\right) = \vartheta^c_i\left(X'^c_{i,j}\right) \quad \text{for } i = 1, \ldots, N; \quad c = 1, \ldots, C \tag{5}$$

In order to construct a recursive function, which calculate $X^c_{i,j}$ according to any of above policies, two binary decision variables $m^{c,k}_{i,j}$ and $r^c_{i,j}$ are defined. These two variables represent the maintenance and replacement states on component c of subsystem i at the end of period j.

$$m^{c,k}_{i,j} = 1 \text{ if component } c \text{ of subsystem } i \text{ maintained in } k\text{th} \\ \text{level at the end of period } j; 0 \text{ otherwise} \tag{6}$$

$$r_{i,j}^{c} = 1 \text{ if component } c \text{ of subsystem } i \text{ replace at the end of period } j; 0 \text{ otherwise} \tag{7}$$

Now according to above definitions and equations from Eq. (2) to Eq. (4), a recursive function between $X_{i,j}^{c}, X_{i,j}^{\prime c}, m_{i,j}^{c,k}, r_{i,j}^{c}$ and α_{i}^{k} can be rewritten as below:

$$X_{i,j}^{c} = \left(1 - r_{i,j-1}^{c}\right)\left[\prod_{k=1}^{K}\left(1 - m_{i,j-1}^{c,k}\right)\right] X'^{c}_{i,j-1} + \sum_{k=1}^{K} m_{i,j-1}^{c,k} \cdot \left(a_{i}^{k} \cdot X'^{c}_{i,j-1}\right)$$
$$\text{for } i = 1,\ldots,N; j = 2,\ldots,T \text{ and } c = 1,\ldots,C \tag{8}$$

Equation (8) presents a closed form to calculate the effective age of component c of subsystem i at the end of period j, according to maintenance and replacement actions are performed in previous period. In this recursive function if a component is replaced in the previous period, then $r_{i,j-1}^{c} = 1$ and $m_{i,j-1}^{c,k} = 0$, where the result will be $X_{i,j}^{c} = 0$. But if a component is maintained in one of maintenance levels, then $r_{i,j-1}^{c} = 0$ and $m_{i,j-1}^{c,k}$ for that maintenance level become one where $X_{i,j}^{c} = \alpha_{i}^{k} \cdot X_{i,j-1}^{\prime c}$ and the improvement factor of that maintenance will be used. Finally, if certain operation doesn't perform and the component continues its working and the equation becomes $X_{i,j}^{c} = X_{i,j-1}^{\prime c}$.

There is a basic assumption that the system starts its working from a completely new state. So the primary lifetime for each component at the start of first period is 0. It is clear that this assumption could be changed according to real system characteristics.

$$X_{i,1}^{c} = 0 \text{ for } i = 1,\ldots N \text{ and } c = 1,\ldots,C \tag{9}$$

4 Costs Related to Maintenance and Replacement Activities

In this section the costs that are necessary to consider in this model will be analyzed.

4.1 Maintenance Cost

It's obvious that performing maintenance on a component imposes a cost to the system. So when a maintenance action performs in level k on component c of susbsystem i in a period, the constant maintenance cost $M_i^{c,k}$ will add to the total system cost at the end of that period.

$$M_{i,j} = \sum_{c=1}^{C}\sum_{k=1}^{K} M_i^{c,k} \cdot m_{i,j}^{c,k} \quad \text{for } i = 1,\ldots,N \text{ and } j = 1,\ldots,T \tag{10}$$

4.2 Replacement Cost

Similar to the maintenance cost, when a component is replaced in period j, a constant replacement cost, R_i^c, that is equal to initial purchase price will add to total cost.

$$R_{i,j} = \sum_{c=1}^{c} R_i^c \cdot r_{i,j}^c \quad \text{for} \quad i = 1,\ldots,N \quad \text{and} \quad j = 1,\ldots,T \tag{11}$$

4.3 Fixed Cost

Imagine a state in which all components of a subsystem is maintained or replaced at the end of a period. It's evident that in this state the total system stops working. Since the considered system is a series system of N subsystems, by failing a subsystem the total system will stop working. In this order the system will contact to a cost related to next system setup. In this purpose, fixed cost Z has been considered in a period when all components of a subsystem is maintained or replaced.

$$\text{Fixed cost} = \sum_{j=1}^{T}\left[Z\left(1 - \prod_{i=1}^{N}\left(1 - \prod_{c=1}^{c}\left(r_{i,j}^c + \sum_{k=1}^{K} m_{i,j}^{c,k}\right)\right)\right)\right] \tag{12}$$

As is obvious in Eq. (12), when all components of a subsystem are maintained or replaced in a period, the expression $\prod_{c=1}^{c}(r_{i,j}^c + \sum_{k=1}^{K} m_{i,j}^{c,k})$ becomes equal to one and the multiply operation on subsystems becomes 0. Then the fixed cost Z will add to total cost.

Considering this cost has two benefits. First, this cost prevents system stops. Second, when a system stops working at the end of a period, other subsystem's components can be maintained or replaced also without any changes in fixed cost. Actually, considering this cost helps centralization in maintenance and replacement activities.

So far it has been found that this model is a mixed integer nonlinear programming model (MINLP). For solving this model using meta-heuristic algorithms is undeniable. So because of wide using of genetic algorithm in preventive maintenance and replacement scheduling, an edition of this algorithm is used which is suitable for multi-objective models. This algorithm is non-dominated sorting genetic algorithm (NSGA-II). This algorithm will be checked in the following.

Up to now the problem is defined, the system is analyzed and primary assumptions are set. Now for system modeling three different system approaches for standby systems with different fundamental assumptions are considered.

- Preventive maintenance and replacement scheduling with failure impossibility assumption
- Preventive maintenance and replacement scheduling with failure possibility assumption

  Non-optional switching
  Optional switching

5 System Modeling

In this section the system that is defined in the second section and is shown in Fig. 1, will be modeled according to above assumptions.

5.1 *Preventive Maintenance and Replacement Scheduling with Failure Impossibility Assumption*

In this model there is no possibility for components failure during the periods. It means that when a component is loaded at the start of a period must continue its working to the end of that period without any failure. In Fig. 4, the system function is illustrated. As shown in this figure the switching operations perform at the end of the periods not during them. Actually in this approach switching operations perform to maintain or replace the components that are not under load.

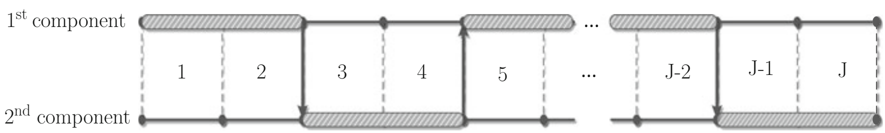


Fig. 4 Schematic view of a subsystem operation in preventive maintenance and replacement scheduling with failure impossibility assumption

5.1.1 System Configuration

It's obvious that if a component is under load in a period, its effective age will increase in amount of *T*/*J* but if the component wouldn't be under load in a period, no changes will happen to its age, because each subsystem is cold standby. In the other words, in each period if a component be under load its age increases otherwise no changes will happen to the component age on that period. So for calculating the age of components a binary variable $l_{i,j}^{c}$ is defined which will be equal to one if component *c* of subsystem *i* is under load at the start of period *j*. So:

$$X'^{c}_{i,j} = X^{c}_{i,j} + l^{c}_{i,j} \cdot T/J \quad \text{for } i = 1, \ldots, N; \; j = 1, \ldots, T; \; c = 1, \ldots, C \tag{13}$$

Another important point is that, in each period and in each subsystem, just one component must be under load. Therefore:

$$\sum_{c=1}^{C} l^{c}_{i,j} = 1 \quad \text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T \tag{14}$$

For simplicity it is assumed that at the start of first period, first component of each subsystem is under load.

$$l^{1}_{i,1} = 1 \quad \text{for } i = 1, \ldots, N \tag{15}$$

5.1.2 System Costs

Failure Cost

Unplanned failures impose a cost to the system. An important point is that at the start of first period, failures occurrence time is unknown. However, it's known that if the component has a high ROCOF in a period, the probability of failure occurrence is more, so higher cost must be considered and vice versa when ROCOF in a period is low, less failure cost will yield. Since in each period just one component of each subsystem is under load and switching operation can be done just at the end of period, so failure of loaded component causes system failure. For this reason, the expected number of failures in each period for each subsystem will be calculated. In this chapter J.S. Usher et al. methodology is used where average of failure rate with a fixed cost has been used (Usher et al. 1998). The expected number of failures in component *c* of subsystem *i* in period *j* can be calculated as below:

$$E\left(N_{i,j}\right) = \sum_{c=1}^{C} l^{c}_{i,j} \cdot \int_{X^{c}_{i,j}}^{X'^{c}_{i,j}} \vartheta^{c}_{i}(t)dt \quad \text{for } i = 1, \ldots, N; j = 1, \ldots, T \tag{16}$$

According to Sect. 2 and by the assumption NHPP for rate of occurrence of failure, expected number of failures in component c of subsystem i in period j will be:

$$E\left(N_{i,j}\right) = \sum_{c=1}^{C} l_{i,j}^{c} \cdot \int_{X_{i,j}^{c}}^{X_{i,j}^{\prime c}} \lambda_i^c \cdot \beta_i^c \cdot t^{\beta_i^c - 1} dt$$

$$= \sum_{c=1}^{C} l_{i,j}^{c} \cdot \left[\lambda_i^c \left(X_{i,j}^{\prime c}\right)^{\beta_i^c} - \lambda_i^c \left(X_{i,j}^{c}\right)^{\beta_i^c}\right]$$

$$\text{for } i = 1, \ldots, N; \quad j = 1, \ldots, T \tag{17}$$

It is assumed that cost of each failure is F_i (a dollar per failure occurrence) which allows the mode to calculate $F_{i,j}$, where $F_{i,j}$ is failure cost of subsystem i in period j.

$$F_{i,j} = F_i \cdot \sum_{c=1}^{C} l_{i,j}^{c} \cdot \left[\lambda_i^c \left(\left(X_{i,j}^{\prime c}\right)^{\beta_i^c} - \left(X_{i,j}^{c}\right)^{\beta_i^c}\right)\right]$$

$$\text{for } i = 1, \ldots, N; j = 1, \ldots, T \tag{18}$$

So regardless of maintenance or replacement action (that is assumed to occur at the end of period) in period j, there is a cost related to failures which may occur in each period.

Switching Cost

If a switching action occurs at the end of period j in subsystem i, then a cost equal to S_i would be imposed to the system. Considering this cost is essential to avoid unnecessary switching. Switching operation can be recognized from the changes in binary variable $l_{i,j}^{c}$. When $l_{i,j}^{c}$ value changes from one to zero or from zero to one means that a switching operation has occurred. So the expression $|l_{i,j+1}^{c} - l_{i,j}^{c}|$ will be equal to one if a switching operation occurs. This expression will be equal to one for two components, one the component that becomes loaded and the component that removes from loading state. So using $1/2S_i$ is inevitable.

$$S_{i,j} = \frac{1}{2} S_i \cdot \left| l_{i,j+1}^{c} - l_{i,j}^{c} \right| \quad \text{for } i = 1, \ldots, N; j = 1, \ldots, T \tag{19}$$

Total Cost

At the start of period $j = 1$, a set of maintenance, replacement, and do nothing actions for all components of each subsystem in each period must be specified to minimize the total cost. According to different costs definition, the total cost can be written as a simple sum on the costs.

$$\text{Total Cost} = \sum_{i=1}^{N}\sum_{j=1}^{T}\left[F_i \cdot \sum_{c=1}^{C} l_{i,j}^{c} \cdot \left[\lambda_i^c \left(\left(X'^{c}_{i,j}\right)^{\beta_i^c} - \left(X_{i,j}^{c}\right)^{\beta_i^c}\right)\right]\right.$$
$$\left. + \sum_{c=1}^{C}\left(\sum_{k=1}^{K}\left(M_i^{c,k} \cdot m_{i,j}^{c,k}\right) + R_i^c \cdot r_{i,j}^c + \frac{1}{2}S_i \cdot \left|l_{i,j+1}^c - l_{i,j}^c\right|\right)\right]$$
$$+ \sum_{j=1}^{T}\left[Z\left(1-\prod_{i=1}^{N}\left(1-\prod_{c=1}^{C}\left(r_{i,j}^c + \sum_{k=1}^{K} m_{i,j}^{c,k}\right)\right)\right)\right] \tag{20}$$

5.1.3 System Reliability

For considering the reliability objective function in this model, the reliability function of subsystem i in period j is defined as Eq. (20) and since the subsystems are series, total reliability could be calculated by multiplying all subsystems in all periods as shown in Eq. (21).

$$R_{i,j} = e^{-\left[\sum_{c=1}^{C} l_{i,j}^c \cdot \int_{X_{i,j}^c}^{X'^{c}_{i,j}} \vartheta_i^c(t)dt\right]} = e^{-\left[\sum_{c=1}^{C} l_{i,j}^c \cdot \left[\lambda_i^c\left(\left(X'^{c}_{i,j}\right)^{\beta_i^c} - \left(X_{i,j}^c\right)^{\beta_i^c}\right)\right]\right]}$$
$$\text{for } i = 1, \ldots N; j = 1, \ldots, T \tag{21}$$

$$\text{Reliability} = \prod_{i=1}^{N}\prod_{j=1}^{T} e^{-\left[\sum_{c=1}^{C} l_{i,j}^c \cdot \left[\lambda_i^c\left(\left(X'^{c}_{i,j}\right)^{\beta_i^c} - \left(X_{i,j}^c\right)^{\beta_i^c}\right)\right]\right]} \tag{22}$$

5.2 Preventive Maintenance and Replacement Scheduling with Failure Possibility Assumption

In this model two approaches can be supposed. In first approach, optional switching is not applicable and by a failure in the loaded component, switching operation performs. In second approach, optional switching is considered. It means that two kinds of switching are conceivable, non-optional switching and optional one according to failures and maintenance-replacement plan, respectively. These two approaches will be surveyed in details.

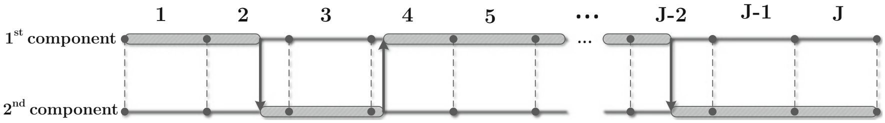


Fig. 5 Schematic view of a subsystem operation in preventive maintenance and replacement scheduling with failure possibility assumption and non-optional switching

5.2.1 Non-optional Switching

In this approach, when the loaded component breaks down, the switching operation performs and the other component of subsystem has to be loaded. In other words, the component that is under load at the start of a period wouldn't be under load at the end of that period necessarily. It is obvious that the time of failure occurrence is unknown. It means that we don't know when a failure occurs so this model deals to a probability which is related to the component failure time. For simplicity just two components are considered for each subsystem as shown in Fig. 5, the switching operations are performed just when a failure occurs. It's obvious that these failures can occur during a period and its time is unknown.

Components Effective Age

Because of this probability approach, the expected effective age of the components must be calculated. It means that in each period a fraction of each period length, T/J, will add to the component effective age. For controlling the model parameters and proper modeling, a variable is defined, $p_{i,j}^{c}$, which indicates the probability of component c of subsystem i loading status at the start of period j. It's clear that sum of these probabilities in each subsystem must be equal to one.

$$\sum_{i=1}^{n} p_{i,j}^{c} = 1 \quad \text{for} \quad i = 1, \ \ldots \ , N \quad \text{and} \quad j = 1, \ \ldots \ , T \tag{23}$$

By considering two components for each subsystem, Eq. (23) could be rewritten as below:

$$p_{i,j}^{1} + p_{i,j}^{2} = 1 \quad \text{for} \quad i = 1, \ \ldots \ , N \quad \text{and} \quad j = 2, \ \ldots \ , T \tag{24}$$

An assumption is considered that at the start of first period, the first component of each subsystem is under load. So:

$$p_{i,1}^{1} = 1 \quad \text{for} \quad i = 1, \ \ldots \ , N \tag{25}$$

$$p_{i,1}^{2} = 0 \quad \text{for} \quad i = 1, \ \ldots \ , N \tag{26}$$

Now, according to the above definitions, the expected effective age of each component at the end of each period is computable. By considering two components for each subsystem, the first component will be under load at the end of a period in two states. These two states are:

1. First component operates from start to end of a period without any failure.
2. Second component is loaded at the start of a period, but by a failure in that component the related subsystem switches to first component.

It's evident that reliability of a component shows the probability of its operation without any failure during a period and the complementary of that reliability expresses the failure probability of that component in a period. Now, the expected effective age of that component at the end of period j can be rewritten as a sum of expected effective age of that component at the start of period j and during that period. So the expected effective age of first component of each subsystem can be formulated as below:

$$X'^{1}_{i,j} = X^{1}_{i,j} + \left[p^{1}_{i,j} \cdot \text{Reliabilty}^{1}_{i,j} + p^{2}_{i,j} \cdot \left(1 - \text{Reliabilty}^{2}_{i,j}\right)\right] \cdot T/J$$
$$\text{for } i = 1, \ \ldots, \ N \ \text{ and } \ j = 1, \ldots \ , \ T \tag{27}$$

Similar to the first component two states of computing the expected effective age for second component of each subsystem are:

1. Second component operates from start to end of a period without any failure.
2. First component is loaded at the start of a period, but by a failure in that component the related subsystem switches to second component.

According to above states the expected effective of the second component of each subsystem can be rewritten as below:

$$X'^{2}_{i,j} = X^{2}_{i,j} + \left[p^{2}_{i,j} \cdot \text{Reliabilty}^{2}_{i,j} + p^{1}_{i,j} \cdot \left(1 - \text{Reliabilty}^{1}_{i,j}\right)\right] \cdot T/J$$
$$\text{for } i = 1, \ \ldots, \ N \ \text{ and } \ j = 1, \ldots \ , \ T \tag{28}$$

It should be noted that just one failure is considered for each subsystem in a period. It's evident that more than one failure in a period in each subsystem causes subsystem and whole system failure finally because two components is considered in each subsystem and maintenance and replacement activities occur at the end of periods.

One of the parameters that should be updated in each period is $p^{c}_{i,j}$. It's obvious that in the subsystems with two components, in two states a component will be under load at the start of period $j+1$, which are:

1. The component is under load at the start of period j and stays loaded during that period without any failure.
2. The other component is under load at the start of period j, but by a failure, a switching operation performs.

According to expressed points, the probability of first component loading status at the end of period $j + 1$ will be:

$$p^1_{i,j+1} = p^1_{i,j} \cdot \text{Reliabilty}^1_{i,j} + p^2_{i,j} \cdot \left(1 - \text{Reliabilty}^2_{i,j}\right)$$
$$\text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T - 1 \quad (29)$$

It is clear that because of being two components in each subsystem, and according to Eq. (24), $p^2_{i,j+1}$ is complementary of $p^1_{i,j+1}$. It means that:

$$p^2_{i,j+1} = 1 - p^1_{i,j+1}$$
$$\text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T - 1 \quad (30)$$

System Costs

Failure Cost

By looking at the foregoing operation periods, the unplanned components failures should be considered. As mentioned before two components are considered for each subsystem in which by a component failure the other one will be loaded. Since the maintenance and replacement operations perform just at the end of periods, if both components of a subsystem fail in a period, the related subsystem and the whole system will fail.

Two approaches can be considered for failure cost. First is calculating the expected failure cost. For this purpose, the subsystem failure probability that is complementary of subsystem reliability is multiplied to constant failure cost F_i.

$$F_{i,j} = F_i \cdot \left(1 - \text{Reliability}^{SS}_{i,j}\right)$$
$$\text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T \quad (31)$$

In the above equation, $\text{Reliability}^{SS}_{i,j}$ shows the reliability of subsystem i in period j. So, its complementary, $(1 - \text{Reliability}^{SS}_{i,j})$, expresses the failure probability of that subsystem.

Second approach is considering expected failure numbers in a subsystem. It's obvious that because of being just two components in each subsystem, at most one failure in each subsystem during a period is reasonable. So, when expected failure number is more than one, subsystem fails. At first the expected number of failures in a component should be calculated:

$$E\left(N^c_{i,j}\right) = \int_{X^c_{i,j}}^{X'^c_{i,j}} \vartheta^c_i(t)dt$$
$$\text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T \text{ and } c = 1, \ldots, C \quad (32)$$

$E(N_{i,j}^c)$ shows the expected number of failures in component c of subsystem i in period j. According to Eq. (1) and by the assumption NHPP for ROCOF, expected number of failures in component c of subsystem i in period j will be:

$$E\left(N_{i,j}^c\right) = \int_{X_{i,j}^c}^{X'^c_{i,j}} \lambda_i^c \cdot \beta_i^c \cdot t^{\beta_i^c - 1} dt = \left[\lambda_i^c \left(X'^c_{i,j}\right)^{\beta_i^c} - \lambda_i^c \left(X_{i,j}^c\right)^{\beta_i^c}\right]$$
$$\text{for} \quad i = 1, \ \ldots, \ N \quad \text{and} \quad j = 1, \ldots, \ T \quad \text{and} \quad c = 1, \ \ldots, C \tag{33}$$

According to component c loading probability, the expected failure numbers of subsystem i will be:

$$E\left(N_{i,j}\right) = \sum_{c=1}^{C} p_{i,j}^c \cdot E\left(N_{i,j}^c\right) = \sum_{c=1}^{C} p_{i,j}^c \cdot \left[\lambda_i^c \left(X'^c_{i,j}\right)^{\beta_i^c} - \lambda_i^c \left(X_{i,j}^c\right)^{\beta_i^c}\right]$$
$$\text{for}\ i = 1, \ \ldots, N \quad \text{and} \quad j = 1, \ \ldots, T \tag{34}$$

As expressed before just one failure in each subsystem is acceptable. So, if the expected failure number is greater than one, failure cost exists. So failure cost of subsystem i in period j will be:

$$F_{i,j} = F_i \cdot \max\left\{0, E\left(N_{i,j}\right) - 1\right\}$$
$$\text{for} \quad i = 1, \ \ldots, N \quad \text{and} \quad j = 1, \ \ldots, T \tag{35}$$

Total Cost

At the start of period $j = 1$, a set of maintenance and replacement activities should be specified for each component in following periods which minimizes the total cost. According to different costs definitions, the total cost function can be rewritten as below:

$$\begin{aligned}\text{Total Cost} = \sum_{i=1}^{N}\sum_{j=1}^{T}\Bigg[F_i \cdot \left(1 - \text{Reliabilty}_{i,j}^{SS}\right) \\ + \sum_{c=1}^{C}\left(\sum_{k=1}^{K}\left(M_i^{c,k} \cdot m_{i,j}^{c,k}\right) + R_i^c \cdot r_{i,j}^c\right)\Bigg] \\ + \sum_{j=1}^{T}\left[Z\left(1 - \prod_{i=1}^{N}\left(1 - \prod_{c=1}^{C}\left(r_{i,j}^c + \sum_{k=1}^{K} m_{i,j}^{c,k}\right)\right)\right)\right]\end{aligned} \tag{36}$$

Above function calculates the total cost as a sum of component costs in each period according to maintenance and replacement cost, down tie fixed cost and expected system failure cost.

System Reliability

In order to calculate the system reliability, at first the reliability of subsystem i in period j should be calculated. It's notable that each subsystem is a cold standby system. In other words, in this subsystems when a component is in standby mode no changes will happen in its age and failure rate. Charles O. Smith defined a general function for reliability of a standby system with two components (Smith 1976).

$$R_{SS} = R_1(t) + Q_1(t_1) \cdot R_2(t - t_1) \quad 0 \le t_1 \le t \tag{37}$$

Above expression shows that the reliability of a standby system is composed of two parts. First part, $R_1(t)$, is related to a case in which the first component is active from time 0 to t without any failure. But the second part, $Q_1(t_1) \cdot R_2(t - t_1)$, represents the state in which first component breaks down in time t_1 and the second component continues its operation from t_1 to t. In the following, each subsystem reliability will be surveyed in details.

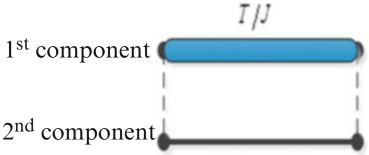
In the above equation, $R_c(t)$ and $Q_c(t)$ represent the reliability of component c from 0 to t and the failure probability of component C at time t, respectively. In a system with C components in each subsystem, $C!\sum_{r=0}^{C-1} \frac{1}{[(C-1)-r]!}$ states can occur for each subsystem in each period. So in this model by considering two components in each subsystem, four states can occur in each period. These states are:

State 1:
In this state, the first component is under load at the start of period j, and will work till the end of this period without any failure (Table 1).

State 2:
In this state, the first component cannot finish the period and because of a failure at time t in first component a switching operation performs. So, the second component will be loaded and the system continues its working from time t to T/J by second component (Table 2).

State 3:
This state is similar to first state, but the second component is under load at the start of period j (Table 3).

Table 1 First state in a standby subsystem



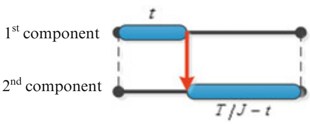
First component works during the period without any failure

$$\text{Reliability}_1^{SS} = Re^1_{i,j}\,(T/J) = e^{-\left[\int_{X^1_{i,j}}^{X^1_{i,j}+{}^{T}\!/_{J}} \vartheta_i^1(t)dt\right]}$$

$$= e^{-\left[\left(\lambda_i^1\left(X^1_{i,j}+T/J\right)^{\beta_i^1}-\left(X^1_{i,j}\right)^{\beta_i^1}\right)\right]}$$

$$\text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \quad (38)$$

Table 2 Second state in a standby subsystem



The first component is loaded at the start of the period and by its failure the second component works till the end of the period

$$\text{Reliability}_2^{SS} = Q^1_{i,j}(t)\cdot Re^2_{i,j}\left(T/J - t\right)$$

$$= \int_{t=0}^{t=T/J} \vartheta_i^1(t)\cdot e^{-\left[\int_{X^1_{i,j}}^{X^1_{i,j}+t} \vartheta_i^1(t)dt\right]}\cdot e^{-\left[\int_{X^2_{i,j}}^{X^2_{i,j}+T/J-t} \vartheta_i^2(t)dt\right]}\cdot dt$$

$$= \int_{t=0}^{t=T/J} \lambda_i^1\cdot\beta_i^1\cdot t^{\beta_i^1-1}\cdot e^{-\left[\lambda_i^1\left(\left(X^1_{i,j}+t\right)^{\beta_i^1}-\left(X^1_{i,j}\right)^{\beta_i^1}\right)\right]}$$

$$\cdot\, e^{-\left[\lambda_i^2\left(\left(X^2_{i,j}+{}^{T}\!/_{J}-t\right)^{\beta_i^2}-\left(X^2_{i,j}\right)^{\beta_i^2}\right)\right]}\cdot dt$$

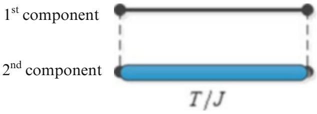
$$\text{for } i = 1,\ldots,N;\ j = 1,\ldots,T \quad (39)$$

State 4:
In this state, second component is loaded at the start of period j, but similar to second state, component failure causes a switching operation and the first component become loaded (Table 4).

Total Reliability

In this section, the above functions for total reliability computation will be combined. Since, at the start of each period, the components loading condition is

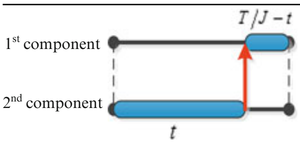
Table 3 Third state in a standby subsystem



Second component works during the period without any failure

$$\text{Reliability}_3^{SS} = Re_{i,j}^2\,(T/J) = e^{-\left[\int_{x_{i,j}^2}^{x_{i,j}^2+T/j} \vartheta_i^2(t)dt\right]}$$
$$= e^{-\left[\lambda_i^2\left(\left(x_{i,j}^2+T/J\right)^{\beta_i^2}-\left(x_{i,j}^2\right)^{\beta_i^2}\right)\right]}$$
$$\text{for } \; i = 1,\ldots,N;\, j = 1,\ldots,T \tag{40}$$

Table 4 Fourth state in a standby subsystem



The second component is loaded at the start of the period and by its failure the first component works till the end of the period

$$\text{Reliability}_4^{SS} = Q_{i,j}^2(t)\cdot Re_{i,j}^1\,(T/J-t)$$
$$= \int_{t=0}^{t=T/J} \vartheta_i^2(t)\cdot e^{-\left[\int_{x_{i,j}^2}^{x_{i,j}^2+t} \vartheta_i^1(t)dt\right]}\cdot e^{-\left[\int_{x_{i,j}^1}^{x_{i,j}^1+T/J-t} \vartheta_i^1(t)dt\right]}\cdot dt$$
$$= \int_{t=0}^{t=T/J} \lambda_i^2\cdot\beta_i^2\cdot t^{\beta_i^2-1}\cdot e^{-\left[\lambda_i^2\left(\left(x_{i,j}^2+t\right)^{\beta_i^2}-\left(x_{i,j}^2\right)^{\beta_i^2}\right)\right]}$$
$$\cdot\, e^{-\left[\lambda_i^1\left(\left(x_{i,j}^1+T/J-t\right)^{\beta_i^1}-\left(x_{i,j}^1\right)^{\beta_i^1}\right)\right]}\cdot dt$$
$$\text{for } \; i = 1,\ldots,N;\, j = 1,\ldots,T \tag{41}$$

unknown, calculating the expected reliability in each period for each subsystem is inevitable. So according to law of total probability, expected reliability will be:

$$\text{Reliability}_{i,j}^{SS} = p_{i,j}^1\cdot Re_{i,j}^{SS,1} + p_{i,j}^2\cdot Re_{i,j}^{SS,2}$$
$$= p_{i,j}^1\cdot\left(R_{i,j}^1\,(T/J) + Q_{i,j}^1\,(t)\cdot R_{i,j}^2\,(T/J-t)\right)$$
$$+\, p_{i,j}^2\cdot\left(R_{i,j}^2\,(T/J) + Q_{i,j}^2\,(t)\cdot R_{i,j}^1\,(T/J-t)\right)$$
$$\text{for } \; i = 1,\ldots,N;\, j = 1,\ldots,T \tag{42}$$

In this equation, $Re_{i,j}^{SS,1}$ illustrates ith subsystem reliability in which the first component is under load at the start of period j and its measure is equal to sum of first and second states reliability as follows:

$$\begin{aligned} Re_{i,j}^{SS,1} &= e^{-\left[\int_{x_{i,j}^1}^{x_{i,j}^1+T/J} \vartheta_i^1(t)dt\right]} \\ &\quad + \int_{t=0}^{t=T/J} \vartheta_i^1(t)\cdot e^{-\left[\int_{x_{i,j}^1}^{x_{i,j}^1+t} \vartheta_i^1(t)dt\right]}\cdot e^{-\left[\int_{x_{i,j}^2}^{x_{i,j}^2+T/J-t} \vartheta_i^2(t)dt\right]}\cdot dt \\ &= e^{-\left[\lambda_i^1\left(\left(x_{i,j}^1+T/J\right)^{\beta_i^1}-\left(x_{i,j}^1\right)^{\beta_i^1}\right)\right]} \\ &\quad + \int_{t=0}^{t=T/J} \lambda_i^1\cdot\beta_i^1\cdot t^{\beta_i^1-1}\cdot e^{-\left[\lambda_i^1\left(\left(x_{i,j}^1+t\right)^{\beta_i^1}-\left(x_{i,j}^1\right)^{\beta_i^1}\right)\right]} \\ &\quad \cdot e^{-\left[\lambda_i^2\left(\left(x_{i,j}^2+T/J-t\right)^{\beta_i^2}-\left(x_{i,j}^2\right)^{\beta_i^2}\right)\right]}\cdot dt \\ &\quad \text{for } i=1,\ldots,N;\ j=1,\ldots,T \end{aligned} \tag{43}$$

Similarly, $Re_{i,j}^{SS,2}$ is referred to the states that the second component is loaded at the start of period j. Sum of third and fourth states reliability equations will be:

$$\begin{aligned} Re_{i,j}^{SS,2} &= e^{-\left[\int_{x_{i,j}^2}^{x_{i,j}^2+T/J} \vartheta_i^2(t)dt\right]} \\ &\quad + \int_{t=0}^{t=T/J} \vartheta_i^2(t)\cdot e^{-\left[\int_{x_{i,j}^2}^{x_{i,j}^2+t} \vartheta_i^1(t)dt\right]}\cdot e^{-\left[\int_{x_{i,j}^1}^{x_{i,j}^1+T/J-t} \vartheta_i^1(t)dt\right]}\cdot dt \\ &= e^{-\left[\lambda_i^2\left(\left(x_{i,j}^2+T/J\right)^{\beta_i^2}-\left(x_{i,j}^2\right)^{\beta_i^2}\right)\right]} \\ &\quad + \int_{t=0}^{t=T/J} \lambda_i^2\cdot\beta_i^2\cdot t^{\beta_i^2-1}\cdot e^{-\left[\lambda_i^2\left(\left(x_{i,j}^2+t\right)^{\beta_i^2}-\left(x_{i,j}^2\right)^{\beta_i^2}\right)\right]} \\ &\quad \cdot e^{-\left[\lambda_i^1\left(\left(x_{i,j}^1+T/J-t\right)^{\beta_i^1}-\left(x_{i,j}^1\right)^{\beta_i^1}\right)\right]}\cdot dt \\ &\quad \text{for } i=1,\ldots,N;\ j=1,\ldots,T \end{aligned} \tag{44}$$

So, according to above descriptions, the expected total reliability for subsystem i in period j is equal to:

$$
\begin{aligned}
\text{Reliability}_{i,j}^{SS} &= p_{i,j}^{1}.Re_{i,j}^{SS,1} + p_{i,j}^{2}.Re_{i,j}^{SS,2} \\
&= p_{i,j}^{1}\cdot\Bigg[e^{-\left[\lambda_i^1\left(\left(x_{i,j}^1+T/J\right)^{\beta_i^1}-\left(x_{i,j}^1\right)^{\beta_i^1}\right)\right]} \\
&\quad + \int_{t=0}^{t=T/J} \lambda_i^1\cdot\beta_i^1\cdot t^{\beta_i^1-1}\cdot e^{-\left[\lambda_i^1\left(\left(x_{i,j}^1+t\right)^{\beta_i^1}-\left(x_{i,j}^1\right)^{\beta_i^1}\right)\right]} \\
&\quad \cdot e^{-\left[\lambda_i^2\left(\left(x_{i,j}^2+T/J-t\right)^{\beta_i^2}-\left(x_{i,j}^2\right)^{\beta_i^2}\right)\right]}\cdot dt \\
&\quad + p_{i,j}^{2}\cdot\Bigg[e^{-\left[\lambda_i^2\left(\left(x_{i,j}^2+T/J\right)^{\beta_i^2}-\left(x_{i,j}^2\right)^{\beta_i^2}\right)\right]} \\
&\quad + \int_{t=0}^{t=T/J} \lambda_i^2\cdot\beta_i^2\cdot t^{\beta_i^2-1}\cdot e^{-\left[\lambda_i^2\left(\left(x_{i,j}^2+t\right)^{\beta_i^2}-\left(x_{i,j}^2\right)^{\beta_i^2}\right)\right]} \\
&\quad \cdot e^{-\left[\lambda_i^1\left(\left(x_{i,j}^1+T/J-t\right)^{\beta_i^1}-\left(x_{i,j}^1\right)^{\beta_i^1}\right)\right]}\cdot dt \\
&\quad \text{for } \; i = 1,\ldots,N;\, j = 1,\ldots,T
\end{aligned} \tag{45}
$$

With regard to considering a series system, the total reliability will be computable by a simple multiply operation on N subsystems and J periods.

$$
\begin{aligned}
\text{Reliability} &= \prod_{i=1}^{N}\prod_{j=1}^{T}\text{Reliability}_{i,j}^{SS} \\
&= \prod_{i=1}^{N}\prod_{j=1}^{T}\Bigg(p_{i,j}^{1}\cdot\Bigg[e^{-\left[\lambda_i^1\left(\left(X_{i,j}^1+T/J\right)^{\beta_i^1}-\left(X_{i,j}^1\right)^{\beta_i^1}\right)\right]} \\
&\quad + \int_{t=0}^{t=T/J} \lambda_i^1\cdot\beta_i^1\cdot t^{\beta_i^1-1}\cdot e^{-\left[\lambda_i^1\left(\left(X_{i,j}^1+t\right)^{\beta_i^1}-\left(X_{i,j}^1\right)^{\beta_i^1}\right)\right]} \\
&\quad \cdot e^{-\left[\lambda_i^2\left(\left(X_{i,j}^2+T/J-t\right)^{\beta_i^2}-\left(x_{i,j}^2\right)^{\beta_i^2}\right)\right]}\cdot dt
\end{aligned}
$$

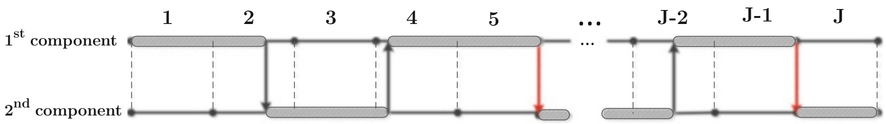


Fig. 6 Schematic view of a subsystem operation in third approach

$$+ p_{i,j}^2 \cdot \left[e^{-\left[\lambda_i^2\left(\left(X_{i,j}^2+T/J\right)^{\beta_i^2}-\left(X_{i,j}^2\right)^{\beta_i^2}\right)\right]} \right.$$
$$+ \int_{t=0}^{t=T/J} \lambda_i^2 \cdot \beta_i^2 \cdot t^{\beta_i^2-1} \cdot e^{-\left[\lambda_i^2\left(\left(x_{i,j}^2+t\right)^{\beta_i^2}-\left(x_{i,j}^2\right)^{\beta_i^2}\right)\right]}$$
$$\left. \cdot e^{-\left[\lambda_i^1\left(\left(x_{i,j}^1+T/J-t\right)^{\beta_i^1}-\left(x_{i,j}^1\right)^{\beta_i^1}\right)\right]} \cdot dt \right] \tag{46}$$

5.2.2 Optional Switching

In this section, optional switching possibility is considered to previous model. It means that two types of switching can perform in this system, optional and non-optional. When a failure occurs non-optional switching performs but, at the end of some periods, optional switching performs to maintain or replace the component that is under load. In Fig. 6, these two types of switching are illustrated.

In Fig. 6, black arrows represent non-optional switching and red arrows show optional switching. By assuming optional switching, it is assumed that by maintaining or replacing a component at the end of period j that component cannot be loaded at the start of period $j+1$. By this assumption, when $\sum_{k=1}^{K} m_{i,j-1}^{c,k} + r_{i,j-1}^{c}$ is equal to one, $p_{i,j}^{c}$ become 0. So:

$$p_{i,j}^c \cdot \left(\sum_{k=1}^{K} m_{i,j-1}^{c,k} + r_{i,j-1}^{c} \right) = 0$$
$$\text{for } \ i = 1, \ldots, N;\ j = 2, \ldots, T \ \text{ and } \ c = 1, 2 \tag{47}$$

Another assumption is that at the start of each period at least one component in each subsystem must be ready to become loaded. In other words, at the end of each period there is at least one component that will not maintain or replace. So:

$$\sum_{c=1}^{c}\left(\sum_{k=1}^{K} m_{i,j}^{c,k} + r_{i,j}^{c}\right) \leq 1$$
$$\text{for } i = 1,\ldots,N;\ j = 1,\ldots,T-1 \text{ and } c = 1,2 \quad (48)$$

By this assumption fixed cost become equal to 0. Because at the end of a period there is no possibility to perform maintenance or replacement activities on all components of a subsystem and at least one component in each subsystem works. So, the total system never stops working for maintenance or replacement.

6 Solution Approach

NSGA-II is one of the most useful and powerful existing algorithms for solving various multi-objective optimization problems that its performance has been proved (Deb et al. 2000). This algorithm combines GA algorithm with dominancy concept to form Pareto front. Deb and his colleague designed first version of this algorithm in 1995 and completed it in 2000 (Srinivas and Deb 1995; Deb et al. 2000). These algorithm parameters are defined as follow:

- Crossover procedure
  In this research, according to specific nature of this problem, two new methods are considered. These two crossover procedures are:

  1. Reverse three points crossover: In this method, at first three elements of $N \times C \times T$ matrix will be selected randomly. By this selection the parent chromosomes will divide into two parts. The procedure of children chromosomes construction is that the arrays out of these three points will select from the first parent and the other arrays will copy from the second parent reversely. By this method, if the parents have been selected similarly, the children will create differently.
  2. NCT points crossover: In this type of crossover, even genes will select from the first parent and the odd genes from the second one.

  So, if the selected solutions be similar, the algorithm uses the reverse three points crossover, otherwise the NCT points crossover will be used.

- Mutation procedure
  It's clear that mutation operator changes the coding design of chromosomes for diversity in solutions. According to the problem construction, if a component is maintained or replaced at the end of a period, the total system contacts to a cost. So a specific mutation procedure is defined for this problem. In this procedure, a number will be generated between one and $N \times C \times T$ randomly. Then if this is a non-zero number, the algorithm changes it to zero and reversely if it is equal to zero changes to a number belongs to {1,2,3,4} randomly.

7 Numerical Example

In this section, three mentioned optimization models will be solved with a set of data (considered in Table 5) to be compared in application area and examining their strength and weaknesses. In addition to considered data set, for each subsystem three similar components is assumed and 24 month is defined for planning horizon. Fixed cost will be 800$. In Table 5 α^1, α^2, and α^3 show three levels of maintenance from overhaul to elementary maintenance and M^1, M^2, and M^3 represent the costs associated to maintenance levels. It's notable that MATLAB R2013a environment is used for model solving.

Table 5 Numerical example parameters

Subsystem	λ	β	α^1	α^2	α^3	Failure cost ($)	Maintenance cost ($)			Replacement Cost ($)	Switching Cost ($)
							M^1	M^2	M^3		
1	0.00022	2.20	0.27	0.62	0.86	250	47	35	22	200	150
2	0.00035	2.00	0.31	0.58	0.82	240	43	32	24	210	142
3	0.00038	2.05	0.22	0.55	0.91	270	78	65	48	245	167
4	0.00034	1.90	0.24	0.50	0.78	210	59	42	28	180	112
5	0.00032	1.75	0.29	0.48	0.83	220	67	50	37	205	187
6	0.00028	2.10	0.33	0.65	0.89	280	51	38	23	235	145
7	0.00015	2.25	0.37	0.75	0.93	200	58	45	31	175	138
8	0.00012	1.80	0.37	0.68	0.90	225	43	30	21	215	129
9	0.00025	1.85	0.26	0.52	0.85	215	56	48	40	210	162
10	0.00020	2.15	0.38	0.67	0.84	255	69	55	43	250	178

In this section, these three models will be checked deeply and compared with each other. In many literature, optimization model is designed as a single objective model by using one limitation formulation, like, Moghaddam and Usher (2011) recent research in which two limitation models have been presented, where the first model minimizes the total cost for a given system reliability and the second one maximizes the total reliability with a budgetary constraint. Of course they completed their research by considering a multi-objective model for a system of several components that are connected in series configuration (Moghaddam and Usher 2011). This chapter presented models are also multi-objective models which are maximizing the total reliability by minimizing the total cost for a series system of several standby subsystems.

7.1 First Model

As surveyed before in this model, preventive maintenance and replacement scheduling with failure impossibility assumption is considered. Final optimization model is:

$$\begin{aligned}\text{Min Total Cost} = \sum_{i=1}^{N}\sum_{j=1}^{T}\Bigg[& F_i \cdot \sum_{c=1}^{C} l_{i,j}^{c} \cdot \left[\lambda_i^c \left(\left(X_{i,j}^{\prime c}\right)^{\beta_i^c} - \left(X_{i,j}^{c}\right)^{\beta_i^c}\right)\right] \\ & + \sum_{c=1}^{C}\left(\sum_{k=1}^{K}\left(M_i^{c,k} \cdot m_{i,j}^{c,k}\right) + R_i^c \cdot r_{i,j}^{c} + \frac{1}{2} S_i \cdot \left| l_{i,j+1}^{c} - l_{i,j}^{c} \right|\right)\Bigg] \\ & + \sum_{j=1}^{T}\left[z\left(1 - \prod_{i=1}^{N}\left(1 - \prod_{c=1}^{C}\left(r_{i,j}^{c} + \sum_{k=1}^{K} m_{i,j}^{c,k}\right)\right)\right)\right]\end{aligned}$$

$$\text{Max Reliability} = \prod_{i=1}^{N}\prod_{j=1}^{T} e^{-\left[\sum_{c=1}^{C} l_{i,j}^{c} \cdot \left[\lambda_i^c \left(\left(X_{i,j}^{\prime c}\right)^{\beta_1^c} - \left(X_{i,j}^{c}\right)^{\beta_1^c}\right)\right]\right]}$$

Subject to:

$$X_{i,1}^{c} = 0 \text{ for } i = 1, \ldots N \text{ and } c = 1, \ldots, C$$

$$\begin{aligned} X_{i,j}^{c} = & \left(1 - r_{i,j-1}^{c}\right)\left[\prod_{k=1}^{K}\left(1 - m_{i,j-1}^{c,k}\right)\right] X_{i,j-1}^{\prime c} \\ & + \sum_{k=1}^{K} m_{i,j-1}^{c,k} \cdot \left(\alpha_i^k \cdot X_{i,j-1}^{\prime c}\right) \\ & \text{for } i = 2, \ldots, N;\ j = 1, \ldots, T \text{ and } c = 1, \ldots, C \end{aligned}$$

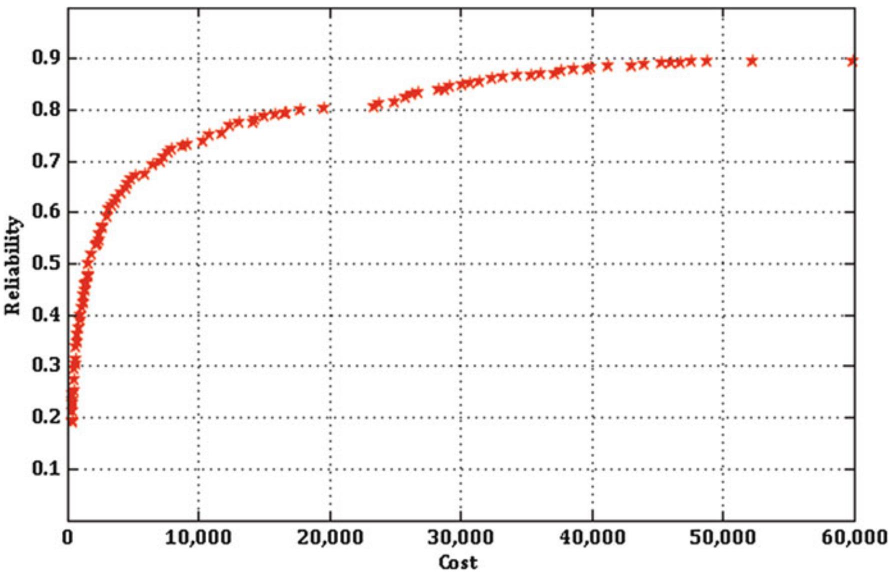


Fig. 7 First model Parto front

$$X'^{c}_{i,j} = X^{c}_{i,j} + l^{c}_{i,j} \cdot T/J \text{ for } i = 1, \ldots, N; j = 1, \ldots, T \text{ and } c = 1, \ldots, C$$

$$\sum_{k=1}^{K} m^{c,k}_{i,j} + r^{c}_{i,j} \leq 1 \text{ for } i = 1, \ldots, N; j = 1, \ldots, T \text{ and } c = 1, \ldots, C$$

$$\sum_{c=1}^{C} l^{c}_{i,j} = 1 \text{ for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T$$

$$l^{c}_{i,j}, m^{c,k}_{i,j}, r^{c}_{i,j}, = 0 \; or \; 1$$
$$\text{for } i = 1, \ldots, N; j = 1, \ldots, T; c = 1, \ldots, C \text{ and } k = 1, \ldots, K$$

$$X'^{c}_{i,j}, X^{c}_{i,j} \geq 0 \text{ for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T \quad (49)$$

The proposed model solved by the considered parameters. The Pareto front related to this problem is shown in Fig. 7. An important point is that all the points in this front provide an approximation of the optimal values and none of them dominate the others. It means that in different strategies different policies can be chosen. In other words the primary points of Pareto front by low costs have less reliability level and reversely by increasing the costs, the reliability levels are also increasing. So according to designers and managers strategies different policies can be implemented.

As is obvious in the components effective age diagrams, effective age of a component can increase, decrease or have no changes during the planning horizon. Increasing in effective age diagram of a component expresses that the component is loaded and operated during the period. No changes in a component effective age shows that the component wouldn't be under load in that period and when an instantaneous drop or decrease occurs in the diagram illustrates the maintenance and replacement activities happened on the component that its measure depends on the maintenance level.

It's notable that in the maintenance and replacement schedule tables, M1 shows overhaul maintenance, M2, intermediate maintenance, M3, primary maintenance, and *R* illustrates replacement activity on a component.

An obvious point about components effective age diagrams is that, in low reliability levels, system prefers to maintain or replace the first component and the second component isn't active so much. But by increases in reliability levels and costs, the second component starts to working (Figs. 8, 9, 10, 11, and 12; Tables 6, 7, 8, and 9).

7.2 Second Model

This model prepared with failure possibility assumption but non-optional switching (Fig. 13). Total mathematical model, its reliability, costs, and components effective age diagrams are shown in the following:

$$\begin{aligned}\text{Min Total Cost} = {} & \sum_{i=1}^{N}\sum_{j=1}^{T}\Bigg[F_i \cdot \left(1 - \text{Reliability}_{i,j}^{SS}\right) \\ & + \sum_{c=1}^{C}\left(\sum_{k=1}^{K}\left(M_i^{c,k} \cdot m_{i,j}^{c,k}\right) + R_i^c \cdot r_{i,j}^c\right)\Bigg] \\ & + \sum_{i=1}^{T}\left[z\left(1 - \prod_{i=1}^{N}\left(1 - \prod_{c=1}^{C}\left(r_{i,j}^c + \sum_{k=1}^{K} m_{i,j}^{c,k}\right)\right)\right)\right]\end{aligned}$$

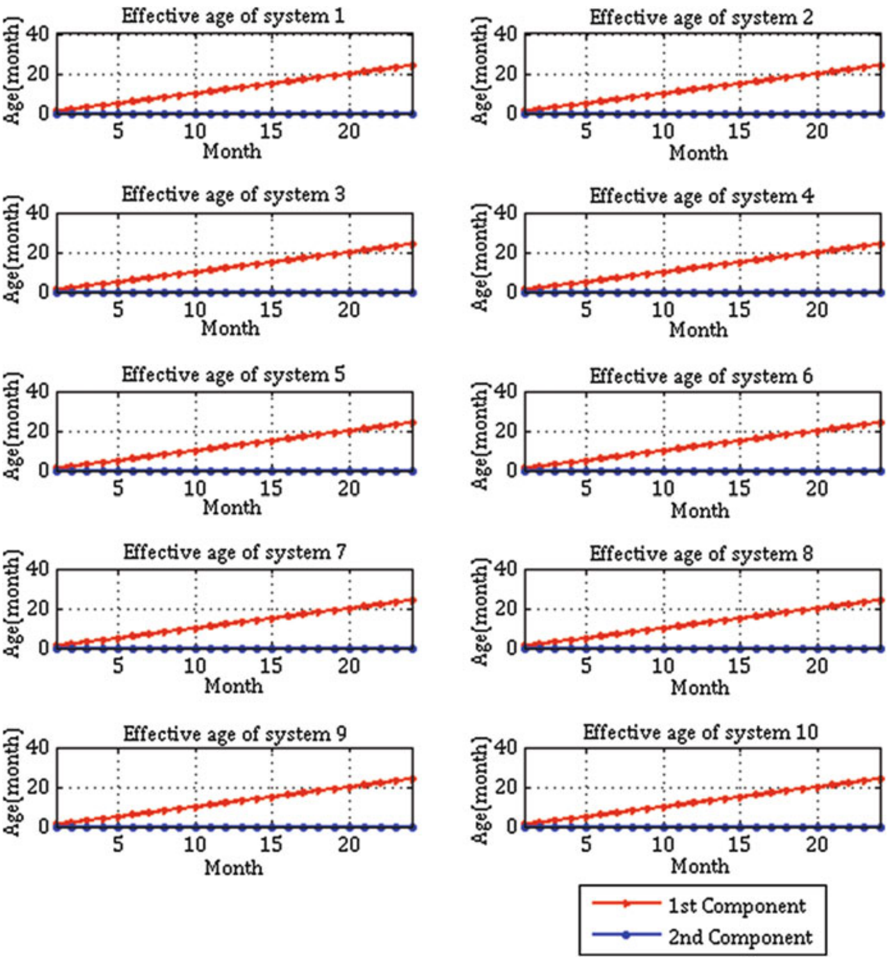


Fig. 8 Components effective age by $R = 19.25\ \%$ and $C = 401\$$

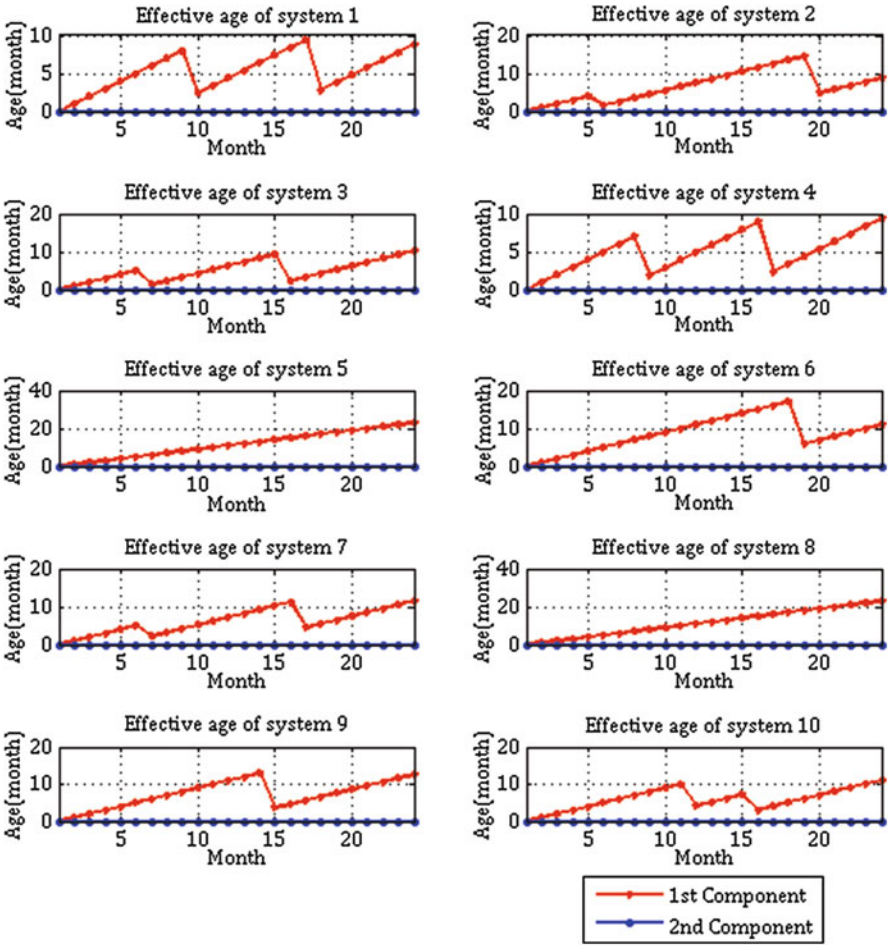


Fig. 9 Components effective age by $R = 39.99\ \%$ and $C = 1037.6\$$

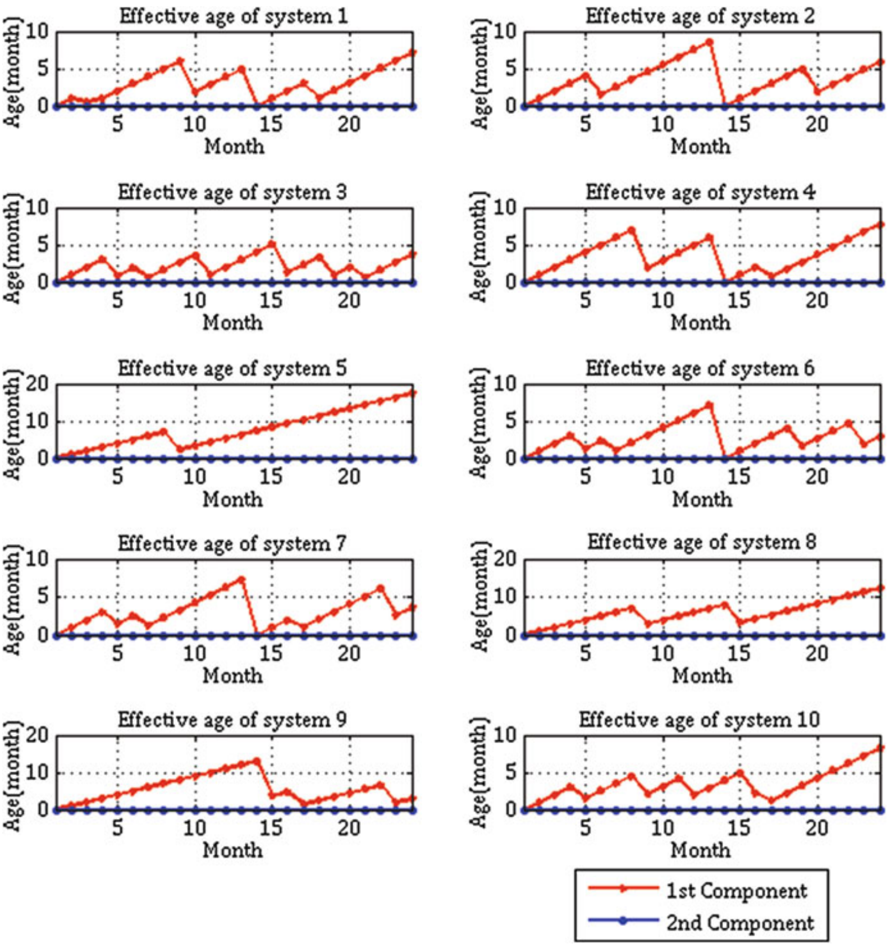


Fig. 10 Components effective age by $R = 60.55\ \%$ and $C = 3116.3\$$

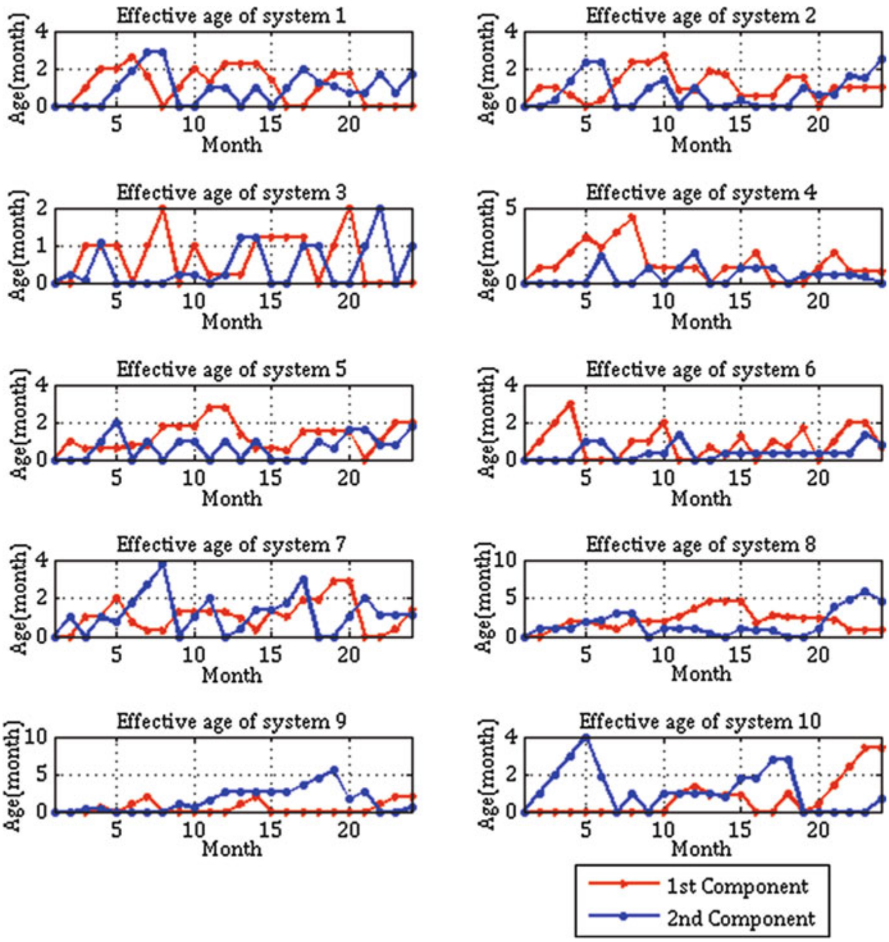


Fig. 11 Components effective age by $R = 85.16$ % and $C = 30700$\$

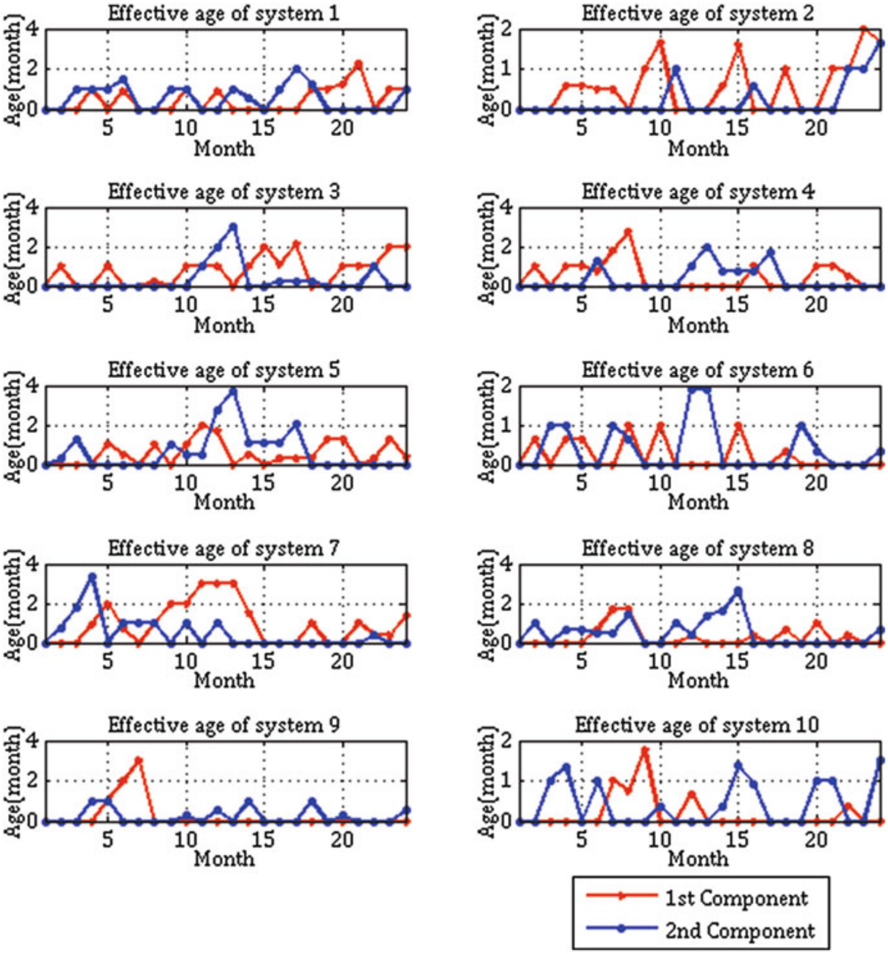


Fig. 12 Components effective age by $R = 89.49$ % and $C = 59930$$

Table 6 Maintenance and replacement schedule by $R = 39.99\ \%$ and $C = 1037.6\$$

Subsytem	1		2		3		4		5		6		7		8		9		10	
Component	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
Maintenance and replacement schedule																				
1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
2	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
3	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
4	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
5	--	--	M1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
6	--	--	--	--	M1	--	--	--	--	--	--	--	M1	--	--	--	--	--	--	--
7	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
8	--	--	--	--	--	--	M1	--	--	--	--	--	--	--	--	--	--	--	--	--
9	M1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
10	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
11	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	M1	--
12	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
13	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
14	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	M1	--	--	--
15	--	--	--	--	M1	--	--	--	--	--	--	--	--	--	--	--	--	--	M1	--
16	--	--	--	--	--	--	M1	--	--	--	--	--	M1	--	--	--	--	--	--	--
17	M1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
18	--	--	--	--	--	--	--	--	--	--	M1	--	--	--	--	--	--	--	--	--
19	--	--	M1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
20	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
21	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
22	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
23	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
24	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

Table 7 Maintenance and replacement schedule by $R = 60.55$ % and $C = 3116.3$$

Subsytem	Component	Maintenance and replacement schedule																							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	-	M1	M2	-	-	-	-	-	M1	-	-	-	R	-	-	-	M1	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	M1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	1	-	-	-	-	M1	-	-	-	-	-	-	-	R	-	-	-	-	-	M1	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	1	-	-	-	M1	-	M1	-	-	-	M1	-	-	-	-	M1	-	-	M1	-	M1	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	1	-	-	-	-	-	-	-	M1	-	-	-	-	R	-	-	M1	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	1	-	-	-	-	-	-	-	M1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	1	-	-	-	M1	-	M1	-	-	-	-	-	-	R	-	-	-	-	M1	-	-	-	M1	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	1	-	-	-	M1	-	M1	-	-	-	-	-	-	R	-	-	M1	-	-	-	-	-	M1	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	1	-	-	-	-	-	-	-	M1	-	-	-	-	-	M1	-	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	1	-	-	-	-	-	-	-	-	-	-	-	-	-	M1	-	M1	-	-	-	-	-	M1	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	1	-	-	-	M1	-	-	-	M1	-	-	M1	-	-	-	M1	M1	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 8 Maintenance and replacement schedule by $R = 85.16$ % and $C = 30700$$

Subsytem	Component	Maintenance and replacement schedule																							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	R	--	--	--	M3	M2	R	--	--	M2	--	--	--	M2	R	R	--	M3	M2	R	--	--	--	M3
	2	--	--	--	--	--	--	--	M3	--	--	--	M3	--	M3	--	--	M2	R	M2	--	--	M1	--	--
2	1	--	--	M2	R	M1	--	--	--	M3	M1	--	--	M2	M1	--	--	--	--	R	--	--	--	--	--
	2	--	M1	--	--	--	M3	--	--	M2	M3	--	R	--	M1	M3	R	M1	--	M2	--	--	M2	--	M3
3	1	--	--	--	--	R	--	--	R	--	M1	--	--	--	--	--	--	R	--	--	R	--	--	--	--
	2	M1	M1	--	M3	M3	--	--	M1	--	M3	M1	--	--	M3	M3	--	--	M3	M2	--	--	M3	--	M2
4	1	--	--	--	--	M3	--	--	M1	M2	--	--	R	--	--	--	R	M1	--	--	--	M1	--	--	--
	2	M3	R	--	--	--	M3	--	--	M3	--	--	R	--	--	--	--	M3	M2	--	--	--	M1	M3	M1
5	1	--	M1	--	--	M2	--	--	--	--	--	--	M2	M2	--	M1	--	--	--	--	R	--	--	--	--
	2	M1	--	--	--	M3	--	M3	--	M2	R	--	M3	--	R	--	--	--	M1	--	--	M2	--	--	--
6	1	--	--	--	R	R	--	--	--	--	R	--	M2	M1	--	R	--	M1	--	R	--	--	--	M1	--
	2	--	--	--	--	--	R	M2	M1	--	--	M3	R	M1	--	--	--	--	--	--	--	--	--	M1	--
7	1	--	--	--	--	M1	M1	--	--	--	--	--	M2	M1	--	M2	M3	--	--	--	R	M1	M1	--	--
	2	--	R	--	M2	--	--	--	R	--	--	R	M1	--	--	M2	M2	M3	--	--	--	M1	--	--	--
8	1	--	--	--	--	M2	M2	--	--	--	M3	--	--	--	--	M1	--	M2	M2	--	M3	M1	--	--	M1
	2	--	--	--	--	M2	--	--	M3	--	M3	--	M1	M3	--	M1	--	R	M1	--	--	--	--	M2	M2
9	1	--	M1	M2	R	--	--	R	--	--	M2	--	--	--	R	R	--	M2	--	--	--	--	--	--	--
	2	M3	M1	--	M3	R	--	--	--	M1	--	--	--	--	--	--	--	--	--	M1	--	R	--	M2	R
10	1	--	--	--	M2	--	--	--	R	--	--	M2	M1	--	--	R	M1	--	R	M1	--	--	--	--	--
	2	--	--	--	--	M1	M3	--	R	--	--	--	--	M1	--	--	--	--	M3	--	M2	--	--	M2	--

Table 9 Maintenance and replacement schedule by $R = 89.49\ \%$ and $C = 59930\$$

Subsytem	Component	Maintenance and replacement schedule																							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	R	R	--	R	M3	R	R	R	--	R	M3	R	M2	R	R	--	--	--	M2	--	R	--	--	--
	2	M2	--	--	--	M2	R	--	--	--	R	M3	--	M1	R	--	--	M2	R	M2	--	R	R	--	R
2	1	R	R	M2	--	M1	--	R	--	M3	R	--	R	M2	--	R	R	--	R	R	--	--	--	M3	--
	2	--	--	--	R	--	R	--	--	M2	--	R	R	--	--	M2	R	M1	--	M2	R	--	--	M3	R
3	1	--	R	--	--	R	--	M1	R	--	--	--	R	--	--	M2	--	R	R	--	--	--	--	--	R
	2	M1	R	M3	R	M3	R	M2	R	R	--	--	--	M3	--	M1	--	--	R	M2	R	--	R	R	--
4	1	--	R	--	--	M3	--	--	R	M2	--	--	R	R	R	--	R	M1	R	--	--	M1	R	R	R
	2	M3	--	--	R	M2	R	--	R	M3	R	--	--	M1	--	--	--	M3	R	M1	R	--	R	M3	--
5	1	--	R	--	--	M2	R	--	R	--	--	M3	R	M2	R	M1	--	--	--	--	R	M1	--	M1	--
	2	M1	--	R	--	M3	--	M3	--	M2	--	--	--	M1	--	--	--	R	--	R	R	M2	--	M3	--
6	1	M2	R	M2	--	R	R	--	R	--	R	M3	R	M1	--	R	R	M1	R	R	R	R	R	M1	--
	2	M1	--	--	R	R	--	M2	R	--	R	--	--	R	R	R	R	--	--	M1	R	--	--	M1	--
7	1	--	R	M3	--	M1	R	--	--	--	--	--	--	M1	R	M2	R	--	R	R	--	M1	--	--	--
	2	M2	--	--	R	--	--	--	R	--	R	--	R	M2	R	M3	R	M3	R	M1	--	M1	R	M2	R
8	1	--	R	M2	R	M2	--	--	R	M1	R	M1	R	M2	--	M1	R	M2	R	--	R	M1	R	--	--
	2	--	R	M2	--	M2	--	--	R	R	--	M1	--	M2	--	M3	R	R	R	M3	R	M2	--	M2	R
9	1	--	--	M2	--	--	--	R	R	R	--	--	R	M2	--	R	R	M2	--	--	R	--	R	M1	--
	2	M3	R	--	--	R	R	R	R	M1	R	M2	R	--	R	R	R	--	R	M1	R	R	R	M2	--
10	1	M2	R	R	R	M2	--	M1	--	R	R	M2	R	M2	--	R	R	--	R	M1	R	M1	R	M3	--
	2	M3	--	M2	R	--	R	M1	R	M1	R	R	R	M1	--	M1	R	R	R	--	--	M3	R	M2	--

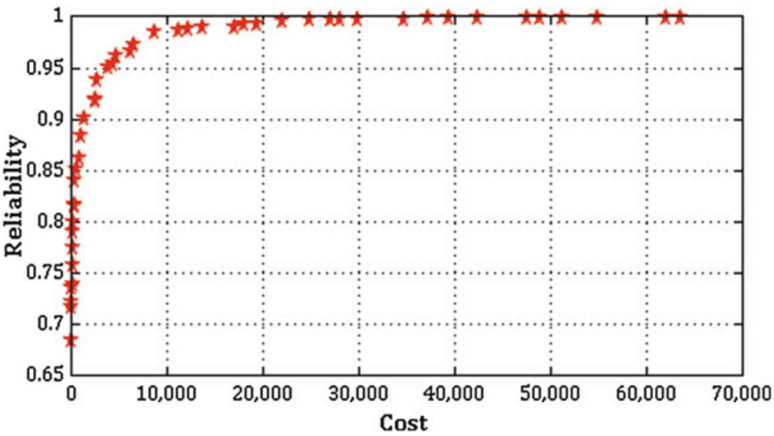


Fig. 13 Second model Parto front

$$\text{Max Reliability} = \prod_{i=1}^{N}\prod_{j=1}^{T}\Bigg(p_{i,j}^{1}\cdot\Bigg[e^{-\left[\lambda_i^1\left(\left(X_{i,j}^1+T/J\right)^{\beta_i^1}-\left(X_{i,j}^1\right)^{\beta_i^1}\right)\right]} + \int_{t=0}^{t=T/J}\lambda_i^1\cdot\beta_i^1\cdot t^{\beta_i^1-1}\cdot e^{-\left[\lambda_i^1\left(\left(X_{i,j}^1+t\right)^{\beta_i^1}-\left(X_{i,j}^1\right)^{\beta_i^1}\right)\right]} \cdot e^{-\left[\lambda_i^2\left(\left(X_{i,j}^2+T/J-t\right)^{\beta_i^2}-\left(X_{i,j}^2\right)^{\beta_i^2}\right)\right]}\cdot dt\Bigg] + p_{i,j}^{2}\cdot\Bigg[e^{-\left[\lambda_i^2\left(\left(X_{i,j}^2+T/J\right)^{\beta_i^2}-\left(X_{i,j}^2\right)^{\beta_i^2}\right)\right]} + \int_{t=0}^{t=T/J}\lambda_i^2\cdot\beta_i^2\cdot t^{\beta_i^2-1}\cdot e^{-\left[\lambda_i^2\left(\left(X_{i,j}^2+t\right)^{\beta_i^2}-\left(X_{i,j}^2\right)^{\beta_i^2}\right)\right]} \cdot e^{-\left[\lambda_i^1\left(\left(X_{i,j}^1+T/J-t\right)^{\beta_i^1}-\left(X_{i,j}^1\right)^{\beta_i^1}\right)\right]}\cdot dt\Bigg]\Bigg)$$

subject to:

$$X_{i,1}^{c} = 0 \text{ for } i = 1, \ldots N \text{ and } c = 1, 2$$

$$X_{i,j}^{c} = \left(1 - r_{i,j-1}^{c}\right)\left[\prod_{k=1}^{K}\left(1 - m_{i,j-1}^{c,k}\right)\right] X_{i,j-1}^{\prime c} + \sum_{k=1}^{K} m_{i,j-1}^{c,k} \cdot \left(\alpha_{i}^{k} \cdot X_{i,j-1}^{\prime c}\right)$$
$$\text{for } i = 1, \ldots, N;\ j = 2, \ldots, T \text{ and } c = 1, 2$$

$$X_{i,j}^{\prime 1} = X_{i,j}^{1} + \left[p_{i,j}^{1} \cdot \text{Reliability}_{i,j}^{1} + \left(1 - p_{i,j}^{1}\right)\left(1 - \text{Reliability}_{i,j}^{2}\right)\right].T/J$$
$$\text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T$$

$$X_{i,j}^{\prime 2} = X_{i,j}^{2} + \left[\left(1 - p_{i,j}^{1}\right) \cdot \text{Reliability}_{i,j}^{2} + p_{i,j}^{1} \cdot \left(1 - \text{Reliability}_{i,j}^{1}\right)\right] \cdot T/J$$
$$\text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T$$

$$p_{i,1}^{1} = 1 \text{ for } i = 1, \ldots, N$$

$$p_{i,1}^{2} = 0 \text{ for } i = 1, \ldots, N$$

$$p_{i,j+1}^{1} = p_{i,j}^{1} \cdot \text{Reliability}_{i,j}^{1} + \cdot \left(1 - p_{i,j}^{1}\right) \cdot \left(1 - \text{Reliability}_{i,j}^{2}\right)$$
$$\text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T - 1$$

$$p_{i,j}^{2} = 1 - p_{i,j}^{1} \text{ for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T$$

$$\sum_{k=1}^{K} m_{i,j}^{c,k} + r_{i,j}^{c} \leq 1 \text{ for } i = 1, \ldots, N;\ j = 1, \ldots, T \text{ and } c = 1, 2$$

$$m_{i,j}^{c,k}, r_{i,j}^{c} = 0 \text{ or } 1 \text{ for } i = 1, \ldots, N;\ j = 1, \ldots, T;\ c = 1, 2 \text{ and } k = 1, \ldots, K$$

$$0 \leq p_{i,j}^{1}, p_{i,j}^{2} \leq 1 \text{ for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T$$

$$X_{i,j}^{\prime c}, X_{i,j}^{c} \geq 0 \text{ for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T \tag{50}$$

This point is notable that second component effective age doesn't change any more and its reason that optional switching isn't possible in this model. Second component effective age changes obviously in the next model (Figs. 14, 15, 16, and 17; Tables 10, 11, and 12).

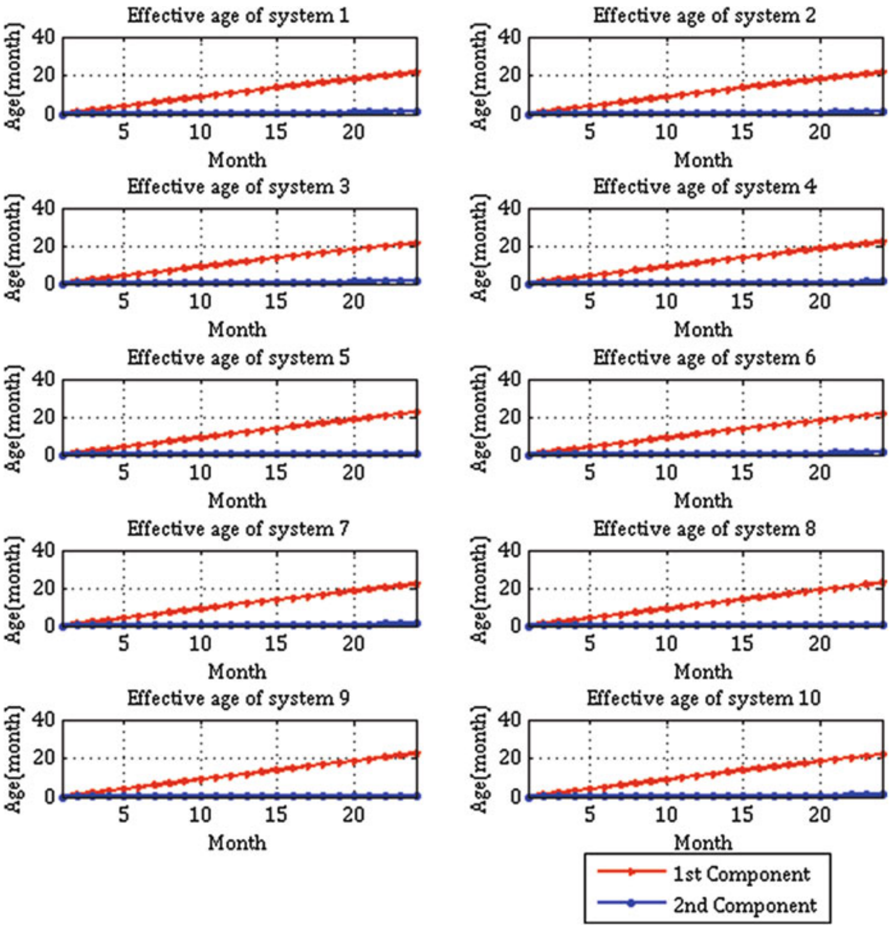


Fig. 14 Components effective age by $R = 68.48$ % and $C = 93\$$

7.3 *Third Model*

This model is designed for a case in which both failure and optional switching is possible (Figs. 18, 19, 20, 21, and 22; Tables 13, 14, and 15). This model formulation and its diagrams are shown as below:

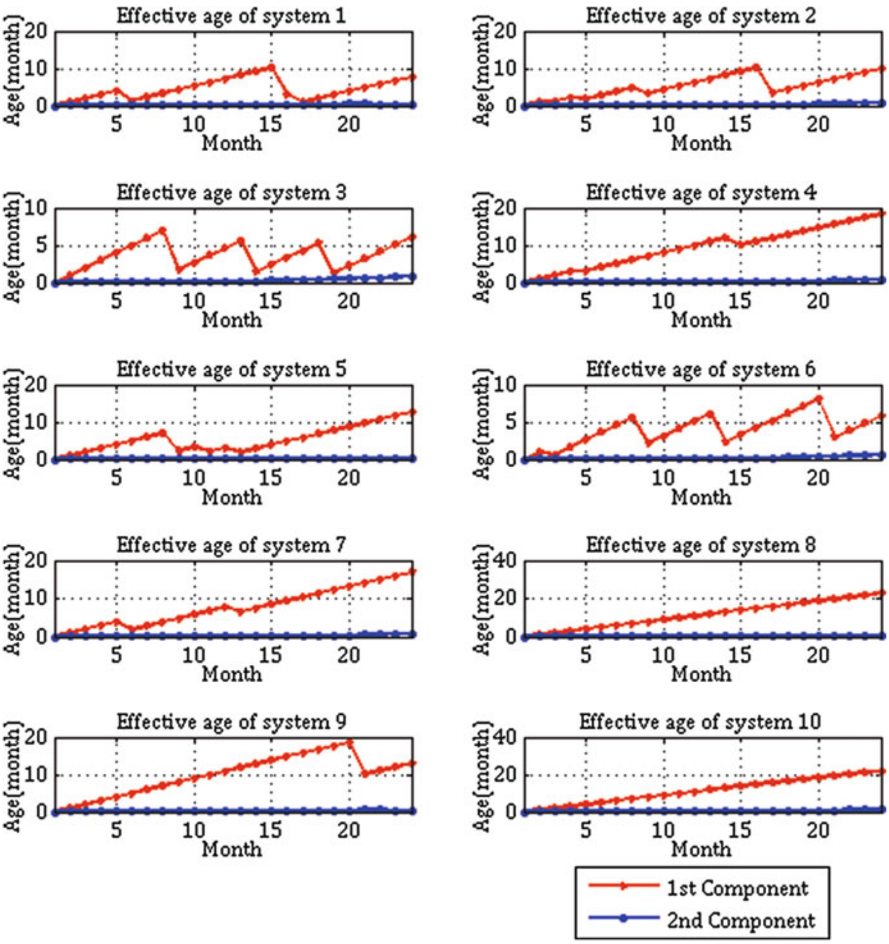


Fig. 15 Components effective age by $R = 90.11$ % and $C = 1481$\$

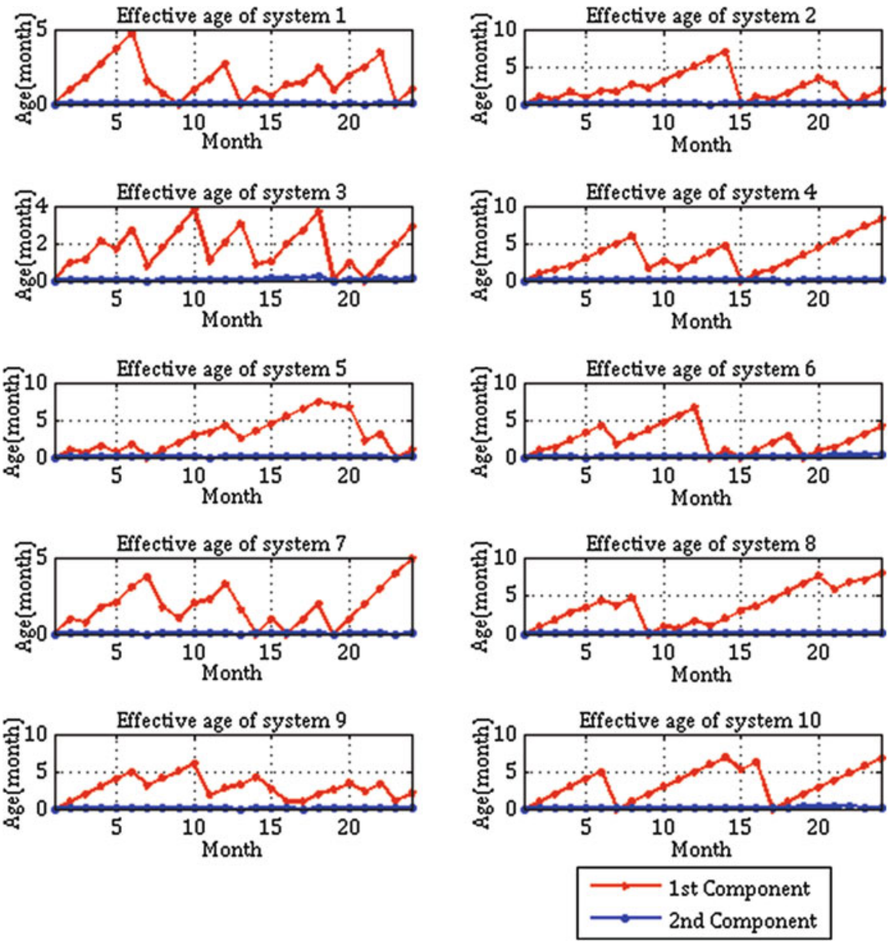


Fig. 16 Components effective age by $R = 99.37$ % and $C = 19431$\$

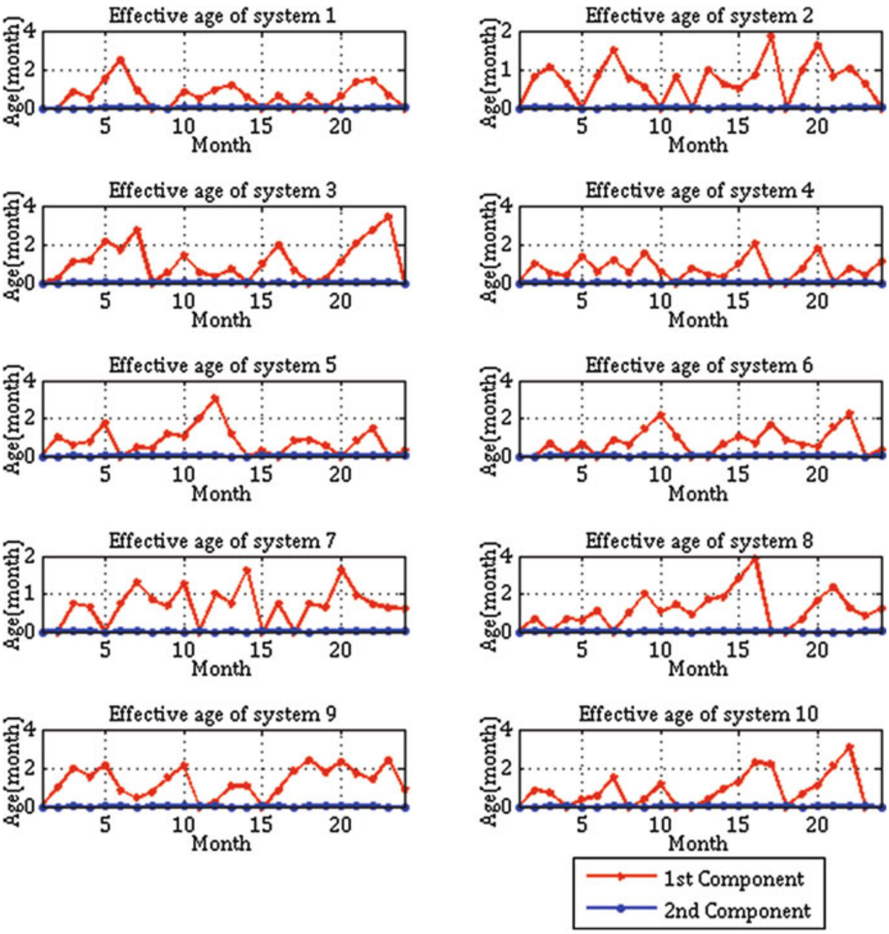


Fig. 17 Components effective age by $R = 99.95$ % and $C = 63583$\$

Table 10 Maintenance and replacement schedule by $R = 90.11\ \%$ and $C = 1481\$$

Subsystem	Component	Maintenance and replacement schedule																							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	-	-	-	-	M1	-	-	-	-	-	-	-	-	-	M1	M1	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	M1	-	-	-
2	1	-	M2	-	M2	-	-	-	M2	-	-	-	-	-	-	-	M1	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	1	-	-	-	-	-	-	-	M1	-	-	-	-	M1	-	-	-	-	M1	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	M2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	1	-	-	-	M3	-	-	-	-	-	-	-	-	-	M3	-	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	M1	-	-	-	-	-	-	-	M2	-	-	-	-	-	-	-	-	-	-
5	1	-	-	-	-	-	-	-	M1	-	M2	-	M2	-	-	-	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	1	-	M1	-	-	-	-	-	M1	-	-	-	-	M1	-	-	-	-	-	-	M1	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	M1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	1	-	-	-	-	M1	-	-	-	-	-	-	M2	-	-	-	-	-	-	-	-	-	-	-	-
	2	-	M2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	M2	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	M1	-	-
10	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 11 Maintenance and replacement schedule by $R = 99.37$ % and $C = 19431$\$

Subsystem	Component	Maintenance and replacement schedule																							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	--	M3	--	--	--	M1	M1	R	--	M3	--	R	--	M1	M3	M2	--	M1	--	M3	--	R	--	--
	2	--	--	--	--	--	--	--	M1	--	--	--	M3	--	--	--	M1	--	R	--	R	--	M1	--	M1
2	1	--	M1	--	M1	--	M2	--	M2	--	--	--	--	--	R	--	M1	--	--	--	M2	R	--	--	--
	2	--	M2	--	M3	--	M1	--	M1	--	--	--	R	--	--	M3	M1	M1	M3	M1	M1	--	M3	--	--
3	1	--	M2	--	M2	--	M1	--	--	--	M1	--	--	M1	M2	--	M3	--	R	--	R	--	--	--	--
	2	--	--	--	M1	--	R	--	--	--	M3	--	M2	--	--	--	--	--	R	--	--	--	M2	--	R
4	1	--	M3	M3	--	--	--	--	M1	--	M2	--	--	--	R	--	M3	--	--	--	--	--	--	--	--
	2	M3	M2	M3	--	--	M1	--	M1	--	--	--	M3	--	M2	--	M2	R	--	--	M2	--	--	--	--
5	1	--	M1	--	M1	--	R	--	--	--	M3	--	M2	--	--	--	--	--	M3	M3	M1	--	R	--	--
	2	--	M3	--	--	--	--	--	M2	--	R	--	M1	--	M3	--	M3	--	--	--	M3	--	R	--	--
6	1	--	M2	--	--	--	M1	--	--	--	--	--	R	--	R	--	--	--	R	--	M2	--	--	--	--
	2	--	M2	--	R	--	M2	--	M1	--	M2	--	--	--	M1	--	--	--	--	--	--	--	--	--	M2
7	1	--	M1	--	M2	--	M3	M1	M1	--	M2	--	M1	R	--	R	--	--	R	--	--	--	--	--	--
	2	--	--	--	--	--	R	--	M3	--	--	--	R	--	--	--	--	--	R	--	--	--	R	--	R
8	1	--	M3	--	M3	--	M2	--	R	--	M1	--	M1	--	--	M3	--	--	--	--	M2	--	M3	--	R
	2	--	--	--	M1	--	M1	--	M3	--	M3	--	M1	--	--	--	M2	--	M3	--	M2	--	--	--	--
9	1	--	--	--	--	--	M2	--	--	--	M1	--	M3	--	M2	M1	M2	--	M3	--	M2	--	M1	--	--
	2	--	M2	M3	--	--	--	--	M2	--	--	--	R	--	M3	--	R	--	--	--	--	--	--	--	M2
10	1	--	--	--	--	--	R	--	--	--	--	--	--	--	M2	--	R	--	--	--	--	--	--	--	M1
	2	--	--	--	M2	--	--	--	--	--	M2	--	--	--	M3	--	--	--	--	--	--	--	M1	--	--

Table 12 Maintenance and replacement schedule by $R = 99.95$ % and $C = 63583\$$

Subsystem	Component	Maintenance and replacement schedule																							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	R	M3	M1	--	--	M1	R	R	M3	M1	M2	M2	M1	R	M2	R	M2	R	M2	M3	M2	M1	R	M3
	2	R	R	R	M2	M1	M2	M3	R	M3	M1	M2	M2	M2	M3	M3	M3	--	--	R	R	M1	M1	--	M1
2	1	M3	M2	M1	R	M3	M3	M1	M1	R	M3	R	--	M1	M1	M2	--	R	--	M3	M1	M2	M1	R	R
	2	M2	M3	M3	R	R	--	M1	M1	M2	M3	M3	R	M3	--	M1	R	M1	--	R	R	M1	M1	M2	M1
3	1	M1	M3	M2	--	M2	--	R	M2	M3	M1	M1	M2	R	--	--	M1	R	M1	M3	--	M3	M3	R	M1
	2	R	--	M2	M3	--	M3	M1	M2	M2	M1	M3	M1	--	R	M1	R	M2	M1	M2	M3	M3	M3	R	R
4	1	--	M1	M1	--	M1	M3	M1	--	M1	R	M3	M1	M1	M3	--	R	R	M3	--	R	M3	M1	M3	R
	2	--	--	--	R	M3	M3	R	--	R	M2	M2	M2	M3	M1	--	--	--	--	--	M2	--	M2	R	M1
5	1	--	M1	M2	--	R	M2	M1	M3	M2	--	--	M1	R	M1	R	M3	M2	M1	R	M3	M3	R	M1	M3
	2	R	M2	R	M3	M2	M2	M2	--	M3	M1	M1	R	R	M1	M3	--	M2	--	--	M2	M2	--	M3	M2
6	1	R	M2	R	M2	R	M3	M1	M3	M3	M1	R	R	M2	M2	M1	--	M1	M1	M1	--	M3	R	M1	M1
	2	R	M2	M1	--	--	M2	R	M2	M1	R	M1	--	R	M3	--	--	--	--	--	M3	M3	R	M3	M2
7	1	R	M2	M1	R	M2	M2	M1	M1	M2	R	--	M1	M3	R	M2	R	M2	M1	--	M1	M1	M1	M1	--
	2	M2	M1	--	R	M2	--	R	M1	R	M2	R	M1	R	--	M2	M1	R	--	M3	M3	M2	M1	M2	M2
8	1	M2	R	M2	M1	M2	R	--	--	M1	M2	M1	M3	M2	--	--	R	R	M2	--	M3	M1	M1	M2	--
	2	M1	M1	M2	M3	--	--	R	M3	--	M2	R	--	M1	--	M1	M3	M2	R	R	R	R	M1	--	M1
9	1	--	--	M2	M3	M1	M1	M2	M3	M3	R	M1	M3	M2	R	M3	--	M3	M2	M3	M2	M2	--	M1	M2
	2	R	M3	R	M1	M2	R	M3	--	M2	--	M3	R	R	M3	M1	R	M1	--	M3	M1	M3	R	R	M2
10	1	M3	M1	R	M1	M1	--	R	M1	M3	R	R	M1	M2	M2	--	M2	R	M2	M2	--	--	R	R	M2
	2	R	--	M1	R	R	M2	M3	R	M2	M2	M2	--	--	--	--	M2	--	R	--	M1	--	--	R	M2

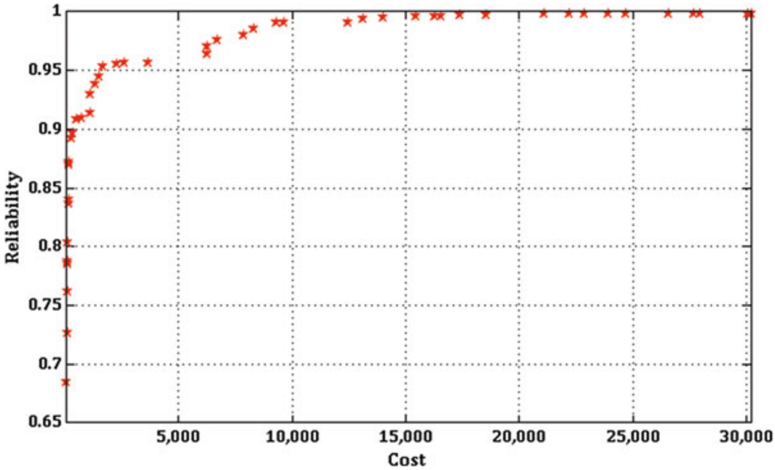


Fig. 18 Third model Parto front

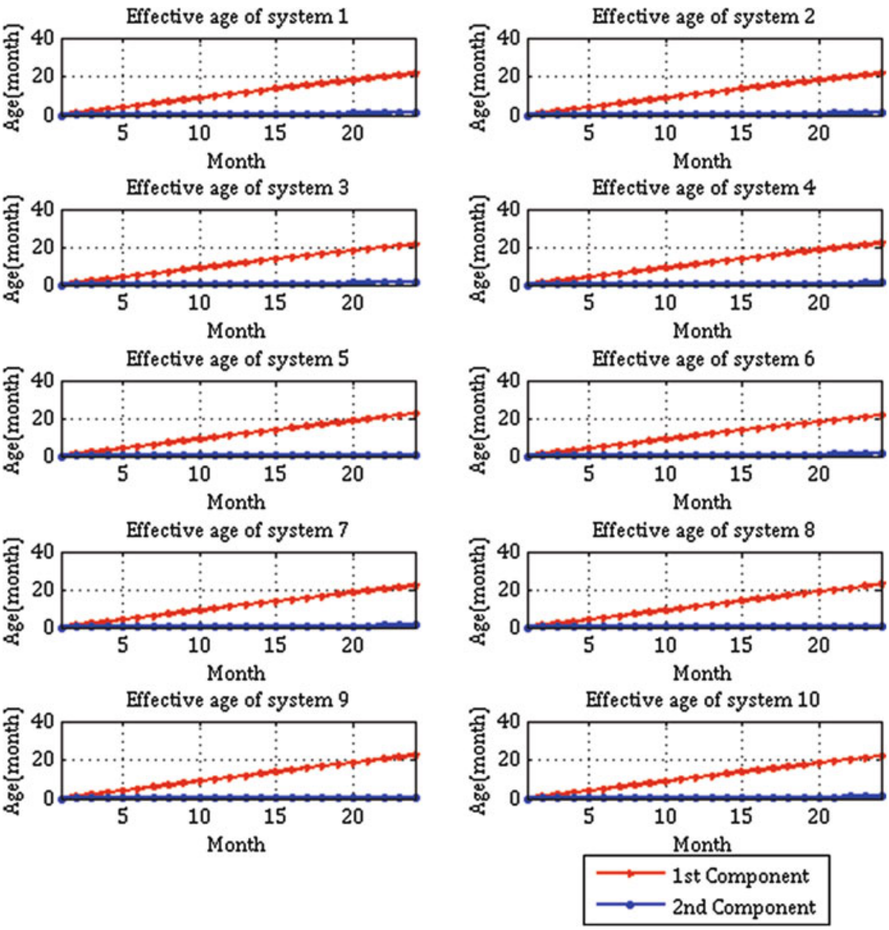


Fig. 19 Components effective age by $R = 68.48$ % and $C = 93\$$

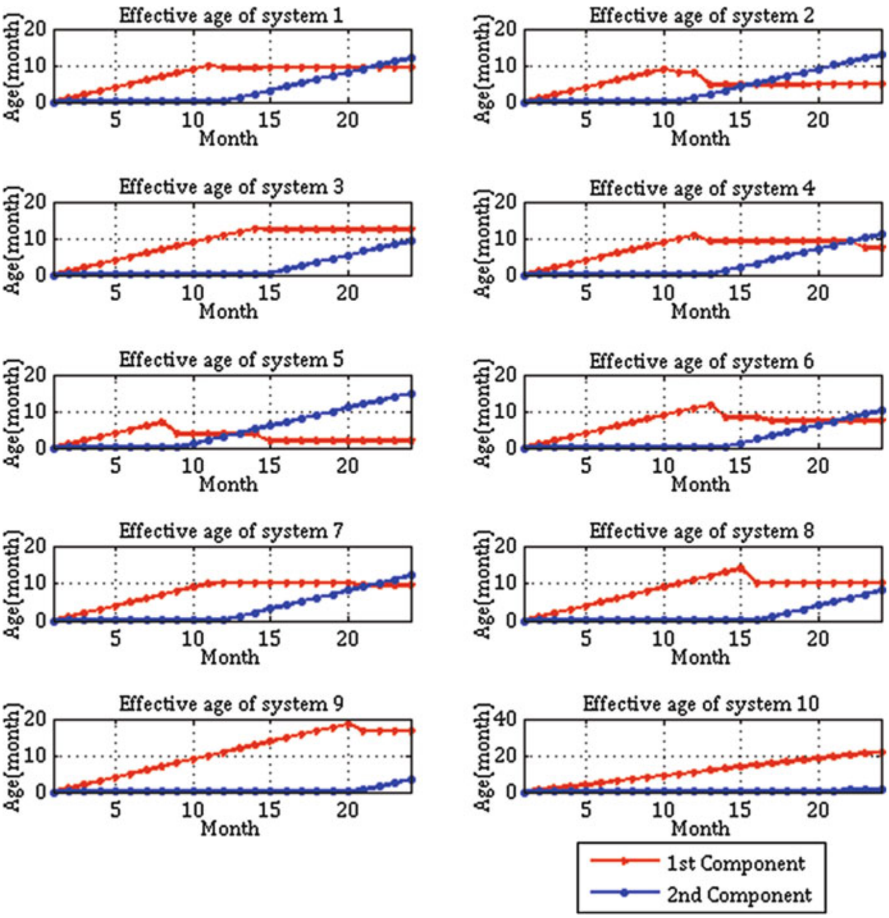


Fig. 20 Components effective age by $R = 90.81$ % and $C = 499\$$

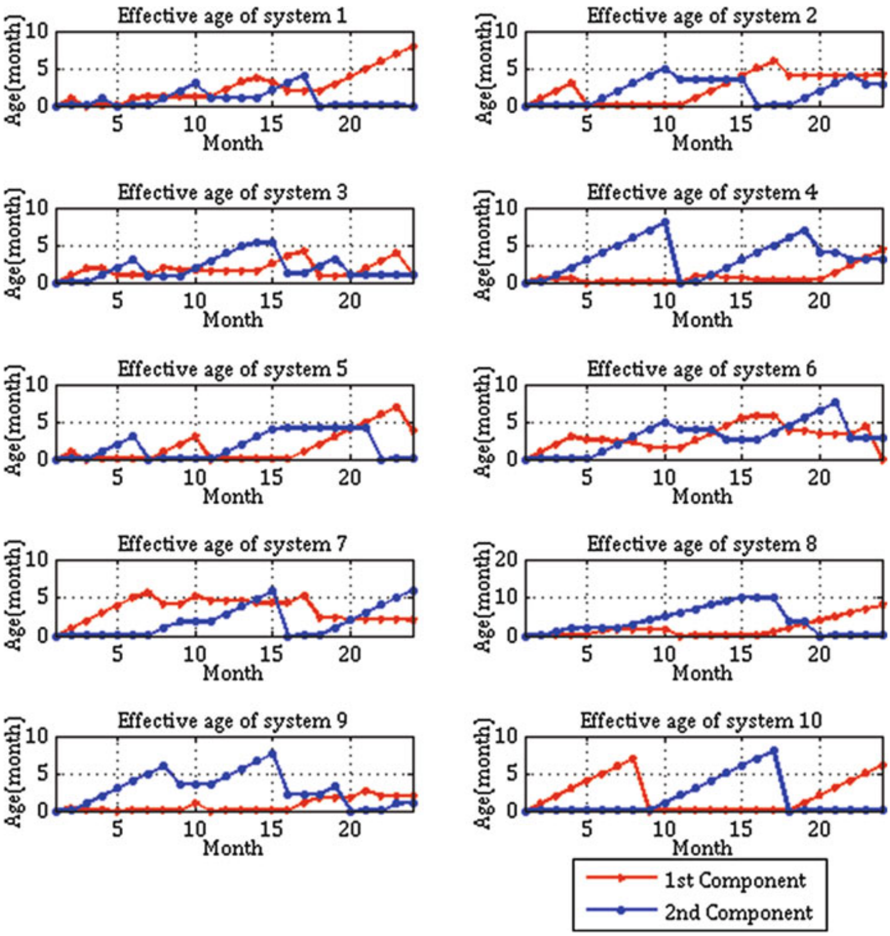


Fig. 21 Components effective age by $R = 99.11$ % and $C = 9611\$$

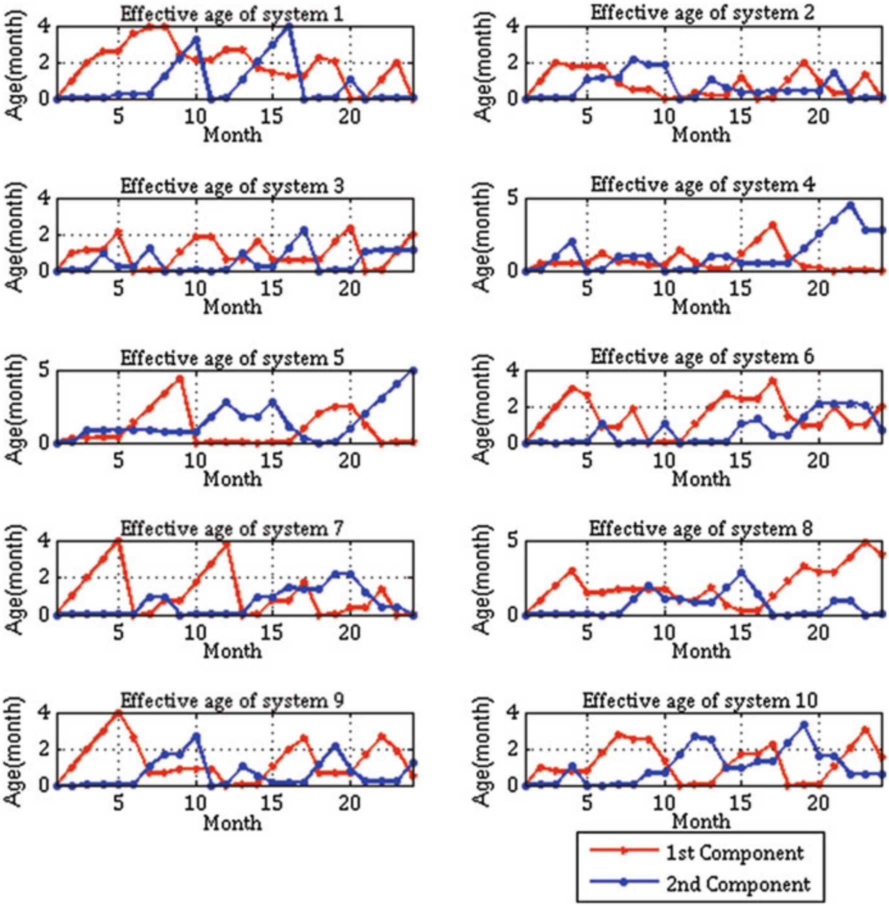


Fig. 22 Components effective age by $R = 99.89$ % and $C = 30184\$$

Table 13 Maintenance and replacement schedule by $R = 90.81$ % and $C = 499$\$

Subsystem	Component	Maintenance and replacement schedule																							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	–	–	–	–	–	–	–	–	–	M3	–	M2	–	–	–	–	–	–	–	–	–	–	–	–
	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
2	1	–	–	–	–	–	–	–	–	–	–	–	–	–	M3	–	–	–	–	–	–	–	–	–	–
	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
3	1	–	–	–	–	–	–	–	–	–	–	–	M3	–	–	–	–	–	–	–	–	–	M3	–	–
	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
4	1	–	–	–	–	–	–	–	M2	–	–	–	–	–	M2	–	–	–	–	–	–	–	–	–	–
	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
5	1	–	–	–	–	–	–	–	–	–	–	–	–	M2	–	–	M3	–	–	–	–	–	–	–	–
	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
6	1	–	–	–	–	–	–	–	–	–	–	M3	–	–	–	–	–	–	–	–	M3	–	–	–	–
	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
7	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	M2	–	–	–	–	–	–	–	–	–
	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
8	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	M3	–	–	–	–
	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
9	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
10	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–

Table 14 Maintenance and replacement schedule by $R = 99.11\ \%$ and $C = 9611\$$

Subsystem	Component	Maintenance and replacement schedule																							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	--	R	--	--	--	M2	--	--	--	--	--	--	M3	M3	M2	--	--	--	--	--	--	--	--	--
	2	M3	--	--	R	--	--	--	--	--	M1	--	--	--	--	--	--	R	--	--	--	M3	--	R	--
2	1	--	--	--	R	--	--	--	--	--	--	--	--	--	--	--	--	M2	--	--	--	--	--	M3	--
	2	M3	--	--	--	--	--	--	--	--	M2	--	--	--	--	R	--	--	--	--	--	--	M2	--	--
3	1	--	M3	--	M2	--	--	--	M2	--	M3	--	--	--	--	--	M3	M1	--	--	--	--	--	M1	--
	2	--	--	--	--	--	M1	--	--	--	--	--	--	M3	--	M1	--	--	--	M1	--	--	--	--	--
4	1	M2	--	--	R	--	M1	--	--	--	--	M3	--	M3	--	M2	--	--	--	--	--	--	--	--	--
	2	--	--	--	--	--	--	--	--	--	R	--	--	--	--	--	--	--	--	M2	--	M3	--	--	--
5	1	--	R	--	--	--	--	--	--	--	R	--	--	--	--	--	--	--	--	--	--	--	--	M2	--
	2	M1	--	--	--	--	R	--	M3	--	--	--	--	--	--	M3	--	--	--	--	--	R	--	--	--
6	1	--	--	--	M2	--	M3	--	M2	--	--	--	--	--	--	M3	--	M2	--	M3	--	--	--	R	--
	2	--	M2	--	--	--	--	--	--	--	M2	--	--	M2	--	--	--	--	--	--	--	M1	--	--	--
7	1	--	--	--	--	--	M3	M2	--	--	M2	--	--	M3	--	--	--	M1	--	M3	--	--	--	M3	--
	2	--	--	--	M2	--	--	--	M3	--	--	--	--	--	--	R	--	--	--	--	--	--	--	--	--
8	1	M1	M3	--	--	--	M2	--	--	--	R	--	--	--	--	--	--	--	--	--	--	--	--	--	--
	2	--	--	--	M2	--	--	--	--	--	--	--	--	--	--	M3	--	M1	--	R	--	M1	--	--	--
9	1	M1	M3	--	R	--	--	--	--	--	R	--	--	--	--	--	--	M3	--	--	--	M2	--	--	--
	2	--	--	--	--	--	--	--	M2	--	--	--	--	--	--	M1	--	--	--	R	--	--	--	M2	--
10	1	--	--	--	--	--	--	--	R	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
	2	--	M2	--	--	--	--	--	--	--	--	--	--	--	--	--	--	R	--	--	--	--	--	--	--

Table 15 Maintenance and replacement schedule by $R = 99.89$ % and $C = 30184$$

Subsystem	Component	Maintenance and replacement schedule																							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	--	--	M3	--	--	M3	--	M2	M3	--	M3	--	M2	M3	M3	--	--	M2	R	--	--	--	R	--
	2	M1	M2	--	M1	M3	--	--	--	--	R	--	--	--	--	--	R	M3	--	--	R	M2	M1	--	M2
2	1	--	--	M2	--	--	M1	M2	--	R	--	M1	M2	--	--	R	--	--	--	M1	M1	--	--	R	R
	2	M3	--	--	--	M2	--	--	M2	--	R	--	--	M1	M2	--	M1	--	--	--	--	R	--	--	--
3	1	--	M2	--	--	R	--	--	--	M3	--	M1	--	--	M1	--	--	--	--	M3	R	--	--	--	--
	2	M2	--	M3	M1	--	--	R	R	--	R	--	M3	M1	--	--	--	R	M3	--	--	M2	--	--	--
4	1	M2	--	--	--	M3	M2	--	M1	--	--	M1	M1	--	--	--	--	M1	M1	M3	R	--	--	R	R
	2	--	--	--	R	--	--	M2	--	R	M2	--	--	M2	M2	--	--	--	--	--	--	--	M2	--	--
5	1	M1	--	M1	--	--	--	--	--	R	--	--	--	R	M3	--	--	--	M3	--	M2	R	M1	M2	M1
	2	--	M3	--	M2	--	--	M3	--	--	--	--	M2	--	--	M1	M1	R	--	--	--	--	--	--	--
6	1	--	--	--	M2	M1	--	--	R	M1	--	--	--	M3	M3	--	--	M1	M2	--	--	M1	--	--	--
	2	--	R	M1	--	--	R	M1	--	--	R	--	M1	--	--	M2	M1	--	--	M3	--	--	M2	M1	M1
7	1	--	--	--	--	R	--	M2	--	--	--	--	R	--	M2	--	--	R	--	M1	--	--	R	--	M3
	2	M1	M3	M1	--	--	M3	--	R	M2	--	--	--	M3	--	M2	M3	--	M3	--	M1	M1	--	R	--
8	1	--	--	--	M1	--	M2	--	--	--	M1	--	M3	M1	M1	--	--	--	--	M2	--	--	--	M2	--
	2	--	M3	--	--	R	--	--	--	M1	--	M1	--	--	--	M1	R	R	M1	--	M3	--	R	--	--
9	1	--	--	--	--	M2	M1	--	M2	--	--	R	M3	--	--	--	M3	M1	--	--	--	--	M2	M1	--
	2	R	M1	M1	M2	--	--	M3	--	--	R	--	--	M1	M1	--	--	--	--	M1	M1	--	--	--	M2
10	1	--	M1	--	--	--	--	M2	--	M1	R	--	--	--	M3	--	M3	R	M3	--	--	--	--	M1	--
	2	--	--	--	R	R	--	--	M2	--	--	--	M2	M1	--	M2	--	--	--	M1	--	M1	--	--	--

$$\text{Min Total Cost} = \sum_{i=1}^{N}\sum_{j=1}^{T}\Bigg[F_i\cdot\left(1-\text{Reliability}_{i,j}^{SS}\right) + \sum_{c=1}^{C}\left(\sum_{k=1}^{K}\left(M_i^{c,k}\cdot m_{i,j}^{c,k}\right)+R_i^c\cdot r_{i,j}^c\right)\Bigg] + \sum_{j=1}^{T}\left[Z\left(1-\prod_{i=1}^{N}\left(1-\sum_{c=1}^{C}\left(r_{i,j}^c+\sum_{k=1}^{K}m_{i,j}^{c,k}\right)\right)\right)\right]$$

$$\begin{aligned}\text{Max Reliability} = \prod_{i=1}^{N}\prod_{j=1}^{T}\Bigg(p_{i,j}^1\cdot\Bigg[e^{-\left[\lambda_i^1\left(\left(X_{i,j}^1+T/J\right)^{\beta_i^1}-\left(X_{i,j}^1\right)^{\beta_i^1}\right)\right]} &\\ +\int_{t=0}^{t=T/J}\lambda_i^1\cdot\beta_i^1\cdot t^{\beta_i^1-1}\cdot e^{-\left[\lambda_i^1\left(\left(X_{i,j}^1+t\right)^{\beta_i^1}-\left(X_{i,j}^1\right)^{\beta_i^1}\right)\right]} &\\ \cdot e^{-\left[\lambda_i^2\left(\left(X_{i,j}^2+T/J-t\right)^{\beta_i^2}-\left(X_{i,j}^2\right)^{\beta_i^2}\right)\right]}\cdot dt\Bigg] &\\ + p_{i,j}^2\cdot\Bigg[e^{-\left[\lambda_i^2\left(\left(X_{i,j}^2+T/J\right)^{\beta_i^2}-\left(X_{i,j}^2\right)^{\beta_i^2}\right)\right]} &\\ +\int_{t=0}^{t=T/J}\lambda_i^2\cdot\beta_i^2\cdot t^{\beta_i^2-1}\cdot e^{-\left[\lambda_i^2\left(\left(X_{i,j}^2+t\right)^{\beta_i^2}-\left(X_{i,j}^2\right)^{\beta_i^2}\right)\right]} &\\ \cdot e^{-\left[\lambda_i^1\left(\left(X_{i,j}^1+T/J-t\right)^{\beta_i^1}-\left(X_{i,j}^1\right)^{\beta_i^1}\right)\right]}\cdot dt\Bigg]\Bigg)\end{aligned}$$

subject to:

$$X_{i,j}^c = 0 \text{ for } i = 1,\ldots N \text{ and } c = 1,2$$

$$\begin{aligned}X_{i,j}^c &= \left(1-r_{i,j-1}^c\right)\left[\prod_{k=1}^{K}\left(1-m_{i,j-1}^{c,k}\right)\right]X_{i,j-1}^{\prime c}\\ &+\sum_{k=1}^{K}m_{i,j-1}^{c,k}\cdot\left(\alpha_i^k\cdot X_{i,j-1}^{\prime c}\right)\\ &\text{for } i=1,\ldots N;\ j=2,\ldots,T \text{ and } c=1,2\end{aligned}$$

$$\begin{aligned}
X'^{1}_{i,j} &= X^{1}_{i,j} + \Big[p^{1}_{i,j} \cdot \text{Reliability}^{1}_{i,j} \\
&\quad + \left(1 - p^{1}_{i,j}\right)\left(1 - \text{Reliability}^{2}_{i,j}\right)\Big] \cdot T/J \\
&\quad \text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T \\
X'^{2}_{i,j} &= X^{2}_{i,j} + \Big[\left(1 - p^{1}_{i,j}\right) \cdot \text{Reliability}^{2}_{i,j} \\
&\quad + p^{1}_{i,j} \cdot \left(1 - \text{Reliability}^{1}_{i,j}\right)\Big] \cdot T/J \\
&\quad \text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T \\
p^{1}_{i,1} &= 1 \text{ for } i = 1, \ldots, N \\
p^{2}_{i,1} &= 0 \text{ for } i = 1, \ldots, N \\
p^{1}_{i,j+1} &= p^{1}_{i,j} \cdot \text{Reliability}^{1}_{i,j} + \cdot \left(1 - p^{1}_{i,j}\right) \\
&\quad \cdot \left(1 - \text{Reliability}^{2}_{i,j}\right) \\
&\quad \text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T - 1 \\
p^{2}_{i,j} &= 1 - p^{1}_{i,j} \text{ for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T \\
\sum_{k=1}^{K} m^{c,k}_{i,j} + r^{c}_{i,j} &\leq 1 \text{ for } i = 1, \ldots, N;\ j = 1, \ldots, T \text{ and } c = 1, 2 \\
\sum_{c=1}^{c}\left(\sum_{k=1}^{K} m^{c,k}_{i,j} + r^{c}_{i,j}\right) &\leq 1 \text{ for } i = 1, \ldots, N;\ j = 1, \ldots, T - 1 \text{ and } c = 1, 2 \\
p^{c}_{i,j} \cdot \left(\sum_{k=1}^{K} m^{c,k}_{i,j-1} + r^{c}_{i,j-1}\right) &= 0 \text{ for } i = 1, \ldots, N;\ j = 2, \ldots, T \text{ and } c = 1, 2 \\
m^{c,k}_{i,j}, r^{c}_{i,j} &= 0 \text{ or } 1 \text{ for } i = 1, \ldots, N;\ j = 1, \ldots, T; \\
&\quad c = 1, 2 \text{ and } k = 1, \ldots, K \\
0 \leq p^{1}_{i,j}, p^{2}_{i,j} &\leq 1 \text{ for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T \\
X'^{c}_{i,j}, X^{c}_{i,j} &\geq 0 \text{ for } i = 1, \ldots, N \text{ and } j = 1, \ldots, T
\end{aligned} \tag{51}$$

Table 16 NSGA-II parameters

NSGA-II parameters	1^{st} model	2^{nd} & 3^{rd} model
Number of Generations	1000	100
Population Size	100	50
Probability of Crossover	0.7	0.7
Probability of Mutation	0.4	0.4

NSGA-II parameters are similar to GA parameters that are set as shown in Table 16.

8 Conclusion

For better conclusion, these three models will be compared. As illustrated in Fig. 23, second and third models with failure possibility are more powerful than first model without failure possibility. As shown in Fig. 24, a brief focus on second and third models shows third model strength. For example, by a simple comparison between points (1) and (2), although the solution reliabilities are similar, the third model cost is 33 % less than the other. Reversely, in the plot primary points (points (8) and (9)), when the costs of these models are close together, the third model reliability is 10 % more than second one. It means that by considering optional switching in a standby system, the total cost reduces effectively without any changes in reliability or in the same costs, optional switching increases total reliability.

Table 17 shows a comparison between one schedule of second and third models, non-optional and optional switching in a same reliability. As is obvious in Table 17 optional switching causes less cost and more applicable scheduling than non-optional switching without any changes in reliability. So it can be found out that, in reality, the third model is more effective and powerful with failure possibility and optional switching.

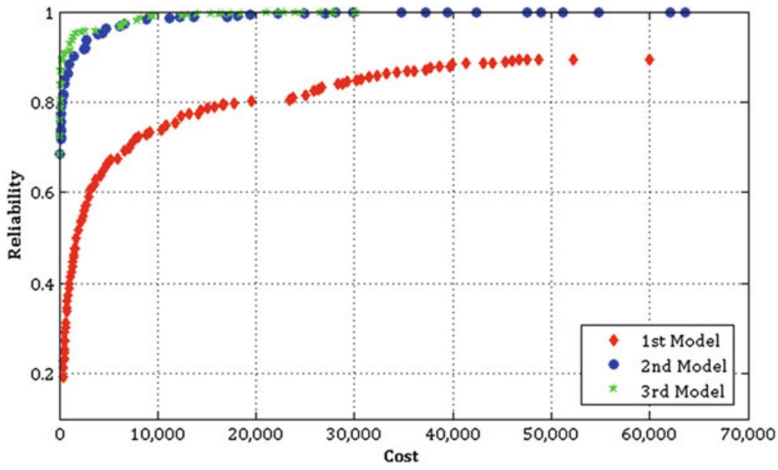


Fig. 23 Models Pareto front comparison

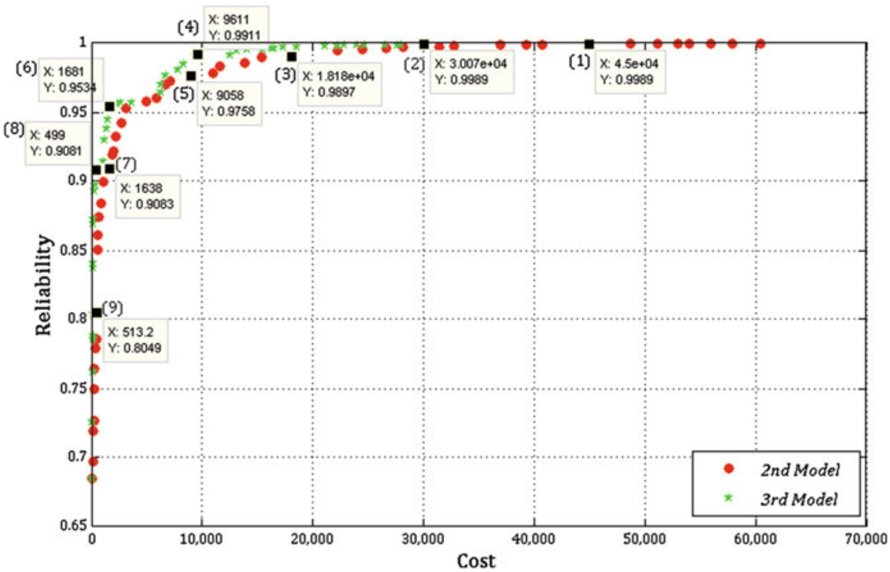


Fig. 24 Second and third model Pareto front comparison

Table 17 Maintenance and replacement scheduling in optional and non-optional switching cases

Subsystem	Component	Mintenance and replacement schedule in non-optional switching									
		1	2	3	4	5	6	7	8	9	10
1	1	M3	M2	M1	M2	M2	--	M1	R	M2	--
	2	R	R	M1	M1	--	--	M3	M3	R	M2
2	1	M3	M1	M2	M1	R	--	M1	M1	R	M1
	2	M3	--	M2	--	R	M3	M1	--	M1	M3
3	1	R	M3	--	M1	R	--	M1	--	--	M1
	2	M2	M2	M3	M2	--	M3	M3	M2	M1	--
4	1	R	--	M3	M1	M1	--	R	M1	M3	M1
	2	M3	--	R	R	M1	M2	--	--	--	--
5	1	M1	M2	M3	--	M2	M3	--	M1	R	M2
	2	M3	--	M2	--	M1	M2	--	--	R	--
6	1	M1	M1	R	--	M1	--	M2	M1	M1	R
	2	--	--	--	M1	M1	--	--	M2	--	--
7	1	M2	M2	R	M3	M3	M3	--	M1	R	--
	2	R	--	--	M3	R	M1	--	M1	M1	M1
8	1	M1	R	R	M2	M3	R	R	R	M3	M2
	2	--	--	--	--	R	M3	M2	--	R	--
9	1	R	M3	M2	M1	M3	--	--	M3	R	R
	2	M1	--	M3	M3	M2	--	--	--	M2	M1
10	1	--	--	--	--	--	--	--	R	M2	M3
	2	--	--	M3	M1	M2	--	R	--	M3	--
Reliability= 99.89%						Cost= 45002$					

Subsystem	Component	Mintenance and replacement schedule in optional switching									
		1	2	3	4	5	6	7	8	9	10
1	1	--	--	M3	--	--	M3	--	M2	M3	--
	2	M1	M2	--	M1	M3	--	--	--	--	R
2	1	--	--	M2	--	--	M1	M2	--	R	--
	2	M3	--	--	--	M2	--	--	M2	--	R
3	1	--	M2	--	--	R	--	--	--	M3	--
	2	M2	--	M3	M1	--	--	R	R	--	R
4	1	M2	--	--	--	M3	M2	--	M1	--	--
	2	--	--	--	R	--	--	M2	--	R	M2
5	1	M1	--	M1	--	--	--	--	--	R	--
	2	--	M3	--	M2	--	--	M3	--	--	--
6	1	--	--	--	M2	M1	--	--	R	M1	--
	2	--	R	M1	--	--	R	M1	--	--	R
7	1	--	--	--	--	R	--	M2	--	--	--
	2	M1	M3	M1	--	--	M3	--	R	M2	--
8	1	--	--	--	M1	--	M2	--	--	--	M1
	2	--	M3	--	--	R	--	--	--	M1	--
9	1	--	--	--	--	M2	M1	--	M2	--	--
	2	R	M1	M1	M2	--	--	M3	--	--	R
10	1	--	M1	--	--	--	--	M2	--	M1	R
	2	--	--	--	R	R	--	--	M2	--	--
Reliability= 99.89%						Cost= 30184$					

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