Preschool Teachers' Knowledge and Self-Efficacy Needed for Teaching Geometry: Are They Related?

Pessia Tsamir, Dina Tirosh, Esther Levenson, Michal Tabach, and Ruthi Barkai

Abstract This chapter focuses on methodological issues related to investigating preschool teachers' self-efficacy for teaching geometry. The first issue discussed is the specificity, as opposed to the generality, of self-efficacy and the need to design instruments which are sensitive to this aspect of self-efficacy. Specificity may be related to content, in this case geometry and the specific figures under investigation. In other words, self-efficacy for teaching triangles may differ from self-efficacy for teaching pentagons. Self-efficacy may also be related to the specific action being performed, such as designing tasks for promoting knowledge versus designing tasks for evaluating knowledge. The chapter also investigates the relationship between preschool teachers' knowledge and self-efficacy for identifying geometrical figures, presenting a method for studying this relationship but also raising questions related to this method.

Keywords Preschool teachers • Teachers' self-efficacy • Teachers' knowledge • Specificity • Geometry

Introduction

Research has shown that promoting young children's mathematics knowledge is important and that the preschool teacher has a significant role in supporting the development of this knowledge (e.g., Ginsburg et al. 2008). Towards the aim of promoting early childhood mathematics education, several position papers have called for advancing preschool teachers' knowledge for teaching mathematics. For example, a joint position paper published in the United States by the National Association for the Education of Young Children (NAEYC) and the National Council for Teachers of Mathematics (NCTM) called for ongoing professional development

P. Tsamir • D. Tirosh • E. Levenson (✉) • M. Tabach • R. Barkai
School of Education, Department of Mathematics, Science, and Technology Education, University of Tel Aviv, Israel, Tel Aviv, Israel
e-mail: pessia@post.tau.ac.il; dina@post.tau.ac.il; levensone@gmail.com; levenso@post.tau.ac.il

© Springer International Publishing Switzerland 2015

B. Pepin, B. Roesken-Winter (eds.), *From beliefs to dynamic affect systems in mathematics education*, Advances in Mathematics Education,
DOI 10.1007/978-3-319-06808-4_16

that would "move beyond the one-time workshop to deeper exploration of key mathematical topics as they connect with young children's thinking and with classroom practices" (NAEYC and NCTM 2002, p. 6). Teachers' knowledge is one of several factors affecting teachers' actions in the classroom. Studies have also shown that teachers with a high self-efficacy are more enthusiastic and more committed to teaching (Allinder 1994; Coladarci 1992); thus, it is also important to investigate and promote preschool teachers' self-efficacy related to the teaching of mathematics.

For the past several years, our research team has investigated preschool teachers' knowledge and self-efficacy for teaching number and geometry concepts. During our investigation, several issues related to the research methods have arisen. One of these issues relates to the specificity of self-efficacy. Research has shown that self-efficacy is content specific. If so, how specific do the content areas have to be? Is it enough to differentiate between preschool teachers' self-efficacy for teaching number concepts and their self-efficacy to teach geometrical concepts or might there be a difference within the domain of number and geometry, for example, between teaching triangles and pentagons? The same question may be asked related to the specificity of the actions being performed. Teaching mathematics in preschool involves the coordination of several activities on the part of the teacher, among them designing mathematical tasks for the children, holding discussions related to some mathematical situation, and answering children's mathematical questions. Is it enough to differentiate between preschool teachers' self-efficacy for designing tasks and their self-efficacy for answering children's mathematical questions? Within the activity of designing tasks for children, might there be a difference between teachers' self-efficacy for designing tasks aimed at promoting children's knowledge versus designing tasks aimed at evaluating children's knowledge? An additional concern is the relationship between teachers' knowledge and self-efficacy. Theoretically, there are four combinations which may occur: high knowledge together with high self-efficacy, high knowledge together with low self-efficacy, low knowledge together with high self-efficacy, and low knowledge together with low self-efficacy. In reality, do all of these combinations exist? Are knowledge and self-efficacy for teaching mathematics in preschool related? Finally, we ask, how might results of such research impact on professional development programs for preschool teachers. These questions will be discussed in this chapter.

Teacher Self-Efficacy, Mathematics Self-Efficacy, and Self-Efficacy for Teaching Mathematics

This paper discusses the study of preschool teachers' self-efficacy for teaching mathematics as well as the relationship between self-efficacy and knowledge for teaching mathematics. In a sense, it draws on a combination of studies: studies related to mathematics self-efficacy and studies related to teachers' self-efficacy, as well as studies related to teachers' knowledge for teaching mathematics. This section reviews studies related to self-efficacy. In the next section, when presenting the framework of program, we refer to studies of teachers' knowledge.

According to Bandura's (1986) social cognitive theory, there is a relationship between psychodynamic and behavioristic influences, as well as personal beliefs and self-perception, when explaining human behavior. Bandura defined self-efficacy as "people's judgments of their capabilities to organize and execute a course of action required to attain designated types of performances" (1986, p. 391). It is different than self-concept, which is more related to judgments about one's attributes, rather than what a person can do. It is also important to stress that self-efficacy cannot be measured by an all-purpose measure (Bandura 2006). Self-efficacy beliefs are not only domain specific (e.g., mathematics, history, science) and content specific (e.g., within the domain of mathematics there is numeracy, patterns, geometry, etc.), but action specific (e.g., is the activity implemented in class, outside, individually, in a group) (Pajares 1996; Zimmerman 2000).

Hackett and Betz (1989) defined mathematics self-efficacy as, "a situational or problem-specific assessment of an individual's confidence in her or his ability to successfully perform or accomplish a particular [mathematics] task or problem" (p. 262). With regard to mathematics self-efficacy, research has shown that regardless of mathematical ability, students with a higher self-efficacy tend to expend more effort on difficult mathematics tasks than students with lower self-efficacy (Collins 1982) and that students' self-efficacy beliefs are positively related to mathematics performance (Bandura 1986; Pajares 1996). Even among 6-year old children, mathematics self-efficacy and behavior were found to be positively related (Davis-Kean et al. 2008). Despite Bandura's (1986) claim that self-efficacy cannot be globally measured and that it is action-specific, and despite Hackett and Betz's (1989) assertion that mathematics self-efficacy is problem-specific, some studies which investigated mathematics self-efficacy included general items such as "I'm doing well in mathematics at school" (Merenluoto 2004, p. 299). Other studies were more specific. For example, Pajares and Miller (1994) used a questionnaire which differentiated between domains of mathematics, cognitive demands, and problem contexts. Pajares and Graham (1999) used an even more problem-specific questionnaire where students were shown specific mathematics questions and were then asked to assess their ability to solve them. Likewise, the survey of the Programme for International Student Assessment (PISA) which took place in 2003 assessed students' mathematics self-efficacy by asking them to what degree they felt confident solving each of eight specific mathematics problems such as calculating how much cheaper a television would be after a 30 % discount (Schulz 2005). In short, different studies included questionnaire items with varying degrees of specificity, regarding both domain and problem specificity.

When relating theories of self-efficacy to teachers, Dellinger et al. (2008) differentiated between teacher efficacy and teacher self-efficacy. The first, teacher efficacy, "assesses teachers' beliefs in their ability to affect student performance (outcome)" (p. 752). These beliefs, however, may be confounded by a teacher's sense of control. Many factors affect students' performance, some not within the teacher's control and not necessarily dependent on the teacher's ability to teach. This study does not focus on teacher efficacy but rather on teacher self-efficacy. Teacher self-efficacy may be conceptualized as "what the individual teacher can accomplish given the limitations caused by external factors" (Skaalvik and Skaalvik 2007, p. 612) or as "a teacher's individual beliefs in their capabilities to perform specific teaching tasks at a specified level of quality in a specified situation" (Dellinger et al. 2008, p. 752).

Several studies have investigated teachers' self-efficacy. When constructing items for questionnaires, some of those studies noted that teachers were consulted with regard to identifying situations and tasks encountered in teachers' daily school activities and which were important to them. For example, in a study which took place in Italy, two of the items included were "I can make my students respect rules and codes of conduct" and "I am capable of engaging even the most reluctant and difficult students in my class activities" (Caprara et al. 2006, p. 481). In Norway, a study which investigated teacher self-efficacy and burnout, took into consideration the national curriculum which stresses differential instruction (Skaalvik and Skaalvik 2007). Thus, test items addressed the teacher's belief in his or her ability to "provide good guidance and instruction to all students regardless of their level of ability" and "provide realistic challenge for all students even in mixed ability classes" (Skaalvik and Skaalvik 2007, p. 624). The above mentioned studies investigated teachers' self-efficacy without regard for a specific content domain. We now turn to studies related to self-efficacy and teaching mathematics.

In order to discuss teachers' self-efficacy for teaching mathematics, we differentiate between teachers' mathematics self-efficacy, i.e., self-efficacy related to solving mathematics problems, and teachers' self-efficacy for teaching mathematics. This differentiation was also pointed out by Bates et al. (2011) who investigated the relationship between early childhood (pre-K to third grade) preservice teachers' mathematics self-efficacy and their mathematics teaching self-efficacy. The instruments used in the study conducted by Bates et al. (2011) were the Mathematics Self-Efficacy Scale developed by Betz and Hackett (1993) and the Mathematics Teaching Efficacy Belief Instrument, developed by Enochs et al. (2000). In general, results of the study showed that teachers who reported higher mathematics self-efficacy were more confident in their ability to teach mathematics than teachers with a lower mathematics self-efficacy. Results also showed that teachers who had a higher mathematics self-efficacy performed better on a basic mathematics skills test than participants with a lower mathematics self-efficacy. However, participants with a high mathematics teaching self-efficacy did not necessarily perform well on the mathematics skills test. In other words, some teachers who scored low on the skills test still felt confident to teach mathematics. While the authors pointed out that these results could be due to the inexperience of the preservice teachers, we raise additional questions. For example, how well did the items on the skill test match the items on the teacher self-efficacy questionnaire. The skills test measured participants' ability to solve problems involving integers, fractions, algebra, and geometry. The mathematics teaching efficacy questionnaire included general statements such as "I will continually find better ways to teach mathematics" (Enochs et al. 2000). It could be that in situations where the items on the two questionnaires are more closely related, a correlation would be found. It could also be that early childhood teachers may know that they cannot solve algebra problems but feel confident in their ability to teach the mathematics necessary for young children. These issues are taken into consideration in the next section which presents the framework of our program.

Framework and Study Background

Our professional development program for preschool teachers is guided by the Cognitive Affective Mathematics Teacher Education (CAMTE) framework (e.g. Tirosh et al. 2011; Tsamir et al. 2014a). This framework takes into consideration the interrelationship between knowledge and beliefs which can affect teachers' proficiency (Schoenfeld and Kilpatrick 2008).

The framework is presented in Table 1. In Cells 1–4, and in Cells 5–8, we address teachers' knowledge and self-efficacy respectively. The same framework guides our research study.

In framing the mathematical knowledge preschool teachers need for teaching, we draw on Shulman (1986) who identified subject-matter knowledge (SMK) and pedagogical content knowledge (PCK) as two major components of teachers' knowledge necessary for teaching. In our previous work with teachers, we found it useful to differentiate between two components of teachers' SMK: being able to produce solutions, strategies and explanations and being able to evaluate given solutions, strategies and explanations (Tabach et al. 2010). Thus our framework takes into consideration both of these aspects of SMK.

Regarding PCK, we draw on the works of Ball and her colleagues (2008) who refined Shulman's theory and differentiated between two aspects of PCK: knowledge of content and students (KCS) and knowledge of content and teaching (KCT). KCS is "knowledge that combines knowing about students and knowing about mathematics" whereas KCT "combines knowing about teaching and knowing about mathematics" (Ball et al. 2008, p. 401). Under this last category, we focus on the design, evaluation, and implementation of mathematical tasks. In Israel, there is a mandatory mathematics preschool curriculum (INMPC 2008), but few curricular materials are available. Teachers often find themselves designing their own tasks to implement in their classes and so it is especially important for them to appreciate the design process and its implication for creating learning opportunities. For each aspect of knowledge in the framework, there is a corresponding aspect of self-efficacy. Thus, the CAMTE framework takes into consideration teachers' mathematics

Table 1 The cognitive affective mathematics teacher education framework

	Subject-matter		Pedagogical-content	
	Solving	Evaluating	Students	Tasks
Knowledge	Cell 1: Producing solutions	Cell 2: Evaluating solutions	Cell 3: Knowledge of students' conceptions	Cell 4: Designing and evaluating tasks
Self-efficacy	Cell 5: Mathematics self-efficacy related to producing solutions	Cell 6: Mathematics self-efficacy related to evaluating solutions	Cell 7: Pedagogical-mathematics self-efficacy related to children's conceptions	Cell 8: Pedagogical-mathematics self-efficacy related to designing and evaluating tasks

self-efficacy (Table 1, Cells 5 and 6) as well as their pedagogical-mathematics self-efficacy, i.e. their self-efficacy related to the pedagogy of teaching mathematics (Table 1, Cells 7 and 8). What we term pedagogical-mathematics self-efficacy corresponds in a way to what was referred to in the previous section as self-efficacy for teaching mathematics (Enochs et al. 2000). However, in accordance with Bandura (1986) we relate to a more action-specific self-efficacy, i.e., self-efficacy is related to specific, as opposed to general actions being performed. This will be illustrated in the following section. In the following section we also show how the framework was used to design tools to investigate knowledge and self-efficacy for teaching geometry concepts.

Over the years we have gathered data from several groups of preschool teachers who have participated in our professional development programs. The teachers were all practicing teachers at the time they participated in the program, teaching children ages 3–6 years old in municipal preschools, sometimes in mixed-aged groups and sometimes in separate-aged groups. In Israel, children begin first grade at age 6, so we consider kindergarten to be the last year before elementary school. All teachers had a B.Ed., specializing in early childhood education, obtained after completing a 4-year course of study in a teacher-education college. Early childhood programs in these colleges focus mainly on psychology, sociology, and general education, with less attention paid to teaching content such as mathematics.

Two questionnaires were used in this study, one focusing on teaching two-dimensional (2-D) shapes and one on three-dimensional (3-D) solids. The mathematical content of each questionnaire and the subsequent items built for each questionnaire were based on the mandatory Israel National Mathematics Preschool Curriculum (INMPC 2008) which provides guidelines and standards for teaching mathematics to children ages 3–6 years old and on our previous research with young children (e.g., Tsamir et al. 2008). In the next sections we describe in more detail different items of the specific questionnaires, how the data was analyzed, and related results. The section "Preschool teachers' pedagogical-mathematical self-efficacy: content specificity" focuses on the question of self-efficacy being content and action specific. The section "Relating self-efficacy to knowledge" focuses on the relationship between self-efficacy and knowledge.

Preschool Teachers' Pedagogical-Mathematical Self-Efficacy: Content Specificity

In order to investigate the question of content and action specificity, we focus on Cell 8 (pedagogical-mathematics self-efficacy related to designing and evaluating tasks) of the CAMTE framework, describing related items and results from the 3-D questionnaire.

Tools and Data Analysis: Teachers' Self-Efficacy for Designing Tasks

Teachers design tasks for many purposes. In this study, we differentiated between tasks used to promote children's knowledge and tasks used to evaluate children's knowledge. A four-point Likert scale was used to rate participants' agreements with self-efficacy statements: (1) I do not agree that I am capable; (2) I somewhat agree that I am capable; (3) I agree that I am capable; (4) I strongly agree that I am capable. The statements which teachers were asked to rate their agreement with were:

1. I am capable of designing tasks to *promote* children's knowledge of *cones*;
2. I am capable of designing tasks to *evaluate* children's knowledge of *cones*;
3. I am capable of designing tasks to *promote* children's knowledge of *cylinders*;
4. I am capable of designing tasks to *evaluate* children's knowledge of *cylinders*.

Altogether, we collected questionnaires from 62 practicing preschool teachers. The data collected from the above four questions led to four very specific self-efficacy scores, referring to specific figures as well as to designing tasks for specific purposes. We then calculated the mean self-efficacy score for questions (1) and (2) and then questions (3) and (4), resulting in more general self-efficacy scores for designing tasks for cones and cylinders. In other words, content specificity (i.e., separation of figures) was preserved but action specificity (i.e., separating designing tasks for promoting knowledge from designing tasks for evaluating knowledge) was generalized. We then calculated mean scores for questions (1) and (3) and then questions (2) and (4), resulting in more general self-efficacy scores for promoting children's knowledge of 3-D figures and evaluating children's knowledge of 3-D figures. In other words, we kept the activity very specific and generalized the content.

Results: Specificity of Self-Efficacy for Designing 3-D Geometry Tasks

Taking into consideration that the self-efficacy scale ran from 1 (lowest) to 4 (very high), in general, teachers did not have a very high self-efficacy when it came to designing tasks related to three-dimensional figures (Table 2). This was true for both cones and cylinders as well as for designing tasks for promoting knowledge and designing tasks for evaluating knowledge. In general, it also seemed that teachers' self-efficacy related to designing cylinder tasks was greater than teachers' self-efficacy for designing cone tasks and that self-efficacy related to designing tasks for promoting knowledge was greater than self-efficacy related to designing evaluation tasks.

Table 2 Self-efficacy for designing different types of tasks per figure

Designing tasks for…	Promoting knowledge		Evaluating knowledge	
Figures	M	SD	M	SD
Cones (N=61)	2.33	.87	2.18	.85
Cylinders (N=60)	2.52	.79	2.45	.81

Table 3 Comparing self-efficacy: different figures and different activities

Cones versus cylinders	Mean difference	t-value	df	p-value
Designing tasks for *promoting* knowledge: cones versus cylinders	−.17	−1.80	59	.077
Designing tasks for *evaluating* knowledge: cones versus cylinders	−.22	−2.43	59	.018
Designing tasks in general: cylinders versus cones	−.17	−1.92	60	.060
Promoting knowledge versus evaluating knowledge				
Cones: designing tasks for *promoting* knowledge versus tasks for *evaluating* knowledge	.15	2.87	60	.006
Cylinders: designing tasks for *promoting* knowledge versus tasks for *evaluating* knowledge	.07	2.05	59	.045
3-D figures: designing tasks for *promoting* knowledge versus tasks for *evaluating* knowledge	.12	2.95	61	.004

In order to analyze if the general results outlined above were significant, paired-samples t-tests were carried out. Results are presented in Table 3. Differentiating between cones and cylinders, we see that teachers had a significantly lower self-efficacy for designing tasks to evaluate children's knowledge of cones than for designing tasks to evaluate children's knowledge of cylinders. However, when it came to designing tasks to promote knowledge or just designing tasks in general, the specific figure, whether it was a cylinder or cone which was the object being discussed, did not seem significant. Focusing on the types of tasks being designed, significant differences were consistently found between teachers' self-efficacy for designing tasks to promote knowledge and their self-efficacy for designing tasks to evaluate knowledge, regardless of the figures being targeted. In other words, for this group of preschool teachers, task-specificity seems to be more of an issue than the specific 3-D figure at stake. Furthermore, it seems that teachers have a higher self-efficacy when it comes to designing tasks for promoting knowledge than they do for designing tasks to evaluate knowledge. That being said, although some of the results were significant, they were relatively small. Thus, teacher educators should consider promoting teachers' self-efficacy for designing both types of tasks.

Relating Self-Efficacy to Knowledge

Tools and Data Analysis: Identifying Two and Three Dimensional Figures

In order to investigate the possible relationships between knowledge and self-efficacy, we focus on items related to Cells 1 and 4 of the CAMTE framework (knowledge and self-efficacy for solving problems) from the 2-D and 3-D questionnaires. Within the context of two-dimensional shapes, we chose to focus on identifying triangles, pentagons, and circles. Within the context of three-dimensional figures, we focused on cones and cylinders. Identifying these two and three-dimensional figures is mentioned specifically in the preschool mathematics curriculum as a competency expected of kindergarten children (INMPC 2008). Both of these questionnaires consisted of two parts. The first part of the 2-D questionnaire began with the following self-efficacy related questions: If I am shown a triangle, I will be able to identify it as a triangle. If I am shown a figure which is not a triangle, I will be able to identify it as not being a triangle. This was repeated for pentagons and circles. Likewise, the 3-D questionnaire inquired about teachers' ability to identify cones and cylinders as well as their ability to identify nonexample of cones and nonexamples of cylinders. As previously described, a four-point Likert scale was used for these questions, 1 meaning the teacher was not in agreement that she was able to identify the figure and 4 meaning that she was in complete agreement that she was able to identify the figure.

When analyzing the data from these items, a mean self-efficacy score was created for each figure from the two self-efficacy questions related to identifying examples and nonexamples of that figure. A more general mean self-efficacy score was then calculated reflecting self-efficacy for identifying two-dimensional figures and for identifying three-dimensional figures.

After the first part of the questionnaire was collected, the second part was handed out. The second part of each questionnaire consisted of a series of examples and nonexamples of different figures. Each figure was accompanied by a question: Is this a triangle (or pentagon or cylinder) Yes/No? Figures 1, 2, and 3 present the figures used when investigating triangles, pentagons, and circles. Figures 4 and 5 present the figures used when investigating cones and cylinders. In choosing the figures, both mathematical and psycho-didactical dimensions were considered. That is, we not only considered whether the figure is an example or a nonexample, but whether or not it would intuitively be recognized as an example or a nonexample (Tsamir et al. 2008). When considering triangles, for example, the equilateral triangle may be considered a prototypical triangle and thus intuitively recognized as a triangle, accepted immediately without the feeling that justification is required (Hershkowitz 1990; Tsamir et al. 2008). The narrow and long scalene triangle may be considered a non-intuitive example because of its "skinniness" (Tsamir et al. 2008). The nonexamples were chosen so that for each figure one critical attribute

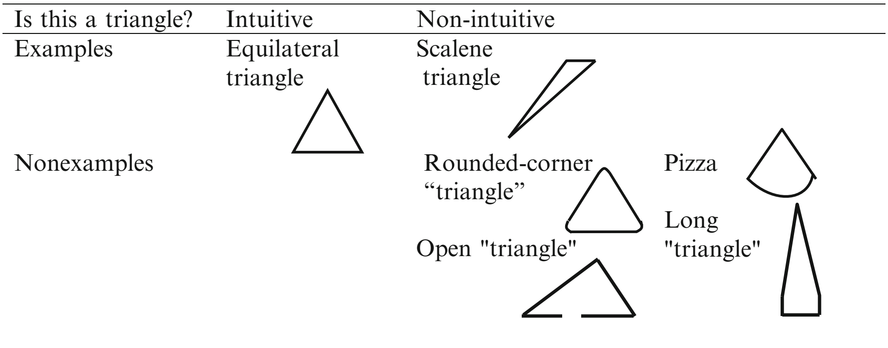


Fig. 1 Is this a triangle?

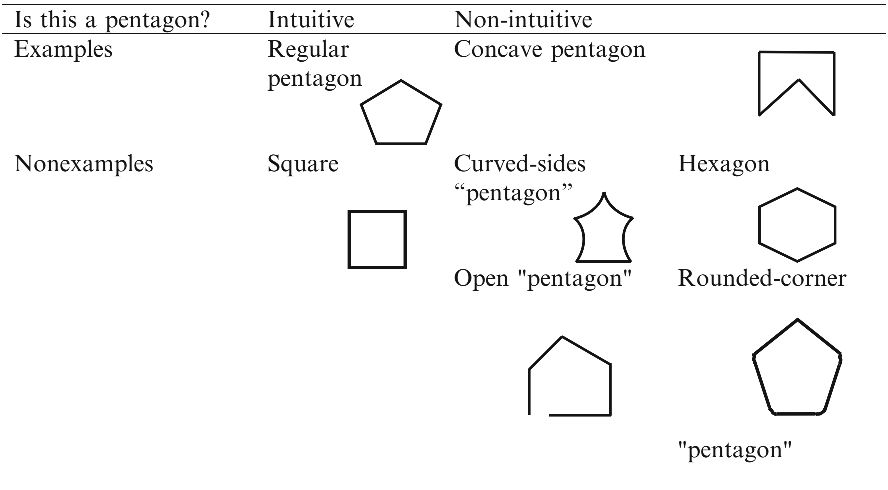


Fig. 2 Is this a pentagon?

is contradicted. Thus, one figure is open, another has five sides; one has a curved side and another has rounded corners. Whereas a circle may be considered an intuitive nonexample of a triangle, the pizza-like “triangle” may be considered a non-intuitive nonexample because of visual similarity to a prototypical triangle. Similarly, the regular pentagon was thought to be easily recognized by children who had been introduced to pentagons whereas, the concave pentagon is more difficult to identify.

As few studies have investigated young children’s knowledge of solids, our differentiation between intuitive and nonintuitive solids is based on our experience and studies regarding how children identify them (Tirosh and Tsamir 2008) and related studies with two-dimensional shapes. For example, studies have shown that

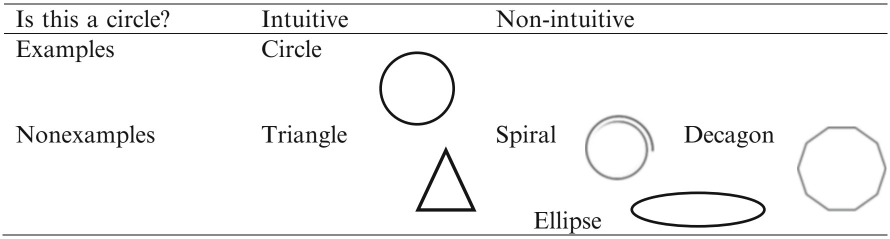


Fig. 3 Is this a circle?

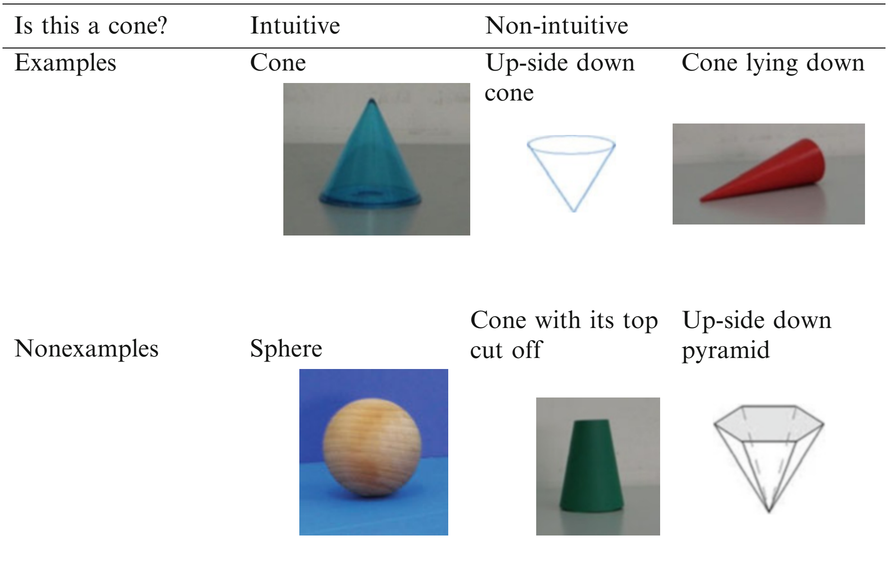


Fig. 4 Is this a cone?

a circle may be considered as an intuitive nonexample of a triangle (Tsamir et al. 2008) and that for many children, being able to give a name to one shape guarantees that it will not be some other shape. Likewise, because most children can name a ball, we classified the ball as an intuitive nonexample of a cone.

When analyzing data from these items, a mean score was configured for identifying each of the different figures. For example, when investigating identification of a cone, six figures were presented. Thus, a participant who correctly identified (either as an example or as a nonexample) three out of the six figures, received a score of 50 %. As with the self-efficacy scores, a general mean knowledge score was configured separately for the two and three-dimensional figures, reflecting teachers' knowledge for identifying two-dimensional figures and their knowledge for identifying three-dimensional figures.

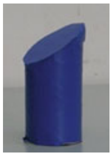
Is this a cylinder?	Intuitive	Non-intuitive	
Examples	Cylinder 	"Coin-like" cylinder 	Cylinder lying  down
Nonexamples	Sphere 	Cone with its top cut off 	Cylinder cut on a slant 

Fig. 5 Is this a cylinder?

Table 4 Mean knowledge scores and self-efficacy scores per 2-D and 3-D figure

	Correct identification		Self-efficacy	
	M	SD	M	SD
Triangle (N = 19)	.95	.14	3.82	.38
Pentagon (N = 18)	.88	.13	3.47	.58
Circle (N = 17)	.98	.07	3.34	.58
General 2-D (N = 19)	.94	.08	3.55	.43
Cone (N = 63)	.93	.14	3.04	.73
Cylinder (N = 62)	.87	.13	3.22	.58
General 3-D (N = 63)	.90	.11	3.10	.65

Results: Relating Knowledge and Self-Efficacy for Identifying Two and Three Dimensional Figures

We begin by presenting overall results of participants' self-efficacy and knowledge for identifying the various specific figures. Recall that self-efficacy was rated on a scale of 1–4, 1 being very low and 4 being very high. Results (Table 4) indicated that in general, participants were able to identify two and three dimensional figures and had a high self-efficacy regarding their ability to do so.

In order to investigate the question of whether preschool teachers' knowledge for identifying some figure is related to their self-efficacy for identifying that figure, Pearson correlations were carried out for each figure. For example, we compared teachers' knowledge of identifying triangles with their self-efficacy for identifying triangles. For the most part, knowledge and self-efficacy were not found to be

Table 5 Levels of knowledge versus self-efficacy – identifying cones

Self-efficacy knowledge	Low	High	Total
Low	12	5	17
High	24	16	40
Total	36	21	57

Table 6 Levels of knowledge versus self-efficacy – identifying cylinders

Self-efficacy knowledge	Low	High	Total
Low	25	8	33
High	12	11	23
Total	37	19	56

related. There were two exceptions. Results indicated a significant positive correlation between teachers' knowledge for identifying cylinders and their self-efficacy for identifying cylinders ($r = .30$, $p = .03$) and for identifying, in general, 3-D figures and their self-efficacy for identifying 3-D figures ($r = .30$, $p = .02$).

Being that significant results were only found related to 3-D figures, we decided to further analyze the distribution of results for the cones and cylinders. Specifically, we were interested in the possibility that teachers who were knowledgeable had a low self-efficacy and/or teachers who were less knowledgeable, nevertheless had a high self-efficacy.

Tables 5 and 6 describe the distribution of low and high knowledge scores for identifying cones and cylinders, respectively, versus low and high self-efficacy, where low and high was determined by the mean score for each variable. We acknowledge that the mean knowledge scores for both the cones and cylinders were above 85 % and that it might seem harsh to claim that a score of less than 85 % is low. However, taking into consideration that all participants were already practicing teachers, and that the means were indeed high, we feel that a score below the mean may be considered in this case, to be low. In general, we see that for cones and cylinders, all four possible combinations of high and low knowledge and self-efficacy exist. We also note that for both figures, few teachers exhibited a low level of knowledge with a high level of self-efficacy, meaning that there were few teachers who could not identify the figures but thought that they could do so. Finally, we note that the phenomenon of being able to correctly identify figures but yet not being aware of this knowledge, was more prevalent for cones than for cylinders.

Discussion

There were two main issues investigated in this chapter: the specificity of self-efficacy and the relationship between knowledge and self-efficacy. When organizing this chapter, the dilemma arose regarding which section should be presented first, the section focusing on specificity or the section focusing on the relationship between knowledge and self-efficacy. On the one hand, we felt that before discussing teachers' self-efficacy for designing geometry tasks, we should first investigate their geometric knowledge. After all, being able to identify cylinders is a prerequisite for being able to design tasks to promote children's knowledge of cylinders. And yet, as we began to design an instrument to investigate different elements of teachers' self-efficacy for teaching geometry, we found ourselves grappling with the question of specificity. That is, yes, investigating knowledge is of prime importance and the relationship between knowledge and self-efficacy is a relevant question. But before this can be investigated, we have to address the issue of how specific the self-efficacy instrument need be. And so, as the focus of this chapter is on methodological questions, we decided to present the sections in the order of which we grappled with the questions.

Two aspects of specificity were discussed in this paper. The first related to the specific figure, cones versus cylinders, and the second related to the specific activity, designing tasks for promoting knowledge versus tasks for evaluating knowledge. Building very specific questions was carried out in accordance with Bandura's (1986) theory that self-efficacy cannot be measured globally. As mentioned previously, Hackett and Betz (1989) asserted that mathematics self-efficacy is problem specific.

The issue of specificity in self-efficacy also arose in other studies we conducted with preschool teachers. For example, when studying teachers' self-efficacy for teaching number concepts, we differentiated between teachers' self-efficacy for teaching verbal counting versus their self-efficacy related to object counting (Tsamir et al. 2014b). While both types of counting are related, they involve different skills. Verbal counting includes being able to say the number words in the proper order and knowing the principles and patterns in the number system as coded in one's natural language. Object counting refers to counting objects for the purpose of saying how many. Gelman and Gallistel (1978) outlined five principles of counting objects: the one-to-one correspondence principle, the stable-order principle, the cardinal principle, the abstraction principle, and the order-irrelevance principle. Recognizing the complexity of counting, one item addressed teachers' self-efficacy to promote children's skill in verbally counting up to 30 while a different item addressed teachers' self-efficacy to promote counting eight objects. Notice also, that in those two questions, the specific number to which children should count to and the specific number of objects to be counted is also related. In other words, the question addressed very specific counting skills and not general ones. In addition to differentiating between self-efficacy related to verbal and object counting, we also related to the specific skills involved with each type of counting. For example, saying which number comes before and after a given number, are two separate important skills related to verbal counting. Thus, one item investigated teachers' self-efficacy related to promoting children's knowledge of which number *follows* each of the numbers from 0 to 9 while a separate item was directed at teachers' self-efficacy for promoting the skill of saying which number comes *before* each of the numbers 1–10. Other number skills promoted during preschool are composing and decomposing numbers and recognizing number symbols. Once again, a separate item on the questionnaire addressed teachers' self-efficacy for teaching each of these skills. Preliminary results indicated that teachers' self-efficacy varied with the items. For example, teachers had a higher self-efficacy for building tasks that assess children's knowledge of enumerating eight items than they did for building tasks that assess children's knowledge related to the counting sequence up till 30.

In the current study, we separated not only between two and three-dimensional figures but investigated self-efficacy related to specific figures, cones versus cylinders. The issue of specificity arose, even in this case, when designing items to investigate teachers' self-efficacy related to identifying each of the figures. One question was directed at teachers' self-efficacy for identifying examples of, for instance, a cone, while a separate question addressed teachers' self-efficacy for identifying nonexamples of a cone. No significant differences were found between teachers' self-efficacy for identifying examples and nonexamples of cones or of cylinders and thus we configured a more general self-efficacy score for identifying each shape. However, at the start, specificity of self-efficacy was taken into consideration. The question which arises from these results is how many items ought to be used in order to insure specificity, taking into consideration, perhaps, that the more items there are on a test, the more general the test might be considered. Hackett and Betz (1989) related to this issue by dividing mathematics self-efficacy into three sub-scales, each containing between 16 and 18 items, in order to measure three sub-constructs of mathematics self-efficacy. This is an open question which needs further investigation.

In general, for this group of preschool teachers, the type of activity (in this case, designing tasks for promoting knowledge versus designing tasks for evaluating knowledge) seemed to have more of an effect on self-efficacy than the specific figure being discussed. Of course, we only differentiated between cones and cylinders. A next step would be to investigate additional 3-D figures. It could also be that the figure is less important when the action being taken is designing tasks, but for other actions, such as responding to children's questions, the specific figure may be very relevant. In the case when a difference was noted between cones and cylinders, designing tasks for promoting knowledge, teachers' had a higher self-efficacy with regard to cylinders. We take note of this as we consider the second issue of this study, the relationship between knowledge and self-efficacy.

The relationship between knowledge and self-efficacy was investigated with regard to identifying 2-D and 3-D figures. No correlations were found within the group of 2-D figures. Within the group of 3-D figures, a significant correlation was found between knowledge and self-efficacy for identifying cylinders. Once again, significant results were found with regard to cylinders but not for cones. In general, it seems that teachers were more aware of their knowledge of cylinders than of cones. Perhaps it was this awareness which affected their higher self-efficacy with regard to designing tasks for promoting knowledge of cylinders than for cones. This is in line with Bates et al. (2011) who found that teachers with a higher mathematics self-efficacy were more confident in their ability to teach mathematics than teachers with a lower mathematics self-efficacy. However, in that study, questionnaire items related to general teaching abilities. In our study, we attempted to pinpoint the different activities a teacher must perform. A next step for us might be to investigate the relationship between mathematics self-efficacy (Cell 5 of the CAMTE framework, Table 1) and teachers' pedagogical-mathematics self-efficacy related to designing tasks (Cell 8 of the framework).

While some significant results were noted, for the most part, knowledge and self-efficacy were not significantly correlated. Non-significant results could mean very plainly that no correlation exists. However, as noted in the background, previous studies found mathematics self-efficacy positively related to performance (Hackett and Betz 1989; Bates et al. 2011). This raises methodological questions. Insignificant correlations may be due to insufficient variance among the variables. While it could very well be that teachers have no difficulties identifying various examples and nonexamples of figures, it could also be that a questionnaire, with more examples and nonexamples of the different figures, would differentiate more clearly between levels of knowledge among teachers. When investigating self-efficacy, nearly all teachers rated their self-efficacy for identifying figures as high (3) or very high (4). It could be that a finer scale is necessary and that the results of this study were limited by a ceiling effect.

Another methodological issue which needs to be investigated is the order of the presentation/administration of the self-efficacy and performance questions. In accordance with previous studies which investigated self-efficacy and performance (e.g. Hackett and Betz 1989) our questionnaire began with self-efficacy questions and then proceeded to performance questions. On the one hand, this makes sense. If I see that I can successfully complete a given activity then I will believe in my ability to complete the same activity again. Thus, if we placed the performance questions first, it could affect how teachers answered the self-efficacy questions. But does that mean that the self-efficacy questions were not influenced by other factors? According to Bandura (1986) one of the sources for self-efficacy beliefs are performance attainments; success raises self-efficacy while failure lowers it. In other words, it is possible that the teachers' past experiences with geometric activities, affected how they responded to the self-efficacy questions. For example, teachers were asked to estimate their ability to identify nonexamples of circles. What nonexamples came to their mind when answering this self-efficacy question? Perhaps they recalled a time when they incorrectly identified an ellipse as a circle. This would then affect their self-efficacy to identify nonexamples of circles. Finally, we also question the assumption that a person's knowledge is unshakeable. Knowledge, or performance on tasks, might be influenced by several factors other than the individual's ability to perform the activity. Might it be that answering the self-efficacy questions affected teachers' performance on the tasks that followed? This needs further investigation.

To summarize, this chapter focused on methodological issues related to investigating teachers' knowledge, self-efficacy, and the relationship between them. We showed how one can design questionnaires that allow for different levels of specificity, both in content and in actions. With relation to content, we began with very specific items to investigate both knowledge and self-efficacy and gradually generalized the investigation. For example, we presented for identification very specific examples and nonexamples of different figures and from the specific items configured a general knowledge score for cones and cylinders. A similar process was carried out with self-efficacy items. Likewise, we differentiated between specific actions, for example, between designing tasks for promoting knowledge and designing tasks for evaluating knowledge. In all cases, there is also the issue of scaling self-efficacy. The scale we used ran from 1 to 4. Perhaps a wider scale would have been more sensitive to differences in self-efficacy. All of these issues influence the results of such an investigation, of knowledge, self-efficacy, and the relationship between them. Our task, as mathematics educators is to design questionnaires that are both specific enough and yet general enough to investigate these issues.

How can the results of this study inform professional development programs for preschool teachers? Teachers in this study were able to identify most of the examples and nonexamples presented to them. This presents a possible starting point from which teachers can begin to explore additional aspects of geometric figures such as definitions, critical and non-critical attributes, and an expanded example space of these figures. In general, teachers' self-efficacy with regard to 3-D figures was lower than their self-efficacy for identifying 2-D figures. This might indicate that during professional development more attention should be paid to promoting teachers' self-efficacy for identifying 3-D figures. Within the group of 3-D figures, a correlation was found between teachers' knowledge and self-efficacy for identifying cylinders but not between their knowledge and self-efficacy for identifying cones or two-dimensional figures. Some teachers were knowledgeable of cones, yet their self-efficacy for identifying cones was low. Studies have shown that mathematics self-efficacy predicts children's choices of the types of problems they prefer to engage (Bandura and Schunk 1981). Likewise, teachers with a low self-efficacy related to cones, may avoid planning activities that involve this figure. Professional development may benefit these teachers by not only increasing their self-efficacy but increasing their self-awareness. This would also benefit those few teachers who had a low self-efficacy but nevertheless thought they were knowledgeable. Wheatley (2002) claimed that teachers' efficacy doubts may cause a feeling of disequilibrium which in turn may foster teacher learning. Results of this study also indicated that teachers had a higher self-efficacy when it comes to designing tasks for promoting knowledge than they did for designing tasks to evaluate knowledge, regardless of the specific figure being addressed. This might indicate that preschool teachers have less experience with designing tasks to evaluate children's knowledge. This issue could be raised and explored during professional development. Teachers can be encouraged, within the supporting environment of professional development programs, to design such tasks, implement them with children in their classes, and discuss together the results. In conclusion, while this paper raised several methodological questions regarding the study of preschool teachers' self-efficacy for teaching geometry, it also led to results which may be used to inform professional development aimed at promoting preschool teachers' knowledge and self-efficacy for teaching geometry.

Acknowledgements This research was supported by THE ISRAEL SCIENCE FOUNDATION (grant No. 654/10). We would also like to thank Dr. Sigal Levy, from The Academic College of Tel Aviv Yaffo, for her assistance and advice regarding statistical analysis.

References

Allinder, R. M. (1994). The relationship between efficacy and the instructional practices of special education teachers and consultants. *Teacher Education and Special Education, 17*, 86–95.

Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching. *Journal of Teacher Education, 59*(5), 389–407.

Bandura, A. (1986). *Social foundations of thought and action: A social cognitive theory*. Englewood Cliffs: Prentice Hall.

Bandura, A. (2006). Guide for constructing self-efficacy scales. In F. Pajares & T. Urdan (Eds.), *Self-efficacy beliefs of adolescents* (pp. 1–43). Greenwich: Information Age.

Bandura, A., & Schunk, G. H. (1981). Cultivating competence, self-efficacy, and intrinsic interest through proximal self-motivation. *Journal of Personality and Social Psychology, 41*(3), 586–598.

Bates, A. B., Latham, N., & Kim, J. (2011). Linking preservice teachers' mathematics self-efficacy and mathematics teaching efficacy to their mathematical performance. *School Science and Mathematics, 111*(7), 325–333.

Betz, N. E., & Hackett, G. (1993). *Mathematics self-efficacy scale*. Palo Alto: Mind Garden Press.

Caprara, G. V., Barbaranelli, C., Steca, P., & Malone, P. (2006). Teachers' self-efficacy beliefs as determinants of job satisfaction and students' academic achievement: A study at the school level. *Journal of School Psychology, 22*, 473–490.

Coladarci, T. (1992). Teachers' sense of efficacy and commitment to teaching. *Journal of Experimental Education, 60*, 323–337.

Collins, J. (1982). *Self-efficacy and ability in achievement behavior*. Paper presented at the Meeting of the American Educational Research Association, New York.

Davis-Kean, P. E., Huesmann, L. R., Jager, J., Collins, W. A., Bates, J. E., & Lansford, J. (2008). Changes in the relation of beliefs and behaviors during middle childhood. *Child Development, 79*, 1257–1269.

Dellinger, A., Bobbett, J., Livier, D., & Ellett, C. (2008). Measuring teachers' self-efficacy beliefs: Development and use of the TEBS-self. *Teaching and Teacher Education, 24*, 751–766.

Enochs, L. G., Smith, P. L., & Huinker, D. (2000). Establishing factorial validity of the mathematics teaching efficacy beliefs instrument. *School Science and Mathematics, 100*, 194–202.

Gelman, R., & Gallistel, C. (1978). *The child's understanding of number*. Cambridge: Harvard University Press.

Ginsburg, H. P., Lee, J. S., & Boyd, J. S. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report, XXII*(I), 1–22.

Hackett, G., & Betz, N. (1989). An exploration of the mathematics self-efficacy/mathematics performance correspondence. *Journal for Research in Mathematics Education, 20*, 261–273.

Hershkowitz, R. (1990). Psychological aspects of learning geometry. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and cognition* (pp. 70–95). Cambridge: Cambridge University Press.

Israel National Mathematics Preschool Curriculum (INMPC). (2008). Retrieved April 7, 2009, from http://meyda.education.gov.il/files/Tochniyot_Limudim/KdamYesodi/Math1.pdf

Merenluoto, K. (2004). The cognitive – Motivational profiles of students dealing with decimal numbers and fractions. In M. J. Høines & A. B. Fuglestad (Eds.), *Proceedings of the 28th conference of the international group for the psychology of mathematics education* (Vol. 3, pp. 297–304).

National Association for the Education of Young Children & National Council of Teachers of Mathematics (NAEYC & NCTM). (2002). *Position statement. Early childhood mathematics: Promoting good beginnings*. Available: www.naeyc.org/resources/position_statements/psmath.htm

Pajares, F. (1996). Self efficacy believes in academic settings. *Review of Educational Research, 66*, 543–578.

Pajares, F., & Graham, L. (1999). Self-efficacy, motivation constructs, and mathematics performance of entering middle school students. *Contemporary Educational Psychology, 24*, 124–139.

Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis. *Journal of Educational Psychology, 86*, 193–203.

Schoenfeld, A. H., & Kilpatrick, J. (2008). Toward a theory of proficiency in teaching mathematics. In D. Tirosh & T. Wood (Eds.), *The international handbook of mathematics teacher education: Tools and processes in mathematics teacher education* (Vol. 2, pp. 321–354). Rotterdam: Sense Publishers.

Schulz, W. (2005, April 11–15). *Mathematics self-efficacy and student expectations*. Results from PISA 2003. Paper prepared for the annual meetings of the American Educational Research Association in Montreal.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Skaalvik, E., & Skaalvik, S. (2007). Dimension of teacher self-efficacy and relations with strain factors, perceived collective teacher efficacy, and teacher burnout. *Journal of Educational Psychology, 99*(3), 611–625.

Tabach, M., Levenson, E., Barkai, R., Tirosh, D., Tsamir, P., & Dreyfus, T. (2010). Secondary school teachers' awareness of numerical examples as proof. *Research in Mathematics Education, 12*(2), 117–131.

Tirosh, D., & Tsamir, P. (2008). *Starting right: Mathematics in preschool*. Unpublished research report. In Hebrew.

Tirosh, D., Tsamir, P., Levenson, E., & Tabach, M. (2011). From preschool teachers' professional development to children's knowledge: Comparing sets. *Journal of Mathematics Teacher Education, 14*, 113–131.

Tsamir, P., Tirosh, D., & Levenson, E. (2008). Intuitive nonexamples: The case of triangles. *Educational Studies in Mathematics, 69*, 81–95.

Tsamir, P., Tirosh, D., Levenson, E., Tabach, M., & Barkai, R. (2014a). Employing the CAMTE framework: Focusing on preschool teachers' knowledge and self-efficacy related to students' conceptions. In C. Benz, B. Brandt, U. Kortenkamp, G. Krummheuer, S. Ladel, & R. Vogel (Eds.), *Early mathematics learning – Selected papers from the POEM 2012 conference* (pp. 291–306). New York: Springer.

Tsamir, P., Tirosh, D., Levenson, E., Tabach, M., & Barkai, R. (2014b). Developing preschool teachers' knowledge of students' number conceptions. *Journal of Mathematics Teacher Education, 17*(1), 61–83.

Wheatley, K. (2002). The potential benefits of teacher efficacy doubts for educational reform. *Teaching and Teacher Education, 18*(1), 5–22.

Zimmerman, B. J. (2000). Attainment of self-regulation: A social cognitive perspective. In M. Boekaerts, P. R. Pintrich, & M. Zeidner (Eds.), *Handbook of self-regulation* (pp. 13–39). San Diego: Academic.