Stellar Atmospheres: Basic Processes and Equations

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Abstract The content of this chapter is a very quick summary of key concepts that concern the interaction between photons created in the stellar interior and plasma, which is the basis of the physical processes occurring in stellar atmospheres. The dominant mechanism of energy transport through the surface layers of a typical star is radiation. This is the reason why radiative transfer is our main focus here. We start by setting up the differential equation describing the flow of radiation through an infinitesimal volume and all the related quantities. We conclude with a generic description of the equations used to compute an atmospheric model.

Keywords Stars: atmospheres · Stars: fundamental parameters

1 Introduction

The main goal of this school was to provide students with the tools to analyse stellar spectra with particular reference to the determination of the atmospheric parameters of B, A, F, and G type stars. It is obvious that a careful spectral analysis is not possible without knowledge of the theory of stellar atmospheres. So the purpose of this introductory lesson on this important subject is to provide students with a refresher on the main equations describing the physical processes that occur when the radiation, generated in the interior of the star, interacts with the stellar matter which composes the atmosphere.

If we consider a star as a succession of layers of gas, we know that going deep in the atmosphere gas becomes opaque and our line-of-sight cannot penetrate into the interior layers. We call the stellar atmosphere the ensemble of the outer layers to which the energy, generated in the nucleus, is carried, either by radiation,

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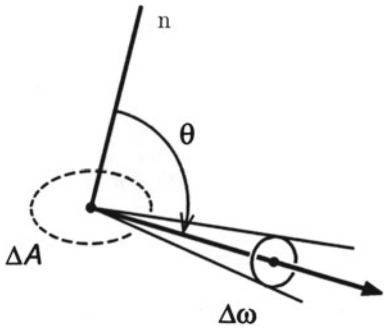

Fig. 1 The increment of area ΔA of a radiating element of material, is seen under an increment of solid angle $\Delta\omega$ and tilted by an angle θ with respect the direction of the normal to the surface.

convection or conduction, before flowing away in the interstellar medium. Interacting with the matter present in the outer layers, this energy finally produces the observed electromagnetic spectrum.

In general, we can say that the theory of stellar atmospheres translates into the study of how the radiation produced in the stellar interior propagates and interacts with the external layers of the star. That is why, during the reading of this introductory lecture on stellar atmospheres we must have always clear in mind this schematic description:

- we call the stellar atmosphere the external layers of a star,
- these are the layers where radiation created in the stellar interior can escape freely into the interstellar medium,
- the atmosphere is the only part from which we receive photons.

Of course, this lecture does not claim to be exhaustive of the topic, but rather a quick recall of the main concepts and definitions. Please refer to specific texts, (i.e. Gray 2005; Hubeny 1996; Mihalas 1978), for a complete and rigorous discussion. In the next sections, before getting to the heart of our topic, we draw some important definitions useful to properly describe light and its interaction with the atmospheric material.

2 Basic Definitions

2.1 Specific Intensity

Looking at the situation represented in Fig. 1, the **specific intensity** is the quantity of energy ΔE_ν that flows through the element ΔA toward the generic direction θ, in the solid angle $\Delta\omega$, during the time Δt, in the interval of frequency $\Delta\nu$. When all these increments become smaller, we can take the limit toward zero:

$$I_\nu = \lim \frac{\Delta E_\nu}{\cos\theta \Delta A\, \Delta\omega\, \Delta t\, \Delta\nu} = \frac{dE_\nu}{\cos\theta dA\, d\omega\, dt\, d\nu}. \tag{1}$$

The right side of this equation is the energy that flows through an element of area dA in the unit of time dt, in the unit of solid angle $d\omega$, and in the unit of frequency $d\nu$. Its physical dimensions are, for example, erg rad^{-1} cm^{-2} s^{-1} Hz^{-1}.

Integrating Eq. 1 over all the directions, we obtain the so-called **mean intensity**:

$$J_\nu = \frac{1}{4\pi} \oint I_\nu d\omega, \tag{2}$$

where the integral is calculated over the whole solid angle.

2.2 *Flux*

Flux represents the total energy passing across an element of area ΔA over the unit of time and frequency. As the specific intensity, we can consider the limit of all the small quantities diminishing toward zero. In this case we will have:

$$F_\nu = \lim \frac{\sum \Delta E_\nu}{\Delta A\, \Delta t\, \Delta\nu} = \frac{\oint \Delta E_\nu}{\Delta A\, \Delta t\, \Delta\nu}, \tag{3}$$

where again we consider a complete integration over all directions. Flux and intensity could be easily related to each other. If we replace in the left side of Eq. 3 the relation for the energy derived from Eq. 1, we obtain:

$$F_\nu = \oint I_\nu \cos\theta d\omega, \tag{4}$$

that represents the component of the net flux in the direction θ.

We can develop this equation for an emitting point on the physical boundary, i.e. the stellar surface. In this case the flux coming in from the outside is null, and if we suppose that there is no azimuthal dependence for F_ν, we get:

$$F_\nu = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_\nu \sin\theta \cos\theta d\theta = 2\pi \int_0^{\pi/2} \sin\theta \cos\theta d\theta = \pi I_\nu. \tag{5}$$

This is the equation that we must solve if we want to compute a theoretical spectrum of a particular star. The importance of this equation is then obvious.

2.3 *K-integral*

It is useful to define another equation using the second moment of θ, that is, the so-called **K-integral**:

$$K_\nu = \frac{1}{4\pi} \oint I_\nu \cos^2 \theta d\omega. \quad (6)$$

It represents the z-component of the radiation stress tensor written in Cartesian coordinates. Physically this integral is linked to the radiation pressure, and it is easy to show the validity of the following equation:

$$P_R = \frac{4\pi}{c} \int_0^\infty K_\nu d\nu. \quad (7)$$

3 Absorption Coefficient and Optical Depth

Let us consider a slab of plasma and let I_ν^0 be the specific intensity of the light before the interaction with the slab and $I_\nu + dI_\nu$ the intensity after the interaction. Let us suppose that only true absorption and scattering give contribution to dI_ν while no emission is present. In this case, we can write:

$$dI_\nu = -\kappa_\nu \rho I_\nu dx, \quad (8)$$

where κ_ν is the absorption coefficient that has units of area per mass ($[\kappa_\nu]=\mathrm{cm}^2\ \mathrm{g}^{-1}$) and is therefore a mass absorption coefficient, ρ is the density in mass per unit volume and dx is the slab thickness, that has units of length. At this point I have to stress an important concept: the way in which the radiation propagates through the stellar material depends both on the physical conditions of the plasma at a given frequency and on the length of the path. We can say that at a given frequency, the radiation sees the combination of these two factors, namely $\kappa_\nu \rho dx$. Define the **optical depth** along the photon direction of propagation as follows:

$$d\tau_\nu = \kappa_\nu \rho dx, \quad (9)$$

which, integrated over some path length L, becomes:

$$\tau_\nu = \int_0^L \kappa_\nu \rho dx, \quad (10)$$

where τ_ν is the optical depth at a given frequency ν and x is the geometrical depth. It measures a characteristic of matter and radiation coupled together, and corresponds, for a given frequency and absorption coefficient, to the distance at which the intensity is reduced by a factor of $1/\mathrm{e}$. Using optical depth, Eq. 8 can be written as:

$$dI_\nu = -I_\nu d\tau_\nu, \quad (11)$$

for which the trivial solution is given by

$$I_\nu = I_\nu^0 \, \mathrm{e}^{-\tau_\nu}. \tag{12}$$

In a plasma of astrophysical interest, we distinguish from optically thick, for which $\tau_\nu \gg 1$, and optically thin, for which $\tau_\nu \ll 1$. I would like to stress again here the importance of frequency: the same plasma (same chemical composition and physical conditions) could be optically thick at a certain frequency, say ν_1, but optically thin for another frequency, say ν_2.

4 Emission Coefficient

Like we did in the previous section, we consider the increase dI_ν undergone by the radiation when passing through a slab of plasma. We suppose now that the processes contributing to dI_ν are true emission and photons scattering into direction of propagation, with no absorption. In this context, scattering refers mainly to photons previously absorbed and then immediately re-emitted in the direction from the same atomic transition.

If we denote by j_ν the emission coefficient (units $[j_\nu]$ = erg rad^{-1} s^{-1} Hz^{-1} g^{-1}), we define the increment of the radiation as:

$$dI_\nu = j_\nu \rho dx. \tag{13}$$

5 Source Function and Its Physical Meaning

We can now introduce a new quantity given by the ratio between the absorption and emission coefficients and called the **source function**:

$$S_\nu = \frac{j_\nu}{k_\nu}. \tag{14}$$

This quantity has the same units of the specific intensity and can be seen as the specific intensity of a radiation emitted in some point in a hot gas.

To better understand the meaning of S_ν, we can refer to Hubeny (1996) and consider this example: Let us write the number of photons emitted in an volume element $dV = dx \cdot dA$, in all directions. From the definition of the emission coefficient, it follows that:

$$N_{em} = \frac{4\pi}{h\nu}(j_\nu \rho \, dx \, dA \, d\nu \, dt), \tag{15}$$

where the quantity in parenthesis represents the energy emitted in the volume dV, the factor 4π comes from an integration over the solid angle, and $h\nu$ transforms energy to the number of photons. By using the definition of the optical depth and the source function, and after some elementary algebra, we obtain

$$N_{em} = S_\nu d\tau_\nu \frac{4\pi}{h\nu} \rho \, dA \, d\nu \, dt. \tag{16}$$

In other words, we have:

$$S_\nu \propto \frac{N_{em}}{d\tau_\nu}. \tag{17}$$

Hence, the source function is proportional to the number of photons emitted per unit of optical depth interval.

5.1 Two Simple Cases

In two "extreme" cases, the algebraic form of S_ν is simple: pure isotropic scattering and pure absorption.

5.2 Pure Isotropic Scattering

All the emitted energy is due to photons being scattered into the direction under consideration. In this case the contribution to the emission dj_ν is proportional to the solid angle $d\omega$ facing the observer and to the energy "absorbed" $\kappa_\nu I_\nu$:

$$dj_\nu = \frac{1}{4\pi} \kappa_\nu I_\nu \, d\omega, \tag{18}$$

where $\frac{1}{4\pi}$ is the normalization factor for unit solid angle, valid under the hypothesis that the energy is isotropically re-radiated.

To obtain all the contributions to j_ν, we proceed with an integration over the solid angle, keeping in mind that κ_ν is independent of ω, and using Eq. 2, we can write:

$$j_\nu = \frac{1}{4\pi} \oint \kappa_\nu I_\nu d\omega = \frac{\kappa_\nu}{4\pi} \oint I_\nu d\omega = \kappa_\nu J_\nu. \tag{19}$$

From this equation it is straightforward to show that:

$$S_\nu = \frac{j_\nu}{k_\nu} = J_\nu. \tag{20}$$

In short, in the simple case of pure isotropic scattering, the source function is the mean intensity. Moreover, when thermodynamic equilibrium holds, the radiation intensity is equal to the Planck function, i.e. $J_\nu = B_\nu$.

5.3 Pure Absorption

Now we are assuming that all the absorbed photons are destroyed and all the emitted photons are newly created with a distribution governed by the physical state of the gas. The source function for this case is given by Planck's radiation law:

$$S_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} = B_\nu(T). \tag{21}$$

This is the specific intensity emitted by a gas of a temperature T and for a given frequency ν.

6 The Transfer Equation

In the previous sections we have discussed separately the cases of radiation travelling in a slab of stellar material in which it is affected either by losses, expressed in the absorption coefficient κ_ν, or gains, expressed in the emission coefficient j_ν. Now we consider the general case in which the change in specific intensity, dI_ν, over an increment of linear path length ds, is the sum of those losses and gains, expressed as:

$$dI_\nu = -\kappa_\nu \rho I_\nu ds + j_\nu \rho ds. \tag{22}$$

This equation can be written in a more useful form, by dividing both sides by $\kappa_\nu \rho ds$, and using the definition of source function (Eq. 14):

$$\frac{dI_\nu}{\kappa_\nu \rho ds} = -I_\nu + \frac{j_\nu}{\kappa_\nu} = -I_\nu + S_\nu. \tag{23}$$

Finally, we have the differential form of the **equation of radiative transfer**

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu. \tag{24}$$

The integration follows from a standard integrating-factor scheme. After some manipulation, we obtain the so-called integral form of the radiative transfer equation:

$$I_\nu(\tau_\nu) = \int_0^{\tau_\nu} S_\nu(t_\nu) e^{-(\tau_\nu - t_\nu)} dt_\nu + I_\nu(0) e^{-\tau_\nu}. \tag{25}$$

The meaning of this equation can be easily understood: radiation along the line at the point τ_ν is composed of the sum of intensities, S_ν, originating at the generic points t_ν along the line, but suffering extinction according to the optical-depth separation $\tau_\nu - t_\nu$ (first term of the sum), plus the radiation due to the original intensity $I_\nu(0)$ that has suffered an exponential extinction $e^{-\tau_\nu}$ (second term of the sum).

Equation (24) holds along a line. In stellar atmospheres applications, it is useful to define the optical depth relative to the star along a stellar radius, and not along the line of sight. We are also assuming, in the following discussion, that as the atmosphere is thin with respect to the radius, a plane-parallel approximation can be used.

Assuming spherical coordinates originating in the centre of the star and with the z axis toward the observer, we write the transfer equation in the form:

$$\frac{1}{\kappa_\nu \rho} \frac{dI_\nu}{dz} = -I_\nu + S_\nu. \tag{26}$$

Let us write $\frac{dI_\nu}{dz}$ according to spherical geometry; if we assume I_ν has no azimuthal dependence, we obtain:

$$\frac{1}{\kappa_\nu \rho} \left(\frac{\partial I_\nu}{\partial r} \frac{dr}{dz} + \frac{\partial I_\nu}{\partial \theta} \frac{d\theta}{dz} \right) = -I_\nu + S_\nu. \tag{27}$$

We know, from geometrical consideration, that, $dr = \cos\theta dz$ and $rd\theta = -\sin\theta dz$. Then, by substitution of these expressions, and keeping in mind that for a plane-parallel atmosphere θ does not depend upon z, the transfer equation becomes:

$$\frac{1}{\kappa_\nu \rho} \left(\frac{\partial I_\nu}{\partial r} \cos\theta \right) = -I_\nu + S_\nu. \tag{28}$$

Adopting the convection of using a new geometrical depth variable, defined as $dx = -dr$ and writing $d\tau_\nu$ for $\kappa_\nu \rho dx$, we have the basic form of the radiative transfer equation used in the stellar atmosphere applications:

$$\cos\theta \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu. \tag{29}$$

6.1 Elementary Solutions

Following the outline depicted in Hubeny (1996), in this section, we describe the simplest solutions of the 1-D plane-parallel radiative transfer equation.

6.2 No Absorption, No Emission

In this elementary case, $\kappa_\nu = j_\nu = 0$, the transfer equation reads $\dfrac{dI}{dz} = 0$, which has a trivial solution:

$$I_\nu = \text{const}. \tag{30}$$

That is, in absence of any interaction with the medium, the radiation specific intensity remains constant.

6.3 No Absorption, Only Emission

In this case, $\kappa_\nu = 0$ and $j_\nu > 0$, the solution is simply:

$$I_\nu(x, \cos\theta) = I_\nu(0, \cos\theta) + \int_0^x j_\nu(x') \sec\theta dx'. \tag{31}$$

This equation is often used for describing an outgoing radiation from an optically thin radiating slab. For instance, a forbidden line radiation from planetary nebulae, or a radiation from the solar transition region and/or corona.

6.4 No Emission, Only Absorption

In this case, $\kappa_\nu > 0$ and $j_\nu = 0$, the transfer equation becomes $\cos\theta \dfrac{dI_\nu}{d\tau_\nu} = I_\nu$, and the solution is simply:

$$I_\nu(0, \cos\theta) = I_\nu(\tau_\nu, \cos\theta)\mathrm{e}^{-\tau_\nu \cdot \cos\theta} \tag{32}$$

6.5 General Case: Absorption and Emission

The full intensity at the position τ_ν on the line-of-sight through the photosphere is factorized in the sum of two terms, radiation coming outward $I_\nu^{\text{out}}(\tau_\nu)$ and radiation going inward $I_\nu^{\text{in}}(\tau_\nu)$:

$$\begin{aligned} I_\nu(\tau_\nu) &= I_\nu^{\text{out}}(\tau_\nu) + I_\nu^{\text{in}}(\tau_\nu) \\ &= \int_{\tau_\nu}^{0} S_\nu e^{-(t_\nu - \tau_\nu)\sec\theta} \sec\theta dt_\nu - \int_{\infty}^{\tau_\nu} S_\nu e^{-(t_\nu - \tau_\nu)\sec\theta} \sec\theta dt_\nu. \end{aligned} \tag{33}$$

An important special case occurs at the stellar surface. In this case $I_\nu^{\mathrm{in}}(\tau_\nu) = 0$ and then:

$$I_\nu(0) = \int_0^\infty S_\nu \mathrm{e}^{-t_\nu \sec\theta} \sec\theta dt_\nu. \tag{34}$$

Here we are assuming that intensity of the radiation coming from other stars, galaxies and so forth, is completely negligible compared to the star's own radiation. For most stars, for which we do not resolve the disk, we must still integrate I_ν^{out} over the star's disk, i.e. we observe the flux.

6.6 Special Case: Linear Source Function

A special case is an emergent intensity from a semi-infinite atmosphere, with a source function being a linear function of optical depth:

$$S_\nu(\tau_\nu) = a + b\tau_\nu. \tag{35}$$

In this case, substituting this form of S_ν in the Eq. 34, we obtain the solution given by:

$$I_\nu(0) = \int_0^\infty (a + b\tau_\nu)\mathrm{e}^{-t_\nu \sec\theta} \sec\theta dt_\nu = a + b\cos\theta, \tag{36}$$

or more simply:

$$I_\nu(0, \cos\theta) = S_\nu(\tau_\nu = \cos\theta). \tag{37}$$

This important expression is called the "Eddington-Barbier relation". It shows that the emergent intensity, for instance in the normal direction ($\cos\theta = 1$) is given by the value of the source function at the optical depth of unity. The values of emergent intensity for all angles θ for which $\cos\theta$ ranges between 0 and 1, map the values of the source function between optical depths 0 and 1. Even though the source function is not a linear function of τ_ν, it can usually be well approximated by it in the vicinity of $\tau_\nu = 1$.

7 The Flux Integral

The transformation in spherical coordinates that we did for specific intensity, can also be done for the integral flux, as already defined in Eq. 3. Assuming there is no azimuthal dependence in I_ν, we can write:

$$F_\nu = 2\pi \int_0^\pi I_\nu \cos\theta \sin\theta d\theta$$
$$= 2\pi \int_0^{\pi/2} I_\nu^{out} \cos\theta \sin\theta d\theta + 2\pi \int_{\pi/2}^{\pi} I_\nu^{in} \cos\theta \sin\theta d\theta. \quad (38)$$

Using the expressions for the specific intensity and assuming that S_ν is isotropic (no θ dependence), we obtain for the flux the following expression:

$$F_\nu(\tau_\nu) = 2\pi \int_{\tau_\nu}^{\infty} S_\nu(t_\nu) E_2(t_\nu - \tau_\nu) dt_\nu - 2\pi \int_0^{\tau_\nu} S_\nu(t_\nu) E_2(t_\nu - \tau_\nu) dt_\nu, \quad (39)$$

where E_2 is the exponential integral of second order.

At the stellar surface, where $\tau_\nu = 0$, we have:

$$F_\nu(0) = 2\pi \int_0^\infty S_\nu(t_\nu) E_2(t_\nu) dt_\nu, \quad (40)$$

which is the theoretical stellar spectrum.

8 Computing a Model Atmosphere

To solve the radiative transfer equation, we must know the source function S_ν that, as we have learnt previously, is the ratio between emission and absorption coefficients. These coefficients have a strong dependence on the physical properties of the atmospheric layers: j_ν and κ_ν depend on temperature, pressure, population of the atomic levels, electronic density and so on. Hence, to compute S_ν, and then solve the radiative transfer equation, we must know the distributions of T, P, n_i, n_e, and other quantities with optical depth. This process is what we commonly refer to as the calculation of a model atmosphere.

By the term "model atmosphere" we indicate a specification of all the atmospheric state parameters as functions of depth. Since the problem is very complex, we cannot construct analytic solutions. Therefore, we discretize the depth coordinate and consider a finite number of depth points (typically of the order of several tens to few hundreds). A model atmosphere is then a table of values of the state parameters in these discrete depth points.

8.1 Basic Equations of Stellar Atmospheres

Let us summarize in this section the basic equations of stellar atmospheres for the case of a horizontally-homogeneous, plane-parallel, static atmosphere.

8.1.1 Radiative Transfer Equation

The radiative transfer equation (see Eq. 29) has been the topic of previous sections, so we do not treat it any more, but we just remember that its solution gives us information on the mean intensity of the radiation.

8.1.2 Hydrostatic Equilibrium Equation

By solving the hydrostatic equilibrium equation we get information on the total gas pressure, and then on total particle density. If P is the total pressure, the equation reads:

$$\frac{dP}{dz} = \rho g. \tag{41}$$

Introducing the optical depth, the previous equation can be written as:

$$\frac{dP}{d\tau_\nu} = \frac{g}{\kappa_\nu}. \tag{42}$$

It should be kept in mind that the total pressure is generally composed of several parts: the gas pressure, P_{gas}, the radiation pressure, P_{rad}, the turbulent pressure, P_{turb}, and if a magnetic field is present also magnetic pressure, P_B, has a contribution. In general, the equation is:

$$P = P_{gas} + P_{rad} + P_{turb} + P_B = NkT + \frac{4\pi}{c}\int_0^\infty K_\nu d\nu + \frac{1}{2}\rho v_{turb}^2 + \frac{B^2}{8\pi}. \tag{43}$$

Neglecting turbulent and magnetic pressure (that in general cases do not give a significant contribution) the hydrostatic equilibrium equation may then be written as:

$$\frac{dP_{gas}}{d\tau_\nu} = \frac{g}{\kappa_\nu} - \frac{dP_{rad}}{d\tau_\nu}. \tag{44}$$

We may think of the right side of this equation as the effective gravity acceleration, since it expresses the action of the true gravity acceleration (acting toward the centre of the star) minus the radiative acceleration (acting outward).

8.1.3 Radiative Equilibrium

The radiative equilibrium equation expresses the fact that the total flux is conserved, and solving it we know the distribution of temperature along the atmosphere:

$$\frac{dF(x)}{dx} = 0 \longrightarrow F(x) = F_0, \tag{45}$$

$$\int_0^\infty F_\nu(\tau_\nu)d\nu = F_0. \tag{46}$$

Other two important equilibrium equations are easily derived from the radiative transfer equation. Let us write Eq. 29 in this way:

$$\cos\theta \frac{dI_\nu}{dx} = \kappa_\nu \rho I_\nu - \kappa_\nu \rho S_\nu. \tag{47}$$

Then, integrating first over solid angle and over frequencies, we have:

$$\frac{d}{dx}\int_0^\infty F_\nu d\nu = 4\pi\rho \int_0^\infty \kappa_\nu J_\nu d\nu - 4\pi\rho \int_0^\infty \kappa_\nu S_\nu d\nu. \tag{48}$$

Considering condition expressed by Eq. 45, we can write this as:

$$\int_0^\infty \kappa_\nu J_\nu d\nu = \int_0^\infty \kappa_\nu S_\nu d\nu. \tag{49}$$

If we multiply by $\cos\theta$ and then we integrate over solid angle and over frequencies, we obtain:

$$\int_0^\infty \frac{dK_\nu}{d\tau_\nu} d\nu = \frac{F_0}{4\pi}. \tag{50}$$

Equations (46), (49), and (50) are the so-called **Milne equations**.

8.1.4 Statistical and Charge Conservation Equations

Two other important ingredients of an atmospheric model are the distribution of the level population, n_i, and electronic density, n_e. Statistical and charge conservation equations are helpful to know how these quantities vary along optical depth.

Let us consider two generic atomic levels, i and j, and if R and C are the radiative and collisional rates, respectively, then the set of equations:

$$n_i \sum_{j \neq i} (R_{ij} + C_{ij}) = n_j \sum_{j \neq i} (R_{ji} + C_{ji}) \tag{51}$$

expresses the equilibrium between the total number of transition out of level i (left hand side) and the total number of transition into level i from all other levels (right hand side).

Another important equation expresses the global electrically neutrality of the medium:

$$\sum_i n_i Z_i - n_e = 0, \tag{52}$$

where Z_i is the charge associated with the level i (i.e. equal to 0 for levels of neutral atoms, 1 for levels for once ionized atoms, etc.). The summation extends over all levels of all ions of all species. This equation is useful to obtain the distribution of the electron density, n_e, along the stellar atmosphere.

8.2 A Pedagogical Example: The Grey Atmosphere

A very simplified case is the so-called grey atmosphere model. The basic assumption of this model is that the absorption coefficient is independent of frequency, that is, $\kappa_\nu = \kappa$. Electron scattering is the only opacity source relevant to stellar atmospheres that is grey, and it is usually a minor contributor to κ_ν, at least in the case of cool stars. The grey case is not very realistic, but nevertheless is useful to understand the interplay between radiative equilibrium and radiative transfer, or in other words, to understand the behaviour of temperature as a function of depth:

$$\cos\theta \frac{dI}{d\tau} = I - S. \tag{53}$$

Using the Milne's equation re-formulated for the grey case, and skipping all the mathematical steps, we arrive at the solution for the source function:

$$S(\tau) = \frac{3F_0}{4\pi}\left(\tau + \frac{2}{3}\right). \tag{54}$$

In this simple case, the source function varies linearly with optical depth. Using the frequency-integrated form of Planck's law, we can write $S(\tau) = \frac{\sigma}{\pi}\,[T(\tau)]^4$ and $F_0 = \sigma T_{\mathrm{eff}}^4$, so the previous equation in LTE becomes:

$$T(\tau) = \left[\frac{3}{4}\left(\tau + \frac{2}{3}\right)\right]^{1/4} T_{\mathrm{eff}}. \tag{55}$$

At $\tau = 2/3$ the temperature is equal to the effective temperature (T_{eff})[1], and $T(\tau)$ scales in proportion to the effective temperature.

9 Conclusions

The conclusions of this lecture can be summarized according to the following outline:

- Modelling stellar spectrum means computing the flux emerging at the stellar surface.
- To accomplish this task we need to know the radiation specific intensity along the atmosphere.
- The calculation of how the radiation propagates within a stellar atmosphere requires knowledge of the source function.
- The source function depends on emission and absorption coefficients.
- Both j_ν and κ_ν depend on the physical conditions of the stellar material: T, P, electronic density and so on.
- We need to solve the equations of the model atmosphere.

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[1] We recall here the definition of effective temperature of a star; it is defined as the temperature of a black body having the same power output per unit area as the star.