Domain-Specific Belief Systems of Secondary Mathematics Teachers

Andreas Eichler and Ralf Erens

Abstract This chapter focuses on belief systems of secondary mathematics teachers as part of teachers' mathematics-related affect. Our particular interest concerns teachers' belief systems that represent the teachers' instructional planning. Further we focus briefly on the impact of the teachers' belief systems on their classroom practice and their professional development. In this paper we discuss our theoretical approach in relation to the international discussion on mathematics-related affect. After a brief outline of methodological considerations, the structure of calculus teachers' belief systems is analyzed with regard to the issue of central and peripheral beliefs and the relationships of belief clusters. Secondly we comment on patterns found in the belief systems of teachers thinking about different mathematical domains. An identification of distinctive features of beliefs regarding different mathematical domains is followed by an analysis of the impact of teachers' beliefs on their classroom practice and their professional development.

Keywords Teachers' beliefs • Teachers' goals • Belief systems • Central and peripheral beliefs

Setting the Field

> How teachers make sense of their professional world [...], and how teachers' understanding of teaching, learning, children, and the subject matter informs their everyday practice are important questions that necessitate an investigation of the cognitive and affective aspects of teachers' professional lives. (Calderhead 1996, p. 709)

A. Eichler (✉)
Institute of Mathematics, University Kassel, Kassel, Germany
e-mail: eichler@mathematik.uni-kassel.de

R. Erens
Institute of Mathematics Education, University of Education Freiburg, Freiburg, Germany
e-mail: erens@ph-freiburg.de

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The vast and still increasing amount of research into teachers' mathematics-related affect, that sometimes seems to fulfill the demand to investigate the cognitive and affective aspects of teachers' professional lives, makes it reasonable to clarify the potential benefit of a further contribution to this field of research. For this reason, we will integrate our research approach that we have pursued for 10 years into the body of research resulting in substantial findings in recent decades. However, we do not want to add a further review of the entire body of research into teachers' mathematics related affect (e.g. Thompson 1992; McLeod 1992; Philipp 2007), but rather highlight three issues to clarify the aim of our contribution, i.e. *models*, *external influences* and *impacts*.

Models of Teachers' Mathematics-Related Affect

In recent years several models have been proposed for positioning the parts of teachers' sense-making. These models enable us to relate a specific research approach to already existing approaches and to describe possible relations among different parts of teachers' sense-making (Schoenfeld 1998). For example Hannula et al. (2007, p. 204) proposed a model in which mathematics-related affect consists of three overlapping constructs, i.e. motivation, cognition and affect, which in turn consist of further constructs. For example the construct of motivation is used to integrate the constructs of goals, needs and, in the intersection of the three overarching constructs, beliefs and belief systems. This model could be understood as further development of McLeod's model (1992), in which emotions, attitudes and beliefs are positioned on a continuum from least stable and cognitive to most stable and cognitive. Further, in the model of Hannula et al. (2007) knowledge and belief are distinct parts of the construct of cognition, which is in line with other models regarding teachers' decision-making (e.g. Ball 1990; Borko and Putnam 1996).

Using the model of Hannula et al. (2007), our own research on upper secondary teachers' mathematics related affect considers the motivational and cognitive aspects, since we investigate primarily teachers' instructional goals and beliefs, which also fits in some sense the approach of Schoenfeld (1998), who proposes a distinction between knowledge, goals and beliefs. However, in contrast to Schoenfeld, we put less emphasis on the teachers' knowledge.

External Influences on Mathematics-Related Affect

A lot of research into teachers' mathematics related affect – particularly when teachers and their thinking began to be an important issue for educational research – does not consider external influences. However, this research yielded important results, e.g. the seminal case study of Thompson (1984), who reported the beliefs of three teachers described as having instrumental, formal and conceptual understanding of mathematics that are immanent to the three teachers beliefs and observable into their classroom practice. Also other researchers provide empirical or theoretical driven categorizations of beliefs about mathematics or mathematics teaching and learning (e.g. Dionne 1984; Ernest 1989; Grigutsch et al. 1998) that still impact on research in teachers' mathematical affect.

However, particularly in the recent decade, researchers increasingly consider external influences on teachers' mathematics-related affect. The broadest scope in this line of research is constituted by the cultural dimension. For instance, the large scale study TALIS (OECD 2009) reported striking differences of different countries referring to teachers' beliefs about their teaching orientation representing a direct transmission and constructivist understanding of teaching mathematics. This finding partly agrees with the results of TEDS-M (Felbrich et al. 2012) that makes a distinction between countries with a culture of individualism and countries with a culture of collectivism.

A further external influence is represented by a social context. For example, the case study of "Larry at Mellemvang" Skott (2009, p. 31) defines the social context as a school specific setting "construed by individuals as they participate in praxis that evolve in interaction". Based on this framework, he explains the influence of social norms in a traditional private school on the beliefs and the classroom practice of Larry. Also Sztajn (2003) explains the differences of the classroom practices of two teachers holding similar beliefs by the social setting of two different schools.

Finally, Schoenfeld (1998) defines the context in a narrow sense regarding teachers' knowledge, beliefs and goals and their teaching. Thus, Schoenfeld analyzed in depth four teachers' "moment-to-moment decision making and acting" (ibid., p. 1) in the context of a specific mathematical "instructional segments" (ibid., p. 78) in a specific class.

Our research approach refers partly to the latter two aspects of influence, while we did not consider cultural differences since we restrict our sample to German teachers. We further acknowledge the influence of the teachers' social context primarily concerning the formation of our sample that we describe in a later section. Secondarily we regard the social context, when we analyze the teachers' professional development to which we refer briefly in the sixth section. Our main focus is, however, on the context that Schoenfeld (1998) describes. In contrast to Schoenfeld, we define context in a broader sense. Thus, we opt for a broad scope regarding the context, i.e. the teachers' beliefs about the entire mathematics curriculum of upper secondary schools lasting from grade 5 to grade 12 or 13. In this broad scope we further define the context by a specific mathematical domain like data and chance (e.g. Eichler 2011).

Impacts of Teachers' Goals and Beliefs

As in the quoted works of Schoenfeld (1998) and Skott (2009), the impact of teachers' beliefs on their classroom practice is an important research question (e.g. Philipp 2007). However, the relation of espoused and enacted beliefs still seems to be far from a conclusion. Thus, it is firstly not clear if the classroom practice impacts on the teachers' beliefs or if the relation is inverse (Franke et al. 1997). Further different researchers reported inconsistencies while others report a consistency between beliefs and classroom practice (e.g.Philipp 2007). From the suggested assertions for observed inconsistencies, i.e. the inexperience of the observed teachers (Artzt and Armour-Thomas 1999), the specific social context of a classroom (Skott 2009) or the grade of intensity with which a teacher holds a belief (Putnam and Borko 2000), we will make a contribution to the latter aspect. For this reason, a main focus in this paper is the identification of central and peripheral beliefs of a teacher about a specific mathematical discipline.

Since there is on the one side a potential impact of teachers' beliefs on their classroom practice, a further function of beliefs is to be a filter that impacts on a person's perception (e.g. Franke et al. 1997; Philipp 2007). For this reason, teachers' beliefs potentially have an impact on their professional development (Chapman 1999). We also refer briefly to this aspect referring to the development from pre-service to in-service teachers.

Concluding Remarks

Based on the brief overview of important issues of the research in teachers' mathematics-related affect the aim of our research approach is to make a contribution to the following research questions:

1. What is the structure of a teachers' system of beliefs and goals referring to a mathematical discipline and the teaching and learning of this discipline?
2. How do these systems of beliefs and goals differ with regard to different mathematical disciplines?
3. How do the teachers' beliefs and goals impact on their classroom practice and their professional development?

We refer to these questions after outlining the central constructs and the method of our research approach. We discuss the results primarily referring to the first two research questions and with less detail on the third.

Theoretical Framework

Stein et al. (2007) define a model to distinguish the possible phases of a curriculum that impact on teachers' beliefs.

The term *written curriculum* involves both instructional content, and teaching objectives, or, more recently, standards, often prescribed by national governments. The teachers' interpretation of the written curriculum – that is, the individual teacher's transformation of the written curriculum – is called the *intended curriculum*. The interactions of a teacher, his students, and the instructional content "bring the curriculum to life and, in the process, create something different than what could exist [...] in the teacher's mind" (Stein et al. 2007, p. 321). This transformation of the intended curriculum is called the *enacted curriculum*.

Finally, the students transform the content addressed in the enacted curriculum into their own personal subjective knowledge and develop their own beliefs about the content. This is the *students' learning*.

A teacher's own experiences with his classroom practice (enacted curriculum) as well as his awareness of the beliefs and knowledge attained by the students (students' learning) in turn have an impact on the teacher's intended curriculum (Hofer 1986) so that it actually develops over time. In this chapter we focus on different parts of the curriculum model. For this reason, a possible aspect of the consistency of teachers' espoused and enacted beliefs could potentially be explained with the teachers' grade of experience (Artzt and Armour-Thomas 1999).

Further, we understand the term *beliefs* as an individual's personal conviction concerning a specific subject, which shapes an individual's ways of both receiving information about a subject and acting in a specific situation (Pajares 1992; Thompson 1992; Furinghetti and Pehkonen 2002). Knowledge and beliefs could be seen as "inextricably intertwined" (Pajares 1992, p. 325). For this reason we distinguish knowledge and beliefs theoretically by understanding beliefs as more individual convictions and by understanding knowledge as more inter-individual (or objective) convictions (Pajares 1992; Borko and Putnam 1996).

An individual's organization of beliefs we call *belief system* following Green (1971) or Thompson (1992). The individual's organization of beliefs involves the distinction of central beliefs, i.e. strongly held beliefs, and peripheral beliefs referring to an individual's belief system of lesser importance. Further, belief systems consist of belief clusters that are quasi-logically interconnected and, thus, different beliefs in an individual's belief system may be contradictory. We discuss the two aspects of centrality and quasi-logicalness later when analyzing mathematics teachers. However, we avoid the theoretical distinction of primary and derivative beliefs, which is the third aspect of the structure of belief systems (Green 1971), since we have no empirical evidence concerning this aspect in our research (c.f. for this aspect also Liljedahl 2010).

As stated in the introductory section, we regard both teachers' beliefs and teachers' goals that are understood as different constructs (e.g. Schoenfeld 1998; Hannula 2012). Schoenfeld describes beliefs as a mental orientation that shape the way of establishing a specific goal. Accordingly, in a further development of his model, Schoenfeld (2010, p. viii) distinguishes *goals* and *orientations* that include beliefs in addition to dispositions or values. This is in line with the consideration of Hannula (2012) about the psychological dimension to state and trait of the motivational aspect of teacher mathematics-related affect. Referring to this distinction, he suggests goals to represent the state and (motivational) beliefs to represent the trait of this motivational aspect. Thus, in both theoretical frameworks goals are necessarily connected with an observable behavior (Cobb 1986). However, our research approach is based on a model of teachers' action that is described in the so-called rubicon-model (Heckhausen and Gollwitzer 1987). In this model, a person defines goals before an observable behavior (pre-behavioral phase; motivation), decides when and how she or he wants to establish the goals (pre-behavioral phase; volition), establish goal-oriented behavior (behavioral phase), and finally evaluate for example if the goals were achieved (post-behavioral phase). Based on this theory, a teacher's intended curriculum consists of goals that are closely connected to his beliefs. For example, a teacher believes that both frequentist and an axiomatic approach to probabilities are important mathematical concepts (belief). However, the teacher plans to achieve his students' understanding of probabilities by choosing the introduction to probabilities according to the frequentist approach (goal).

Teaching goals could represent overarching beliefs representing "world views" (Grigutsch et al. 1998) or epistemological beliefs (Hofer and Pintrich 1997) about mathematics (or different mathematical disciplines), about school mathematics or about teaching and learning mathematics (Grossman 1990). However, teaching goals could also concern, for instance, specific content or issues of a mathematical discipline, representations of mathematical objects or students' difficulties with specific content. Thus teaching goals exist with different grain sizes (Schoenfeld 1998, p. 21) or rather ranges of influence. We discuss teaching goals of a lower range of influence later. For analyzing overarching teaching goals, we use the construct of mathematical world views proposed by Grigutsch et al. (1998):

- a formalist (world) view that stresses that mathematics is characterized by a strongly logical and formal approach. Accuracy and precision are most important and a major focus is put on the deductive nature of mathematics.
- a process-oriented view that is represented by statements about mathematics being experienced as a heuristic and creative activity that allows solving problems using different and individual ways.
- an instrumentalist view that places emphasis on the "tool box"-aspect which means that mathematics is seen as a collection of calculation rules and procedures to be memorized and applied according to the given situation.
- an application oriented view that accentuates the utility of mathematics for the real world and the attempts to include real-world problems into mathematics classrooms.

Concluding our theoretical framework (c.f. Eichler 2011), we understand a teacher's *intended curriculum* as an individual's belief system including

- an individual's world view consisting of beliefs about the nature of mathematics or a mathematical discipline represented by overarching teaching goals,
- beliefs represented by teaching goals of different ranges of influence that a teacher takes into account when planning (in his view) appropriate classroom practices. These goals (beliefs) might concern content, the best way to teach mathematics or a specific mathematical discipline, or the way students learn mathematics.

Further, teachers' *enacted curricula* involve the observable part of the teachers' intended curricula transformed by the interaction of teachers, their students, and the content within the classroom practice. Finally, *students' learning* is represented by students' knowledge and beliefs concerning mathematics.

Method

For different parts of our research program, we used different methods. We briefly discuss these methods structured by the curriculum model (Fig. 1).

In this report, we refer to a sample of 51 secondary teachers. 30 teachers' were interviewed in respect to calculus, 13 teachers were interviewed in respect to stochastics (statistics and probability), and 8 teachers were interviewed in respect to geometry. Regarding the selection of teachers, different degrees of teaching experience were considered as well as a balanced proportion concerning gender (Hannula 2012). Teachers who were interviewed about geometry or stochastics are all in-service teachers. The "calculus sample" consists of 30 teachers divided into three subsamples: pre-service teachers, teacher trainees and experienced teachers. The first subsample includes 10 experienced teachers who have been teaching calculus for at least 5 years. Data concerning the intended curricula of experienced teachers that are assumed to be relatively stable (McLeod 1992) were collected once. The other subsamples consist of each 10 prospective teachers. The data for these subsamples were collected twice within one and a half years in a quasi-longitudinal design.

In order to capture both the need of contextualizing beliefs and the notion of belief enactment in a locally social approach (c.f. Skott 2009, p. 29), the teachers who participated in this study were recruited from different universities, teacher training colleges and secondary schools across Germany. Every (in-service) teacher in this study teaches all domains of mathematics from grade 5 to 12. The domains of stochastics and calculus are a central part of the curriculum at upper secondary level (grade 10–12). However, our sample is a theoretical, not a representative sample.

To investigate *teachers' intended curricula* referring to one discipline we use intensive semi-structured interviews (Witzel 1982) lasting about 2 h following a qualitative case study approach and questionnaires for a quantitative analysis.

The interviews consist of several clusters of questions that mostly concern intended curricula referring to a specific mathematical discipline (e.g. calculus) but also to mathematics in general, e.g. instructional content, teaching objectives, reflections on the nature of mathematics (as a discipline generally) and of school mathematics, the students' views, or textbook(s) used by the teachers. Further, we use prompts to

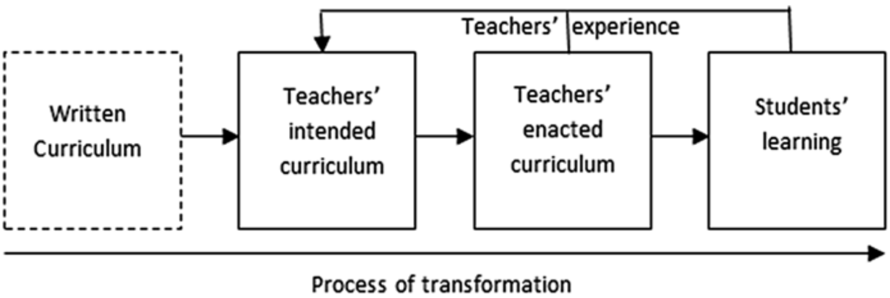


Fig. 1 Model of a curriculum

(a) I like calculus, because there is a connection to real life problems.
(b) I like calculus, because hard nuts must be cracked and difficult problems can be solved.
(c) I like calculus, because many exercises can be solved by similar procedures and patterns.
(d) I like calculus, because the logic is clear and it follows strict mathematical rules.

Fig. 2 Fictive statements of students concerning calculus

provoke teachers' beliefs, e.g. by making teachers comment on different parts of a textbook. We show one of these prompts in the results sections. In other parts of the interviews, the teachers were asked to comment on fictive or real statements of teachers or students. One of these prompts is shown in Fig. 2.

Each of the various prompts represents a specific view concerning mathematics or a mathematical discipline, e.g. a formalist view (see above). Further we employed two different questionnaires including adapted scales concerning mathematical beliefs (Grigutsch et al. 1998) and teaching orientation (Staub and Stern 2002) of which we refer to the former in the results sections.

For analysis of qualitative and quantitative data we used mixed methods including coding methods (Strauss and Corbin 1998; Mayring 2003), and statistical methods. A qualitative coding method was used for analysis of the interview data that is close to grounded theory (Glaser and Strauss 1967). The codes gained by interpretation of each episode of the verbatim transcribed interviews indicate goals of calculus teaching. We used deductive codes to the above mentioned mathematical views as well as the teachers' teaching orientation. The latter is not the focus of this report and will therefore not be discussed in the results section. Inductive codes for those goals we did not deduce from existing research such as the integration of technology into classrooms or the impact of authorities were developed from the interview data. The codes were conducted by at least two persons. The interrater reliability shows an accordance above 80 %.

To investigate *teachers' enacted curricula*, we used videography and protocols to document the teachers' classroom practices. In this chapter, we refer to a subsample of stochastics teachers that were observed in their classroom practice for half a year. These teachers were selected due to the differences that these teachers show referring to their intended curricula.

Structure of Teachers' Intended Curricula: Calculus Teachers

In this paragraph, we analyze calculus teachers' belief systems representing their intended curricula. For this, we firstly discuss the issue of central and peripheral beliefs (Green 1971), and afterwards the issue of (quasi-)logical relationships of clusters of beliefs.

Central and Peripheral Beliefs

In order to categorize and illustrate teachers' beliefs concerning the planning and teaching of calculus by means of qualitative analysis, the deductive aspects of four different views (see above) were chosen. This involves the subjective teachers' definition of a specific view that represents the teachers' overarching teaching objectives.

First, we illustrate a coherent view, in this case a formalist view concerning the subjective definition of Mr. C_{Calc}.

> Mr. C_{Calc}: In general, exactness is crucial for me. That means to fit a necessary formalism as I know from my university studies. This also means that it must be possible to recognise a logical rigor. Sometimes I do more in that sense than the textbook actually demands.

Taking this teacher as a paradigmatic example, he did not mention aspects such as to apply mathematics in real world problems or to learn problem solving, which means to emphasize the process of developing mathematical concepts. By contrast, for Mr. C_{Calc}, the main goal of calculus teaching seems to be emphasizing the stringent and logical construction of a mathematical domain.

The identification of specific teachers' views is always established in various parts of a single interview with either questions regarding the teaching of calculus or teaching orientation in general or prompts to provoke teachers' beliefs (see section "Method") and we report only teachers' views that are in some sense coherent throughout the whole interview. We illustrate this concerning this exemplified teacher. When Mr. C_{Calc} was asked to regard the expectations and needs of his students, he agrees consistently with a formalist view.

Mr. C_{Calc} further explains his goals concerning his students' beliefs towards calculus:

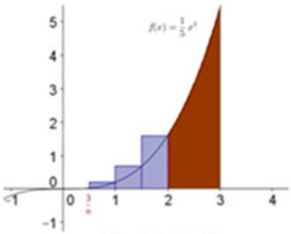
> Interviewer: How should your students characterize calculus?
> Mr. C_{Calc}: Precise mathematics. Thus, on the one side that it is possible to understand how one develops mathematical ideas and how it is possible to build up a theory on the foundation of few basic ideas.

The coherence of the beliefs of Mr. C_{Calc} is also apparent in his responses to several prompts used in the interview regarding decisions on instructional content and the above described views concerning teaching calculus. For example, when Mr. C_{Calc} was asked to evaluate four tasks that represent the four different views, he valued the task representing the formalist view (Fig. 3) higher than the other tasks.

Summarizing the beliefs of Mr. C_{Calc} concerning the teaching and learning of calculus, there exist a lot of other unambiguous examples of evidence for Mr. C_{Calc}'s formalist view. The high degree of coherence in different parts of the interview leads to the hypothesis that this formalist view is dominant and thus *central* in the belief system on calculus. This hypothesis is supported by reported examples and tasks of Mr C's classroom practice. Furthermore the hypothesis of centrality is supported by the evaluation of the questionnaires which consistently confirm the qualitative codings (Erens and Eichler 2013a).

Calculate the area between the graph of the function $f(x) = \frac{1}{5} \cdot x^3$ and the x-axis between $0 \le x \le 3$.

Solution:
As the function f is continuous, we obtain the same result for the limit of upper sum and lower sum of the area of rectangles. It is thus sufficient to regard the upper sum of rectangles.



The given interval is divided into n parts of length $\frac{3}{n}$ and the corresponding area of the sum of upper rectangles is

$$O_n = \frac{3}{n}\left[\frac{1}{5}\cdot\left(\frac{3}{n}\right)^3 + \frac{1}{5}\cdot\left(2\cdot\frac{3}{n}\right)^3 + \ldots + \frac{1}{5}\left(n\cdot\frac{3}{n}\right)^3\right]$$

$$= \frac{3^4}{n^4}\cdot\frac{1}{5}\cdot\left[1^3 + 2^3 + 3^3 + \ldots + n^3\right]$$

Because

$$1^3 + 2^3 + 3^3 + \ldots + z^3 = \frac{1}{4}\cdot z^2 \cdot (z+1)^2$$

one can simplify O_n to

$$O_n = \frac{81}{n^4}\cdot\frac{1}{5}\cdot\frac{1}{4}\cdot n^2(n+1)^2 = \frac{81}{20}\cdot\frac{(n+1)^2}{n^2}$$

$$= \frac{81}{20}\cdot\left(1+\frac{1}{n}\right)\cdot\left(1+\frac{1}{n}\right)$$

The limit of O_n for $n \to \infty$ is $\lim_{x\to\infty} O_n = \frac{81}{20}$

which is the area we wanted to calculate.

Fig. 3 Task representing the formalist view

In addition to central beliefs that teachers like Mr. C_{Calc} show in different parts of the interview in a coherent way, most of the teachers also provide insights into some peripheral goals. For example, some teachers indicate a *peripheral* goal that calculus and the teaching of calculus is a collection of rules and procedures although their beliefs can neither qualitatively nor quantitatively be categorized globally as an instrumentalist view.

> Mr. F_{Calc}: The main goal of every student is to perform well in his final exams – therefore calculation rules and procedures have to be thoroughly practiced in class. Especially the calculus part of final exam tasks are alike in some respect, so practicing is a substantial guideline for my course.

Like Mr. F_{Calc}, several calculus teachers show a connection of an instrumentalist view and considerations referring to normative aspects such as final exam tasks that represent the teacher's acknowledgement to a social context (c.f. Skott 2009). However, it is apparent that for most of the teachers in our sample teaching goals representing an instrumentalist view are not central for mathematics or calculus instruction per se, but are important in terms of preparing students for the final exam, which supposedly is a particularity due to the situation of German teachers (cultural dimension; e.g. OECD 2009).

(Quasi-)Logical Relations into Belief Systems

In contrast to Mr. C_{Calc}, most of the teachers show a mixture of different views that individually are also coherent. In particular, if teachers' hold beliefs that represent different views, we analyze relationships within and among the different views. The codes gained by interpretation of each episode of the interviews allow a more differentiated analysis of the different views and can be warranted with substantial reactions to content and teaching goals. We describe this analysis exemplarily by considering Mr. A_{Calc} and Mr. B_{Calc} starting at the subjective definitions of their possibly central beliefs. In contrast to a (central) formalist view, these two teachers delivered an insight into their views on applications:

> Mr A_{Calc}: I quite agree with the emphasis on applications in the given example. That is certainly a way to motivate them (students), but nevertheless one should not reduce genuine calculus or the teaching of calculus to that topic.
> Mr. B_{Calc}: Examples for applications are quite suitable here, and with applications I always associate modeling of real data, [...] increasingly introducing relevant applications into lessons may, for the students, succeed in a deeper insight into the concepts and ideas of calculus.

For Mr. B_{Calc}, beliefs representing an application oriented view seem to be central, since other teaching goals are peripheral or of no importance if school mathematics is regarded:

> Mr B_{Calc}: "…because I think that the formal derivation of integrals by limits is of no avail for secondary level students. It's just too complex for most of them."

By contrast Mr. A_{Calc} supports the integration of applications as a principle of learning calculus at school for reasons of (student) motivation. Using applications in his teaching represents an additive goal to achieve a teaching goal of higher importance, i.e. students' motivation. Consistently Mr. A_{Calc} mentions other possibly more central teaching goals that represent a formalist view illustrated in the following quotation:

> Mr. A_{Calc}: "Calculus is more than just dealing with application-oriented tasks. Then, for example, one would not regard the precision and exactness of calculus and use applications as a means to an end."

The difference between the instructional goals of motivation on the one side, and solving real problems on the other, is stated by Förster (2011) concerning teachers who teach modeling. Both views on applications can be found several times in our sample.

Our hypothesis on the basis of the present data is the following: If teachers hold a consistent formalist view on calculus like Mr. C_{Calc}, they do not mention any applications. The converse conclusion, however, is not possible. Teachers who favour applications in their calculus courses, e.g. Mr. A_{Calc} and Mr. B_{Calc} (see above), cannot necessarily be described as non-formalist. This example already demonstrates the abundance of calculus teachers' beliefs and the need to differentiate the views of teachers on calculus as well as relations between different views. It demonstrates further, that qualitative analysis of the data hence enables us to discern these relations in a sophisticated manner.

Referring to the teachers regarded so far, both the formalist view and the application-oriented view is identified to be central for some of the teachers. By contrast, for a majority of teachers in our sample, the process-oriented view is subordinated to other views, namely an application-oriented view. Thus, many teachers in our sample noted application-oriented tasks as illustrative approaches for relevant mathematical methods, often manifested by giving appropriate examples from their own lessons. In close connection with these evidential classroom episodes teachers often used key words such as "understanding", "comprehension", "problem-solving strategies" and "students finding out" or "discovering by themselves". The emphasis on problem-solving strategies and student activity in the classroom discourse suggests a rather close connection between an explicit preference of experiencing calculus methods as a heuristic and creative activity (process-oriented view) and attempting to accentuate the utility of calculus for the real world. This result agrees with the findings of Felbrich et al. (2012) referring to the teaching orientation of German mathematics teachers.

While we have described above relations between beliefs that are logically connected, we also found contradictory belief clusters that we call conflicts of instructional goals that represent the quasi-logicalness of a belief system. We illustrate contradictory clusters of beliefs with the paradigmatic example of Mrs. E_{Calc}. Throughout the whole interview she speaks about the central role of logic in calculus lessons offering her perspective that exactness and logical rigour are necessary ingredients of secondary level calculus courses. Again, the degree of coherence of favoring formalist elements could provide an indication for a central belief. Yet, as she describes representative classroom situations, her subjective experience surfaces a conflict between her belief system about calculus teaching and pedagogical processes in her calculus course.

> Mrs. E_{Calc}: In my view it is quite important that there are formal definitions of concepts because you need them for proofs later on and it's the tiny details that are particularly important.
>
> In my class I clearly notice that students come to their limits concerning the degree of abstraction. [...] Remembering my own calculus course at school I can't remember any bad experience with these formal aspects. So far I haven't seen such a mismatch between teacher and students in maths.

Mrs E_{Calc} can be identified favoring a formalist view, but probably will not enact her formalist view on calculus teaching in the classroom in a predominant way because there is a conflict with the real situation she encounters in the classroom i.e. the students' ability to understand the formal way of developing calculus ideas. This teacher shows a high awareness and consciousness of how these conflicting forces are affecting her curricular and pedagogical decisions with respect to the differences between teaching and learning calculus. Therefore this situation can be characterized as a conflict of goals between her view on calculus and her teacher authority and responsibility. In particular when teachers are asked to reflect on representative examples of their actual teaching processes of specific elements of their calculus courses, the interview transcripts provide a deep and concrete insight into teachers' subjective notions of their intended curricula and sometimes yield conflicts of a teacher's system of instructional goals. We hypothesize that a conflict of teaching objectives gives evidence for a central belief since peripheral beliefs might be superimposed if they show a conflict with central beliefs.

Differences Among teachers' Belief Systems of Different Mathematical Disciplines

We firstly illustrate patterns found in the belief systems of teachers thinking about specific mathematical disciplines structured by the mathematical views outlined above. However, the teachers sometimes compared the focused mathematical discipline like calculus with other disciplines like geometry. We emphasize these comparisons at the end of this section.

Teaching Mathematics with an Application Oriented View

The eight teachers in our sample mostly tended to neglect application-oriented goals when they think about geometry. One paradigmatic example for this assertion is the case of Mr. B_{Geo}:

> Mr. B_{Geo}: Geometry as a tool to get access to the real world is not fundamentally important, and it is deservedly not in the first place. An application is useful to introduce a new subject, to legitimize it, and to test the competencies of this field by realistic tasks in the end. But in between, a lot has to be done without any reference to the real world, detached from these accessory parts which are not important to the mathematical model.

Like Mr. B_{Geo}, most of the investigated geometry teachers understood real applications at most as a strategy to motivate students, but not as an important aspect of this mathematical discipline. By contrast, for geometry teachers, geometry is rather seen as a language that can be used to describe reality, but doesn't have to. The predominant goal of teaching geometry is to learn this language, wherefore real situations are mainly used just as illustrations, and not as interesting occasions to gain insight into realistic problems and to learn model building. Hence, the real situations, their data and empirical challenges are of minor interest and in principle interchangeable: The context is suspended in favor of the theory (Girnat 2009).

In contrast to the geometry teachers, the *role of the context* (Shaughnessy 2007) is omnipresent for the teachers interviewed on their intended stochastics curricula. Whereas the geometry teachers doubted whether geometry is an adequate discipline to emphasize the applied aspect of mathematics, the participating German stochastics teachers did not question that applications play a significant role in stochastics teaching (Eichler 2011):

> Mrs. B_{Stoch}: Which objectives I have? That students were enabled not to fail when they were confronted with challenges or allurements in their daily life, but to develop the possibility to evaluate things for themselves.

The consideration of Mrs. B_{Stoch} represents a common idea concerning the application-oriented view of the stochastics teachers: Applications are used to gain the insight that mathematics can be useful for real world problems; and therefore, the real situations have to be treated more seriously than in geometry. However, the stochastics teachers differ in their way to highlight stochastics as an applied domain of mathematics (e.g. Eichler 2011).

Since both geometry teachers evaluate an application-oriented view to be peripheral and stochastics teachers evaluate this view to be central, the findings referring calculus teachers are ambiguous. Thus, we found some teachers like Mr. C_{Calc}, who neglect an application oriented view (see above), as well as some teachers, who stress real applications as tool for motivating students (e.g. Mr. A_{Calc}, see above) or stress calculus as discipline to emphasize modeling (e.g. Mr. B_{Calc}, see above).

Teaching Mathematics with a Process Oriented View

Most of the teachers articulated a process-oriented view by thinking about problem solving according to the approach of Pólya (1949). Referring to this orientation, geometry teachers tended to emphasise problem solving as the main idea of a geometry curriculum in school. We illustrate this orientation by quoting the typical statement of Mrs. G_{Geo}:

> Mrs. G_{Geo}: Besides proof abilities, problem solving is in fact the most important thing I want to convey in my lessons on geometry. To pose students problems.

For geometry teachers "problems" mean mathematical problems that need not have a connection to a real world situation and that are posed to enhance properties in reasoning, and not to gain empirical knowledge or to conceive mathematics as being useful.

By contrast, stochastics teachers also mentioned that to learn problem-solving strategies has to be a teaching objective. However, these teachers identified the problem of stochastics tasks to find an appropriate model for a realistic situation:

> Mr. E_{Stoch}: To learn problem-solving in stochastics is to learn to argue mathematically on the basis of a specific realistic context.

In the same way, calculus teachers showed a process-oriented view in connection with other views, e.g. a formalist view or an application-oriented view emphasizing creativity in the students' individual ways of modeling real situations, working on main concepts of calculus, and, more peripheral, to solve mathematical problems.

Thus, since problem solving predominantly appears to be a central goal for geometry teachers, it seems to be a more peripheral or subordinated one for both stochastics teachers and calculus teachers. For stochastics teachers in our sample the process oriented view is subordinated to the main objective to translate real world problems into stochastics and to interpret stochastical results by referring to a real world situation. For calculus teachers the process-oriented view is subordinated to different main objectives that represent a formalist or an application-oriented view.

Teaching Mathematics with a Formalist View

Some geometry teachers tended to emphasize the formalist view mentioning for example the integration of phenomena investigated in mathematics lessons into a formal and abstract mathematical structure following a deductive approach that Girnat (2009) calls classical Euclidean view on geometry. Mr. C_{Geo} formulates this view mentioning a meaningful example:

> Mr. C_{Geo}: If someone asserted in case of the Pythagorean Theorem "Proved by measuring, the theorem holds", then something valuable would disappear, something which is genuinely mathematical. […] If geometry just consisted of measuring, calculations, drawing, constructing, and land surveying, then I would regard it as poor.

Although some of the stochastics teachers hold beliefs representing a formalist view, they mostly seem to understand these goals as peripheral ones. The case of Mrs. B_{Stoch} shows a paradigmatic example of more or less neglecting the formalist view:

> Mrs. B_{Stoch}: Formalism is out. Indeed, there are some colleagues, who say that it is not the right way to show, for instance, the theorem of Bayes by using an example. I think let them teach in this way. In my opinion, for students it is better to show them the theorem of Bayes using examples or using a probability tree.

At first, beliefs representing a formalist view seem to be central for geometry teachers. However, taken into account all interviewed geometry teachers the formalist view seems to be subordinated in comparison to the process-oriented view. By contrast, our stochastics teachers mostly neglected the formalist view in favor of the application-oriented view.

Although the calculus teachers differed concerning their beliefs representing a formalist view, it is striking that only calculus teachers like Mr. C_{Calc} hold a coherent belief system that represents a nearly pure formalist view.

Teaching Mathematics with an Instrumentalist View

None of the geometry teachers and the stochastics teachers emphasized an instrumentalist view, i.e. highlighting teaching formulas and rules to enable students to solve a category of specific tasks. Only the calculus teachers tended to value an instrumentalist view in respect to their students' final exams and, thus, refer to the social context (Skott 2009). So, do secondary teachers mostly neglect the instrumentalist view that Thompson (1992) described? We hypothesize that the absence of this view is the consequence of the mathematical domains we are focused on. If we regard the beliefs of teachers thinking about mathematics instruction in primary schools and the first grades of secondary schools (Bräunling and Eichler 2011), in which arithmetics is the core subject, these beliefs represent in their majority an instrumentalist view.

Differences of Teachers or Differences of Mathematical Disciplines?

Since we investigated the teachers' beliefs only referring to one mathematical domain, the differences between the teachers have to be interpreted carefully. However, our purpose in this report is to illustrate the fundamentally distinct views towards the teaching and learning of mathematics in different mathematical domains. Further, almost all the teachers were asked to comment on the comparison of different mathematical disciplines to highlight characteristics of that discipline. We illustrate three of the mentioned comparisons:

> Mr. A_{Geo}: I think the better applications can be found in algebra or stochastics, per cent calculations, linear optimization. It is important to get a deeper insight into reality by modeling. In geometry, there are such things as dividing a pizza by a compass. I saw a trainee teacher do so. That's ridiculous.
>
> Mr. J_{Stoch}: One goal is to know that stochastics has a high relevance in real life […]. I have to say, we have neglected this aspect of mathematics for a long time. We have emphasized geometry and transformation geometry and have put application to the side. However application oriented mathematics is very important and more important for stochastics than calculus.
>
> Mr. T_{Calc}: I think in geometry it is just a different, constructional kind of approach: vectors, lines, reflection with respect to a plane and so on. […] Stochastics is rather based on our living environment, statistical investigations, polls, all of these topics that come from real life […]. Of course that is more challenging for students as they can't apply the schematic tools from calculus.

These three quotations provide evidence that teachers have different views regarding different mathematical domains. Particularly, teachers seem to emphasize an application-oriented view when they consider stochastics. By contrast they seem to emphasize a process oriented view when they consider geometry. Since both stochastics teachers and geometry teachers showed a consistent predominance referring to one view, calculus teachers differed concerning their predominance in respect to an application view or a formalist view.

Possible Impacts of Teachers' Beliefs on Their Classroom Practice and Professional Development

In the last two paragraphs, we discussed on the one side findings referring to the identification of central and peripheral beliefs. On the other side we provided evidence that the teachers' central beliefs vary when different mathematical disciplines are regarded. We took these findings into account when discussing possible impacts of the teachers' beliefs on their classroom practice and on their professional development. Due to the limited space in this report, we restrict discussion on results that we reported elsewhere (Erens and Eichler 2013b).

Impact of Teachers' Beliefs on Their Classroom Practice

From the sample of stochastics teachers, of which we analyzed their intended curricula and observed these teachers in their classroom practice in a stochastics course lasting a half year, we examine only the case of Mr. D_{Stoch} (for greater detail see Eichler 2008). For this teacher the application-oriented view is central. We illustrate this view only by the following episode of the interview:

> Mr. D_{Stoch}: That's what I am trying to illustrate, that you will of course get quite far with relative frequency, but that if you have similar situations afterwards, such as elections or opinion polls, you will need to develop the use of confidence intervals. This means showing them [students], as well, that mathematics really has applications that there are quite often problems which you can solve with maths. Students should be enabled to better categorize mathematical models which determine our economic condition.

Actually, Mr. D_{Stoch} did not show his central goal (or belief) in every lesson or instructional segment (Schoenfeld 1998). However, he enacted his central belief over the period of half a year consistently. Thus, his students predominantly worked on realistic problems comprising real data sets. The students were asked to look at statistics-related broadcasts on TV, e.g. concerning polls. Afterwards, Mr. D_{Stoch} discussed the main information in his lessons and often introduced new concepts from these discussions. It is further interesting that Mr. D_{Stoch} also referred to a formalist view concerning his intended curriculum that is a central goal in calculus or analytical geometry for him. However, except for a brief oral presentation referring to Kolmogoroff's axioms, there is no evidence that Mr. D_{Stoch} enacted his peripheral beliefs in his stochastics course. Thus, Mr. D_{Stoch} enacts his central goals but not his peripheral ones if the entire course lasting half a year is regarded.

Impact of Teachers' Beliefs on Their Professional Development

One of the main questions in teachers' professional development was the potential change of central and peripheral goals. We investigated this question concerning the 'calculus sample' referring to teachers from their final exams at university and their 2-year-period as teacher trainees until their start as qualified teachers (for greater details see Erens and Eichler 2013b). The teacher trainees were strongly schooled over a period of about 2 years and assessed after this period by their trainers. The grade in this final exam may determine the teachers' possibility of getting employment. Accordingly, all the teacher trainees like Mr. G_{Calc} tried to meet the demands of their trainers:

> Mr. G_{Calc}: In conceptualizing new content I always use a task-oriented approach, which is a guideline given by our teacher trainers. In my opinion it's not bad, but I think it's too stringently guided like our trainers want it to be implemented. […] From time to time I vary a little bit, but at the moment I must keep in mind my demonstrative exam lessons with my students.

However, the exemplary quotation of Mr. G_{Calc} involves an illustration of a striking result: The teachers in our subsample tend to retain their central beliefs regardless of the influences of either teacher trainers or the first intense classroom experience. Of course, we will neither suggest that it is impossible to change teachers' central beliefs nor suggest that trainee teachers' beliefs show no changes at all. We find, for example, considerable changes in the teachers' rationales of their beliefs, e.g. a change from justifying their beliefs by considering their university studies to justifying their beliefs by the needs of their students. We further find that these teachers seem to adopt many aspects of teaching and learning referring to their peripheral beliefs. However, analyzing the intended curricula of these teacher trainees after their teacher training phase, we did not find any fundamental change in their previously held central beliefs (c.f. ibid.).

Discussion and Conclusion

In this report, we focused on different parts of a research program aiming to investigate mathematics teachers' beliefs referring to different mathematical domains. The main aim of this report was not to give a deep insight into the teachers' beliefs concerning a specific discipline – we reported about this aspect elsewhere (e.g. Eichler 2011), but to emphasize several aspects that might be important for research in teachers' mathematics-related affect in general. We will highlight three aspects in this concluding section.

A Qualitative Interview Design Enables an Identification of the Structure of Teachers' Belief Systems The research-approach we reported in this chapter facilitates the identification of a teacher's belief system including beliefs representing overarching teaching objectives (world views). In the case of Mr. C_{Calc}, we identified his coherent formalist view. This qualitative result agrees with the result of the teachers' individual responses to questionnaires (Erens and Eichler 2013a). Thus, a predominant view could also be gained through a quantitative survey. However, in addition to overarching teaching goals, the qualitative approach could disclose teaching goals of a lower range of influence including even the selection of specific content or a specific task. For the calculus teachers (like Mr. C_{Calc}), the selection of tasks used in prompts were consistent with their predominant view. Further, this approach enables us to identify predominant views or central beliefs on the one side, but also to analyze relationships among different beliefs or belief clusters that sometimes match each other, but sometimes are contradictory (quasi-logicalness; Green 1971).

Teachers' Beliefs Seem to Differ Referring to Different Mathematical Domains In fact, the comparison of mathematics teachers' thinking about various mathematical disciplines gave evidence that teachers hold different beliefs about different mathematical domains. Although these teachers were mainly interviewed concerning one specific domain, i.e. calculus, geometry or stochastics, the differences in the teachers' belief systems are striking: It seems that the teachers- each of them teaches all the mentioned mathematical disciplines in upper secondary school- think considerably differently about mathematics when a specific discipline is concerned. Whereas an application oriented view seems to characterize teachers' beliefs concerning stochastics, it seems to be a process oriented view concerning geometry and, less specific, a formalist view concerning calculus. It is possible that this finding is a particular characteristic of German secondary teachers, who teach different mathematical disciplines. However, for these teachers, it is hard to claim for mathematical beliefs in general, but only for beliefs concerning a specific mathematical domain (c.f. Franke et al. 2007).

Teachers' Central Beliefs Impact on Their Enacted Curricula and Influence Their Professional Development We do not suggest clarifying completely the difficult relation between the teachers' espoused and enacted beliefs. However, we hypothesize that under specific conditions the teachers' espoused beliefs could explain the teachers' enacted beliefs. The first condition concerns the distinction between central and peripheral beliefs, since central beliefs seems to be more clearly enacted than peripheral beliefs (c.f. Putnam and Borko 2000). The second condition concerns a global perspective on a teacher's intended curriculum instead of a local perspective referring to one or few lessons. For instance, a teacher like Mr. D_{Stoch} does not enact his central beliefs in every lesson. However, regarding a teaching period of a half year, this teacher showed predominately the enacting of those beliefs that we identified to be central in his intended curriculum. A third condition is to analyze conflicts of goals represented by contradictory beliefs about mathematics and mathematics teaching. Actually, the formalist view of calculus is central for Mrs. E_{Calc}. However, we expect she will not enact this view in her classroom practice, since it is in contradiction to further beliefs referring to the teaching and learning of calculus that might be more relevant for her actual teaching.

The third condition seems to us close to Skott (2009) since the teaching-related beliefs of Mrs. E_{Calc} refer to the social context of her teaching practice.

Finally, the brief discussion of the professional development of teacher trainees highlights the robustness of deep-seated central beliefs. Whereas peripheral beliefs seemed to be modified in the period of a teacher traineeship, partly caused by teacher educators, partly caused by the first intense practical experience of these teachers, the central beliefs of these teachers seem to be stable. This result is partly in line with Franke et al. (1997) and is also in compliance with theoretical considerations about the stability of teachers' beliefs (e.g. McLeod 1992).

To conclude, the careful examination of mathematics teachers' beliefs is on the one hand a crucial challenge of educational research to understand "the cognitive and affective aspects of teachers' professional lives" (Calderhead 1996, p. 709), it is, on the other hand, a mandatory research field since "the nature of mathematics teachers' thinking becomes a key factor in any movement to reform the teaching of mathematics" (Chapman 1999, p. 185). For both teachers' professional lives, and a change of teachers' beliefs, a long-term and discipline-specific approach referring to teachers' intended curricula – involving the investigation of teachers' systems of beliefs including central and peripheral beliefs, coherent and contradictory belief clusters – could be a reasonable contribution to the research in teachers' mathematics-related affect.

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