The Construct of Attitude in Mathematics Education

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Abstract This chapter addresses a number of crucial theoretical issues about research on attitude towards mathematics, a field that has a very long tradition in mathematics education, with early studies on attitude being published more than 60 years ago. Over time, research on attitude in mathematics education has developed a range of perspectives and methodologies, dealing with a variety of questions concerning the construct of attitude: discussion and development of tools for measuring/assessing/observing it; analysis of the relationship with other affective constructs and with cognition; investigation of the relationship with achievement; critique of the lack of a suitable theoretical framework. The chapter traces the 'story' of the construct of attitude, providing a theoretical discussion of the issues mentioned above that are crucial to understanding the mosaic of relationships and interactions within the affect field. Through the theoretical debate, the aim of the chapter is to highlight new directions for research on attitude in mathematics education.

Keywords Attitude towards mathematics • Affect in mathematics education • Emotions • Beliefs

Introduction

Research on attitudes towards mathematics can be viewed as paradigmatic of research in mathematics education. This research field lies at the intellectual crossroads of many different domains (e.g. mathematics, psychology, cognitive science, epistemology, semiotics, anthropology), and often deals with constructs that have been developed in those domains to face (new) emerging issues in mathematics education (Sierpinska et al. 1993). The construct of *attitude* was introduced in the first decades of the nineteenth century in the context of social psychology in order

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to foresee individuals' choices in contexts such as voting or buying goods. *Attitude* is seen as a trait of an individual that has a direct influence upon his/her behaviour:

> An attitude is a mental and neural state of readiness, organized through experience, exerting a directive and dynamic influence upon the individual's response to all objects and situations with which it is related. (Allport 1935, p. 810)

In mathematics education, early studies about attitude appear in the middle of the twentieth century. These pioneering studies were deeply affected by the field (social psychology) in which the construct was born, both regarding the characterization of attitude, seen as an individual's trait capable of influencing his/her own behaviour (Aiken 1970), and the methods used to assess and measure it.

In this context, the main goal was the search for a measurement of attitude: Dutton (1951), in one of the first studies concerning attitude and mathematics, stated his aim to measure pupils' and teachers' attitude towards arithmetic using Thurstone scales. As a matter of fact, following the trend in social psychology, the measurement of attitude was mainly carried out by the means of unidimensional ad hoc scaling methods, such as Thurstone and Likert scales.

Many things have changed in the field of research on attitude from those early studies up to now; some of those changes have been deeply influenced by a change of perspectives in mathematics education. At present, attitude is considered (together with beliefs, emotions and values) as one of the constructs that characterize a new field of research: that of affect.

Research on attitude, as often happens, has not followed a linear path. Over the years, the researchers' position on basic issues such as the definition itself of attitude and the instruments used to assess the construct has dramatically changed and new issues and goals have been identified.

This feature of research on attitude has increased the need for a clear theoretical systematization of research results, which has also emerged as a priority in the whole mathematics education field in the last two decades. As a matter of fact, this need has now become a necessity in mathematics education, due to the considerable development of the research field in the last few years and, in particular, to the identification of its *cumulative* and *universal* characters (Boero and Szendrei 1998). This view of the field is strictly linked with the characterization of the nature of research findings:

> Researchers in education have an intellectual obligation to push for greater clarity and specificity (…) [in mathematics education] findings are rarely definitive; they are usually suggestive. Evidence is not on the order of proof, but is cumulative. (Schoenfeld 2000, pp. 647–648)

Therefore, coherently with the *cumulative* characterization of research in mathematics education, we believe that tracing, with critical eyes, the history of research on attitude may bring forward an understanding – through a theoretical lens – of the mosaic of the relationships and interactions between definitions of attitude and instruments to measure it, and of the influence the shift from a normative to an interpretive paradigm had on both these issues.

Moreover, this systematization is necessary to map out the future of research on attitude, including the identification of new issues, the development of suitable methods, and a warning against repeating the same old mistakes.

Early Studies of Attitude in Mathematics Education: The Problematic Relationship Between Attitude and Achievement

During its early period (ranging from the first half of the twentieth century to the end of the 1980s), research on attitude within mathematics education followed the trend of research in social psychology. The definition of attitude was rarely made explicit, and implicitly it seemed to refer to the tendency to behave in a certain way. A central research topic was the development or refinement of measuring instruments and sampling methods:

> The search for more adequate questionnaire and sampling techniques and factors underlying attitudes toward these subjects [arithmetic and mathematics] continues to be an important area for research. (Dutton 1951, p. 418)

In this period, the predominant methodology was quantitative and statistical: as a matter of fact the quantitative and statistical approach seems to have been considered a sort of warrant for the scientific nature of the discipline.

Research on attitude at this stage reflects the evolution of the field of mathematics education: an in-depth discussion about the very *nature* of this emerging field had not yet been developed. According to Kilpatrick (1992, p. 15), in that period "the measurement movement begins". The quantitative primacy in the methods used had its roots in the search for scientific acceptance of a young discipline that was just beginning to take its first steps:

> From the beginnings of the century through its three-quarter point, such inquiry [inquiry in math education] becomes increasingly "scientific", that is, ostensibly objective and rigorously quantified. (Schoenfeld 1994, p. 698)

On the other hand, the attention paid to measurement instruments was also linked to the main goal of early studies on attitude, which was the identification of causal correlations between attitude and other significant factors.

In the first review of the construct of attitude within mathematics education, Feierabend (1960) highlighted two main reasons for the increasing academic interest in this construct. Drawing on the development of such construct in social psychology, the first reason was related to the view of attitude as a *selective factor* because of its correlation with the choice of enrolling/not enrolling in advanced mathematical courses:

> Mathematics, geometry, and algebra are the courses which, when disliked in high school, have the highest percentage of students who never take a course in this area again. This implies

> the operation of such a strong selective factor that by the time students reach college, only the students with a strong positive attitude will still be taking mathematics; the rest have negative attitudes which may increase in strength with the operation of time and the lack of counteracting influences. (Feierabend 1960, p. 19)

The second reason concerned the relationship between attitude and mathematical achievement:

> A series of recent investigations have attempted to explain differences in school performance among students of equal abilities on the basis of their attitudes. (Feierabend 1960, p. 11)

This point also implied taking into account gender differences in mathematics achievement and in problem-solving ability:

> There are sex differences in problem-solving ability unrelated to general mental ability, special abilities, or specific knowledge (…) [he] attempted to show that the differential performance of the two sexes was due to a difference in attitude toward problem-solving. (Feierabend 1960, p. 17)

In his review Feierabend advanced some criticism towards research on attitude, but his criticism was limited to some aspects related to the development of instruments and to the statistical analysis. There was no reference to the lack of theoretical clarity and in particular no explicit definition of attitude was provided: a naïve view of the construct emerges. The term 'attitude' was used to address different constructs, such as preference, interest, motivation.

Ten years later Aiken (1970) summarized early research on attitude as follows:

> The major topics covered were: methods of measuring attitudes towards arithmetic and mathematics; the distribution and stability of mathematics attitudes; the effects of attitudes on achievement in mathematics; the relationship of mathematics attitudes to ability and personal factors. (Aiken 1970, p. 592)

It is interesting to notice that Aiken's list also does not include reference to the topic 'nature of the construct of *attitude*' (that would become a major topic in research on attitude in the early 1990s).

What emerges from the reviews carried out by Feierabend and Aiken and from the analysis of other literature of that period (Reyes 1984) is that most studies were focused on the search for evidence of a causal relationship between "something called *attitude*" (Neale 1969, p. 631) and other variables, in particular mathematical achievement. This causal relationship is even seen as a hypothesis of the aetiology of attitude towards mathematics (Aiken and Drager 1961). The search for a causal relationship reveals a normative approach, that seems to drive research on attitude and provide a justification, and in some way a reinforcement, for the great attention paid to measurement instruments, rather than to the theoretical clarification of the construct.

Despite its theoretical limitations, this first phase of research on attitude was fruitful and produced meaningful results that, coherently with a *cumulative view* of research, contributed significantly to the new research era that would follow. The most significant contribution was what became the initial assumption of this kind of research, that is, that non-cognitive factors strictly interact with cognitive factors and have a crucial role in the learning of mathematics. This assumption is a sort of

break in the wall of the purely cognitive approach to mathematics education, and was to be decisive in the development of the specific field of affect in mathematics education: not purely cognitive factors – and in particular attitude – would become a relevant topic in the study of mathematical learning:

> The attitudes of students toward mathematics play a vital part in their learning (…) Important for the study of attitudes toward mathematics is the idea that an attitude involves both cognitive and non-cognitive aspects. (Corcoran and Gibb 1961, p. 105)

In addition to this, the great emphasis placed on methods caused a refinement of many observational instruments. This brings to light important issues related to the observation of attitude (and more in general of affective constructs), as for example the tendency of individuals to reply to questionnaires according to what is socially accepted and valued, rather than expressing their own thoughts – the so-called social desiderability phenomenon (Kloosterman and Stage 1992).

Moreover, in this early period, research on attitude consolidated two significant findings. A first result – confirmed by many studies – was the relationship between attitude towards mathematics and the choice of mathematics courses. For instance, in his literature review of research on attitude, Aiken (1970) stated that there is a good body of evidence showing that the choice of enrolling in advanced mathematics courses is significantly affected by attitude towards mathematics. A second important finding refers to gender differences in mathematical achievements. In particular, the valuable work of Elizabeth Fennema and Julia Sherman highlighted the differences in attitude towards mathematics between males and females, offering a new and important key for the interpretation of gender differences in mathematics achievements:

> Since the study of mathematics appears not to be sex-neutral, attitudes toward mathematics may reflect cultural proscriptions and prescriptions (…) These data certainly indicate that many females have as much mathematical potential as do many males. The generalized belief that females cannot do well in mathematics is not supported. (Fennema and Sherman 1977, p. 69)

This result, that may now appear unquestionable, was not so obvious before the work of Fennema and Sherman.

Even if the first period of research on attitude provides several important findings and suggests a number of research hypotheses, the above discussion has also shown its strong limitations from the very beginning. The identification and analysis of these limitations has been crucial for the development of research on attitude in the following years.

According to Bishop (1992), carrying out a research study in mathematics education requires taking into account three components: enquiry (which concerns the reason for the research activity), evidence and theory. The initial studies on attitude are motivated by the assumption of the existence of a causal relationship between attitude and achievement in mathematics, and seem to be focused on searching for evidence of this relationship rather than developing a theoretical framework or clarifying the nature of the construct. But in spite of the efforts devoted to developing measuring instruments, research fails to show a causal relationship in the direction attitude → achievement, or a clear correlation between them.

Aiken (1970) reported the results of several studies in which this correlation is far from being clear, highlighting the need for clarifying its very nature. Almost 30 years later, Ma and Kishor (1997), analysing the results of 113 different studies, conclude that this correlation is not statistically significant. Assuming that this correlation does exist, Ma and Kishor identify the cause of the failure to prove it in the inappropriateness of the observing instruments used in the research on attitude towards mathematics up to that point. At that stage, the instruments used to measure attitude towards mathematics have been criticized by many researchers, because their nature is considered "exceptionally primitive" (Leder 1985).

However, starting from the 1980s, researchers increasingly acknowledge that the major weakness of this kind of research lies in the lack of clarity at the theoretical level and in the definition of the construct itself. Kulm (1980) suggests the existence of a trend that tends to avoid an explicit definition of attitude towards mathematics and instead adopts operational definitions determined by the types of instruments used to measure attitude. This lack of interest in characterizing the construct produces a gap between the definition of attitude and its measurement (Leder 1985), and results in the lack of reliability of the observational instruments.

Germann's words below summarize the criticism towards the first phase of research about attitude:

> First, the construct of attitude has been vague, inconsistent, and ambiguous. Second, research has often been conducted without a theoretical model of the relationship of attitude with other variables. Third, the attitude instruments themselves are judged to be immature and inadequate. (Germann 1988, p. 689)

In other words, the naïve theoretical approach that characterizes early studies on attitude appears to be inadequate within the normative-positivistic paradigm in which those studies were conducted. As a matter of fact, this paradigm demands isolating and clearly identifying variables in order to interpret statistical results and to be able to compare them across studies:

> Sometimes no description or definition of what is meant by a particular variable is even included in the research report. This makes interpretation of results difficult and detracts from efforts to compare results across studies. (Hart 1984, p. 573)

For this reason, the process of re-thinking research on attitude began at the end of the 1980s, addressing many aspects such as: the paradigm in which it is framed, the goals that it pursues, the construct definition, the relationship between the construct and other (affective and cognitive) factors, the development of observational tools and the discussion about methods for analysing data.

The Theoretical Debate About *Attitude* in Mathematics Education

In 1992, in the well-known *Handbook of research on mathematics teaching and learning*, McLeod traces the way for a reconceptualization of research on affect in mathematics education. He identifies three different constructs – beliefs, attitudes

and emotions[1] – that, in his view, vary in stability and differ in the degree of the role played by cognition. McLeod's work starts with a crucial premise:

> Affective issues play a central role in mathematics learning and instruction (…) If research on learning and instruction is to maximize its impact on students and teachers, affective issues need to occupy a more central position in the minds of researchers. (McLeod 1992, p. 575)

He emphasizes the relationship between the newly acknowledged role assigned to affective factors and the constructivist view of mathematics learning:

> If we believe that the learner is someone who only receives knowledge rather than someone who is actively involved in constructing knowledge, our research program could be entirely different in terms of both the affective and the cognitive domain. (McLeod 1992, p. 576)

The need for reconceptualization is strictly connected with the criticism of the previous research on attitude:

> Research on affect has been voluminous, but not particularly powerful in influencing the field of mathematics education. It seems that research on instruction in most cases goes on without any particular attention to the affective issues (…) A major difficulty is that research on affect has not usually been grounded in a strong theoretical foundation. (McLeod 1992, p. 590)

Therefore, McLeod highlights that research on affect has to pay particular attention to three strictly intertwined aspects: the discussion of theoretical issues, the development of a wider variety of methods, and the analysis of the relationships among affective constructs and between affect and cognition.

Once again, the development of research on attitude is deeply influenced by the simultaneous development of the field of mathematics education at the end of the 1980s. In this period, many scholars debate on the nature of mathematics education and on the criteria for establishing quality of research in this field. In particular, consistently with the goal of universalization of research results, the request for a theoretical clarification of the constructs used in research is emphasized:

> A community of scholars engaged in the research of common areas with common themes, however, has responsibility to communicate ideas and results as clearly as possible using common terms. For these reasons, it is important to use the terms consistently, accurately, and appropriately once their definitions have been agreed on. (Pajares 1992, p. 315)

What Is Attitude Towards Mathematics?

The discussion about the theoretical aspects of research on attitude starts with a 'definition problem': what is attitude towards mathematics?

[1] Later, De Bellis and Goldin (1999) propose 'values' as the fourth construct of the affective domain.

A first critical issue relates to the object that attitude is oriented towards – that is, mathematics. Some researchers refer to a 'unique' attitude toward mathematics (Haladyna et al. 1983), while others claim that many different attitudes exist according to the different topics and activities that are considered (Tirosh 1993); still other scholars propose to distinguish between attitude towards mathematics seen as a branch of scientific knowledge and towards mathematics as school subject (Schoenfeld 1989), or even that attitude can refer to different objects and situations, such as mathematical content, characteristics of mathematics, kind of teaching, mathematical activities in the classroom and mathematics teacher (Kulm 1980).

Moreover, this complexity grows when, in addition to the variety of objects attitude is oriented towards, we also consider the variety of subjects: whose attitude? Research on attitude has dealt with a large variety of individuals: students, prospective and in-service teachers, students' parents, and, more in general, adults.

But the most significant aspect of the complexity regarding the 'definition problem' is that it involves not only the characterization of the construct 'attitude', but also that of positive/negative attitude, a dichotomy that pervades research, both implicitly and explicitly. Classic studies regarding the relationship between attitude and achievement in practice investigate the correlation between *positive* attitude and success. In the same way, studies aiming to change attitude actually end up in setting the objective of transforming a *negative* attitude into a *positive* one.

As already mentioned, a large portion of studies show the lack of a clear definition of the construct: attitude tends to be defined implicitly and a posteriori through the instruments used to measure it (Kulm 1980; Leder 1985; Daskalogianni and Simpson 2000).

In social psychology, the most recent theories agree on the multidimensionality of the construct, and make reference to a *tripartite model*, according to which attitude has a cognitive, an affective, and a behavioural component (Eagly and Chaiken 1998). Within the field of mathematics education many explicit definitions of attitude refer to this tripartite model, describing attitude by means of three components: the emotional disposition towards mathematics, the set of beliefs regarding mathematics, and the behaviour related to mathematics (Hart 1989; Leder 1992; Ruffel et al. 1998). However, some studies – generally in the earliest period of research – adopt a 'simple' characterization, seeing attitude as a general emotional disposition (Haladyna et al. 1983).

Both definitions show their theoretical but also operational and didactical limitations (Di Martino and Zan 2001). The *simple* definition does not make explicit reference to cognitive aspects, although many researchers who subscribe to this definition use models (see Mandler 1984; Ortony et al. 1988) that emphasize the relationship between emotion and cognition, describing emotional experience as the result of a combination of cognitive analyses and physiological responses. In this framework, it is the interpretation given by an individual to an experience that elicits the emotion, and not the experience itself:

> First, the meaning comes out of the cognitive interpretation of the arousal. This meaning will be dependent on what the individual knows or assumes to be true. In other words, the

> individual's knowledge and beliefs play a significant role in the interpretation of the interruption. (McLeod 1992, p. 578)

According to the simple definition, the characterization of positive/negative attitude is clear: a positive (negative) attitude is a positive (negative) emotional disposition towards the subject.

This characterization can be useful when dealing with issues such as the choice of mathematics courses or the comparison between different groups of individuals, but it seems inadequate to deal with complex issues such as success in mathematics. In this context, the idea of positive attitude that emerges from the simple definition is not considered very significant by many mathematics education researchers, who underline the importance of linking a positive emotional disposition with an epistemologically correct view of the discipline (Ernest 1988). In the same vein, the crucial issue of promoting a positive attitude risks losing its significance if the goal of developing a positive emotional disposition toward mathematics is not associated to the goal of promoting a *positive* view of the discipline. Considering only the emotional aspects poses an even greater didactical threat, since teachers may choose to avoid complex tasks in order to prevent producing negative emotions.

Kulm (1980) discusses similar issues about the attitude definition in the early research period and concludes:

> It is probably not possible to offer a definition of attitude towards mathematics that would be suitable for all situations, and even if one were agreed on, it would probably be too general to be useful. (Kulm 1980, p. 358)

The awareness that the *appropriateness* of the construct depends on the studied issues will lead to the idea of a 'working definition' (Daskalogianni and Simpson 2000).

As regards the *tripartite* model, the main critical aspect is that the implicit assumption of a link between attitude and behaviour becomes part of the construct definition itself. This theoretical choice exposes research to the risk of circular reasoning, as eloquently described by Lester (2002) in relation to the belief-construct:

> A central difficulty is that the fundamental assumption undergirding much of this research rests on a shaky logical foundation. Specifically, a basic assumption is that beliefs influence peoples' thinking and actions. However, it is also often assumed that beliefs lie hidden and so can be studied only by inferring them from how people think and act. For researchers to claim that students behave in a particular manner because of their beliefs and then infer the students' beliefs from how they behave involves circular reasoning. (Lester 2002, p. 346)

In the light of these critical aspects, a third definition of attitude emerges in which behaviours are not explicitly mentioned: attitude towards mathematics is described as the pattern of beliefs and emotions associated with mathematics (Daskalogianni and Simpson 2000).

This choice overcomes the risk of circularity, but the theoretical problem of identifying a positive/negative attitude according to a multidimensional definition still remains (Di Martino and Zan 2003). As a matter of fact, there is not only a need for characterization of the positive/negative dichotomy for each dimension (emotions, beliefs, possibly behaviour), but it is also essential to identify if and how the dichotomies

related to the single components can result in a unique characterization of positive/negative attitude. This issue is strictly connected with the choice of the instruments used to measure attitude.

Instruments Used to Measure Attitude

As Leder (1985) claims, the lack of interest in characterizing the construct produces a gap between the definition of attitude and its measurement: as a matter of fact the instruments traditionally used to assess and measure attitudes are not consistent with the different definitions and with whether an explicit definition of attitude is given or not.

The instruments used are almost exclusively self-report scales (Kulm 1980; Leder 1985; McLeod 1987) such as Thurstone or Likert scales. These instruments propose items that take into consideration beliefs and behaviours as well as emotions: for example 'Mathematics is useful', 'I think about arithmetic problems outside school', 'I like problem solving'. Therefore, they make implicit reference to the tripartite model, regardless of whether this definition is explicitly selected as a starting point or not. Even if the instruments used appear to be increasingly sophisticated, the measurement generally results in a reduction to the positive/negative bipolarity, which is obtained by summing up the scores related to each of the three dimensions: cognitive, affective and behavioural.

While some scholars underplay this operation by observing that 'the correlation among measures of the three components, although leaving room for some unique variance, are typically of considerable magnitude' (Ajzen 1988, p. 22), others consider this reduction as contradicting the recognized complexity of the tripartite model (Eagly and Chaiken 1998). Reducing the description of attitude to a single score is also in contrast with the original idea of Thurstone and Chave (1929) who claim that attitude is a complex construct that cannot be measured by a single score, but requires several indices. Thurstone and Chave underline that the choice of the characteristics (indices) to be measured depends on the context – in the same way as when measuring a physical object like a table one can decide whether to measure length, width or height.

But the theoretical debate about research on attitude highlights other critical issues in the *measurement process*. First, the separate measurement of each component presents significant problems, due to the limitations of questionnaires. As far as beliefs are concerned, the mismatch between exposed beliefs and beliefs-in-action is well known (Schoenfeld 1989), just like the already mentioned *social desiderability* phenomenon (Kloosterman and Stage 1992). Regarding emotions, researchers have discussed the difference between an *opinion* about an emotion and the *emotion* itself (Ruffel et al. 1998) and the limitations of instruments such as questionnaires and interviews in capturing emotional reactions that are not conscious (Schlöglmann 2002).

A second critical point concerns the choice of items that, in the case of questionnaires, is fully determined by the researchers, while respondents are only asked to express their agreement/disagreement with these items: how can we be sure that the topic of the items is relevant to the respondent? In other words, using the terminology introduced by Green (1971), how can we be sure that the corresponding beliefs are psychologically central in the respondent's belief system?

A third critical aspect concerns the choice of the score to be attributed to each of the items, requiring identification of what a *positive* emotion/belief/behaviour is (this shows the strong relationship that exists between the definition and the measurement problem). Essentially:

- When *positive* refers to an emotion, it normally means 'perceived as pleasurable'. So anxiety when confronting a problem is seen as negative, while pleasure in doing mathematics is evaluated as positive.
- When *positive* refers to beliefs, it is generally used with the meaning 'shared by experts'. The first limitation of this approach is brought to light by a number of studies which highlight that there is no single pattern of beliefs shared by experts in mathematics (Mura 1993, 1995; Grigutsch and Törner 1998). In light of this, identifying several different typical patterns of beliefs towards mathematics shared by experts becomes necessary. At present, this still remains an issue for discussion that could lead to the definition of a number of different patterns to act as models of 'successful views of mathematics'.
- When it refers to a specific behaviour, *positive* generally means 'successful'. In the school context, a successful behaviour is generally identified with high achievement. This characterization leads to the problem of how to assess achievement (Middleton and Spanias 1999).

A further problem is that the differences between the various meanings of positive attitude are rarely made explicit. If the researcher does not declare his/her initial choices, interpreting the results of a study and comparing different studies becomes problematic.

Even if this ambiguity is overcome by making explicit the initial choices and assumptions, in our opinion other problems remain. In some studies the three meanings for 'positive' (related to emotion, belief and behaviour) overlap thanks to implicit assumptions: for example, that a 'positive' belief (i.e. shared by experts) is associated with a successful behaviour and elicits a pleasurable emotion; or that a pleasurable emotion is necessarily associated with a positive behaviour in mathematics, and vice versa for negative emotion.

Evaluating a belief (or an emotion) as 'positive' or 'negative' according to the emotion and behaviour related to it implies using a cause/effect model, according to which the same belief *causes* the same emotion or the same behaviour in all individuals. Moreover, this evaluation not only assumes that a certain belief has an emotional component, but also looks at the significance of that emotional component, that is, not just that it is linked to a behaviour, but also which type of behaviour.

In this case, the cure seems worse than the disease, since this approach does not take into account the very complex nature of the relationship among beliefs, emotions and behaviour.

As a matter of fact, a number of studies about emotions (Evans 2000) suggest the possibility that for certain subjects, an optimal level of anxiety exists, above which, but also below which, performance is reduced.

The relationship between beliefs and emotions was investigated in a study with 211 high school students aged between 14 and 18 (Di Martino and Zan 2002) in the case of the belief 'In mathematics there is a reason for everything', which is an item that is also used in many scales for measuring attitude towards mathematics. Students are asked to fill in a questionnaire including the following item:

Choose the option you most agree with:
☐ In mathematics there is always a reason for everything (B)
☐ It is not true that in mathematics there is always a reason for everything (not B)
And: ☐ I like ☐ I don't like ☐ I am indifferent to this characteristic of mathematics

The findings show that only 51.7 % of the sample fell in the two *expected* groups (i.e. 'B – I like' and 'not B – I don't like'). But overall there was no difference in the percentage of belief B-holders between the groups of high achievers and low achievers. The distinction between these two groups is related to the emotion associated with this belief: 76 % of the high-achievers who are belief B-holders liked this characteristic of mathematics, while this percentage dramatically decreased to 28 % within the low-achievers group.

About the combination 'epistemological correct belief – negative emotion', we suggest two possible interpretations. The first interpretation is that the negative emotion is *directly* related to the belief. On the other hand, we also need to consider the possibility that the emotional disposition is not directly linked to that single belief, but to its interaction with other beliefs. This remark questions the possibility of characterizing a single belief as positive or negative, without considering its connection with other beliefs an individual may have (belief system):

> Because they [single beliefs] offer a limited glimpse into a much broader system and because understanding their connections and centrality is essential to understanding the nature of their effect, researchers must study the context-specific effects of beliefs in terms of these connections. (Pajares 1992, p. 326)

More specifically, when describing belief systems Rokeach (1968) recognizes the dimension of *centrality* for a particular belief, highlighting that not all beliefs have the same importance for an individual. Central beliefs play a prominent role in people's belief systems, and consequently in influencing their behaviour. As Eagly (1967) observes, beliefs about self are generally considered more central than other ones.

Consider for example the relationship between belief B used in our study ('In mathematics there is a reason for everything') and the (likely) central self-belief 'I am not able to understand these reasons': the presence of such relationship may result in unproductive behaviours such as avoiding answering a question or giving random answers (Di Martino 2004).

The discussion above highlights that the assumption of the existence of a cause-effect relationship between a specific belief and emotion or behaviour is inadequate. The interaction is more complex, since it involves an individual's belief system (and not only the single belief) and is strongly dependent on the individual.

Following the results of this debate, a movement towards the overcoming of the normative approach and the use of an interpretive approach for research on attitude emerges with the aim of attending to the complexity of the issues at stake.

The Attitude Construct in the Reconceptualization of the Affective Domain in Mathematics Education

Once again, the history of research on attitude reflects the evolution of the mathematics education field: the theoretical debate about attitude develops in parallel with a new interpretive perspective that begins to emerge within the field of mathematics education. This perspective, in contrast with the normative-positivistic one, significantly affects the discussion about the theoretical characterization of constructs.

The gradual affirmation of the interpretive paradigm in the social sciences, including a greater attention paid to the complexity of human behaviour, leads researchers in mathematics education to abandon the attempt of explaining behaviour through measurements or general rules based on a cause-effect model, and to search instead for new interpretive tools (once again drawing on other domains):

> The purpose of doing interpretivist research (…) is to provide information that will allow the investigator to "make sense" of the world from the perspective of participants. (Eisenhart 1988, p. 103)

This implies a significant shift in focus: an emerging attention to the understanding of a phenomenon ('making sense of the world') replaces the description of the phenomenon itself, which could be seen as a shift from product to process (Schoenfeld 1994).

The interpretive approach has a direct influence on the process of re-thinking research methods since the limitations of the statistical methods become evident:

> Through the 1980s and into the 1990s (…) with a shift in focus there was a concomitant shift in methods (including the reporting of clinical interviews, process and simulation models, field observations and participant observations), because a new class of phenomena required a new set of explanations a new set of tools to uncover them. (Schoenfeld 1994, p. 703)

This shift of perspectives gives new strength to research on attitude that until this point had remained stuck in the causal-relationship paradigm. In particular, attitude

gains renewed popularity in the studies about problem-solving activities aimed at interpreting the failure of students who seem to have the required cognitive resources.

The book *Affect and mathematical problem solving* (McLeod and Adams 1989), collecting contributions by several authors, represents the turning point for research on affective constructs, and in particular on attitude. For the first time, affective constructs are used not only to prove the existence of a numerical correlation with an outcome (mathematical achievement), but also to interpret a process (the interactions between affective and cognitive aspects in problem-solving activities). Therefore, the need for a theoretical clarification in mathematics education (that is also related to the possibility and the intention for a cumulative development in the field) appears to become a fundamental issue also specifically for research on affect:

> There was a lack of definition, lack of clarity, and lack of connections to mathematics. It is possible to avoid making the same mistakes again as new ideas and research methodologies are employed. It is hoped that new researchers on affect will be clear about what is being studied, precise in definition, and respectful of what has been learned previously. (Fennema 1989, p. 209)

The double occurrence of the adjective 'new' in Fennema's words is not casual: it shows awareness of the fact that new perspectives and new more complex issues force a rethinking of the affective constructs. In particular, the shift from a normative paradigm to an interpretive one provokes a discussion (re-definition) of goals, definitions and methods.

The belief that research on attitude towards mathematics may offer interpretive instruments to understand the reasons for an individual's intentional actions in the mathematical context grows (Zan et al. 2006). This *belief* is supported by the initial evidence coming from related research in the field of neuroscience:

> There is apparently some neurological basis for asserting a link between affective and cognitive aspects of human functioning. (Silver 1985, p. 253)

More recently, Damasio (1996) highlighted the close relationship between affect and decision-making processes.

The theoretical construct of 'attitude towards mathematics' is no longer a construct aimed at explaining causes of behaviour, thus enabling researchers to predict it, but instead it becomes a flexible and multidimensional interpretive tool, aimed at describing the interactions between affective and cognitive aspects in mathematical activity. In particular, attitude becomes a tool to interpret people's decisions in mathematical activities, and, if necessary, suggest strategies to modify them. In this context, particularly significant is Ruffel, Mason and Allen's position about the definition of the construct of attitude itself:

> Reflecting on them [some previous studies about attitude] led us to challenge the very construct of attitude. We are also led to challenge the cause-and-effect model underlying much attitudinal research. We now see *attitude* as at best a complex notion, and we conjecture that perhaps it is not a quality of an individual but rather a construct of an observer's desire to formulate a story to account for observation. (Ruffel et al. 1998, p. 1)

It could be argued that the same thing can be said about every theoretical construct, not only in mathematics education. In fact, in our opinion, this position shows awareness of the fact that any phenomenon can only be observed from a

particular point of view and thus highlights the role of the researcher/observer, who cannot be a mere measurer. This position represents the overcoming of a naïve approach, in which attitude is seen as an objectively measurable quality of an individual, and the transition to a theoretical approach.

In line with this perspective, Daskalogianni and Simpson (2000) assume that the definition of attitude becomes a *working definition*, which is functional to the research questions that researchers pose in each study. Therefore, having different definitions of the construct appears natural, and a definition is no longer evaluated in terms of *correctness* (is it the *right* definition?) but in terms of *suitability* to address a specific research problem in mathematics education (Di Martino and Zan 2010). According to the classification of research proposed by Bishop (1992), this kind of approach characterizes the new trend of research on attitude as *problem-led*.

The theoretical re-thinking of research on attitude leads to the exploration of new methods of inquiry in the field. Coherently with their position, Ruffel et al. (1998) emphasize the inadequacy of the *measurement approach* by replacing the verb 'measuring' with the verb 'probing'.

At the beginning of the new millennium, a strong criticism of the use of quantitative methods in the research on attitude emerged, and a movement towards the use of qualitative approaches has begun. It is understood that qualitative methods, and in particular the use of narratives, enable researchers to take into account those beliefs and emotions which are psychologically central for the respondents. A number of studies using essays, diaries, interviews and also the observation of behaviour in natural settings or in structured situations appear (Karsenty and Vinner 2000; Hannula 2002; Zan and Di Martino 2007; Kaasila 2007).

Differently from what happens with the traditional attitude scales, the respondents are not requested to express agreement/disagreement with respect to items chosen by others, but are asked to tell their mathematical 'stories', through which they can recount all the aspects that they consider relevant in their relationship with mathematics. As a matter of fact, the pivotal motivation for using narratives in educational research is the following:

> Humans are storytelling organisms who, individually and socially, lead storied lives. The study of narrative, therefore, is the study of the ways humans experience the world. (Connelly and Clandinin 1990, p. 2)

As a consequence, almost 10 years after McLeod's *manifesto*, the shift in focus in research on attitude provokes a shift in perspectives and methods: a real *revolution*.

The TMA Model: A Definition of Attitude Grounded in Students' Narratives

Within the described framework, we have investigated how mathematics teachers use the diagnosis 'this student has a negative attitude' to interpret students' mathematical difficulties in the context of an Italian National Project. The results of the

study (Polo and Zan 2006) show that this diagnosis is frequently used (at all school levels) by mathematics teachers to explain students' difficulties, and, above all, that in the majority of cases it represents a case of surrender instead of being used as an interpretive step capable of steering future action.

These findings persuade us that, in order to turn the 'negative attitude diagnosis' into a useful instrument for both practitioners (teachers) and researchers, it is necessary to link the theoretical construct of attitude to practice. This fits with the strong incentive put forward by Phillip "to develop constructs that might be applied to help make sense of teaching and learning environments" (Philipp 2007, p. 264).

Therefore we have designed a study based on the collection and analysis of students' autobiographical narratives and aimed at constructing a characterization of students' attitude towards mathematics in relation to their experience (Di Martino and Zan 2010).

Our reason for choosing to use autobiographical essays is that through this format pupils tend to explicitly evoke events about their past that they deem important and also to paste fragments by introducing causal links, not in a logical perspective but rather in a social, ethical and psychological one (Bruner 1990). We believe that in order to describe the kind of relationship an individual has with mathematics, and consequently to suggest a characterization of attitude towards mathematics strictly linked to experience, this pasting process is more important than an *objective* report of one's experience with the discipline at school. As Bruner claims:

> It does not matter whether the account conforms to what others might say who were witnesses, nor are we in pursuit of such ontologically obscure issues as whether the account is 'self-deceptive' or 'true'. Our interest, rather, is only in what the person thought he did, what he thought he was in, and so on. (Bruner 1990, pp. 119–120)

In doing so, a theoretical model for attitude emerges from the data collected through a cyclical analytical process, that is, through what Glaser and Strauss (1967) call *grounded theory*. In this kind of process, the autobiographical texts are analysed in order to systematically make meaning out of the individuals' narrations: the final outcome is the identification of a set of categories and relationships aimed at understanding and interpreting different behaviours (Demazière and Dubar 1997).

We have collected and analysed 1662 anonymous essays entitled "Maths and me: my relationship with maths up to now", written by students whose school levels ranged from grade 1 to grade 13.[2] The results of our study show that when students describe their relationship with mathematics, almost all of them refer to one (or more) of the following three dimensions:

- emotional disposition towards mathematics,
- vision of mathematics,
- perceived competence in mathematics.

This result suggests the Three-dimensional Model for Attitude (TMA) represented in Fig. 1.

[2] The sample of the study was not chosen on a statistical basis, but we relied on the collaboration of teachers who voluntarily agreed to participate in our research.

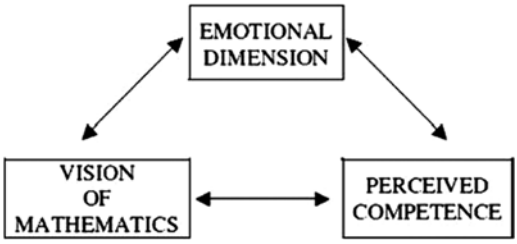

Fig. 1 The three-dimensional model for attitude

TMA takes explicitly into account the close relationship amongst the three dimensions. The research study also highlights the subjectivity of these interactions, confirming the need for designing suitable observational tools to track it:

> The proposed model of attitude acts as a *bridge* between beliefs and emotions, in that it explicitly takes into account beliefs (about self and mathematics) and emotions, and also the interplay between them. However, in order for it to become effective theoretical and didactical instruments, the construction and use of consistent instruments for observation, capable of taking into account its complexity, is needed. (Di Martino and Zan 2011, p. 479)

The analysis of the students' autobiographical essays also suggests the need for the development of a new approach to the positive/negative characterization of attitude, confirming that the reduction of the dichotomy positive/negative attitude to the emotional dimension is questionable. As a matter of fact, we find that negative emotional dispositions towards mathematics may be associated with different patterns of attitude, depending on the student's perceived competence and vision of mathematics as well as on the relationships amongst the three dimensions. Coherently with this observation, and with the multidimensional characterization of the construct in TMA, we have developed a definition of 'negative attitude' that explicitly makes reference to the negativity of at least one of the three dimensions:

> The multidimensionality of the model underlines the inadequacy of the positive/negative dichotomy for attitude referred only to the emotional dimension (like/dislike), and rather suggests considering an attitude as *negative*, when at least one of the dimensions is *negative*. In this way, we can outline *profiles* of negative attitude, depending on the dimension that appears to be *negative*. (Di Martino and Zan 2010, p. 44)

We identify two polarities for each dimension, and define as negative an emotional disposition resulting in a dislike for mathematics, a low perceived competence, and – according to the characterization of Skemp (1976) – an instrumental vision of mathematics. This definition of negative profiles of attitude within TMA suggests two new interrelated research avenues. One the one hand, the development of observational tools aimed at identifying a student's profile of attitude towards mathematics, in particular at recognizing a possible negative component in this profile. On the other hand, the theoretical construction and implementation of didactical interventions, aimed at preventing or overcoming a negative attitude towards mathematics and differentiated according to the different profiles of negative attitude identified in TMA.

The TMA model, originally created as a model for students' attitudes towards mathematics, also appears suitable for characterizing attitudes towards specific mathematics topics (geometry, algebra, etc.) and for investigating the attitudes towards mathematics held by different groups of people (teachers, adults, etc.). For this reason, the TMA model has recently been used to study and analyse in-service and pre-service primary teachers' attitude towards mathematics and its teaching (Coppola et al. 2012).

Summing Up and Looking Ahead

In mathematics education, research on attitude has a very long tradition, based on the interest, shared by mathematicians, teachers and mathematics educators, in identifying a causal relationship between something called 'positive attitude' and achievement. In the first period of the research most studies aimed at refining or developing measuring instruments, rather than at clarifying theoretical aspects.

With the evolution of mathematics education as a research field, and with the development of a specific research field on affect, research on attitude towards mathematics has evolved by identifying critical points in the previous phase and setting the need for a theoretical framework as a crucial item in the research agenda. This change has also provoked a shift from a normative paradigm to an interpretive one. Attitude is no longer seen as an individual's trait, useful for predicting his/her behaviour, but as an observer's construct, capable of suggesting an understanding of the individual's intentional actions in a complex context, as is the learning of mathematics: a multidimensional construct that involves beliefs and emotions and acts as a bridge between them (Di Martino and Zan 2011).

The development of research on attitude also suggests new issues to be explored, such as: constructing new observation tools that are consistent with the interpretive approach and the multidimensional characterization of attitude; investigating attitude toward mathematics of different groups of individuals; identifying possible motives underlying a change of attitude; designing and testing didactical paths to prevent or modify attitude.

But the theoretical debate about the quality of research about attitude persists. As a matter of fact, the need for comparing results from different studies and different theoretical frameworks is still a crucial issue, even when studies using questionnaires and statistical analysis have been replaced by qualitative case studies. New paradigms and new methods require the identification of new criteria for research quality: this is an important topic for future research in the affect field.

Despite the fact that many studies on attitude 'look ahead', drawing on the most important findings produced so far, in our opinion some critical issues still remain. The gap between the definition of the construct of attitude and the methods used to assess it is far from being bridged: many studies still use the term 'attitude' without defining it, or propose questionnaires that are not consistent with the chosen characterization of attitude, and, in particular, without clarifying the theoretical choices

underlying the studies themselves. Moreover, although the normative approach in the research on attitude has showed all its theoretical limitations, many recent studies place themselves in a normative paradigm, even if, perhaps, this is not a conscious choice made by the researcher.

This lack of a cumulative character in research on attitude is, in our opinion, one of its main weaknesses, a historical weakness that has not yet been overcome. In 1976, in his update on research on attitude, Aiken wrote:

> Regardless of the efforts of this writer and others to bring to the educational research community periodic reviews of studies concerned with attitudes and anxiety toward mathematics, many investigators in this area continue to be unaware or unappreciative of previous research on the topic (…) This oversight is almost certainly due to a failure to search the relevant literature, the first step in any scientific inquiry (Aiken 1976, p. 293)

More than 30 years later, we notice exactly the same phenomenon, in a sort of theoretical and meta-theoretical déjà vu that, we are sure, has limited the development of stronger results in the field. For that reason, we believe that tracing the 'story' of the construct of attitude and discussing the results obtained so far is a very significant step in the development of research in this field.

References

Aiken, L. (1970). Attitudes toward mathematics. *Review of Educational Research, 40*, 551–596.

Aiken, L. (1976). Update on attitudes and other affective variables in learning mathematics. *Review of Educational Research, 46*(2), 293–311.

Aiken, L., & Drager, R. (1961). The effect of attitudes on performance in mathematics. *Journal of Educational Psychology, 52*, 19–24.

Ajzen, I. (1988). *Attitudes, personality, and behavior*. Chicago: Dorsey Press.

Allport, W. (1935). Attitudes. In C. A. Murchison (Ed.), *A handbook of social psychology*. Worcester: Clark University Press.

Bishop, A. (1992). International perspectives on research in mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 710–723). New York: Macmillan.

Boero, P., & Szendrei, J. (1998). Research and results in mathematics education: Some contradictory aspects. In J. Kilpatrick & A. Sierpinska (Eds.), *Mathematics education as a research domain: A search for identity* (pp. 197–212). Dordrecht: Kluwer.

Bruner, J. (1990). *Acts of meaning*. Cambridge, MA: Harvard University Press.

Connelly, F. M., & Clandinin, D. J. (1990). Stories of experience and narrative inquiry. *Educational Researcher, 19*(5), 2–14.

Coppola, C., Di Martino, P., Pacelli, T., & Sabena, C. (2012). Primary teachers' affect: A crucial variable in the teaching of mathematics. *Nordic Studies in Mathematics Education, 17*(3–4), 101–118.

Corcoran, M., & Gibb, G. (1961). Appraising attitudes in the learning of mathematics. In *Evaluation in mathematics: Twenty-sixth yearbook of the NCTM*. Reston: NCTM.

Damasio, A. (1996). *Descartes' error: Emotion, reason and the human brain*. Basingstoke: Papermac.

Daskalogianni, K., & Simpson, A. (2000). Towards a definition of attitude: The relationship between the affective and the cognitive in pre-university students. In T. Nakahara & M. Koyama (Eds.), *Proceedings of the 24th conference of the IGPME* (Vol. 2, pp. 217–224). Hiroshima, Japan.

De Bellis, V., & Goldin, G. (1999). Aspect of affect: Mathematical intimacy, mathematical integrity. In O. Zaslavsky (Ed.), *Proceedings of the 23rd conference of the IGPME* (Vol. 2, pp. 249–256). Haifa, Israel.

Demazière, D., & Dubar, C. (1997). *Analyser les entretiens biographiques*. Paris: Édition Nathan.

Di Martino, P. (2004). From single beliefs to belief systems: A new observational tool. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of the 28th conference of the IGPME* (Vol. 2, pp. 271–278). Bergen, Norway.

Di Martino, P., & Zan, R. (2001). Attitude toward mathematics: Some theoretical issues. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th conference of the IGPME* (Vol. 3, pp. 351–358). Utrecht, The Netherlands.

Di Martino, P., & Zan, R. (2002). An attempt to describe a 'negative' attitude toward mathematics. In P. Di Martino (Ed.), *Proceedings of the MAVI XI European workshop* (pp. 22–29). Pisa: Università di Pisa Press.

Di Martino, P., & Zan, R. (2003). What does 'positive' attitude really mean? In N. Pateman, B. Doherty, & J. Zilliox (Eds.), *Proceedings of the 27th conference of the IGPME* (Vol. 4, pp. 451–458). Honolulu, Hawaii.

Di Martino, P., & Zan, R. (2010). 'Me and maths': Towards a definition of attitude grounded on students' narratives. *Journal of Mathematics Teacher Education, 13*(1), 27–48.

Di Martino, P., & Zan, R. (2011). Attitude towards mathematics: A bridge between beliefs and emotions. *ZDM – The International Journal on Mathematics Education, 43*(4), 471–482.

Dutton, W. (1951). Attitudes of prospective teachers toward arithmetic. *The Elementary School Journal, 42*, 84–90.

Eagly, A. (1967). Involvement as a determinant of response to favorable and unfavourable information. *Journal of Personality and Social Psychology, 7*, 1–15.

Eagly, A. H., & Chaiken, S. (1998). Attitude structure and function. In D. T. Gilbert, S. T. Fiske, & G. Lindzey (Eds.), *The handbook of social psychology* (4th ed., Vol. 1, pp. 269–322). New York: McGraw-Hill.

Eisenhart, M. (1988). The ethnographic research tradition and mathematics education research. *Journal for Research in Mathematics Education, 19*(2), 99–114.

Ernest, P. (1988). The attitudes and practices of student teachers of primary school mathematics. In A. Barbos (Ed.), *Proceedings of the 12th conference of the IGPME* (Vol. 1, pp. 288–295). Veszprém, Hungary.

Evans, J. (2000). *Adults' mathematical thinking and emotions: A study of numerate practices*. London: Routledge Falmer.

Feierabend, R. (1960). Review of research on psychological problems in mathematics education. *Research Problems in Mathematics Education, 3*, 3–46.

Fennema, E. (1989). The study of affect and mathematics: A proposed generic model for research. In D. McLeod & V. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 205–219). New York: Springer.

Fennema, E., & Sherman, J. (1977). Sex-related differences in mathematics achievement, spatial visualization and affective factors. *American Educational Research Journal, 14*(1), 51–71.

Germann, P. (1988). Development of the attitude toward science in school assessment and its use to investigate the relationship between science achievement and attitude toward science in school. *Journal of Research in Science Teaching, 25*(8), 689–703.

Glaser, B., & Strauss, A. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Chicago: Aldine.

Green, T. (1971). *The activities of teaching*. New York: McGraw-Hill.

Grigutsch, S., & Törner, G. (1998). *World views of mathematics held by university teachers of mathematical science*. Duisburg: Gerhard Mercator University, Preprint 420.

Haladyna, T., Shaughnessy, J., & Shaughnessy, M. (1983). A causal analysis of attitude toward mathematics. *Journal for Research in Mathematics Education, 14*, 19–29.

Hannula, M. (2002). Attitude toward mathematics: Emotions, expectations and values. *Educational Studies in Mathematics, 49*, 25–46.

Hart, L. (1984). Affective variables and mathematics education. *The Elementary School Journal, 84*(5), 558–581.

Hart, L. (1989). Describing the affective domain: Saying what we mean. In D. McLeod & V. Adams (Eds.), *Affect and mathematical problem solving* (pp. 37–45). New York: Springer.

Kaasila, R. (2007). Using narrative inquiry for investigating the becoming of a mathematics teacher. *ZDM – The International Journal of Mathematics Education, 39*(3), 205–213.

Karsenty, R., & Vinner, S. (2000). What do we remember when it's over? Adults' recollections of their mathematical experience. In T. Nakahara & M. Koyama (Eds.), *Proceedings of the 24th conference of the IGPME* (Vol. 4, pp. 451–458). Hiroshima, Japan.

Kilpatrick, J. (1992). A history of research in mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 3–38). New York: Macmillan.

Kloosterman, P., & Stage, F. (1992). Measuring beliefs about mathematical problem solving. *School Science and Mathematics, 92*(3), 109–115.

Kulm, G. (1980). Research on mathematics attitude. In R. J. Shumway (Ed.), *Research in mathematics education*. Reston: NCTM.

Leder, G. (1985). Measurement of attitude to mathematics. *For the Learning of Mathematics, 5*(3), 18–22.

Leder, G. (1992). Measuring attitudes to mathematics. In W. Geeslin & K. Graham (Eds.), *Proceedings of the 16th conference of the IGPME* (Vol. 2, pp. 33–39). Durham, USA.

Lester, F. (2002). Implications of research on students' beliefs for classroom practice. In G. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* Dordrecht: Kluwer.

Ma, X., & Kishor, N. (1997). Assessing the relationship between attitude toward mathematics and achievement in mathematics: A meta-analysis. *Journal for Research in Mathematics Education, 30*, 65–88.

Mandler, G. (1984). *Mind and body: Psychology of emotion and stress*. New York: Norton.

McLeod, D. (1987). A constructivist approach to research on attitude toward mathematics. In L. Streefland (Ed.), *Proceedings of the 9th conference of the IGPME* (Vol. 1, pp. 133–140). Montreal, Canada.

McLeod, D. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York: Macmillan.

McLeod, D., & Adams, V. (Eds.). (1989). *Affect and mathematical problem solving: A new perspective*. New York: Springer.

Middleton, J. A., & Spanias, P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticism of the research. *Journal for Research in Mathematics Education, 30*, 65–88.

Mura, R. (1993). Images of mathematics held by university teachers of mathematical sciences. *Educational Studies in Mathematics, 25*(4), 375–385.

Mura, R. (1995). Images of mathematics held by university teachers of mathematics education. *Educational Studies in Mathematics, 28*(4), 385–399.

Neale, D. (1969). The role of attitudes in learning mathematics. *The Arithmetic Teacher, 16*, 631–641.

Ortony, A., Clore, G., & Collins, A. (1988). *The cognitive structure of emotions*. Cambridge: Cambridge University Press.

Pajares, F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307–332.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Reston: NCTM.

Polo, M., & Zan, R. (2006). Teachers' use of the construct 'attitude'. Preliminary research findings. In M. Bosch (Ed.), *Proceedings of the 4th CERME*. Barcelona: FundEmi.

Reyes, L. (1984). Affective variables and mathematics education. *The Elementary School Journal, 84*(5), 558–581.

Rokeach, M. (1968). *Beliefs, attitudes and values*. San Francisco: Jossey-Bass.

Ruffel, M., Mason, J., & Allen, B. (1998). Studying attitude to mathematics. *Educational Studies in Mathematics, 35*, 1–18.

Schlöglmann, W. (2002). Affect and mathematics learning. In A. D. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th conference of the IGPME* (Vol. 4, pp. 185–192). Norwich, UK.

Schoenfeld, A. (1989). Exploration of students' mathematical beliefs and behavior. *Journal for Research in Mathematics Education, 20*, 338–355.

Schoenfeld, A. (1994). A discourse on methods. *Journal for Research in Mathematics Education, 25*(6), 697–710.

Schoenfeld, A. (2000). Purposes and methods of research in mathematics education. *Notices of the American Mathematical Society, 47*(6), 641–649.

Sierpinska, A., Kilpatrick, J., Balacheff, N., Howson, G., Sfard, A., & Steinbring, H. (1993). What is research in mathematics education, and what are its results? *Journal for Research in Mathematics Education, 24*(2), 274–278.

Silver, E. (1985). Research on teaching mathematical problem solving: Some underrepresented themes and needed directions. In E. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 247–266). Hillsdale: Lawrence Erlbaum.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77*, 20–26.

Thurstone, L., & Chave, E. (1929). *The measurement of attitude: A psychophysical method and some experiments with a scale for measuring attitude toward the Church*. Chicago: University of Chicago Press.

Tirosh, D. (1993). Affective and cognitive variables: Students' perspectives. In I. Hirabayashi, N. Nohda, K. Shigematsu, & F. L. Lin (Eds.), *Proceedings of the 17th conference of the IGPME* (Vol. 1, pp. 11–13). Tsukuba, Ibaraki, Japan.

Zan, R., & Di Martino, P. (2007). Attitude toward mathematics: Overcoming the positive/negative dichotomy. *The Montana Mathematics Enthusiast, Monograph 3*, 157–168.

Zan, R., Brown, L., Evans, J., & Hannula, M. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics, 63*(2), 113–121.