Kinematics, Dynamics, Control and Accuracy of Spherical Parallel Robot

Sergey Kheylo and Victor Glazunov

Abstract This paper deals with the kinematical and dynamical properties of parallel spherical manipulators with three degrees of freedom. Accelerations, dynamics, control and accuracy are considered. Algorithm of control is based on the inverse problem of dynamics and allows minimizing deviations of coordinates, velocities and accelerations.

Keywords Spherical parallel mechanism · Dynamics · Control · Accuracy

1 Introduction

Spherical mechanisms are used as orienting devices, machine tools, medical equipments, etc. The first works were focused on the mechanisms of one degree of freedom and open kinematic chain [1]. It was one of the first monographs dealing with the spherical mechanisms.

Furthermore these mechanisms were applied in parallel robots that contain several kinematic chains [2–6]. A lot of publications describe 3-dof spherical parallel mechanisms. Such 3-dof spherical parallel mechanism is composed of three limbs connecting a moving platform (end-effector) to a fixed base and all the axes of kinematic pairs intersect at one point [7]. But not all the mentioned axes

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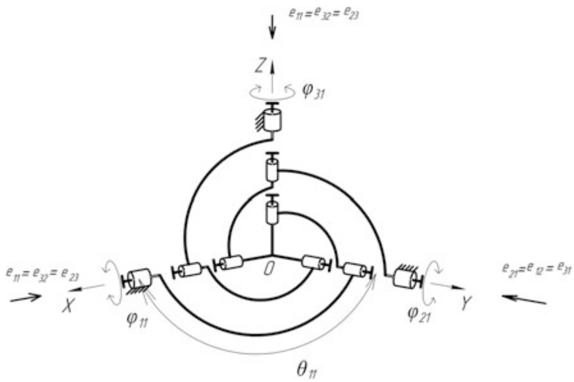

Fig. 1 Spherical mechanism with three kinematic pair in each chain

may intersect at a single point. An intermediate rotating kinematic pair can be replaced by three pairs with parallel axes. [8–10].

Spherical mechanisms were subjects of many publications dealing with the structure [6, 7], the problem of positions and velocities and workspace modeling [11], problems of optimization and design [12, 13], dynamical analysis and singularity analysis [14].

However, not all the important problems have been addressed earlier. These concern the problem of accelerations and accuracy. Besides, the problem of control is very important [15]. This article is devoted to these issues. Proposed algorithm is based on concept of minimized deviations of coordinates, velocities and acceleration, using inverse problems of dynamics

2 Mechanism Configuration

In the considered spherical mechanism with three kinematic pairs in each chain (Fig. 1), the input link is connected to the engine. The output link is a platform that revolves around three axes intersecting at a point O. The output coordinates are angles of rotation of the platform α, β, γ around the axes, whose relative positions are described by a fiction kinematic chain (Fig. 2a). The generalized coordinates are angles φ_{11}, φ_{21}, φ_{31}. Each of the three kinematic chains has three joints with intersecting axes.

Let us associate the output link of the mechanism with a moving coordinate system ξ, η, ζ, whose axes are situated along the main inertia axes of this link. Therefore for the orientation angles ($\alpha = \beta = \gamma = 0$) the directions of the axes ξ, η, ζ coincide with directions of the axes x, y, z, respectively. The constrain equations are derived from geometry of mechanism, transfer matrixes absolute and moving coordinate systems.

The constrain equations for a spherical mechanism with three kinematic chains can be represented by the following system:

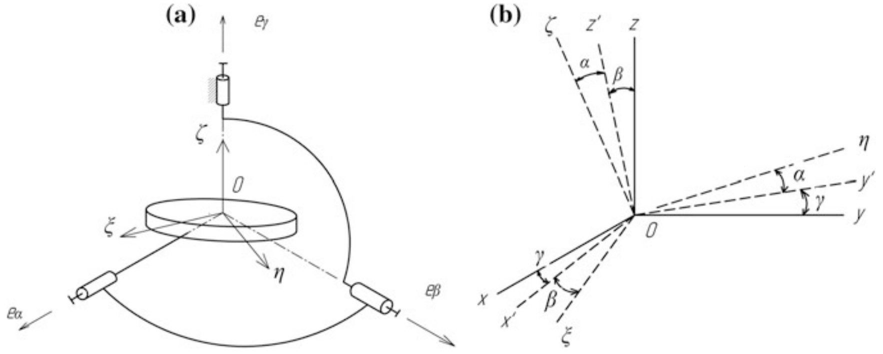

Fig. 2 The angles α, β, γ: **a** fiction kinematic chain, **b** coordinate systems

$$F_1 = \mathrm{tg}\varphi_{11} + \frac{\cos\alpha\cdot\sin\gamma\cdot\sin\beta - \cos\gamma\cdot\sin\alpha}{\cos\alpha\cdot\cos\beta} = 0 \quad ; \quad F_2 = \frac{\sin\beta}{\cos\gamma\cdot\cos\beta} - \mathrm{tg}\varphi_{21} = 0;$$
$$F_3 = \mathrm{tg}\varphi_{31} + \frac{\cos\gamma\cdot\sin\alpha\cdot\sin\beta - \cos\alpha\cdot\sin\gamma}{\cos\alpha\cdot\cos\gamma + \sin\alpha\cdot\sin\beta\cdot\sin\gamma} = 0 \tag{1}$$

3 Dynamical Analysis

Differentiating these expressions (1) with respect to t, we obtain a system of equations of velocities of the input and output links:

$$\frac{\partial F_i}{\partial\alpha}\dot{\alpha} + \frac{\partial F_i}{\partial\beta}\dot{\beta} + \frac{\partial F_i}{\partial\gamma}\dot{\gamma} + \frac{\partial F_i}{\partial\varphi_{i1}}\dot{\varphi}_{i1} = 0 \tag{2}$$

Differentiating the Eq. (1) again with respect to t, we obtain equations of accelerations:

$$\frac{\partial F_1}{\partial\varphi_{11}}\ddot{\varphi}_{11} = \frac{\partial^2 F_1}{\partial\alpha^2}\dot{\alpha}^2 + 2\frac{\partial^2 F_1}{\partial\alpha\partial\beta}\dot{\alpha}\dot{\beta} + 2\frac{\partial^2 F_1}{\partial\alpha\partial\gamma}\dot{\alpha}\dot{\gamma} + \frac{\partial^2 F_1}{\partial\beta^2}\dot{\beta}^2 + 2\frac{\partial^2 F_1}{\partial\beta\partial\gamma}\dot{\beta}\dot{\gamma} + \frac{\partial^2 F_1}{\partial\gamma^2}\dot{\gamma}^2$$
$$+ \frac{\partial^2 F_1}{\partial\varphi_{11}^2}\dot{\varphi}_{11} + \frac{\partial F_1}{\partial\alpha}\ddot{\alpha} + \frac{\partial F_1}{\partial\beta}\ddot{\beta} + \frac{\partial F_1}{\partial\gamma}\ddot{\gamma}\ldots \tag{3}$$

The equation of motion of a spherical mechanism with three degrees of freedom has the following form (mass of the arc shaped links are ignored):

$$J_{\xi} \cdot \ddot{\underset{\xi}{\varphi}} = M_1 \cdot \frac{\partial \varphi_{11}}{\partial \varphi_{\xi}} + M_2 \cdot \frac{\partial \varphi_{21}}{\partial \varphi_{\xi}} + M_3 \cdot \frac{\partial \varphi_{31}}{\partial \varphi_{\xi}} + \dot{\varphi}_{\eta} \cdot \dot{\varphi}_{\zeta} \cdot \left(J_{\zeta} - J_{\eta}\right) \tag{4}$$

where $J_{\xi} = J_{\eta}$, J_{ζ} are the inertia moments in respect to the axes ξ, η, ζ; M_1, M_2, M_3, are the moments in the drives, $\frac{\partial \varphi_{ij}}{\partial \varphi_{\xi}}$ are variable coefficients, $\ddot{\underset{\xi}{\varphi}} = \dot{\omega}_{\xi}$, $\dot{\varphi}_{\xi} = \omega_{\xi}$, $\ddot{\underset{\eta}{\varphi}}$, $\dot{\varphi}_{\eta}$, $\ddot{\underset{\zeta}{\varphi}}$, $\dot{\varphi}_{\zeta}$ are the projections of accelerations and velocities on the axes ξ, η, ζ. Other equations are similar. The torques are determined using the algorithm represented in [15]. Let $\mathbf{r_i}$ be the vector perpendicular to the axes of the passive joints of the *i*-th chain. This vector has coordinates r_{ix}, r_{iy} r_{iz} and $r_{i\zeta}$, $r_{i\eta}$, $r_{i\xi}$ in the absolute and moving coordinate systems correspondingly. It may be shown that:

$$\omega_{\xi} \cdot r_{i\xi} + \omega_{\eta} \cdot r_{i\eta} + \omega_{\zeta} \cdot r_{i\zeta} = \omega_{i1} \cdot \left(x_{i1} r_{ix} + y_{i1} r_{iy} + z_{i1} r_{iz}\right) \tag{5}$$

where ω_{ξ}, ω_{η}, ω_{ζ} are projections of velocity of the output link to the axes ξ, η, ζ, x_{i1}, y_{i1}, z_{i1} are the coordinates of the unite vector $\mathbf{e_{i1}}$ (Fig. 1).

Then from (5) the variable coefficients can be determined. By this we take into account that $y_{i1} = 0$, $z_{i1} = 0$ (Fig. 1).

$$\frac{\partial \varphi_{11}}{\partial \varphi_{\xi}} = \frac{\omega_{11}}{\omega_{\xi}} = \frac{r_{1\xi}}{r_{1x}}; \quad \frac{\partial \varphi_{11}}{\partial \varphi_{\eta}} = \frac{\omega_{11}}{\omega_{\eta}} = \frac{r_{1\eta}}{r_{1x}}; \quad \frac{\partial \varphi_{31}}{\partial \varphi_{\eta}} = \frac{\omega_{31}}{\omega_{\eta}} = \frac{r_{3\eta}}{r_{3z}} = 0 \tag{6}$$

Other coefficients have similar forms.

The velocities $\dot{\alpha}$, $\dot{\beta}$, $\dot{\gamma}$ may be obtained from linear equations:

$$\omega_{\xi} = \dot{\alpha} \cdot \alpha_{\xi} + \dot{\beta} \cdot \beta_{\xi} + \dot{\gamma} \cdot \gamma_{\xi}, \ldots, \omega_{\zeta} = \dot{\alpha} \cdot \alpha_{\zeta} + \dot{\beta} \cdot \beta_{\zeta} + \dot{\gamma} \cdot \gamma_{\zeta} \tag{7}$$

where (α_{ξ}, α_{η}, α_{ζ}) are the projections of the unit vector $\mathbf{e_{\alpha}}$ of the joint corresponding to the velocity $\dot{\alpha}$ (Fig. 2a), (β_{ξ}, β_{η}, β_{ζ}) and (γ_{ξ}, γ_{η}, γ_{ζ}) are defined analogously.

4 Control of the Manipulator

Control of parallel manipulators is one of the most important problems. There are different approaches to solve this problem. The applied algorithm is based on the inverse problems of dynamics [15].

The desired laws of the coordinates of the mobile platform are described by equations $\alpha_T(t)$, $\beta_T(t)$, $\gamma_T(t)$.

The task is to minimize the errors $\Delta\alpha = \alpha_T(t) - \alpha(t)$, $\Delta\beta = \beta_T(t) - \beta(t)$, $\Delta\gamma = \gamma_T(t) - \gamma(t)$, where $\alpha(t)$, $\beta(t)$, $\gamma(t)$ are the actual coordinates of the mobile platform.

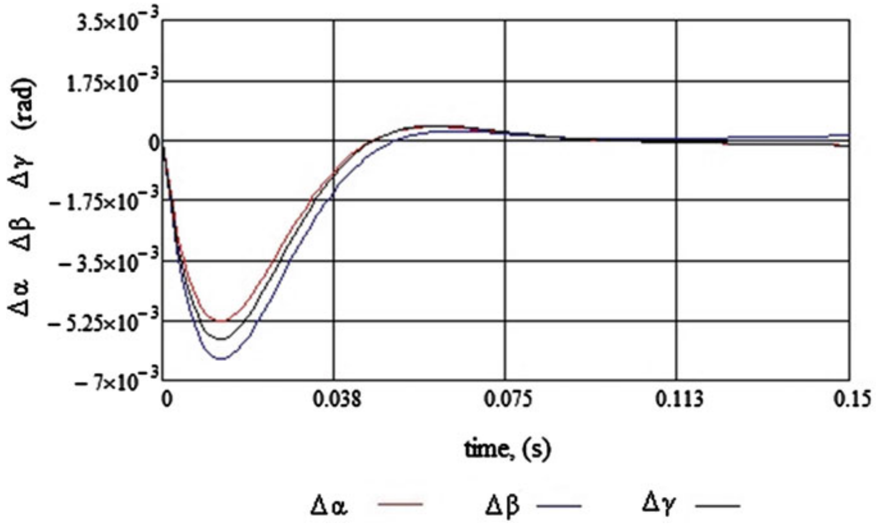

Fig. 3 The simulation result of displacement

Equations of errors are

$$\Delta\ddot{\alpha} + K_D\Delta\dot{\alpha} + K_P\Delta\alpha = 0, \ldots, \Delta\ddot{\gamma} + K_D\Delta\dot{\gamma} + K_P\Delta\gamma = 0 \tag{8}$$

where K_D, K_P are the feedback coefficients. The value feedback gains coefficients are determined according the theory robotic control [15].

According to Eq. (8) the actual accelerations become:

$$\ddot{\alpha} = \ddot{\alpha}_T + K_D(\dot{\alpha}_T - \dot{\alpha}) + K_P(\alpha_T - \alpha), \ldots, \ddot{\gamma} = \ddot{\gamma}_T + K_D(\dot{\gamma}_T - \dot{\gamma}) + K_P(\gamma_T - \gamma) \tag{9}$$

The laws of errors is described oscillatory second-order systems, which control time is minimize. Let us consider an example: the desired laws of the coordinates of the moving platform are $\alpha_T(t) = 0.1\cdot\sin(\omega\ t)$, $\beta_T(t) = 0.11\cdot\sin(\omega\ t)$, $\gamma_T(t) = 0.12\cdot\sin(\omega\ t)$. Moment of inertia $J_\xi = J_\eta = 0.0012$ kg·m^2, $J_\zeta = 0.002$ kg·m^2, $K_D = 120$, $K_P = 7200$. Result of the simulation is presented on Fig. 3. The maximal errors are about $6\cdot10^{-3}$ rad.

5 Accuracy of the Manipulator

The full differential of input-output equation can be written:

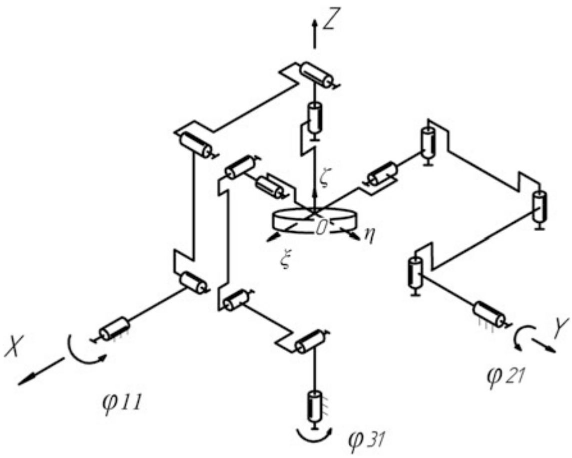

Fig. 4 Spherical mechanism with five kinematic pair in each chain

$$\frac{\partial F_i}{\partial \alpha}\delta\alpha + \frac{\partial F_i}{\partial \beta}\delta\beta + \frac{\partial F_i}{\partial \gamma}\delta\gamma + \frac{\partial F_i}{\partial \theta_{i1}}\delta\theta_{i1} + \frac{\partial F_i}{\partial \theta_{i2}}\delta\theta_{i2} + \frac{\partial F_i}{\partial \varphi_{i1}}\delta\varphi_{i1} = 0 \tag{10}$$

where θ_{i1}, θ_{i2} are the angels between the axes of kinematic pairs (Fig. 1).

According to the linear theory of accuracy, we assume increment in the actuators are equal to zero: $\delta\varphi_{11} = \delta\varphi_{21} = \delta\varphi_{31} = 0$. Therefore the system of equations can be represented as:

$$\frac{\partial F_i}{\partial \alpha}\delta\alpha + \frac{\partial F_i}{\partial \beta}\delta\beta + \frac{\partial F_i}{\partial \gamma}\delta\gamma + \frac{\partial F_i}{\partial \theta_{11}}\delta\theta_{i1} = -\left(\frac{\partial F_i}{\partial \theta_{12}}\delta\theta_{i2} + \frac{\partial F_i}{\partial \varphi_{11}}\delta\varphi_{i1}\right) \tag{11}$$

Thus, knowing the deviations of angles between the axes of kinematic pairs we can determine the deviations of the position of the output link.

For example, if errors are: $\theta_{11} = 0.01$ rad, $\theta_{12} = 0.005$ rad, $\theta_{31} = 0.01$ rad, $\theta_{i2} = 0.0075$ rad, $\theta_{12} = 0.005$ rad, $\theta_{22} = 0.005$ rad, $\theta_{23} = 0.005$ rad, then the deviations of the position of the output link are $\delta\alpha = 0.012$ rad, $\delta\beta = 0.017$ rad, $\delta\gamma = 0.005$ rad.

6 Prototyping of the Manipulator

We considered the mechanism where all the axes of the kinematic pairs intersect at a single point. But one rotating kinematic pair can be replaced by three pairs with parallel axes. [8–10]. Thus, we introduce a mechanism with five kinematic pairs in each kinematic chain [9, 10] (Fig. 4).

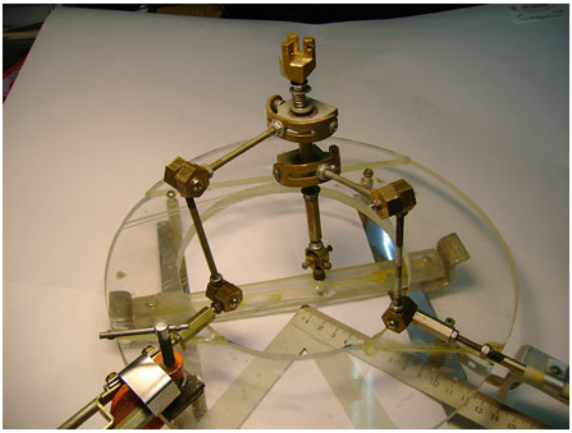

Fig. 5 Prototype of spherical mechanism with three kinematic chains

One of versions of spherical mechanisms with five kinematic pairs in any chain is prototyped One of the versions of spherical mechanism with five kinematic pairs in two kinematic chains and three kinematic pairs in one kinematic chain is prototyped (Fig. 5).

7 Conclusion

In this article, the problem of kinematics and dynamics of the spherical parallel manipulator is considered. Dynamical properties are analyzed applying the virtual work principle. Algorithm of control is based on the inverse problem of dynamics and allows minimizing deviations of coordinates, velocities and accelerations. Our further work will focus on determination of workspace of the prototype (Fig. 5).

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