Investigation of System Reliability Depending on Some System Components States

Elena Zaitseva, Vitaly Levashenko, and Miroslav Kvassay

Faculty of Management Science and Informatics,
University of Zilina, Univerzitna 8215/1, 01026 Zilina, Slovakia
{Elena.Zaitseva,Vitaly.Leavshenko,
Miroslav.Kvassay}@fri.uniza.sk

Abstract. The system reliability depending on some system components states changes is investigated in this paper. This investigation assumes the representation of the initial system by the structure function. This function definition agrees to the Boolean function. Therefore the mathematical approach of Logical Differential Calculus is used in the analysis of the system reliability change depending on the changes of components states. Based on this mathematical approach, calculation of two measures is considered – Dynamic Reliability Indices and Birbaum's Importance Measure. These measures are indices of importance analysis, that allow estimating the system reliability depending on components states changes.

Keywords: Reliability, binary-state system, logical differential calculus.

1 Introduction

Reliability evaluation methods exploit a variety of tools for system modeling and reliability indices calculation. A discrete model has been used in reliability analysis frequently. There are two mathematical types of this model: a *Binary-State System* (BSS) and a *Multi-State System* (MSS). The system and its components are allowed having only two possible states (completely failed and perfect functioning) in a BSS. This approach is well known in reliability analysis, but can prevent the examination of many situations where the system can have more than two distinct states [1-3]. MSS reliability analysis is a more flexible approach for evaluation of system reliability. A MSS and its components are allowed having more than two levels of working efficiency. These levels are interpreted as states of reliability of the system and its components. However, when only consequences of the failure have to be identified, then BSSs are more suitable for this task. In what follows, we assume that the system is presented as a BSS.

Principal measures of BSSs are reliability function that defines probability that system is functional. There are other measures in reliability analysis. Importance measures are one of groups of these measures [4]. Principal goal of the importance analysis is the investigation of influence of different components states changes on the system

W. Zamojski et al. (eds.), *Proceedings of the Ninth International Conference DepCoS-RELCOMEX*, Advances in Intelligent Systems and Computing 286,
DOI: 10.1007/978-3-319-07013-1_49,

reliability. Researchers have developed various methods to calculate importance indices. For example, Markov processes are used to analyze the system state transition process [5] or the structure function approach is used to investigate the system topology [6]. We propose another method that is based on system description by the structure function. This function defines the correlation between system reliability and its components states. The mathematical background of this method is Logical Differential Calculus. This mathematical approach investigates the influence of the variables values on the function value [7].

Two importance measures are considered in this paper. *Component Dynamic Reliability Indices* (CDRIs) allow measuring an influence of each individual component or a fixed group of components on the system reliability. Another importance measure that is very often used in reliability analysis is the *Birnbaum's Importance Measure* (BIM), which defines the probability that given component is critical for system operation [4]. The calculation of these measures for one system component using Logical Differential Calculus has been considered in [8, 9].

In this paper, we develop the investigation of the influence of state changes of more than one component on the system reliability by Direct Partial Logic Derivatives (as the part of Logical Differential Calculus). System failure and its repair caused by changes of some components states are defined in Direct Partial Logic Derivative terminology for two variants of components states changes: for simultaneous and successive changes in the states of components group.

2 Direct Partial Logic Derivatives in Reliability Analysis

2.1 Direct Partial Logic Derivatives

Mathematical approach of Logical Differential Calculus has been introduce for the investigation of the Boolean function value changes depending on the changes of the variables of this function [7]. Therefore, Logical Differential Calculus can be used in applications, where an investigated object is defined and presented by the Boolean function. One of such application problems is reliability analysis. As a rule in reliability analysis, the investigated object is defined as a system with two states that allows analyzing the condition of the system failure and functioning [6, 10]. Such system is named as a *Binary-State System* (BSS).

Consider the system of n components with states x_i ($i = 1,2,..., n$). A system in stationary state will be considered in this paper. At that, the value $x_i = 1$ corresponds to the operable state of the i-th component and $x_i = 0$ to its failure. The correlation of the system state in the fixed time (system availability) and components states is defined by the structure function $\phi(\boldsymbol{x})$:

$$\phi(x_1, x_2, \dots, x_n) = \phi(\boldsymbol{x}): \ \{0,1\}^n \rightarrow \{0,1\}, \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2,\dots, x_n)$ is a state vector.

Every system component is characterized by probabilities of its working and failed state:

$$p_i = \Pr\{x_i = 1\},\ \ q_i = \Pr\{x_i = 0\},\ \ p_i + q_i = 1. \tag{2}$$

The system availability A and unavailability U are defined based on the structure function in the following way:

$$A = \Pr\{\phi(\boldsymbol{x}) = 1\},\ U = \Pr\{\phi(\boldsymbol{x}) = 0\},\ \ A + U = 1. \tag{3}$$

In reliability analysis, the following assumptions are used for the system that is coherent [4]:

(*a*) The structure function can be interpreted as Boolean function,
(*b*) The structure function is monotone,
(*c*) All components are *s*-independent and are relevant to the system.

The assumption (*a*) is very important because it allows using the mathematical approaches of Boolean algebra for the investigation of the system availability. One of such approaches is Logical Differential Calculus [7].

The structure function (1) is a Boolean function; therefore, the mathematical approach of Logical Differential Calculus can be used for the analysis of the influence of component state changes on the system availability. Direct Partial Logic Derivatives are part of Logic Differential Calculus. These derivatives reflect the change in the value of the underlying function when the values of variables change and can be applied for analysis of dynamic behavior of a system that is declared by the structure function (1) according to the assumption (a).

A Direct Partial Logic Derivative with respect to variables vector for the structure function allows estimating the change of system reliability caused by state changes of some system components. These components are interpreted as the vector of components. A Direct Partial Logic Derivative with respect to variables vector for the structure function permits to analyze the system availability change when values of every variable of this vector changes from $\boldsymbol{a}^{(m)}$ to $\overline{\boldsymbol{a}^{(m)}}$. The Direct Partial Logic Derivative of the structure function $\phi(\boldsymbol{x})$ of n variables with respect to vector of components $\boldsymbol{x}^{(m)} = (x_{i_1}, x_{i_2}, \dots, x_{i_m})$ is defined as follows [10]:

$$\partial\phi(j \to \bar{j})/\partial\boldsymbol{x}^{(m)}\left(\boldsymbol{a}^{(m)} \to \overline{\boldsymbol{a}^{(m)}}\right) = \begin{cases} 1, & \text{if } \phi\left(\boldsymbol{a}^{(m)}, \boldsymbol{x}\right) = j \text{ AND } \left(\overline{\boldsymbol{a}^{(m)}}, \boldsymbol{x}\right) = \bar{j} \\ 0, & \text{otherwise} \end{cases}, \tag{4}$$

where $\phi\left(\boldsymbol{a}^{(m)}, \boldsymbol{x}\right) = \phi\left(a_{i_1}, a_{i_2}, \dots, a_{i_m}, \boldsymbol{x}\right)$ is the value of the structure function, when $x_{i_1} = a_{i_1}$, $x_{i_2} = a_{i_2}, \dots,$ $x_{i_m} = a_{i_m}$ and $\phi\left(\overline{\boldsymbol{a}^{(m)}}, \boldsymbol{x}\right) = \phi\left(\overline{a_{i_1}}, \overline{a_{i_2}}, \dots, \overline{a_{i_m}}, \boldsymbol{x}\right)$ is the value of the structure function, when $x_{i_1} = \overline{a_{i_1}}$, $x_{i_2} = \overline{a_{i_2}}, \dots,$ $x_{i_m} = \overline{a_{i_m}}$.

Equation (4) for $m = 1$ is Direct Partial Logic Derivative $\partial\phi(j \to \bar{j})/\partial x_i(a \to \bar{a})$ of the structure function $\phi(\boldsymbol{x})$ with respect to variable x_i. This derivative reflects the fact of changing of function from j to $\bar{j}$ when the value of the variable x_i is changing from a to $\bar{a}$:

$$\partial\phi(j \to \bar{j})/\partial x_i(a \to \bar{a}) = \begin{cases} 1, & \text{if } \phi(a_i, \boldsymbol{x}) = j \text{ AND } \phi(\bar{a}_i, \boldsymbol{x}) = \bar{j} \\ 0, & \text{in other cases} \end{cases}, \tag{5}$$

where $\phi(a_i, \boldsymbol{x}) = \phi(x_1, \dots, x_{i-1}, a, x_{i+1}, \dots, x_n)$ and $\phi(\bar{a}_i, \boldsymbol{x}) = \phi(x_1, \dots, x_{i-1}, \bar{a}, x_{i+1}, \dots, x_n)$.

Direct Partial Logic Derivative (4) is a mathematical description of the system reliability change depending on state changes of the fixed system components. This derivative is used to investigate system reliability change when states of the fixed system components change.

2.2 System Failure

Consider the system failure and repair depending on some system components states changes in terms of Direct Partial Logic Derivatives. There are two types of changes of components states for fixed system components: simultaneous states changes and state changes one by one.

The first variant, the simultaneous state changes, is represented using Direct Partial Logic Derivative terminology as follows [11, 12]:

$$\partial\phi(1 \to 0)/\partial\boldsymbol{x}^{(m)}\left(\mathbf{1}^{(m)} \to \mathbf{0}^{(m)}\right), \tag{6}$$

where $\mathbf{1}^{(m)} = \left(1_{i_1}, 1_{i_2}, \ldots, 1_{i_m}\right)$ and $\mathbf{0}^{(m)} = \left(0_{i_1}, 0_{i_2}, \ldots, 0_{i_m}\right)$.

The assumption (b) that the structure function is monotone is used in (6). Therefore, the system failure is declared by change of function $\phi(\boldsymbol{x})$ from state 1 to state 0 and only decreases of every of m system components from state 1 to state 0 is taken into account. This assumption is also used in the second variant of mathematical description of the system failure.

The second variant of the system failure (system components fail one by one, if they can) is represented as m-times Direct Partial Logic Derivative:

$$\partial^m\phi(1 \to 0)/\partial x_{i_1}(1 \to 0)\partial x_{i_2}(1 \to 0) \ldots \partial x_{i_m}(1 \to 0). \tag{7}$$

The m-times derivative (7) is calculated by successive computation of Direct Partial Logic Derivatives with respect to variable x_{i_s}, for $s = 1,2,\ldots, m$:

$$\partial\phi_{s-1}(1 \to 0)/\partial x_{i_s}(1 \to 0),$$

where the initial function for s-step is defined as $\phi_s(\boldsymbol{x}) = \phi\left(0_{i_1}, 0_{i_2}, \ldots, 0_{i_s}, \boldsymbol{x}\right)$ and $\phi_0(\boldsymbol{x}) = \phi(\boldsymbol{x})$.

For example, consider the failure of system that is presented in Fig. 1. This system consists of three components ($n = 3$) and its structure function is defined as $\phi(\boldsymbol{x})$ = OR(AND(x_1, x_2), x_3). The two aforementioned variants of the failure of this system when component 1 or 2 failed are presented in Table 1, where symbol '-' means that the Direct Partial Logic Derivative does not exist.

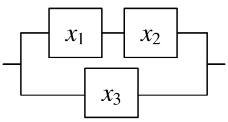

Fig. 1. Series-parallel system

In Table 1, Direct Partial Logic Derivative $\partial\phi(1 \to 0)/\partial\boldsymbol{x}^{(2)}\big((1_1, 1_3) \to (0_1, 0_3)\big)$ models situations, in which the simultaneous breakdown of the 1-st and the 3-rd component causes the system failure. In this case, the system has two boundary states for components $x_1x_2x_3$: {101, 111}. The second Direct Partial Logic Derivative $\partial^2\phi(1\to0)/\partial x_3(1\to0)\partial x_1(1\to0)$ describes system failure when the 3-rd component breaks firstly and then the 1-st component (if can, i.e. if the first component is not failed). The system for this variant of the failure has the following boundary states $x_1x_2x_3$: {001, 011, 101, 111}. The last Direct Partial Logic Derivative $\partial^2\phi(1\to0)/\partial x_1(1\to0)\partial x_3(1\to0)$ describes system failure when component 1 breaks as first and component 3 as the next one. There are three situations $x_1x_2x_3$: {110, 101, 111} in which successive failures of the first and the third components cause the system breakdown (if the third component can fail).

Table 1. The structure function of the system in Fig. 1 and two variants of its failure

x_1	x_2	x_3	$\phi(\boldsymbol{x})$	$\frac{\partial\phi(1 \to 0)}{\partial\boldsymbol{x}^{(2)}\big((1_1, 1_3) \to (0_1, 0_3)\big)}$	$\frac{\partial^2\phi(1 \to 0)}{\partial x_3(1 \to 0)\partial x_1(1 \to 0)}$	$\frac{\partial^2\phi(1 \to 0)}{\partial x_1(1 \to 0)\partial x_3(1 \to 0)}$
0	0	0	0	-	-	-
0	0	1	1	-	1	-
0	1	0	0	-	-	-
0	1	1	1	-	1	-
1	0	0	0	-	-	0
1	0	1	1	1	1	1
1	1	0	1	-	-	1
1	1	1	1	1	1	1

2.3 System Repair

The system repair, using Direct Partial Logic Derivatives, has been defined for state change of one system component in [10] and for state changes of fixed system components in [11]:

$$\partial\phi(0 \to 1)/\partial\boldsymbol{x}^{(m)}\big(\boldsymbol{0}^{(m)} \to \boldsymbol{1}^{(m)}\big), \tag{8}$$

where $\boldsymbol{0}^{(m)} = \big(0_{i_1}, 0_{i_2}, \dots, 0_{i_m}\big)$ and $\boldsymbol{1}^{(m)} = \big(1_{i_1}, 1_{i_2}, \dots, 1_{i_m}\big)$.

The system repair in Direct Partial Logic Derivative terminology (8) is declared as the structure function change from value 0 into 1, when states of m failed system components change from 0 into 1.

Here, we present state changes of fixed system components one by one for system repair in Direct Partial Logic Derivative terminology as m-times Direct Partial Logic Derivative:

$$\partial^m\phi(0 \to 1)/\partial x_{i_1}(0 \to 1)\partial x_{i_2}(0 \to 1) \dots \partial x_{i_m}(0 \to 1). \tag{9}$$

The computation of derivative (9) is similar to calculation of derivative (7).

3 Importance Measures

Importance measures are very often used in reliability analysis. They estimate coincidence between component failure (repair) and system failure (repair). In this section, we generalize some of them for the simultaneous or successive failure (repair) of components group.

3.1 Component Dynamic Reliability Indices

In this paper, the concept of the CDRIs is generalized for the reliability changes that are caused by state changes (simultaneous or successive) of a group of system components. The basic mathematic model for these indices is based on (6) – (9).

Definition 1. The CDRI of the first type of a group of *m* components for the system failure is probability of the system failure caused by simultaneous failure of given system components:

$$P_{1f}\left(\boldsymbol{x}^{(m)}\right) = \frac{\rho_{1f}}{\rho_1} \prod_{j=1}^{m} q_{i_j}, \tag{10}$$

where ρ_{1f} is a number of situations when the breakdown of m system components results the failure of the system; ρ_1 is a number of operational system states when $\phi\left(1_{i_1}, 1_{i_2}, \ldots, 1_{i_m}, \boldsymbol{x}\right) = 1$ (it is computed by the structure function); and q_{i_j} is the failed state of component i_j (2).

Number ρ_{1f} is calculated as a number of nonzero elements of Direct Partial Logic Derivative (6):

$$\rho_{1f} \equiv \partial\phi(1 \to 0)/\partial\boldsymbol{x}^{(m)}\left(\mathbf{1}^{(m)} \to \mathbf{0}^{(m)}\right) \neq 0. \tag{11}$$

Definition 2. The CDRI of the second type of a group of *m* components for the system failure is probability of the system failure caused by successive failure of given system components:

$$P_{2f}\left(\boldsymbol{x}^{(m)}\right) = \sum_{s=1}^{m} \frac{\rho_{f,i_s}}{\rho_{1,i_s}} p_{i_s} \prod_{j=1}^{s-1} q_{i_j}, \tag{12}$$

where ρ_{1,i_s} is a number of system states when $x_{i_1} = x_{i_2} = \cdots = x_{i_s} = 1$ and $\phi(\boldsymbol{x}) = 1$; q_{i_j} is the i_j-th component unreliability and p_{i_s} is the i_s-th component reliability according to (2); ρ_{f,i_s} is a number of system states for which breakdown of s components, from the i_1-th to the i_s-th, cause system failure and it is calculated by the structure function as a number of nonzero elements of Direct Partial Logic Derivative (7).

Definition 3. The CDRI of the first type of a group of *m* components for the system repair is probability of the system repair caused by simultaneous replacements of *m* failed system components:

$$P_{1r}\left(\boldsymbol{x}^{(m)}\right) = \frac{\rho_{1r}}{\rho_0}\prod_{j=1}^{m} p_{i_j}, \tag{13}$$

where ρ_{1r} is a number of system states when the simultaneous replacements of m failed system components causes the system repair and it is calculated by Direct Partial Logic Derivative (8):

$$\rho_{1r} \equiv \partial\phi(0 \to 1)/\partial\boldsymbol{x}^{(m)}\left(\boldsymbol{0}^{(m)} \to \boldsymbol{1}^{(m)}\right) \neq 0, \tag{14}$$

and ρ_0 is a number of zero system states for which $\phi\left(0_{i_1}, 0_{i_2}, \dots, 0_{i_m}, \boldsymbol{x}\right) = 0$ and q_{ij} is the probability that component i_j is failed (2).

Definition 4. The CDRI of the second type of a group of m components for the system repair is probability of system repair caused by successive replacements of m failed system components:

$$P_{2r}\left(\boldsymbol{x}^{(m)}\right) = \sum_{s=1}^{m} \frac{\rho_{r,i_s}}{\rho_{0,i_s}} q_{i_s} \prod_{j=1}^{s-1} p_{i_j}, \tag{15}$$

where $\rho_{0,is}$ is a number of situations when $x_{i_1} = x_{i_2} = \cdots = x_{i_s} = 0$ and $\phi(\boldsymbol{x}) = 0$; p_{i_j} is the i_j-th component reliability and q_{i_s} is the i_s-th component unreliability according to (2); ρ_{r,i_s} is a number of system states for which replacements of system components, from the i_1-th to the i_s-th, cause the system repair and ρ_{r,i_s} is calculated by the system structure function and Direct Partial Logic Derivative (9) as a number of its nonzero elements.

3.2 The Birnbaum's Importance Measure

The *Birnbaum's Importance Measure* (BIM) defines probability that given component is critical for system operation, i.e. its failure causes the system failure [4]. In papers [8, 11], there has been proposed the definition of the BIM for component i as the probability that Direct Partial Logic Derivative $\partial\phi(1 \to 0)/\partial x_i(1 \to 0)$ contains nonzero values:

$$BIM(x_i) = \Pr\{\partial\phi(1 \to 0)/\partial x_i(1 \to 0) = 1\}. \tag{16}$$

Generalizations of the BIM for two system components have been considered in paper [13]. Those generalizations are known as Joint Reliability (Failure) Importance Measures and they represent the degree of interactions between two system components. However, they do not estimate the probability that simultaneous or successive failures of some components cause the system failure. Therefore, in this paper, the definition of the BIM is generalized for these cases. We denote these generalizations as the *Modified BIM* (MBIM).

Definition 5. The MBIM of the first type of a group of m components for the system failure is probability that the simultaneous failure of the fixed group of system components results the failure of the system:

$$MBIM_1\left(\boldsymbol{x}^{(m)}\right) = \Pr\left\{\partial\phi(1 \to 0)/\partial\boldsymbol{x}^{(m)}\left(\mathbf{1}^{(m)} \to \mathbf{0}^{(m)}\right) = 1\right\}. \quad (17)$$

Definition 6. The MBIM of the second type of a group of m components for the system failure is probability that the successive failures of the fixed system components cause the system failure:

$$MBIM_2\left(\boldsymbol{x}^{(m)}\right) = \sum_{s=1}^{m} \Pr\left\{\frac{\partial\phi_{s-1}(1\to0)}{\partial x_{is}(1\to0)} = 1\right\} \prod_{j=1}^{s-1} \Pr\left\{\frac{\partial\phi_{j-1}(1\to0)}{\partial x_{ij}(1\to0)} = 0\right\}, \quad (18)$$

where $\phi_s(\boldsymbol{x}) = \phi\left(0_{i_1}, 0_{i_2}, \dots, 0_{i_s}, \boldsymbol{x}\right)$ and $\phi_0(\boldsymbol{x}) = \phi(\boldsymbol{x})$. The assumption ($c$) for the structure function, i.e. all components are independent and relevant to the system, must hold for the definition (18) of the MBIM of the second type.

3.3 Examples for Calculation of the CDRIs and the BIM

Consider the example in Fig. 1 and calculation of the CDRIs and the MBIM for it. Let the probabilities of the states of components be the ones in Table 2. The structure function of this system is:

$$\phi(\boldsymbol{x}) = \mathrm{OR}(\mathrm{AND}(x_1, x_2), x_3). \quad (19)$$

Let us calculate the probabilities of this system failure if two components of it fail. The CDRIs for this system failure are determined according to (10) and (12).

Table 2. State probabilities of components of the system in Fig. 1

	Components		
	x_1	x_2	x_3
q_i	0.03	0.12	0.03
p_i	0.97	0.88	0.97

The CDRIs of the first type $P_{1f}(\boldsymbol{x}^{(2)})$ (for two simultaneous components breakdown) for this system are: $P_{1f}(x_1, x_2) = 0.4268$, $P_{1f}(x_1, x_3) = 0.9409$ and $P_{1f}(x_2, x_3) = 0.8536$.

Numbers ρ_{1f} for different changes of system components (11) are determined by derivatives $\partial\phi(1 \to 0)/\partial\boldsymbol{x}^{(2)}\left(\left(1_{i_1}, 1_{i_2}\right) \to \left(0_{i_1}, 0_{i_2}\right)\right)$ and number ρ_1 is computed by the structure function of this system.

The CDRIs of the second type for the system in Fig. 1 are calculated for successive components breakdowns by (12) and this equation in this case is:

$$P_{2f}\left(\boldsymbol{x}^{(2)}\right) = \frac{\rho_{f,i_1}}{\rho_{1,i_1}} p_{i_1} + \frac{\rho_{f,i_2}}{\rho_{1,i_2}} q_{i_1} p_{i_2}, \quad (20)$$

where ρ_{1,i_1} is a number of system states where $x_{i_1} = 1$ and $\phi(\boldsymbol{x}) = 1$; ρ_{1,i_2} is a number of system states where $x_{i_1} = x_{i_2} = 1$ and $\phi(\boldsymbol{x}) = 1$; ρ_{f,i_1} is a number of state vectors for which the i_1-th component breakdown causes system failure and ρ_{f,i_2} is a number of state vectors for which the i_1-th and the i_2-th components breakdowns cause system failure; p_{i_1} (q_{i_1}) and p_{i_2} are the i_1-th and the i_2-th components reliabilities (unreliability) according to (2).

Numbers ρ_{1,i_1} and ρ_{1,i_2} in (20) are computed by structure function (19) of the system. Numbers ρ_{1,i_1} and ρ_{1,i_2} are determined by Direct Partial Logic Derivative (7) as a number of its nonzero elements.

The CDRIs of the second type $P_{2f}(\boldsymbol{x}^{(2)})$ for this system are calculated for successive components breakdowns by (20) as follows: $P_{2f}(x_1, x_2) = 0.3233$, $P_{2f}(x_2, x_1) = 0.2933$, $P_{2f}(x_1, x_3) = 0.3524$, $P_{2f}(x_3, x_1) = 0.7421$, $P_{2f}(x_2, x_3) = 0.4097$ and $P_{2f}(x_3, x_2) = 0.7407$. These numbers show that the most probable scenarios which lead into the failure of the system are successive failures of the 3-rd and the 1-st component or successive failures of components 3 and 2.

The MBIM of the first type (for two simultaneous components breakdown) for this system are: $MBIM_1(x_1, x_2) = 0.03$, $MBIM_1(x_1, x_3) = 1$ and $MBIM_1(x_2, x_3) = 1$. These numbers mean that the simultaneous failure of component 1 and 3 or 2 and 3 causes the failure of the system regardless to state of another component.

The MBIM of the second type are computed for this system according to (18) as follows:

$$MBIM_2\left(\boldsymbol{x}^{(2)}\right) = \Pr\left\{\frac{\partial\phi(1\to 0)}{\partial x_{i1}(1\to 0)} = 1\right\} + \Pr\left\{\frac{\partial\phi(1\to 0)}{\partial x_{i1}(1\to 0)} = 0\right\}\Pr\left\{\frac{\partial\phi_1(1\to 0)}{\partial x_{i2}(1\to 0)} = 1\right\}, \quad (21)$$

where $\phi_1(\boldsymbol{x}) = \phi\left(0_{i_1}, \boldsymbol{x}\right)$.

Using (21), we get the next values of the MBIM of the second type for the system in Fig. 1: $MBIM_2(x_1, x_2) = 0.0264$, $MBIM_2(x_1, x_3) = 1$ and $MBIM_2(x_2, x_3) = 1$. These numbers mean that the successive failure of component 1 and 3 or 2 and 3 causes the failure of the system every time, i.e. regardless to state of another component.

4 Conclusion

Two new importance measures are proposed in this paper: extension of the CDRI and the BIM. The CDRI is used to investigate the system availability depending on the components states changes. New type of the BIM has been denoted as the Modified BIM, because it does not estimate the overall criticality of given group of components for the system activity but only the influence of their simultaneous or successive failures.

The novelty of the proposed measures consists of two aspects. One of them is investigation of influence of some fixed components states changes on the system availability. As a rule, importance measures are defined for only one system component. In this work, the simultaneous or successive components states changes are considered.

Another aspect of the novelty is using of the Direct Partial Logic Derivatives with respect to the vector of variables for calculation of these measures. The advantages of this mathematical approach are: (a) the application for analysis of system with any complexity and (b) the simplicity of the calculation. The computational complexity of the Direct Partial Logic Derivatives calculation depends on the number of system components only and is not influenced by other topological complexity of the system. The Direct Partial Logic Derivatives are calculated based on the comparison of the values of the structure function only, and therefore, their calculation is very simple.

Logical Differential Calculus is very perspective for analysis of dynamic properties of the investigated system. Its potential in reliability analysis can increase in combination with Logical Integral Calculus [14] that allows revealing the structure function when it is not completely defined. However, this idea needs another research.

Acknowledgement. This work was partially supported by the grant of Slovak Research and Development Agency SK-PL-0023-12 and the grant of 7th RTD Framework Program No 610425 (RASimAs).

References

1. Lisnianski, A., Levitin, G.: Multi-State System Reliability. Assessment, Optimization and Applications. World Scientific (2003)
2. Xie, M., Dai, Y.-S., Poh, K.-L.: Multi-State System Reliability. In: Xie, M., Dai, Y.-S., Poh, K.-L. (eds.) Computing System Reliability. Models and Analysis, pp. 207–237. Kluwer Academic Publishers (2004)
3. Shooman, M.L.: Reliability of Computer Systems and Networks: Fault Tolerance, Analysis, and Design. John Wiley & Sons, Inc. (2002)
4. Kuo, W., Zhu, X.: Importance Measures in Reliability, Risk and Optimization. John Wiley & Sons, Ltd. (2012)
5. Fricks, R.M., Trivedi, K.S.: Importance analysis with Markov chains. In: Proc. of Reliability and Maintainability Annual Symposium, pp. 89–95 (2003)
6. Armstrong, M.J.: Reliability-importance and dual failure-mode components. IEEE Trans. Reliability 46, 212–221 (1997)
7. Akers, S.B.: On a Theory of Boolean Functions. J. Soc. Ind. Appl. Math. 7, 487–498 (1959)
8. Zaitseva, E., Levashenko, V.: Importance Analysis by Logical Differential Calculus. Automation and Remote Control 74(2), 171–182 (2013)
9. Moret, B.M.E., Thomason, M.G.: Boolean Difference Techniques for Time-Sequence and Common-Cause Analysis of Fault-Trees. IEEE Trans Reliability R-33, 399–405 (1984)
10. Zio, E.: An Intriduction to the Basics of reliability and Risk Analysis. World Scientific (2007)
11. Zaitseva, E., Puuronen, S.: Estimation of Multi-State system reliability depending on changes of some system component efficiencies. In: Proc. of European Safety and Reliability Conference (ESREL 2007), pp. 253–261. Taylor & Francis Group (2007)
12. Kvassay, M., Levashenko, V.: Birnbaum Importance for Estimation of Multi-state and Binary-state Systems. Radioelectronic and Computer Systems 64(5), 261–266 (2013)
13. Hong, J.S., Lie, C.H.: Joint Reliability-importance of Two Edges in an Undirected Network. IEEE Trans. on Reliability 42(1), 17–23, 33 (1993)
14. Tucker, J.H., Tapia, M.A., Bennett, A.W.: Boolean Integral Calculus for Digital Systems. IEEE Trans. on Computers C-34(1), 78–81 (1985)