A Degenerate Parabolic Logistic Equation

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Abstract We analyze the behavior of positive solutions of parabolic equations with a class of degenerate logistic nonlinearity and Dirichlet boundary conditions. Our results concern existence and strong localization in the spatial region in which the logistic nonlinearity cancels. This type of nonlinearity has applications in the nonlinear Schrödinger equation and the study of Bose–Einstein condensates. In this context, our analysis explains the fact that the ground state presents a strong localization in the spatial region in which the nonlinearity cancels.

1 Introduction

In this paper we analyse the behavior of positive solutions of parabolic equations with a degenerate logistic nonlinearity and Dirichlet boundary conditions

$$\begin{cases} u_t - \Delta u = \lambda u - n(x)u^\rho & \text{in } \Omega,\ t > 0, \\ u = 0 & \text{on } \partial\Omega,\ t > 0, \\ u(0) = u_0 \geq 0, \end{cases} \tag{1}$$

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where $\Omega \subset \mathbb{R}^N$, $N \geq 1$, is a bounded domain, $\rho > 1$, $\lambda \in \mathbb{R}$ and $n(x) \geq 0$ in Ω. Assume also that $n(x)$ remains strictly positive near the boundary of Ω and therefore

$$K_0 = \{x \in \Omega : n(x) = 0\} \subset \Omega \qquad \text{is a nonempty compact set.} \tag{2}$$

The parabolic problem (1) degenerates into a linear equation on K_0, there the growth rate is exponential and a solution could be expected to be unbounded. In the region where $n(x) > n_0 > 0$, the growth is logistic, and a solution could be expected to be bounded. The question is what kind of behavior could be expected in the whole domain Ω, and how the solution will 'glue' the different behavior in those subregions. Hence K_0 plays a crucial role in the dynamical properties and the asymptotic behavior of solutions of (1), as we will show below.

There is a large amount of mathematical literature in this kind of logistic equations, see below. This type of nonlinearity has also applications in the nonlinear Schrodinger equation and the study of Bose-Einstein condensates. In this context, assumption (2) implies the fact that the *ground state* presents a strong localization in the spatial region K_0, see [12] and references therein.

Throughout this paper we shall assume that the compact set K_0 and the function $n(x)$ satisfy the following hypotheses

(H1) $K_0 = K_1 \cup K_2 \subset \Omega$, where K_1 and K_2 are compact sets and

$$K_1 = \overline{\Omega}_0, \qquad \text{is the closure of a regular connected open set} \quad \Omega_0 \neq \emptyset,$$

$$K_2 \qquad \text{has zero Lebesgue measure.}$$

In some cases (H1) will be strengthened to
(H1′) K_0 satisfies (H1) and

$$K_2 \qquad \text{is a closed regular } d\text{−dimensional manifold, with} \quad d \leq N-1.$$

(H2) $n(x)$ is a Hölder continuous function and

$$n(x) \geq C\big(d_0(x)\big)^{\gamma} \quad \text{for some } \gamma > 0, \qquad \text{where} \quad d_0(x) := \text{dist}(x, K_0).$$

When the set K_0 is empty, that is, if $n(x)$ is strictly bounded away from zero, the parabolic problem (1) is classical and well understood, see e.g. [13] and references therein. Also, when K_0 is "smooth" in the sense that in (H1) we have $K_0 = K_1 = \overline{\Omega}_0$ where Ω_0 is a smooth open set, and $K_2 = \emptyset$, this problem has also been studied in [3–5, 9, 11] and further developments in [6, 8], see also references therein. Therefore here we focus on the effect on the solutions of the presence of the part with empty interior K_2.

Let us consider the stationary associated problem, see [1]. We will denote by $\lambda_1(\omega)$ the first eigenvalue of the Laplace operator defined in an open set ω, with Dirichlet boundary conditions on $\partial\omega$. As λ crosses the value $\lambda_1(\Omega)$, a bifurcation phenomena takes place and a unique positive solution emanates from the trivial one. This solution can be continued in λ up until it reaches a critical value $\lambda_c = \lambda_1(\Omega_0)$, see [1, Theorem 2.3]. Note that this is precisely the same situation as when K_0 is "smooth", i.e. $K_2 = \emptyset$. On the other side, when K_0 is empty, the picture is also as above, with $\lambda_c = \infty$. In [1] we give a detailed description of the behavior of this branch of solutions for $\lambda \in (\lambda_1(\Omega), \lambda_1(\Omega_0))$ and specially as $\lambda \to \lambda_1(\Omega_0)$.

For any $\lambda \in (\lambda_1(\Omega), \lambda_1(\Omega_0))$, there exists a unique classical positive stationary solution, denoted by φ_λ, which is globally asymptotically stable for positive solutions of (1). Moreover, inside Ω_0, the pointwise limit of φ_λ as $\lambda \uparrow \lambda_1(\Omega_0)$ is unbounded, see Theorem 1 for a precise statement and see [1, Theorem 1.1] for a proof. This result is already know in the particular case when $n(x)$ is a smooth function, $K_2 = \emptyset$, and $K_0 = K_1 = \overline{\Omega_0}$, an open set with regular boundary, see [3, 4, 11].

In K_2 we have two competing mechanisms: on one hand the fact that $n(x) \equiv 0$ in K_2 "pushes" the solution towards $+\infty$ while the fact that K_2 is not "fat" enough means that this effect may not have enough room to force the solution to go to infinity.

Roughly speaking, our main result state that if

$$\gamma + 2 < (\rho - 1)(N - d) \tag{H3}$$

then any positive equilibrium remains bounded on compact sets of $\Omega \setminus K_1$ and, in particular, at each point of $K_2 \setminus K_1$, see Theorem 1 below, see also [1, Theorem 1.1].

We will distinguish two situations for which we will be able to show that the solutions remain bounded in K_2. In case $K_2 \cap K_1 = \emptyset$, any solution will be bounded in K_2, actually it will be so in a neighborhood of K_2. In case $K_2 \cap K_1 \neq \emptyset$, it will turn out that a balance between the geometry of K_2 and the strength of the logistic term, given by the exponent ρ and the behavior of the function $n(x)$ near K_2, will determine the behavior of the solution, see the following theorem.

Theorem 1 *Assume K_0 satisfies (H1) and $n(x)$ satisfies (H2). Then for any $\lambda \in (\lambda_1(\Omega), \lambda_1(\Omega_0))$ there exists a unique classical positive equilibrium, denoted by φ_λ, which is globally asymptotically stable for nonnegative nontrivial solutions of* (1)*, that is, for every $u_0 \gneqq 0$, the solution of* (1) *satisfy*

$$\lim_{t\to\infty} u(t, x; u_0) = \varphi_\lambda(x).$$

Also we have

$$\lim_{\lambda\uparrow\lambda_1(\Omega_0)} \varphi_\lambda(x) = \infty, \quad \textit{for all } x \in \Omega_0, \tag{3}$$

with uniform limit in compact sets of Ω_0. *Moreover, we have the following two cases:*

(i) If $K_1 \cap K_2 = \emptyset$, *then there exists a* $\delta > 0$ *and* $M > 0$ *such that*

$$|\varphi_\lambda(x)| \leq M, \quad \forall x : d(x, K_2) \leq \delta, \quad \forall \lambda \in (\lambda_1(\Omega), \lambda_1(\Omega_0)).$$

(ii) If $K_1 \cap K_2 \neq \emptyset$, K_0 *satisfies* (H1′) *and hypothesis* (H3) *holds, then* φ_λ *remains uniformly bounded on compact sets of* $\Omega \setminus K_1$. *In particular it remains bounded at each point of* $K_2 \setminus K_1$.

Turning back to the parabolic problem, we have the following result:

Theorem 2 *Assume* (H1)–(H2) *hold. Let* $u_0 \geq 0$ *be a bounded initial data for (1). Then for any* $\lambda > \lambda_1(\Omega_0)$ *any positive solution of (1) satisfy*

$$\lim_{t \to \infty} u(x, t) = \infty, \quad \textit{for all } x \in \Omega_0, \tag{4}$$

and the limit is uniform in compact sets of Ω_0. *Moreover, we have the following:*

(i) If $K_1 \cap K_2 = \emptyset$, *then there exists a* $\delta > 0$ *and* $M = M(u_0, \lambda, \delta) > 0$ *such that*

$$|u(x, t; u_0)| \leq M, \quad \forall x : dist(x, K_2) \leq \delta, \quad \forall t > 0.$$

(ii) If $K_1 \cap K_2 \neq \emptyset$, K_0 *satisfies* (H1′), *and hypothesis (H3) holds, then for any* $\lambda \geq \lambda_1(\Omega_0)$ *any solution of (1) remains uniformly bounded on compact sets of* $\Omega \setminus K_1$ *as* $t \to \infty$. *In particular it remains bounded at each point of* $K_2 \setminus K_1$.

The proof of this result relies on the following argument. If we denote by u a nonnegative solution of (1), then we obtain first an upper bound of u, independent of λ, in compact sets of $\Omega \setminus K_0$. If $\bar{B}(x_0, a) \subset \Omega \setminus K_0$, where $n(x) \geq \beta$ in this ball, we may compare the solution u with radial solutions of singular Dirichlet problems, posed in $B(x_0, a)$, going to infinity at the boundary, see [5, 7, 10]. By radial symmetry, the minimum of the singular solution is attained at the center of the ball (that is in x_0), and can be estimated in terms of β, a, ρ and the dimension N. Translating this result to our problem, we can move those balls for points in $\Omega \setminus K_0$ next to the boundary of K_0, and state some rate for the upper bounds in terms of some inverse power of the distance to the boundary of K_0. This estimates provide an upper rate at which the solution may diverge to infinity as we approach K_0. See Lemma 2, Proposition 1 and Lemma 3.

Once this estimate is obtained, we realize that the rate obtained with the argument above may imply that the solution u is a solution of a parabolic problem with an L^r trace at the boundary. Parabolic regularity will imply that the solution u is bounded, independent of λ, in compact sets of $\Omega \setminus K_1$. Therefore, we may obtain conditions on ρ, the dimensions N and d and the rate γ at which $n(x)$ approaches to zero, see (H2), which may guarantee that the solution is bounded in $K_2 \setminus K_1$, see Theorem 2.(ii).

This paper is organized as follows. We first show that the solutions are uniformly bounded in compact sets of $\Omega \setminus K_0$ (see Proposition 1 below). Next, we prove that for $\lambda \geq \lambda_1(\Omega_0)$, any solution of the parabolic problem (1) start to grow up in K_1 as $t \to \infty$, see Theorem 2. Also, if the two parts K_1 and K_2 of K_0 are disjoint, then all solutions remain globally bounded on K_2 as $t \to \infty$, see Theorem 2.(i). Finally, when $K_1 \cap K_2 \neq \emptyset$, provides sufficient conditions ensuring that all solutions of (1) remain bounded in $K_2 \setminus K_1$, see Theorem 2.(ii).

2 Boundedness and Unboundedness of Solutions

We analyze where and how solutions of (1) become unbounded. The first thing we can say is that the blow-up is a complete blow-up at every point in Ω_0.

Lemma 1 *Assume K_0 satisfies (H1). Let u be a solution of the parabolic problem (1). If $\lambda > \lambda_1(\Omega_0)$, then*

$$\lim_{t\to\infty} u(x,t) = \infty, \quad \textit{for all } x \in \Omega_0.$$

Proof Let $z(x,t)$ be the solution of

$$\begin{cases} z_t - \Delta z = \lambda z, & \text{in } \Omega_0, \quad t > 0, \\ z = 0 & \text{on } \partial\Omega_0, \quad t > 0, \\ z(0) = z_0 \geq 0 & \text{in } \Omega_0 \end{cases}$$

with $z_0 \leq u_0$. Then, by comparison and due to $n(x) \geq 0$ in K_0, $z(x,t) \leq u(x,t)$ for $x \in \Omega_0$. Since $\lambda > \lambda_1(\Omega_0)$ then $z(x,t)$ grows exponentially in Ω_0.

To get upper bounds on the solutions outside Ω_0 we will use the following Lemma, see [5]. This Lemma analyzes the minimum of a radially symmetric solution of a singular logistic equation with constant coefficients and going to infinity at the boundary, see [7, 10].

Lemma 2 *Assume $\rho > 1$ and $\lambda, \beta > 0$ and consider a ball in $\mathbb{R}^N$ of radius $a > 0$ and the following singular Dirichlet problem*

$$-\Delta z = \lambda z - \beta z^\rho \quad \textit{in } B(0,a); \qquad z = \infty \quad \textit{on } \partial B(0,a).$$

Then, there exists a unique positive radial solution, $z_a(x)$. Moreover z_a satisfies

$$\left(\frac{\lambda}{\beta}\right)^{\frac{1}{\rho-1}} \leq z_a(0) = \inf_{B(0,a)} z_a(x) \leq \left(\frac{\lambda(\rho+1)}{2\beta} + \frac{B}{\beta a^2}\right)^{\frac{1}{\rho-1}}$$

for some constant $B = B(\rho, N) > 0$, B independent of λ.

The above Lemma gives a local upper bound for the parabolic problem, out of K_0.

Proposition 1 *Let $x_0 \in \Omega \setminus K_0$ and let $u_0 \geq 0$ be a bounded initial data for (1). Then for any given $\lambda \geq \lambda_0(K_0)$ there exists $b > 0$ and $M > 0$ such that*

$$0 \leq u(t,x;u_0) \leq M, \quad x \in B(x_0,b), \quad t > 0,$$

where $B(x_0,b)$ denotes the ball centered at x_0 with radius b.

Proof Let $x_0 \in \Omega \setminus K_0$ and let $a > 0$ be such that $B(x_0,a) \subset \Omega \setminus K_0$. Denote $\beta = \inf\{n(x),\ x \in B(x_0,a)\} > 0$ and consider $z(x)$ the translation to $B(x_0,a)$ of the function in Lemma 2.

Given u_0, for a sufficiently small we have that $u_0(x) \leq z(x_0) \leq z(x)$ for $x \in B(x_0,a)$. Hence $z(x)$ is a supersolution for $u(x,t)$ and then

$$u(x,t) \leq z(x), \quad x \in B(x_0,a), \quad t > 0.$$

Now in $B(x_0,a/2)$, $z(x)$ remains bounded and we conclude the proof with $b = a/2$.

Next we discuss the behavior of the solutions in K_0. First we give a universal (and singular) bound.

Lemma 3 *Let $u_0 \geq 0$ be a bounded initial data for (1). Then, there exists a constant $A = A(u_0,\lambda)$ such that the following holds*

$$0 \leq u(t,x;u_0) \leq h(x) = \left(\frac{A}{d_0^2(x)\ \inf_{x\in B_0}\ n(x)}\right)^{\frac{1}{\rho-1}},$$

where $B_0 := B\left(x_0, \frac{d_0(x)}{2}\right)$, and $d_0(x) = dist(x, K_0)$.

Proof Let $x_0 \in \Omega \setminus K_0$, hence $B_0 \subset \Omega \setminus K_0$. Denote $\beta(x_0) = \inf\{n(x),\ x \in B_0\} > 0$ and consider $z(x)$ the translation to B_0 of the function in Lemma 2.

Let $u_0 \leq M$ in $\overline{\Omega}$. Using the continuity of $n(x)$, we can assume that $\beta(x_0) \leq \frac{\lambda}{M^{\rho-1}}$ for all x_0 close enough to K_0. Then, using Lemma 2 we have

$$u_0(x) \leq M \leq \left(\frac{\lambda}{\beta(x_0)}\right)^{\frac{1}{\rho-1}} \leq z(x_0) \leq z(x), \qquad \forall x \in B_0.$$

Hence $z(x)$ is a supersolution for $u(x,t)$ and then $u(x,t) \leq z(x)$, for all $x \in B_0$, $t > 0$. In particular, for $x = x_0$ we get, from Lemma 2 that for all $t > 0$,

$$u(x_0,t) \leq z(x_0) \leq \left(\frac{\lambda(\rho+1)}{2\beta(x_0)} + \frac{B}{\beta(x_0)d_0(x_0)^2}\right)^{\frac{1}{\rho-1}}$$

for some constant $B > 0$. Since x_0 is close enough to K_0 we can assume

$$u(x_0, t) \leq z(x_0) \leq \left(\frac{A}{\beta(x_0) d_0(x_0)^2}\right)^{\frac{1}{\rho-1}}$$

for all $t > 0$ and some $A > 0$.

From previous results, far from K_0, $u(x, t)$ remains bounded, and for $x_0 \in K_0$ the result is obvious.

Next we want to distinguish the behavior of the solutions in K_1 and on K_2. The following result gives a criteria to check whether a function that is infinity on a compact set of measure zero is integrable. As shown below, this criteria depends on the dimension of the set and on the form the function diverges on the compact set.

Lemma 4 *Assume $K \subset \mathbb{R}^N$ is a compact set with zero Lebesgue measure and dimension $d \leq N - 1$ and consider a function defined on a bounded neighborhood ω of K of the form*

$$f(x) = (dist(x, K))^{-\alpha}, \qquad \textit{for some} \quad \alpha > 0, \qquad f|_K = \infty.$$

If $r\alpha < N - d$ for some $r \geq 1$, then $f \in L^r(\omega)$.

Proof Note that

$$\int_\omega |f(x)|^r \, dx = \int_0^\infty |A_s| \, ds$$

where $A_s = \{x \in \omega, \ |f(x)|^r \geq s\}$. But

$$|f(x)|^r \geq s \qquad \text{iff} \quad dist(x, K) \leq s^{-\frac{1}{\alpha r}}.$$

Therefore $|A_s| = |\omega_{\delta(s)}|$ where

$$\omega_\delta = \{x \in \omega, \quad dist(x, K) \leq \delta\} \qquad \text{and} \quad \delta(s) = s^{-\frac{1}{\alpha r}}.$$

From the assumption on the dimension of K we get $|\omega_\delta| \leq C\delta^{N-d}$. Moreover, due to $|A_s| \leq |\omega|$, $\int_0^\infty |A_s| \, ds \leq |\omega| + \int_1^\infty |A_s| \, ds$. Therefore,

$$\int_\omega |f(x)|^r \, dx \leq |\omega| + C \int_1^\infty \left(\frac{1}{s}\right)^{\frac{N-d}{\alpha r}} ds < \infty \qquad \text{whenever} \quad 1 < \frac{N-d}{\alpha r},$$

and the result follows.

We prove now Theorem 2.

Proof of Theorem 2 From Lemma 1, any positive solution is unbounded in Ω_0, and so (4) holds. Moreover, with the comparison argument used in the proof of Lemma 1 we get that the limit is uniform in compact sets of Ω_0.

(i) Since $K_1 \cap K_2 = \emptyset$ and $|K_2| = 0$, we can construct a set of the form $V_\delta = \{x \in \Omega : d(x, K_2) < \delta\}$ with $\delta > 0$ small enough so that $K_1 \cap \bar{V}_\delta = \emptyset$ and $\lambda_1(V_\delta)$ is large enough, say $\lambda_1(V_\delta) > \lambda$. Moreover, from Proposition 1, $|u|$ is bounded uniformly in $t > 0$ in ∂V_δ, by a constant, say M.

Hence, the solution U of

$$\begin{cases} U_t - \Delta U = \lambda U & \text{in } V_\delta, \quad t > 0, \\ U = M & \text{on } \partial V_\delta, \quad t > 0, \\ U(0) = u_0 \geq 0 & \text{in } V_\delta \end{cases}$$

becomes a supersolution of $|u(x,t)|$ in V_δ. Since $\lambda < \lambda_1(V_\delta)$ then $U(x,t)$ and therefore $|u(x,t)|$, remains bounded in V_δ.

(ii) From Proposition 1, for any given solution of (1) we have L^∞ bounds on compact sets of $\Omega \setminus K_0$.

Let K be an arbitrary compact set in $\Omega \setminus K_1$, such that $K \cap K_2 \neq \emptyset$. Let B be a "transversal isolating box" for K, that is B is an open bounded set such that $K \subset \overline{B} \subset \Omega \setminus K_1$ and $dim(K_2 \cap \partial B) \leq d - 1$. Then, from Lemmas 3, 4 and condition (H3), we have that there exists a function $h \in L^r(\partial B)$ such that $|u(x,t)| \leq h(x)$ for all $x \in \partial B$. Hence, the solution of

$$\begin{cases} U_t - \Delta U = \lambda U & \text{in } B, \quad t > 0, \\ U = \tilde{h}(x) & \text{on } \partial B, \quad t > 0, \\ U(0) = u_0 \geq 0 & \text{in } B \end{cases}$$

becomes a supersolution of $|u(x,t)|$ in B.

Now, if $\lambda \geq \lambda_1(\Omega_0)$ we can shrink B to be close enough to K_2 such that $\lambda < \lambda_1(B)$. Then, standard parabolic regularity gives L^∞ bounds for $U(x,t)$ for all time, on compact subsets of B. Hence, $u(x,t)$ remains bounded on K_2 as $t \to \infty$. □

Remark 1 It is an interesting open problem to determine whether we always obtain that the solution of the parabolic problem (1) are bounded in compact sets of $\Omega \setminus K_1$ or, in the contrary, that we have cases in which u becomes infinity in K_2 as $t \to \infty$.

Remark 2 This work is still in progress, and we refer to [2] for details and more general results, including more general configurations for the set K_0.

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