Reaction to Section 2: The Relevance of Affective Systems and Social Factors: A Commentary

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Abstract This commentary of the six chapters in this section will address three issues. First, a metatheoretical framework for research on mathematics-related affect will be presented. It consists of three dimensions: (1) emotional, motivational and cognitive components of affect, (2) state and trait aspects of affect, and (3) theories of affect as a social, psychological and physiological phenomenon. Secondly, there will be a discussion on the structure of affect. Lastly, there will be a reflection on social influences on individual affect.

Keywords Emotion • Motivation • Beliefs • Affect • State and trait • Social influences • Structure of affect • Mathematics

Relevance

The first part of this section's title, "Relevance in the field", relates in my mind to two things. Firstly, research in mathematics-related affect is relevant when it identifies which of the many affective components of the individual are the most important cornerstones of their view of mathematics. As an educator, I would love to know which set of affective components is such that, when challenged and changed, would cause a chain reaction of permanent changes throughout the person's view of mathematics. Secondly, it would be important to identify which of the affective components can be changed and how. Knowledge of cornerstones of affect is not helpful, unless there are ways to shake them.

My commentary will consist of three sections. The first part focuses on paving the way through establishing a metatheoretical framework and a vocabulary to discuss the different chapters. The second part of this commentary looks at what the six chapters reveal about the systemic nature of affect. The last part this commentary, will focus on what the chapters are able to tell about the social aspects influencing affect.

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A Metatheory of Affect

Ambiguous terminology is a known problem in research of mathematics-related affect (Furinghetti and Pehkonen 2002; Hannula 2011, 2012; Goldin 2004). Most notably, some researchers use attitude as the umbrella concept consisting of cognitive (beliefs), affective (emotions) and conative (behaviour) dimensions (e.g. Hart 1989) while several others define attitudes, beliefs and emotions as three dimensions of affect (e.g. McLeod 1992). This problem is not severe in this section, as all chapters give sufficient theoretical background and define their concepts (norm, motivation, goal, view, attitude, affective pathway, local and global beliefs, and belief system) clearly. Yet, the reader may find it difficult to relate these studies to some others using a different terminology.

In this commentary, we will be using the terminology by Hannula (2011, 2012). The terminology is related to a metatheory that aims at linking and contextualizing theories for mathematics related affect (Fig. 1). This terminology distinguishes not only the cognitive, emotional and motivational aspects of affect, but also separates the relatively stable traits and the more dynamically changing states in all three. The *cognitive traits* include beliefs and other mental representations to which it makes sense to attribute a truth value (c.f. Goldin 2002). The *emotional traits* include emotional dispositions, i.e. tendencies to feel joy, anxiety or other emotions in relation to certain objects or situations (such as mathematics). The third category, *motivational traits* include personal preferences. The distinction from the cognitive aspect is that preferences are subjective and it is not possible to attribute truth value to them. Respectively, the dynamically changing mental states, such as thoughts, feelings and motives, refer to the respective cognitive, emotional and motivational states. In addition, the metatheory identifies three levels of theorizing affect; one focusing on affect as a social phenomenon, the second looking at affect as part of individual psychology and the third that looks at affect as a biological/physiological phenomenon.

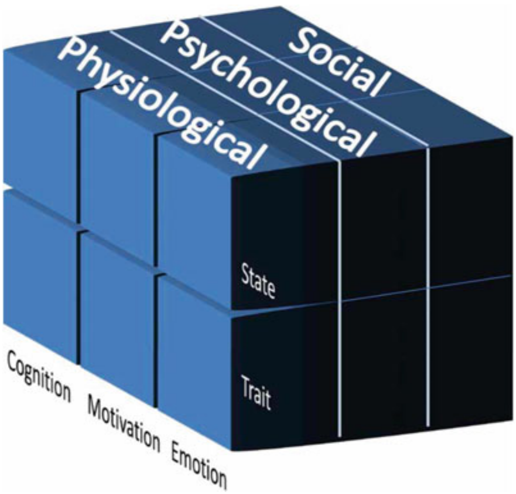

Fig. 1 Illustrating the three dimensions for a metatheory of mathematics-related affect (Hannula 2011, 2012)

When we examine the six chapters of this section using the metatheoretical framework for affect, we see some areas that are hardly, or not at all, touched in them. Firstly, none of the chapters discusses affect as a biological/physiological phenomenon. Secondly, only Gómez-Chacón discusses the dynamics of affective states. Hence, this commentary will focus mainly on two aspects of affective systems. First there will be an analysis of what the six chapters tell about the structure of different cognitive, motivational and emotional traits, i.e. how different aspects of these traits relate to each other. Secondly there will be an analysis of how different aspects of the social level may influence student and teacher affect.

The Structure of Student Affect

The unsurprising conclusion from the affect-related research is that those students who do well in mathematics tend to have a more positive affect towards mathematics than those who do less well. However, it has been more problematic to establish the direction of causality (see Hannula 2011 for a detailed discussion). Taken together, studies suggest a reciprocal rather than unidirectional causality between achievement and affect (Ma and Kishor 1997a, b; Ma 1999; Ma and Xu 2004; Williams and Williams 2010; Minato and Kamada 1996).

The tendency for positive variables to correlate positively seems to hold true also for relations between different affective variables, but this is only a tendency. In their chapter, Ding, Pepin and Jones observed that, in Shanghai, Chinese students' liking of mathematics (an emotional trait) and their perceived competence (a cognitive trait) were positively related. However, cases "I like it but can't do it" and "I can do it but I dislike it" were much more frequent (over 20 % of the responding students) than previously observed in Italy (Zan and Di Martino 2007). Ding et al. provide also an interesting preliminary glimpse of the reasons these Chinese students provide for their attitudes. Some of these reasons seem to relate to an internal motivation (e.g. it's interesting or they like the teacher), while some seem to relate more to an external motivation (e.g. it is a key subject in examination or it is important for future career). While the positive relation between internal motivation and positive emotions is a well known phenomenon, motivation theories and related research do not suggest a general positive relationship between external motivation and positive emotions (e.g. Deci et al. 1991; Pekrun et al. 2006).

One reason to be interested in affect is its assumed relation with metacognition and self-regulation (McLeod 1992; Goldin 2002). The chapter of Gómez-Chacón explores affect in the context of teacher students solving geometry problems using a dynamic geometry software. In addition to analysing the emotional, cognitive and motivational traits in this situation, she also explores how these traits and student meta-emotions (awareness and control of emotions) influence their cognitive-affective pathways (including emotional states) during the instrumental genesis. Her analysis exemplifies how specific can the relations between affective traits and states be: students who preferred visualizing (motivational trait) tended to encounter different types of difficulties than students who had a preference for non-visual approaches. Moreover, her report illustrates the highly individual patterns related to dynamics of affect.

The systemic nature of beliefs was first discussed by Green (1971), who identified three characteristics of belief systems. Firstly, primary beliefs are used as arguments to reason for derivative beliefs (quasi-logical ordering). Secondly, psychologically central beliefs are held more strongly than peripheral beliefs. And thirdly, beliefs are situated, i.e., belief clusters relate to specific situations and contexts. The contextual nature of beliefs has been frequently acknowledged in the elaborations of beliefs specific to content areas. Also in this section there are chapters that pay attention to the contextual nature of beliefs. Depaepe, De Corte and Verschaffel focus on teacher and students beliefs in the specific area of word problems. The way they focus on beliefs in this very specific context, they are implicitly acknowledging the contextual nature of beliefs. Gómez-Chacón addresses both general beliefs (e.g. self-confidence in mathematics) and more specific beliefs (e.g. emotions regarding use of GeoGebra). The contextual nature of beliefs is the explicit focus of the Eichler and Erens chapter, where they explore teacher beliefs in context of calculus, statistics and geometry, analysing how teachers' beliefs are different in these different contexts.

In a similar way that direction of causality is problematic between affect and achievement, it is also problematic between different affective variables. There is often little empirical evidence to assume a direction of causality between any two affective variables, and yet, any advanced theorizing or developed methodology forces us to assume causality. For example, McLeod's theory (1992) assumed a direction of causal effects, where student beliefs have their origin in individual experiences and the social context, these beliefs would influence the onset of emotions, which, when repeated, would lead to attitudes. However, Bandura's self-efficacy theory (1978) assumes a reciprocal relationship between the individual and the social level; emotions are known to influence the interpretation of experiences through directing attention and biasing memory (Power and Dalgleish 1997; Linnenbrink and Pintrich 2004) and the narrow definition of attitude used by McLeod defines attitudes as a tendency to feel certain emotions. Hence, all of the causal directions McLeod suggest, are likely to be reciprocal and we should be cautious with any assumed causalities. While Gómez-Chacón explores the possible causalities empirically using a data mining method and Depaepe et al. are explicit that they are not making any "hard causal statements", Blömeke and Kaiser have assumed a direction of causality from motivation to beliefs.

How the Social Influences the Individual

The main contribution of these six chapters is in the richness of how different social aspects influence student and teacher affect. The importance of the social influences on affect has been long recognized, for example with respect to gender differences (McLeod 1992; Frost et al. 1994) and social norms of the classroom (Cobb et al. 1989).

In their chapters, Depaepe et al., and Forgasz et al. report cross-national differences in the strength of different affective traits, while Ding et al. and Blömeke and Kaiser also report differences in the structure of affect across countries studied. We already mentioned above that in the Ding et al. study the connection between liking mathematics and self-confidence was weaker than in previous study in Italy. The authors discuss the specific features of Chinese culture for learning, where education is the key to mobility and success and schools produce high level learning outcomes although they fail to fulfil features identified in (Western) research as characteristics of good learning environment. In fact, the response pattern where students say that they like mathematics although they have low self-confidence, fits together with previous results where Chinese students in Hong Kong and Macao were found to perceive their competencies (a cognitive trait) to be low and yet their levels of anxiety (emotional trait) were only of medium level (Lee 2009). This pattern is interestingly different from the patterns in Japan and Korea, where perceived competences were even lower than in China and student anxiety was very high (Lee 2009).

In addition to cross-national comparisons, the influence of the teacher is a repeated issue in this section. Most thoroughly the influence of teacher beliefs is discussed in the chapter by Dapaepe et al., who also observe a number of other influencing factors. They reviewed evidence on how certain students' beliefs explain their poor performance in realistic word problems and where these beliefs might originate from. The studies show that students interpret mathematical tasks in the context of schooling, suspending real-life information when it conflicts the "culture and practices of school mathematics". Student beliefs about word problems are influenced by the unrealistic tasks they encounter in textbooks and by the way teachers treat word problems in class. They observe that mathematics classrooms have shifted towards more realistic tasks and that most of the word problems in textbooks are still "stereotyped, easy and non-challenging". Their study exemplifies how their teacher's beliefs and other features of the learning environment can promote student beliefs that are counter to the explicit aims of the curriculum but also how educational initiatives do have an influence on educational practice.

Also Ding et al. mention that some of the observed effects may be influenced by features of the 11 schools in their study. The unusual combinations of high perception of confidence and negative affective relation with mathematics in their study were more frequent in high-achieving schools. Moreover, the response patterns across grade levels varied a lot between the schools, responses being quite uniform in some schools (e.g. in school 18 there were strikingly few students disliking mathematics on all grades) and very varied in some schools (e.g. in school 15 one third of grade 6 students disliked mathematics and on grade 8 nobody disliked mathematics). These suggest that both the school level and the level of classroom/teacher might be influential to student responses.

As teacher's affect seems to be highly relevant for the instructional choices they make, it is important that attention is paid also to teacher affect. Two of the chapters focus on teacher affect, both examining the relationship between teacher motivation and their beliefs about the nature of mathematics (cognitive trait). These two studies exemplify how qualitative and quantitative studies have different strengths.

Eichler and Erens explore teacher motivation and beliefs qualitatively within the framework of intended curriculum. Their study explores the structure of teacher affect with respect to context, the quasi-logical ordering of teachers' beliefs, and also the psychological centrality of different beliefs (c.f. Green 1971). Such a rich and detailed description would not have been possible using a quantitative approach. However, we can not know how generalizable these observations are. On the other hand, Blömeke and Kaiser used a large international data set (TEDS-M) to test the causal relationships from teachers' intrinsic professional motivations to a more dynamic view of mathematics and from their extrinsic professional motivation to more transmission oriented teaching style. Moreover, they chose two Western and two East Asian countries. The choice of four different countries and the statistical power of the large data provide strong evidence for the findings that are similar across all four countries: intrinsic motivation to become a teacher is related to an epistemological belief that mathematics is a dynamically developing science, while extrinsic professional motivation is related to a belief that transmission-oriented teaching style is efficient.

While the social structures of nations, schools, and classrooms are neatly nested, there are also social identities that cut across all these social groups, such as social class, gender and ethnicity. While Depaepe et al. and Ding et al. provide examples for how different features of learning environment promote certain affective traits in students, the study by Forgasz et al. reminds us that these influences are not the same for all students. In the same class, boys and girls are facing different expectations and interpretations by their teachers. Their article discusses also ethnicity in relation to gender and how students are affected by the beliefs of their parents and other members of society. However, none of the chapters in this section has studied the influences of social class, although Ding et al. discuss school socio-economic status as a variable to pay attention in their future analysis.

Although the six chapters in this section address a variety of social influences from textbooks and teachers to gender, ethnicity and nationality, they fail to address the agency of the student and their role in negotiating the social norms of the classroom. Discussion of classroom norms is reduced to norms of teaching. Hannula (2012) discusses the importance of agency, not only in the context of school, but also in all the multiple social roles that students and teachers take as family members, friends, citizens and as members of different social groups.

> Each group and each role requires building interpersonal relations and negotiating about shared norms, values and understandings, i.e., learning in the community of practice (Wenger 1998). For this negotiation, it is not necessary to explicate values and norms. Rather, norms and values become established as participants enact them. In this process of negotiation, both the individual and the social system change (Bandura 1978). Even a passive adaptation to existing rules and norms influences the system, validating the status quo. (Hannula 2012, p. 151)

Mathematics education has already made the social turn (Lerman 2000) and many researchers have taken the strong social position (Lerman 2006) to study the discourse, classroom climate, and other social phenomena emerging in classrooms, schools and more broadly in society. Such studies have observed how school culture and broader socio-cultural situation penetrate to the classroom microculture (e.g. Cobb and Yackel 1996; Partanen 2011), and how the microculture of the classroom may also build resilience against overall educational policy (e.g. Ciani et al. 2010). Although the chapters in this section were rich in their discussion of the social influences of affect, we should also note that the strong social position was not present.

Conclusions

In the introduction, I framed the relevance of research on mathematics-related affect to consist of two components: to identify the most important aspects of affect, and to identify how to influence them. What I observed in the six chapters, was that it seemed to be more important for the students and teachers what they want (motivational trait) than what they believe to be true (cognitive trait) or what they tend to feel (emotional trait). When the importance of sense making is being emphasized, the teachers shift towards this way of teaching word problems (Depaepe et al.). When students see mathematics to be important for personal future, they continue to like mathematics even if they lose their confidence (Ding et al.). Students' preference for visual or non-visual approaches predicts what kind of problems they will encounter when solving problems in a dynamic geometry environment (Gómez-Chacón). Eichler and Erens discuss the psychological centrality of beliefs explicitly, and in their study these beliefs relate mostly to teachers' instructional goals (motivational trait). Although this is mainly based on personal impression, I would conclude that motivational traits are in the centre of mathematics-related affect.

However, when it comes to changing the affect, the picture becomes much more complex. We see a number of social factors that influence student and teacher affective traits. Teacher beliefs and practices influence student approaches to and beliefs about non-realistic word problems (Depaepe et al.). The student gender, age, school and class all seem to influence how student emotional and cognitive traits develop (Ding et al.). The subject to teach influences the instructional goals of the teacher (Eichler and Erens). And also the country where you are influences how student and teacher affect is developing (Ding et al., Blömeke and Kaiser). However, the big variation in student affect across the grade levels of the same school (Ding et al.) suggests that the classroom level is an important factor in this development. Depaepe et al., Eichler and Erens, Blömeke and Kaiser and Forgasz et al. show that there is great variation between teachers and their teaching across a number of factors. There is temptation to conclude that the different national educational policies are reflected in teacher affect, and they would implement in their classrooms teaching that reflects their beliefs and instructional goals. This would highlight the importance of top-down interventions. However, that would ignore the agency of teachers and students. Although none of the chapters in this section addressed the complex processes of negotiating new norms in the classrooms, this certainly is an issue to be taken into account.

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