Solution Method for the Inventory Distribution Problem

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Abstract Previous research on inventory distributions between local warehouses or retailers (bases) has focused separately on either of two types of stock transshipment policies: preventive lateral transshipments or emergency lateral transshipments. Each of these has its advantages and disadvantages, and combining these policies may well enable merchandisers to achieve higher service levels. Thus, the combined use of these policies is the focus of the present study. A stochastic programming problem is formulated with demand as a stochastic variable, and the policy of using both preventive and emergency lateral transshipment is examined for its effectiveness while solution methods are examined for their efficiency.

1 Introduction

The approach to supply chain issues in recent years has been for suppliers to seek to improve service levels while satisfying a broad spectrum of consumer needs and at the same time to reduce inventory amounts and their associated expenses. However, there is a trade-off between inventory volume and service levels. To improve both at the same time, a supply chain must be carefully constructed from the planning stage, which may involve a large investment.

Lateral transshipments between retail bases are viewed as effective method for improving both inventory volume and service levels, and has come into use in some operating businesses. Two inventory transfer policies have been investigated in previous research on distribution between bases: preventive lateral transshipment [5] and emergency lateral transshipment [7]. Each has its own advantages and disadvantages, and so it is reasonable to expect that combining these will allow higher service

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levels to be provided. For this reason, examining the combination of these policies is the focus of the present study. Specifically, a stochastic programming problem is formulated with demand as a stochastic variable, and the policy of combined preventive and emergency lateral transshipment is examined for its effectiveness and solution methods for the formulated problem are examined for their efficiency.

2 Lateral Transshipments

In most supply chains, when a warehouse faces a stock-out situation, or when it expects a stock-out situation, it sends an order upstream. It is possible, however, that this order will have repercussions throughout the supply chain. Lateral transshipments, which regularize the risk of stock-outs by transferring inventory between bases at the retail level, are employed to reduce orders to the distribution center and improve service levels. The following two policies for lateral transshipments exist.

Preventive transshipments: Made in response to future demand expected due to inventory fluctuations prior to detecting demand increases.

Emergency transshipments: Made in response to emergencies occurring because of empty inventories, after detecting demand increases.

According to Herer, Tzur and Yucesan [3], research on problems in transferring inventory is classified into that on preventive lateral shipments, in which stock is supplied when the demand is known in advance, and that on emergency lateral shipments, in which urgent transfers are made after demand is known. Research on the former has been carried out by Karmarkar and Patel [5] and others, whereas the latter is has been studied by Tagaras [7] and others.

3 Stochastic Programming Formulation

Stochastic programming [2, 4] deals with optimization under uncertainty. A stochastic programming problem with recourse is referred to as a two-stage stochastic problem. To solve the problem, an L-shaped method [9] has been used. This approach is based on Benders [1] decomposition. The expected recourse function is piecewise linear and convex, but it is not given explicitly in advance. The L-shaped method was used to solve stochastic programs having discrete decisions in the first stage [6, 8]. The following notations are employed in the problem.

Variables

o_i	Volume of order sent to the distribution center for base i
x_{ij}	Volume of preventive lateral transshipment from base i to base j
s_i	Intended inventory volume at base i
u_i	1 if order is sent from base i to the distribution center, otherwise 0
y_{ij}^k	Volume of emergency lateral transshipment from base i to base j in scenario k

z_i^{+k} Inventory at period end at base i in scenario k
z_i^{-k} Shortage in inventory at base i in scenario k

Parameters

R_i Variable costs of orders to distribution center at base i
C_{ij} Variable costs of preventive lateral transshipment from base i to base j
S_i^0 Initial inventory at base i
E_{ij} Variable costs of emergency lateral transshipments from base i to base j
L_i Losses due to inventory outage at base i
H_i Inventory storage cost at base i
W_i Fixed order cost at base i
p^k Probability of scenario k
ξ_i^k Demand at base i in scenario k
K Total number of scenarios
I Total number of bases

The stochastic programming problem is formulated as follows.

$$\begin{aligned}
&\min \sum_{i=1}^{I} W_i u_i + \sum_{i=1}^{I} R_i o_i + \sum_{i=1}^{I}\sum_{j\neq i}^{I} C_{ij}x_{ij} + \sum_{k=1}^{K} p^k Q(s,\xi^k)\\
&\text{subject to } S_i^0 + o_i + \sum_{j=1}^{I} x_{ji} - \sum_{i=1}^{I} x_{ij} = s_i, \quad i=1,\ldots,I\\
&o_i \le Mu_i, \quad i=1,\ldots,I \ (M\text{: positive large number})\\
&s_i \ge 0, o_i \ge 0, x_{ij} \ge 0, u_{ij} \in \{0,1\}, \quad i=1,\ldots,I, j=1,\ldots,I, i\neq j
\end{aligned}$$

$$\begin{aligned}
Q(s,\xi^k) = \min\Bigg\{ &\sum_{i=1}^{I}\sum_{j\neq i}^{I} E_{ij} y_{ij}^k + \sum_{i=1}^{I} L_i z_i^{k-} + \sum_{i=1}^{I} H_i z_i^{k+} \Bigg|\\
&z_i^{k+} + \sum_{j=1}^{I} y_{ij}^k - (z_i^{k-} + \sum_{j=1}^{I} y_{ji}) = s_i - \xi_i^k, \quad i=1,\ldots,I\\
&z_i^{k+}, z_i^{k-} \ge 0, y_{ij}^k \ge 0, \quad i=1,\ldots,I, j=1,\ldots,I, i\neq j \Bigg\}, \quad k=1,\ldots,K
\end{aligned}$$

In the L-shaped algorithm, the following problem Master uses θ as the upper bound of the expected value for the recourse function.

$$\begin{aligned}
\text{(Master): } &\min \sum_{i=1}^{I} W_i u_i + \sum_{i=1}^{I} R_i o_i + \sum_{i=1}^{I}\sum_{j\neq i}^{I} C_{ij}x_{ij} + \theta\\
&\text{subject to } S_i^0 + o_i + \sum_{j=1}^{I} x_{ji} - \sum_{i=1}^{I} x_{ij} = s_i, \quad i=1,\ldots,I\\
&o_i \le Mu_i, \quad i=1,\ldots,I\\
&s_i \ge 0, o_i \ge 0, x_{ij} \ge 0, u_{ij} \in \{0,1\}, \quad i=1,\ldots,I, j=1,\ldots,I, i\neq j
\end{aligned}$$

L-shaped algorithm for approximate solution

Step 1: Solve the continuous relaxation of Master, providing a solution in terms of $(\hat{u}, \hat{x}, \hat{s}, \hat{o}, \hat{\theta})$.

Step 2: Solve the second stage problem for each scenario. Because the second stage problem is feasible, the upper bound of the optimal value of the recourse function is found as $Q(\hat{s}, \xi^k), k = 1, \ldots, K$.

Step 3: If $\hat{\theta} < \sum_{k=1}^{K} p^k Q(\hat{s}, \xi^k)$, the optimality cut $\theta \geq \sum_{k=1}^{K} p^k \sum_{i=1}^{I} (s_i - \xi_i^k) \hat{\mu}_i^k$ is generated from the optimal dual solution $\hat{\mu}^k$ and added to the Master problem. Return to Step 1.

Step 4: Find the solution $(\bar{u}, \bar{x}, \bar{s}, \bar{o}, \bar{\theta})$ for the MIP problem Master. Given this solution, calculate $\sum_{i=1}^{I} W_i \bar{u}_i + \sum_{i=1}^{I} R_i \bar{o}_i + \sum_{i=1}^{I} \sum_{j \neq i}^{I} C_{ij} \bar{x}_{ij} + \sum_{k=1}^{K} p^k Q(\bar{s}, \xi^k)$ and the upper bound for the value of the optimal objective function of the original problem can be obtained.

In order to find an optimal solution with integer constraints of the original problem, the recourse function must be approximated in a feasible solution to a first stage problem satisfying the integer constraints. This must be done by solving the MIP problem Master repeatedly, and so the calculation time is potentially extremely long; however, an optimal solution is being sought for the original problem. Since the solution method shown in this paper does not necessarily approximate a recourse function completely, it provides an approximate solution for the original problem. And, it can be expected to have advantages from the viewpoint of calculation time.

4 Numerical Experiments

This experiment employed examples of lateral shipments between 20 and 25 bases. The bases were generated from a uniform distribution on a [0, 100] × [0, 100] grid. The variable cost C_{ij} of a preventive lateral transshipment from base i to base j was defined as 0.1 × (the distance between the bases), and the variable cost of an emergency lateral transshipment was defined as $E_{ij} = 1.5 \times C_{ij}$. The variable costs of orders were set at $R_i = 5$, and other parameters were set with random numbers obeying a normal distribution. Specifically, the demand at base i in scenario k, ξ_i^k, had mean 100 and variance 10; the fixed order cost at base i, W_i, had mean 200 and variance 10; the losses due to inventory outage at base i, L_i, had mean 10 and variance 1; and the inventory storage cost at base i, H_i, had mean 4 and variance 0.4.

The data sets for the different numbered scenarios (indicating problem scale) were supplied for solution by deterministic equivalent MIP conversion and by the L-shaped algorithm and the calculation times were compared. The computer used for this experiment had a 3.2 GHz Core i7-2600K (8.0 GB of memory) main processor and ran the IBM ILOG AMPL-CPLEX System 11.0 branch-and-bound solver. Both methods showed calculation times increasing with the problem scale, but the

Table 1 Results of experiment (computing time)

Base locations I	Scenarios K	L-shaped Optimal objective function value	L-shaped Computing time (s)	Branch-and-bound Optimal objective function value	Branch-and-bound Computing time (s)	Relative error (%)
20	10	12,420	8	12,298	28	0.99
20	20	12,424	12	12,298	63	1.02
20	30	12,502	18	12,390	233	0.90
25	10	15,574	11	15,454	747	0.77
25	20	15,449	35	15,385	1,759	0.42
25	30	15,426	51	15,365	5,979	0.40

Table 2 Results of experiment (comparing transshipment policies)

Base locations I	Scenarios K	Variance Var[$\tilde{\xi}$]	Preventive only Optimal cost	Preventive only Shortage ratio (%)	Emergency only Optimal cost	Emergency only Shortage ratio (%)	Combined policy Optimal cost	Combined policy Shortage ratio (%)
20	10	10	12,738	9.9	13,865	4.6	12,298	3.3
20	10	20	13,367	23.4	13,723	14.1	12,381	5.8
20	10	30	14,012	29.7	13,779	10.0	12,608	7.5
20	20	10	12,877	9.0	13,999	5.4	12,476	3.2
20	20	20	13,502	20.6	13,993	8.0	12,650	6.0
20	20	30	14,426	33.3	13,840	16.0	12,680	10.5
20	30	10	12,862	9.5	13,937	5.7	12,420	2.9
20	30	20	13,579	19.4	14,020	7.7	12,720	7.1
20	30	30	14,054	32.1	13,803	18.0	12,706	9.7

L-shaped algorithm had shorter times. As shown, solving the problem using the direct branch-and-bound algorithm for a deterministic equivalent MIP required a quite long calculation time. Thus, the L-shaped algorithm is advantageous in terms of calculation time for large-scale problems. Also, the calculation errors in this method were kept within almost 1 %, so the L-shaped method clearly provides highly accurate solutions.

Next, the difference between the costs of sending emergency and preventive lateral transshipments independently or together was compared and the effectiveness of the policy of combining emergency and preventive shipments was validated (Tables 1, 2).

For comparison with the policy of combining emergency and preventive lateral transshipments, the transfer policies restricting transshipments to either the emergency or the preventive types were reformulated, and the effectiveness of the two lateral transshipment policies was shown by comparing with the total costs of the policy of combining transshipments. The reformulation of the policy of restricting transshipments to preventive was obtained from the formulation of the policy of combining transshipments, and then eliminating the two-stage variable y_{ij}^k. The reformulation of the policy of restricting transshipments to emergency shipments was obtained from the formulation of the policy of combining transshipments, eliminating the firts stage variable x_{ij}.

The optimal costs of the above policies and the policy of combining transshipments were compared. The numbers of demand scenarios and the standard deviations were varied in a comparison experiment. The policy of combining transshipments exhibited lower total costs than exercising policies independently, regardless of the number of scenarios or the variance. When the variance was small, the "preventive lateral transshipments only" policy had lower total costs than the "emergency lateral transshipments only" policy, and the opposite was true at high variances. This was due to the fact that the mean shortage ratio, which was defined as given below, was high when there were large fluctuations in demand. In turn, this raised shortage costs, making more emergency shipments required in order to avoid shortages.

$$\text{Mean shortage ratio (\%)} = \sum_{k=1}^{K} p^k \left(\sum_{i=1}^{I} z_i^{k+} / \sum_{i=1}^{I} \xi_i^k \right) \times 100. \qquad (1)$$

5 Summary

In the present study, stochastic programming was employed to formulate a lateral transshipment problem, and two solution methods were examined for their efficiency in providing solutions and in combining policies enforcing preventive or emergency lateral transshipments.

The L-shaped algorithm and the direct branch-and-bound algorithm for an equivalent MIP were compared in a numerical experiment. The L-shaped algorithm was found to be advantageous in terms of calculation time for large-scale problems. It was also shown that the total costs are lowered if preventive and emergency lateral transshipment policies are combined, rather than exercising them independently.

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