Branch-and-Price on the Split Delivery Vehicle Routing Problem with Time Windows and Alternative Delivery Periods

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Abstract In this article we address the Split Delivery Vehicle Routing Problem with Time Windows and alternative Periods (SDVRPTWA). The consideration of multiple delivery periods per customer and the possibility of splitting deliveries across different periods makes it a relaxation of the well-known Vehicle Routing Problem with Time Windows and Split Deliveries (VRPTWSD). The problem is solved by a branch-and-price method. The opportunity for freight forwarders is to plan more efficient tours by exploiting alternative delivery periods. The contribution of this article is to prove the potential of this approach for cost savings and to demonstrate the decomposition of a SDVRPTWA in a demand focused master problem and period related pricing problems.

1 Introduction

Orders exceeding the vehicle capacity and customer induced time windows are common challenges in transport planning. As long as time windows are fix and the customer does not offer alternatives the problem can be solved with the well known VRPTWSD. In case that alternative non excluding delivery periods are offered, it would be possible to split the delivery across different periods. To our knowledge, this scenario has not been considered in the literature yet. Indeed, this methodology becomes more and more relevant as a growing number of companies introduces time window management systems requiring freight forwarders to book binding time windows. To face this requirement, the SDVRPTWA considers the possibility of serving

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D. Huisman et al. (eds.), *Operations Research Proceedings 2013*,
Operations Research Proceedings, DOI: 10.1007/978-3-319-07001-8_9,

a customer either in one or in multiple periods. Thereby, it makes the selection of delivery periods subject of the optimization and offers the opportunity to plan tours with visits in alternative periods.

2 Related Problems

Research on the Vehicle Routing Problem with Time Windows (VRPTW) came up with [14] and [5] and with [9] as one of the most recent publications on solving the problem by column generation.

Splitting of deliveries was introduced by Dror and Trudeau [8]. A worst case analysis of split delivery problems was performed by Archetti et al. [1]. Archetti et al. [2] developed an efficient procedure to solve the Vehicle Routing Problem with Split Deliveries by column generation.

Frizzell and Giffin [10] were among the first authors examining the combination of split deliveries and time windows. The most relevant results to solve this problem by Branch-and-Price were presented by Gendreau et al. [11] and Desaulniers [6]. The two approaches differ mainly in regard to the decision, if quantities are considered within the pricing problem.

A good overview in general on the VRPTW can be found in [7] and in [3].

Cordeau [4] presented a Periodic Vehicle Routing Problem (PRVP). Here, each customer selects an individual combination of days. Visiting this customer is only possible at the selected combination of days. So, the possible combinations are input parameters for the program an not part of the optimization itself. Pirkwieser and Raidl [12] solved this problem by a column generation approach. In addition to a set-partioning formulation they introduce multiple pricing problems, one for each period. Their approach, anyhow, does not ensure integer solutions.

3 Problem Formulation

Given a set $\mathbb{V}$ which represents the nodes of the problem. Node 0 is the depot where all tours start and end. Set $\mathbb{I}$ represents the customers to deliver with $\mathbb{I} = \mathbb{V} \backslash 0$. $\mathbb{A}$ is the set of all arcs $(i, j) \in \mathbb{V} \times \mathbb{V}$. With $\mathbb{V}$ and $\mathbb{A}$ we can define a directed Graph $\mathbb{G} = (\mathbb{V}, \mathbb{A})$.

Additionally each customer defines which periods are valid to deliver goods. The set of periods is $\mathbb{P}$ and a customer i can choose any combination of periods $\mathbb{P}_i \subseteq \mathbb{P}$ to deliver him. Any delivery can have a split either within a time window in the same period or within time windows in different periods. These customer specific periods specify real alternatives. In an example this means: if $\mathbb{P} = [1, 2, 3, 4, 5]$ a customer i can choose $\mathbb{P}_i = [2, 3, 4]$ as alternative periods to receive deliveries. A possible alternative could be to split up the delivery and to visit this customer in period 2 with the first part of the delivery and visiting him in period 4 to bring the last part of the delivery. This formulation is a relaxation of [4] and adds also the opportunity of split deliveries.

The problem can be written as a decomposed problem with a single Master Problem 3.1 and Sub Problems 3.2 for each period $p \in \mathbb{P}$.

All Sub Problems have to be solved separately for each period.

3.1 Master Problem

$\mathbb{R}$ is the set of available routes r in the Master Problem defined. $\mathbb{T}$ is the set of available delivery patterns t.

The decision variable is λ_{rt}^{p} which indicates the usage of a route $r \in \mathbb{R}$ in $p \in \mathbb{P}$ and a delivery pattern $t \in \mathbb{T}$. Each route has a length e_r. Constraints 2 ensure that each customer receives the complete demand d_i. ρ_{it} represents the delivery for customer i is visited in tour r with pattern t. The delivery pattern t is related to λ_{rt}^{p}. In constraints 3 we substitute each λ_{rt}^{p} with an variable y_{ij}^{p}, introduced by Desaulniers [6]. β_{ijr}^{p} is set to 1 if arc (i, j) is used in λ_{rt}^{p}, it is set to 0 otherwise. Variables y_{ij}^{p} will be used to perform branches (see Sect. 3.3). These variables represent how often an arc (i, j) is used in total or by period p as well as how often a customer i is visited in total or by period p. For all arcs between customers the usage is limited to $y_{ij}^{p} \leq 1$. For all arcs leaving/arriving at the depot usage of y_{ij}^{p} is unlimited and any value $\geq$ 0.4 is non-negative.

$$\min \left(\sum_{r\in\mathbb{R}, p\in\mathbb{P}, t\in\mathbb{T}} e_r \cdot \lambda_{rt}^{p} \right) \tag{1}$$

$$\text{s.t.} \sum_{r\in\mathbb{R}, p\in\mathbb{P}, t\in\mathbb{T}} \rho_{it} \cdot \lambda_{rt}^{p} \geq d_i \qquad \forall i \in \mathbb{I} \tag{2}$$

$$\sum_{r\in\mathbb{R}, t\in\mathbb{T}} \beta_{ijr}^{p} \lambda_{rt}^{p} = y_{ij}^{p} \quad \forall (i, j) \in \mathbb{V}, p \in \mathbb{P} \tag{3}$$

$$\lambda_{rt}^{p} \geq 0 \; \forall r \in \mathbb{R}, p \in \mathbb{P}, t \in \mathbb{T} \tag{4}$$

We start the solution procedure with a small set of columns in the Master Problem. These columns represent single trips from the depot to one customer and back to the depot. Also we add corresponding delivery patterns to those initial variables. With this first set of variables we start to generate additional variables λ_{rt}^{p} in a column generation process.

3.2 Pricing Problem

To get new valid routes for the Master Problem we solve an Elementary Shortest Path Problem with Ressource Constraints (ESPPRC) and include in the objective function the dual values of the Master Problem solved. We stop generating new λ_{rt}^{p} when no more columns with negative reduced cost can be found.

The objective of the problem is to find a route with lowest costs. α_{ij}^{p} and δ_i are the dual variables from the constraints 3 and 2 from the Master Problem presented. Variables x_{ij}^{p} decide if arc (i, j) is used in period p and are binary. Variables d_i^p decide which amount of the complete order d_i is delivered to customer i in period p and is integer. s_i^p and s_j^p, respectively, decide in which period p customers i, j are visited and are real values.

The distance between two nodes is c_{ij}. We have an unlimited set of identical vehicles T with capacity Q. Each customer $i \in \mathbb{I}$ has a demand $d_i > 0$ and a time window to deliver the goods. The customer specific interval to deliver starts at s_i^{start} and ends at s_i^{end}.

$$\min \left(\sum_{(i,j)\in\mathbb{V}} (c_{ij} + \alpha_{ij}^{p}) \cdot x_{ij}^{p} - \sum_{i\in\mathbb{I}} (\delta_i \cdot d_i^p) \right) \tag{5}$$

$$\text{s.t.} \sum_{i\in\mathbb{I}} x_{oj}^{p} = 1 \tag{6}$$

$$\sum_{j\in\mathbb{V}|i\neq j} (x_{ij}^{p} - x_{ji}^{p}) = 0 \qquad \forall i \in \mathbb{V} \tag{7}$$

$$\sum_{j\in\mathbb{I}} x_{i0}^{p} = 1 \tag{8}$$

$$\sum_{i\in\mathbb{I}} d_i^p \leq Q \tag{9}$$

$$\min(d_i; Q) \sum_{j\in\mathbb{V}|i\neq j} x_{ij}^{p} \geq d_i^p \qquad \forall i \in \mathbb{I} \tag{10}$$

$$s_i^{start} \leq s_i^p \leq s_i^{end} \qquad \forall i \in \mathbb{V}, p \in \mathbb{P}, t \in \mathbb{T} \tag{11}$$

$$s_i^p + b_{ij} - M(1 - x_{ijt}^{p}) \leq s_j^p \; \forall (i, j) \in \mathbb{V}, p \in \mathbb{P}, t \in \mathbb{T} \tag{12}$$

$$x_{ij}^{p} \in \{0; 1\} \qquad \forall (i, j) \in \mathbb{V} \tag{13}$$

$$d_i^p \in \{0, 1, \ldots\} \qquad \forall i \in \mathbb{I} \tag{14}$$

3.3 Branching Rules

When no more columns with negative reduced costs can be found, we test the solution of integrity. The solution is integral when the following branching rules deliver values which are integral. (i) branch on the vehicles used in total: $\sum_{r\in\mathbb{R},p\in\mathbb{P},t\in\mathbb{T}}\lambda_{rt}^{p}$ (ii) branch on the customers visited in total: $\sum_{p\in\mathbb{P},j\in\mathbb{V}}y_{ij}^{p}$ (iii) branch on the arcs used in total: $\sum_{p\in\mathbb{P}}(y_{ij}^{p}+y_{ji}^{p})$ (iv) branch on the vehicles used by period: $\sum_{r\in\mathbb{R},t\in\mathbb{T}}\lambda_{rt}^{p}$ (v) branch on the customers visited by period: $\sum_{j\in\mathbb{V}}y_{ij}^{p}$ (vi) branch on the arcs used by period: $(y_{ij}^{p}+y_{ji}^{p})$ (vii) branch on consecutive arcs: in [13] and [6] it is stated that also consecutive arcs are possible which generate non-integer solutions. In this case we branch on those consecutive arcs. However, those branches are only rarely necessary.

4 Results

The results are based on self generated instances for the problem. We created instances for up to 17 customers. The characteristics of the instances are specified by the available alternative periods and the demand of a customer.

Regarding the periods there are instances which allow (i) only a delivery at a single period p, (ii) at periods p or p − 1 and/or (ii) at periods p, p − 1 and/or p − 2. These instances are denoted as **s**ingle, **t**ight or **w**ide period instances.

Regarding the demand we created instances where (i) all demands do not exceed the capacity of a vehicle ($d_i \leq Q$), (ii) all demands exceed the capacity of a vehicle ($d_i > Q$) and (iii) the demands are a mix of the former ones. This is denoted by **d**eceeding, **e**xceeding or **m**ixed.

In an example instance 05dw is an instance with 5 customers, where demand does not exceed the capacity of a vehicle and three alternative periods for delivery are given.

Each of the instances was created for 4 different customer locations with different time windows, denoted by testset a–d.

The results are summarized in Table 1. The basis for the comparision of the savings is the related instance with a single period for delivery. For testinstance 07 dt (7 customers with a demand less or equal to the capacity of the vehicle and alternative periods for a delivery in p or p − 1 there are minimal savings of 5 % possible and maximal 15 % compared with instance 07 ds, where no alternative periods are allowed.

Further, the results show decreasing savings from deceeding instances to exceeding instances. The results show also that more alternative periods promise more savings. This is due to the fact that the forwarder has more alternatives to schedule an optimal transport plan.

Detailed results can be found in the appendix of this article.

Table 1 Minimal and maximal possible savings of the traveling distance

Demand	Periods		Customers						
			5 (%)	7 (%)	9 (%)	11 (%)	13 (%)	15 (%)	17 (%)
Deceeding	Tight	Min	0	5	8	2	7	15	0
		Max	13	15	20	20	19	22	10
	Wide	Min	6	11	9	2	14	21	1
		Max	24	20	23	20	32	32	23
Mixed	Tight	Min	0	4	6	3	6	10	/
		Max	9	12	15	16	14	17	/
	Wide	Min	4	5	10	3	7	10	/
		Max	14	15	22	19	20	26	/
Exceeding	Tight	Min	0	0	2	4	/	/	/
		Max	6	10	9	5	/	/	/
	Wide	Min	0	1	5	4	/	/	/
		Max	6	13	11	6	/	/	/

5 Conclusion

We presented in this article a Vehicle Routing Problem which includes Time Windows, Split Deliveries and alternative delivery periods. To the best of our knowledge this combination of restrictions/relaxations was never reviewed before.

Compared with problems without alternative delivery periods, using alternative delivery periods can achieve savings up to 32 % in our test instances. The idea of route scheduling with alternative periods is relevant in practice when freight forwarders have to book binding time windows at their destination

Appendix

In this section we present our solutions to the problem. We created instances with up to 17 customers

Table 2 Traveling distance for instances with 5–9 customers

Customers	Scenario	Testset a		Testset b		Testset c		Testset d	
		Distance	Savings (%)	Distance	Savings (%)	Distance	Savings (%)	Distance	Savings (%)
5	ds	216.56	0.00	229.65	0.00	158.21	0.00	253.83	0.00
	dt	187.68	13.34	229.65	0.00	158.21	0.00	223.16	12.08
	dw	164.77	23.91	193.59	15.70	148.9	5.88	222.72	12.26
	ms	366.5	0.00	340.59	0.00	231.15	0.00	364.75	0.00
	mt	335.32	8.51	340.59	0.00	231.15	0.00	333.64	8.53
	mw	314.71	14.13	304.53	10.59	221.84	4.03	333.64	8.53
	es	547.14	0.00	523.39	0.00	427.59	0.00	678.17	0.00
	et	546.4	0.14	523.39	0.00	427.59	0.00	637.73	5.96
	ew	523.82	4.26	505.27	3.46	425.9	0.40	637.73	5.96
7	ds	336.95	0.00	241.89	0.00	270.42	0.00	326.6	0.00
	dt	306.3	9.10	229.4	5.16	256.54	5.13	276.89	15.22
	dw	270.13	19.83	215.79	10.79	240.19	11.18	265.22	18.79
	ms	464.01	0.00	325.93	0.00	366.22	0.00	416.92	0.00
	mt	434.1	6.45	310.08	4.86	352.34	3.79	367.21	11.92
	mw	397.19	14.40	285.03	12.55	347.23	5.19	355.54	14.72
	es	814.76	0.00	667.92	0.00	717.65	0.00	745.4	0.00
	et	800.81	1.71	640.27	4.14	717.65	0.00	669.24	10.22
	ew	752.83	7.60	640.27	4.14	710.3	1.02	649.28	12.90
9	ds	319.16	0.00	393.64	0.00	409.67	0.00	342.89	0.00
	dt	270.56	15.23	358.53	8.92	377.58	7.83	275.25	19.73
	dw	253.51	20.57	358.53	8.92	353.85	13.63	264.19	22.95
	ms	607.82	0.00	622.14	0.00	713.35	0.00	544.61	0.00
	mt	557.53	8.27	587.03	5.64	668.01	6.36	461.83	15.20
	mw	511.05	15.92	553.14	11.09	641.91	10.01	424.8	22.00
	es	889.31	0.00	938.62	0.00	1,026.37	0.00	899.98	0.00
	et	807.35	9.22	918.22	2.17	979.8	4.54	849.77	5.58
	ew	792.1	10.93	890.47	5.13	976.34	4.87	824.04	8.44

Table 3 Traveling distance for instances with 11–17 customers

Customers	Scenario	Testset a		Testset b		Testset c		Testset d	
		Distance	Savings (%)	Distance	Savings (%)	Distance	Savings (%)	Distance	Savings (%)
11	ds	333.42	0.00	342.21	0.00	417.31	0.00	369.88	0.00
	dt	324.84	2.57	274.87	19.68	377.99	9.42	362.97	1.87
	dw	296.64	11.03	274.73	19.72	354.84	14.97	362.97	1.87
	ms	497.43	0.00	450.73	0.00	611.99	0.00	549.74	0.00
	mt	469.51	5.61	378.31	16.07	568.13	7.17	533.88	2.89
	mw	446.32	10.27	365.37	18.94	552.9	9.66	533.88	2.89
	es	940.98	0.00	863.31	0.00	1,163.74	0.00	962.35	0.00
	et	905.23	3.80		–	1,105.36	5.02		–
	ew	905.98	3.72		–	1,093.1	6.07		–
13	ds	588.4	0.00	500.96	0.00	587.13	0.00	471.3	0.00
	dt	492.46	16.31	438.46	12.48	545.1	7.16	380.24	19.32
	dw	401.11	31.83	438.46	12.48	505.71	13.87	340.11	27.84
	ms	978.39	0.00	920.02	0.00	968.78	0.00	863.41	0.00
	mt	863.25	11.77	861.29	6.38	877.6	9.41	746.12	13.58
	mw	789.41	19.32	858.37	6.70	818.2	15.54	723.11	16.25
	es	1,500.27	0.00	1,370.7	0.00	1,494.91	0.00	1,195.62	0.00
	et		–	1,345.31	1.85		–		–
	ew		–		–		–		–
15	ds	514.1	0.00	606.84	0.00	448.31	0.00	647.74	0.00
	dt	439.23	14.56	483	20.41	375.73	16.19	507.72	21.62
	dw	408.28	20.58	425.94	29.81	356.61	20.45	439.5	32.15
	ms	822.84	0.00	983.33	0.00	761.53	0.00	1,090.57	0.00
	mt	744.65	9.50	820.99	16.51	642.4	15.64	923.34	15.33
	mw		–	773.46	21.34	612.92	19.51	802.37	26.43
	es		–		–		–		–
	et		–		–		–		–
	ew		–		–		–		–
17	ds	511.73	0.00	564.42	0.00	536.06	0.00	534.07	0.00
	dt	466.71	8.80	508.67	9.88	481.06	10.26	530.56	0.66
	dw	454.45	11.19	433.85	23.13	467.72	12.75	527.24	1.28
	ms		–		–		–		–
	mt		–		–		–		–
	mw		–		–		–		–
	es		–		–		–		–
	et		–		–		–		–
	ew		–		–		–		–

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