Singularity Analysis of 3-DOF Translational Parallel Manipulator

Pavel Laryushkin, Victor Glazunov and Sergey Demidov

Abstract In this paper we analyze singularities of the 3-DOF translational parallel mechanism with three kinematic chains, each consisting of five revolute joints. Both Jacobian and Screw theory methods are used to determine singular points of different types. Constraint singularity is also studied.

Keywords Parallel mechanism · Three degrees of freedom · Singularity · Screw theory · Constraint singularity

1 Introduction

For last decades, the class of parallel robots is certainly attracting much attention of researchers and manufacturers. For instance, Gogu [1] has over 700 pages concerning only 2- and 3-DOF translational parallel mechanisms' topologies. However, along with undoubted advantages, several typical problems also exist for this class of manipulators.

The problem of singularities is a very common issue in parallel robots. There are some points within the workspace of a mechanism where losing a degree of freedom or uncontrollable motion of a moving plate is possible. These points are

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called singular [2]. Minimizing the number of such points [3], finding reasonable ways to avoid them [4] and determining singularity-free zones [5] are among the main problems in parallel mechanism design.

It is also possible to design singularity free manipulators. For translational parallel mechanisms this topic was deeply studied in Carricato [6].

There exist several approaches in finding and classifying singularities, for instance [7, 8]. The most popular method of finding singular points and zones is based on Jacobian analysis [9], and Screw theory approaches [10, 11] is also used quite often. Here we will be using a classification proposed in Gosselin and Angeles' paper.

In this paper we present a singularity analysis of 3-DOF translational parallel mechanism with three RRRRR legs. The mechanism was mentioned earlier as a part of a family of translational parallel mechanisms, for instance in [12] and [13]. Later it was analyzed in detail in [14] where results of the overall kinematics, dynamics and workspace analysis of the mechanism were presented.

2 Mechanism Overview

In this section we provide a quick overview of the analyzed parallel mechanism and the results obtained during initial singularity analysis presented in our previous paper.

The mechanism has three legs (or kinematic chains) with five revolute joints each (Fig. 1).

Let $l_1 = A_iB_i,\ l_2 = B_iC_i,\ l_3 = C_iD_i,\ l_4 = D_iE_i,\ l_5 = E_iF,\ l_A = OA_i$ then the relationship between Cartesian coordinates x, y, z and generalized coordinates (rotation angles) θ_1, θ_2, θ_3 can be expressed using system of three equations which we can treat as three implicit functions

$$\begin{aligned} f_1(x,y,z,\theta_1) &= (y - l_x \sin\theta_1)^2 + (z + l_x \cos\theta_1)^2 - l_2^2 = 0 \\ f_2(x,y,z,\theta_2) &= (z - l_y \sin\theta_2)^2 + (x + l_y \cos\theta_2)^2 - l_2^2 = 0 \\ f_3(x,y,z,\theta_3) &= (x - l_z \sin\theta_3)^2 + (y + l_z \cos\theta_3)^2 - l_2^2 = 0 \end{aligned} \tag{1}$$

where

$$l_x = l_2\sqrt{1 - \left(\frac{x + l_2}{l_2}\right)^2},\ l_y = l_2\sqrt{1 - \left(\frac{y + l_2}{l_2}\right)^2},\ l_z = l_2\sqrt{1 - \left(\frac{z + l_2}{l_2}\right)^2}$$

Note that we assume $l_4 = l_2$ and $l_A + l_2 = l_1 + l_3 + l_5$. These conditions ensure that in initial position (Fig. 1) all angles between links are right angles.

Previously we have conducted a singularity research based on Jacobian and Screw theory methods.

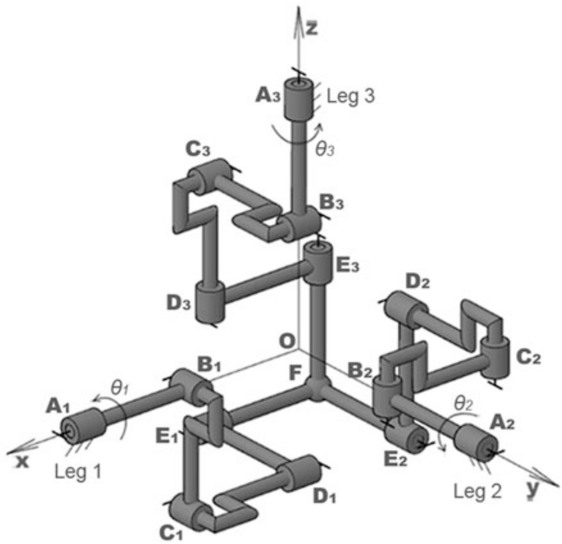

Fig. 1 Scheme of the mechanism

Using both methods, it was analytically discovered that Type 1 singularity (loss of moving platform mobility) can occur only at the theoretical edge of the workspace. For Type 2 singularities (uncontrollable motion of the moving platform) no analytical conditions were found, and iteration analysis was used instead. Analytical conditions for Type 3 singularities (can be treated as a manifestation of both Type 1 and Type 2 singularities simultaneously) were proven to be included in conditions for Type 1 singularities.

Iteration analysis of Type 2 singularities resulted into a statement that the mechanism has no such singularities. Later we found that is only true for two of eight possible mechanism configurations, corresponding to the number of inverse kinematics solutions, and for iteration step $0.05l_2$ that we used in our previous paper during singularity analysis.

In next two sections we present a more detailed iteration analysis of Type 2 singularities followed by analysis of constraint singularities. All numerical calculations are done in MATLAB (using built-in functions if needed), and took no longer than 787 s on a laptop with average characteristics.

3 Type 2 Singularities Analysis

We start with Jacobian analysis of Type 2 singularities. We can find partial derivatives of (1) and form the following matrix.

$$\mathbf{A} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial f_1(x,y,z,\theta_1)}{\partial x} & \frac{\partial f_1(x,y,z,\theta_1)}{\partial y} & \frac{\partial f_1(x,y,z,\theta_1)}{\partial z} \\ \frac{\partial f_2(x,y,z,\theta_2)}{\partial x} & \frac{\partial f_2(x,y,z,\theta_2)}{\partial y} & \frac{\partial f_2(x,y,z,\theta_2)}{\partial z} \\ \frac{\partial f_3(x,y,z,\theta_3)}{\partial x} & \frac{\partial f_3(x,y,z,\theta_3)}{\partial y} & \frac{\partial f_3(x,y,z,\theta_3)}{\partial z} \end{pmatrix} \tag{2}$$

Type 2 singularity occurs when matrix (2) is singular, i.e. $det(\mathbf{A}) = 0$.

As we mentioned earlier, we use an iteration method to find such singularities. The idea is to scan a workspace of the mechanism with a relatively small step and find a value of matrix determinant in each point. Then we assume that if for two neighbor points the determinant value have different signs ("+" and "−") than there is a some point between those two where $det(\mathbf{A}) = 0$, so Type 2 singularity occurs.

With this said and assuming iteration step is equal to $0.025l_2$ and $l_2 = 20$, we have found that for two of eight possible configurations of the mechanism's legs approximately 0.27 % of analyzed workspace points have $det(\mathbf{A}) > 0$ while all other points have $det(\mathbf{A}) < 0$. These points form three small volumes lying very close to the edge of the workspace (Fig. 2). Thus, these volumes are separated from the workspace by surfaces consisting of Type 2 singular points.

Mechanism configuration is determined by choosing the solution of inverse kinematics problem. Figure 3 illustrates two possible configurations of Leg 1 with two different values of rotation angle θ_1.

Thus having two different possible configurations for each leg with corresponding θ_i one can easily see that for any point in the workspace (except Type 1 singular points) there are eight possible configurations and eight different sets of ($\theta_{1,j}$, $\theta_{2,j}$, $\theta_{3,j}$) in total where $j = 1, 2$. For example, configuration in Fig. 1 corresponds to a set ($\theta_{1,2}$, $\theta_{2,2}$, $\theta_{3,2}$).

Figure 2 corresponds to a set ($\theta_{1,1}$, $\theta_{2,1}$, $\theta_{3,1}$). Using set ($\theta_{1,2}$, $\theta_{2,2}$, $\theta_{3,2}$) also produces only 0.27 % loss of total workspace volume, however volumes shown in Fig. 2 will be rotated by 90°.

For other six possible configurations, Type 2 singularities divide the workspace into several significantly large zones. Shape of these zones varies depending on the configuration. In Fig. 4 two "slices" of the workspace are shown. Slice (a) corresponds to a set of angles ($\theta_{1,1}$, $\theta_{2,1}$, $\theta_{3,1}$) and slice (b) corresponds to a set ($\theta_{1,2}$, $\theta_{2,1}$, $\theta_{3,1}$). Here $l_2 = 20$ and coordinate z is fixed at $z = -18$.

Darker areas present points where $det(\mathbf{A}) > 0$ and lighter areas present points where $det(\mathbf{A}) < 0$. The lightest area around does not belong to the workspace.

One can see that for set ($\theta_{1,1}$, $\theta_{2,1}$, $\theta_{3,1}$) the area separated by Type 2 singularities from the main workspace is quite insignificant while for set ($\theta_{1,2}$, $\theta_{2,1}$, $\theta_{3,1}$) it appears to be much bigger, and Type 2 singularity can occur in the middle part of the workspace. Furthermore, moving plate of real mechanism most likely will not be capable to reach the volumes shown in Fig. 2 because it requires A_iB_i, B_iC_i, and C_iD_i to be on the same line and at the same place.

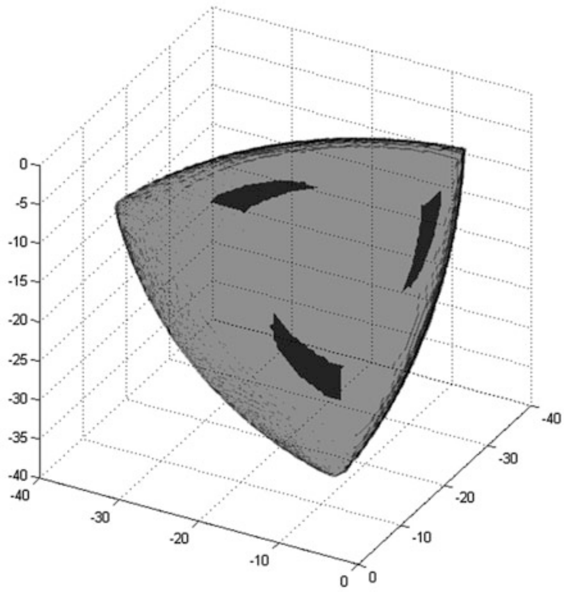

Fig. 2 Volumes separated from the workspace by Type 2 singularities

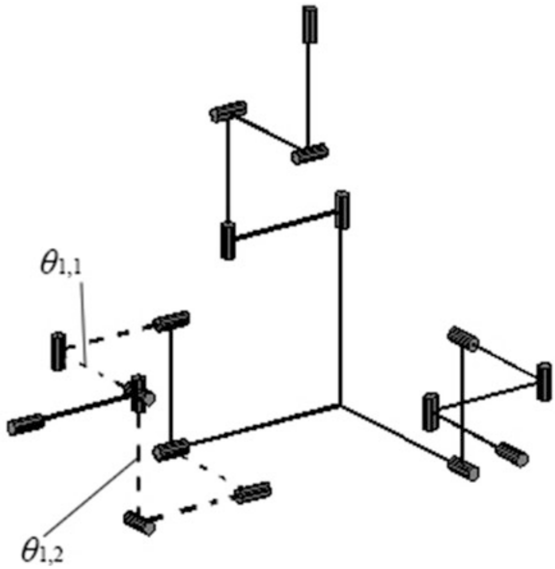

Fig. 3 Two possible configurations of Leg 1

Thus, Type 2 singularities depend on mechanism configuration but undesirable configurations can be easily avoided by choosing one of two optimal configurations and corresponding set of rotation angles: ($\theta_{1,1}$, $\theta_{2,1}$, $\theta_{3,1}$) or ($\theta_{1,2}$, $\theta_{2,2}$, $\theta_{3,2}$).

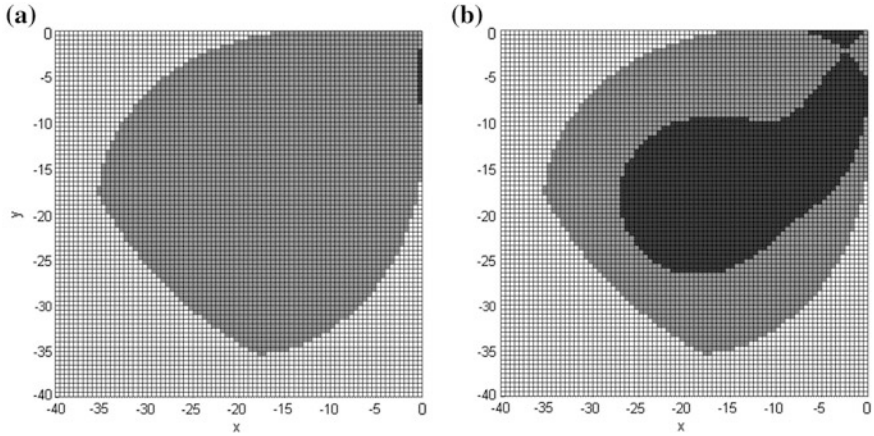

Fig. 4 Slices of the workspace at z = −18

4 Constraint Singularities Analysis

Constraint singularity is a phenomenon that arises for mechanisms with less than six degrees of freedom while legs of the mechanism have more degrees of freedom than a mechanism itself [15]. This type of singularity cannot be analyzed by Jacobian method because it deals with degrees of freedom that a mechanism normally does not have. For instance, a translational parallel mechanism can obtain a rotational degree of freedom when such singularity occurs.

In order to analyze constraint singularities, Screw theory approach must be used. We need to find screws that form a matrix of constraints.

First step is to find corresponding twists for each leg. Using Plücker coordinates, such twists for Leg 1 can be written in the form of a matrix as follows

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 & 0 & -\sin\theta_1(l_A - l_1) & \cos\theta_1(l_A - l_1) \\ 0 & \cos\theta_1 & \sin\theta_1 & l_x & -\sin\theta_1(x + l_5 + l_3) & \cos\theta_1(x + l_5 + l_3) \\ 1 & 0 & 0 & 0 & -l_x\cos\theta_1 & -l_x\sin\theta_1 \\ 1 & 0 & 0 & 0 & z & -y \end{pmatrix}$$

For these twists we can now find a reciprocal wrench. Doing this for each leg, we obtain three wrenches and can form a matrix of wrenches. For the discussed mechanism this matrix is

$$\begin{pmatrix} 0 & 0 & 0 & 0 & \sin\theta_1 & -\cos\theta_1 \\ 0 & 0 & 0 & -\cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 0 & 0 & \sin\theta_3 & -\cos\theta_3 & 0 \end{pmatrix}$$

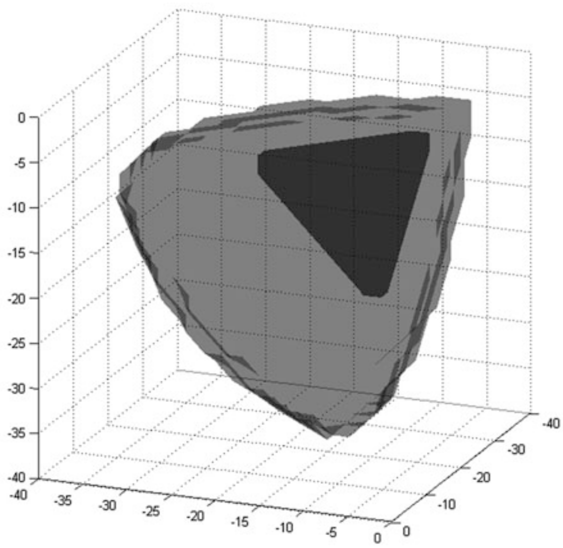

Fig. 5 Volume separated from the workspace by constraint singularities

When this matrix loses its maximum rank (3 in this case) a constraint singularity occurs.

Using the same iteration approach as for Type 2 singularities analysis we have found that for sets ($\theta_{1,1}$, $\theta_{2,1}$, $\theta_{3,1}$) or ($\theta_{1,2}$, $\theta_{2,2}$, $\theta_{3,2}$) approximately 4.48 % of total workspace volume will be separated by a surface consisting of points of constraint singularity. This volume is situated at the top part of the workspace (Fig. 5).

The research shows that for other six possible configurations the workspace divided into several parts. Though, the shapes of these parts are different, such situation is clearly as unacceptable as it was with undesirable mechanism configurations when we analyzed Type 2 singularities.

The volumes shown in Figs. 2 and 5 intersect, and total theoretical workspace volume loss is 4.69 % due to both Type 2 and constraint singularities.

Thus, constraint singularities, just like Type 2 singularities, depend on mechanism configuration. Undesirable configurations can be avoided by choosing between two optimal configurations which are the same for Type 2 singularity avoidance.

5 Conclusion

In this paper the analysis of 3-DOF translational parallel mechanism (focused on Type 2 and constraint singularities) is presented. The two optimal configurations of mechanism links and corresponding sets of input rotation angles were discovered, so the total loss of the theoretical workspace volume is about 4.69 %. This means that for actual mechanism, if designed with taking these results into account, at least 95.31 % of the workspace volume will be singularity-free with singular points of any kind only at the edge of this volume.

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