Meta-emotion and Mathematical Modeling Processes in Computerized Environments

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Abstract Integrating technology into teaching mathematics is a complex issue whose inter-related components must be addressed holistically. The research on the interaction between affect and cognition proposed in this chapter focuses on a number of understudied areas in problem-solving: visualization, affect, meta-emotion and the identification of students' affective pathways. The two studies described revealed the existence of several emotional phenomena associated with technology-assisted learning: (a) an initially positive attitude toward computer-aided mathematics learning and a preference for visual reasoning; (b) instrumental genesis associated with social and contextual dimensions of emotion and cognition; and (c) the effect of meta-emotion on task performance and the development of visual processes.

Keywords Visual thinking • Teacher training • Geometry • Technology • Beliefs • Meta-emotion • Mathematical modeling

Introduction

In recent decades, building on McLeod's groundbreaking studies (1992) on affect in mathematical education, a number of authors have conducted in-depth research on the definition of affect with a view to developing more consistent theoretical frameworks (Zan et al. 2006; Leder et al. 2002; Goldin et al. 2009), while others have analyzed the interaction between cognition and affect (DeBellis and Goldin 2006; Goldin 2000; Gómez-Chacón 2000a, b; Malmivuori 2001, 2006; Hannula 2002; Schlöglmann 2002).

Scientific output on the latter issue has been less abundant, although the findings are promising and encourage further research. For example, studies on learning and affect tend to refer either to the affective reactions that may have a bearing on

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cognitive and conative processes (DeBellis and Goldin 1997; Goldin 2000, 2004; Gómez-Chacón 2000, 2011; McLeod 1994; Liljedahl 2005) or to the so-called directive processes (metacognitive and meta-affective processes) involved in the development of mathematical thinking (creativity and intuition, attribution, visualization, generalization processes and similar) (De Corte et al. 2011; Gómez-Chacón 2008). Others address the ways that emotions impact cognitive processing, such as the bias introduced in attention and memory and the encouragement of a tendency to act (Schlöglmann 2002). Emotions have also been seen to play a key role in human coping and adaptation (Evans 2000; DeBellis and Goldin 2006; Hannula 2002; Gómez-Chacón 2011).

These studies focus, on the one hand, on students' emotions during problem solving and on the other on the importance of cognition-affect interaction pathways in the construction of mathematical knowledge. Nonetheless the conceptual structure underlying this interaction has yet to be addressed in any depth. The present research sought just such depth, focusing on the cognition-affect inter-relationship and an understanding of the role played by emotion and meta-emotion in personal learning in which technology and visualization are involved. Affect is believed to play an important role in the conversion of artifact into mathematical instrument, inasmuch as a positive or negative attitude toward computers (for instance) may influence how cognitive and instrumental schemes develop. Whereas most studies adopting an instrumental approach analyze the technological use of such schemes in their cognitive and institutional dimensions (Artigue 2002; Monaghan 2004), the present research stresses the individual and affect in their generation. The study hinged on the observation of cognitive-affect processes in learning situations involving Dynamic Geometry Systems (DGS) that prioritize visualization.

Two studies, whose subjects were Spanish mathematics undergraduates planning to become secondary school math teachers are described in this chapter. The question posed was: what affective or belief systems inform mathematical visualization processes when using DGS in mathematical learning? The first study characterizes cognitive-emotional interactions in a context of technology-assisted learning, identifying the emotional typologies and phenomena experienced by subjects. The second focuses on meta-emotion and the cognitive-emotional processes that characterize interactive visualization in technology-assisted problem-solving situations.

The specific research questions posed in each study were as follows. In Study I: What are subjects' initial attitudes toward technology-mediated mathematics teaching? What cognitive-emotional processes lead to subjects' positive or negative appraisal of the use of GeoGebra to learn mathematics? And in Study II, focusing on visualization processes: What conceptual structures underlie visualization and affect during the use of DGS to learn mathematics? What information on meta-emotion and visualization can be gleaned from the productive affective pathways reported by students in locus problems? (See items 3.1. and 3.2 for further information).

The present research is primarily exploratory for two reasons: (1) meta-emotion has been scantly analyzed in mathematics classroom contexts; and (2) previous studies on visualization and technology have yielded divergent findings (e.g. Eisenberg 1994; McCulloch 2011; Presmeg and Bergsten 1995; Stylianou 2001).

The section below reviews the theoretical background and literature related to the subject addressed. This is followed by a description of the methodology, the results and discussion, and the conclusions.

Theoretical Considerations

Given the complexity of the subject, a number of theoretical considerations were addressed to establish a consistent interpretative framework: integration of technology, visualization processes and the emotional dimension.

Technology and Instrumental Genesis

Over the last two decades the impact of technologies on learning and teaching processes in mathematics has been widely studied, as attested to by the ICMI Study on the subject. According to some of these studies, attitudes toward mathematics and technology occupy different domains, while other reports contend that even when students hold very positive attitudes toward technology, they find that it tends to interfere with mathematical understanding (Galbraith and Haines 2000; Forgasz 2006; Pierce et al. 2007). New approaches suggest that much remains to be learned about the human side of technology-assisted mathematics learning, and especially about cognitive, metacognitive and affective interaction.

The theory of instrumentation (Rabardel 1995) has been acknowledged by several authors (Drijvers et al. 2010) to be an assistive framework for research. Set into a socio-cultural framework, the instrumental approach (Artigue 2002) combines the anthropological approach in didactics with the theory of instrumentation developed in cognitive ergonomics.

For Rabardel, the individual plays a key role in the process of conceiving, creating, transforming and using instruments. This process evolves in terms of both behavior and knowledge. Instruments are not spontaneously generated, but are rather the outcome of instrumental genesis. Each subject constructs artifact usage schemes for the task at hand at any given time. A scheme is defined as the invariable organization of an activity for a specific series of situations.

Artigue (2002) stressed the need for instrumental genesis that transforms artifacts into tools in two directions relevant to the present study. Upward transition, from artifact to the construction of geometric configuration, known as instrumentation, describes users' manipulation and mastery of drawing tools. The downward process, called instrumentalization, runs from configuration to the proper choice and use of an instrument and refers to geometric construction.

Research has shown that instrumental genesis is a complex, (Guin et al. 2004), primarily individual process in which cognitive, metacognitive and affective channels carry heavy specific weight. Nonetheless, it is also characterized by a social dimension, because students develop their mental schemes in the context of their class community.

This chapter analyzes instrumental genesis in subjects in a DGS environment and the impact of affect on the manner that such genesis integrates artifact and visualization in mathematical learning. The stress is on instrumental phenomena, analyzing the interaction and overlapping between the development of mathematical knowledge (visualization processes) and the understanding of artifact during teaching experiments conducted over long genesis periods.

Visualization

Any analysis of the psychological (cognitive and affective) processes involved in working with (internal and external) representations or images in reasoning and problem solving requires a holistic definition of the term visualization. Hence, Arcavi's (2003: 217) proposal was adopted here: "the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings". In the present study the focus was on interactive visualization, a term used to mean an interactive approach to learning concepts such as DGS in which users receive feedback within a few seconds of entering their input. This term stresses the object's changed appearance and the dynamic view of functional dependencies, which are readily attained when the user creates figures using interactive tools in DGS.

Image typology and the use of visualization were analyzed as per Presmeg (2006) and de Guzmán (2002). Presmeg describes images as both functional distinctions between types of imagery and products (concrete pictorial, kinesthetic, dynamic and patterned imagery; memory images of formulae). In Guzmán they are categorized conceptually: the use of visualization as a reference and its role in mathematization, and the heuristic function of images in problem solving (isomorphic, homomorphic, analogical and diagrammatic visualization). This final category was adopted in the present study in connection with the use of tools in problem solving and research and the precise distinction between the iconic and heuristic functions of images (more closely related to non-iconic visualization) (Duval 2006) in analyzing students' performance (see Table 7). According to Duval, iconic visualization, the recognition of what forms represent, is based on resemblance to the (real) object represented, or by comparison with a type-model. Non-iconic visualization, in turn, is a heuristic series of operations through which geometric properties are recognized when certain configurations cannot be obtained or the configuration obtained cannot be varied.

Emotion and Meta-emotion in Cognition-Affect Interaction

If learning is perceived essentially as interaction among cognition, conation and emotion, emotion must be broached multi-dimensionally (Op't Eynde and Turner 2006). The nature of these interactions varies, rendering their interpretation highly complex.

On the one hand, emotional experience consists of multiple inter-relations among affect, cognition (appraisal) and motivation. And on the other, in learning situations, emotional experience is closely linked to learning objectives and the control of behavior (i.e., volition), and most particularly to cognitive and metacognitive knowledge acquisition processes and heuristic problem-solving strategies.

The approach adopted here was to view affect through the lens of a representational system. The reference framework for studying affective processes has been described by a number of authors (Debellis and Goldin 2006; Goldin 2000; Gómez-Chacón 2000a, 2011), who propose regarding affect as one of several internal, mutually-interacting systems of human representation that encode meaning for the individual and can be externalized to communicate meaning to others. Affect includes changing states of emotional feeling during mathematical problem solving (*local affect*). It also includes more stable, longer-term constructs that establish contexts for and can be influenced by local affect. Known generically as *global affect*, such constructs include attitudes, beliefs and values.

The present hypothesis is that affect is fundamentally representational, rather than a system of frequently involuntary effects on cognition. Affective pathways are sequences of (local) emotional reactions that interact with cognitive configurations in problem solving. Such pathways provide solvers with useful information, favoring the learning process and suggesting heuristic problem-solving strategies. Prior research (Gómez-Chacón 2000b) identified interactions between affect and reasoning (geometrical visualization as an aspect of mathematical reasoning). The potential for affective pathways was shown to be at least partially inherent in the individual, although the effect of social and cultural conditions was also discussed. The present study focuses on the individual and any local or global affect appearing in classroom mathematical problem solving or observed by questionnaires. That same procedure was adapted for use in technological environments, where characterizing affective competencies is meaningful (affective competencies = capabilities that depend on appropriate affective encoding of strategically significant information and instrumental and cognitive skills).

Cognitive and affective self-control constitute another key factor in the cognition-affect interaction. Meta-emotion, along with meta-cognition, is regarded as "an organized and structured set of emotions and cognitions about the emotions, both one's own emotions and the emotions of others" (Gottman et al. 1997). In mathematics education, this term can be found in recent papers by the De Corte team (such as De Corte et al. 2011), and in earlier proposals under the term meta-affect (DeBellis and Goldin 1997, 2006; Goldin 2000; Gómez-Chacón 1997, 2000; Schlöglman 2005).

Meta-affect is a conceit introduced by DeBellis and Goldin (1997). Goldin describes it as: "a central notion … to refer to affect about affect, affect about and within cognition that may again be about affect, monitoring of affect both through cognition and affect" (Goldin 2002: 62).

Several studies have improved this initial intuitive definition by refining factors referring to meta-emotional understanding and meta-emotional skills. This has contributed conceptually to how meta-affect arises in the formation of an individual's cognitive and affective schemes (Gómez-Chacón 2000a, b, 2008; Malmivuori 2001; Schlöglmann 2005). Cognitive understanding of affect enables individuals to control their actions in affective situations. Successful handling of affective situations stabilizes affect schemata, and consequently beliefs, through simulation, as a cognitive window to emotions (Schlöglmann 2005). Prior research has shown that the stability of the individuals' beliefs is closely related to the interaction among belief structures. These include not only affect (feelings, emotions) but also and especially meta-affect (emotions about emotional states, emotions about cognitive states, thinking about emotions and cognitions, regulation of emotions) (Gómez-Chacón 2000b). Those findings reveal the personal and social dimensions of the affective constructs and self-control of emotions.

Research Design and Methodology

The decision to discuss two studies here was informed by questions of approach and methodology.

1. *Comprehension of the subject of the research at different levels:* the first study, which explored cognitive processes and their interaction with affect through surveys (attitude scales and questionnaires), revealed a positive relationship among attitudes, emotions and visualisation processes in computer-aided mathematics learning. The second aimed to explore the Study I findings in greater depth, in particular the conceptual structure underlying visualisation processes and productive routes and the role of meta-emotion in the solution of problem types using DGS.
2. *The combination of qualitative and quantitative methods* to broach the subject of the study: while survey studies and questionnaires may be suitable methods for measuring trait-like variables, design experiments provide the means for addressing the complexity of educational settings. They afford a fuller understanding of learning ecology (a complex, interacting system involving many elements of different types and at different levels) by designing the elements involved and anticipating how they interact to support learning (Cobb et al. 2003: 10). Qualitative research was conducted by observing the subjects as they solved a problem during training. Students were asked to discuss their approach to solving the problem on protocols covering the following items: problem-solving process, step-by-step, description of the difficulties they might face and strategies deployed. They were also asked to record the emotions and difficulties experienced in writing. Their motivational and emotional processes were assessed via performance analysis and video-recorded, semi-structured interviews.

The methodology deployed in each study is described below.

Methodology of Study I

The research questions posed in the study were as follows. What are subjects' initial attitudes toward technology-mediated mathematics teaching? What cognitive-emotional processes lead to subjects' positive or negative appraisal of the use of GeoGebra to learn mathematics?

The subjects surveyed in this study were 98 (65 women and 33 men) Spanish undergraduates working toward a B.Sc. in mathematics with a view to becoming secondary school math teachers.[1] They comprised four classroom groups engaging in training in the analysis and instrumental aspects of teaching situations. This is what is known as a convenience sample, a category of selected samples in which the accessible population is representative of the theoretical population (Gliner et al. 2009). In this case, the population included all the students in the Faculty of Mathematics taking this subject, from which a smaller group was selected and asked by the researcher to participate in the study.

The survey used an adapted version of the instruments (Likert-type attitudes scales) developed by other researchers (Galbraith and Haines 2000) to evaluate attitudes toward mathematics and technology, along with a specific questionnaire to determine preferences for visual reasoning and feelings about computers. These instruments covered both feelings and opinions about the use of technology to learn and use mathematics. The questionnaire posed questions such as the following. Is visual reasoning central to mathematical problem solving? Justify your reply and provide examples. Describe your feelings about the use of problem representations or visual imagery. Describe your emotional reactions and specify whether you hit a mental block when doing the problem with pencil and paper or with a computer. Do computer graphics help you learn mathematics?

Different groups of items required different statistical methods. (1) Likert-type scale attitudes were analyzed with SPSS software, which computed the means, standard deviation and internal consistency (Cronbach's α) for each of these sub-scales of the survey (based on a 5-point Likert scale, from 1 to 5); the correlation between attitude scales; the factor pattern matrix; and clusters. (2) The open-ended questions concerning the most and least preferred method of visual reasoning, computer-related emotions and cognitive learning difficulties in technology-assisted mathematics work were coded by qualitative data processing using content analysis to define the categories listed below. Frequency values were computed by two researchers. (3) Similarly, all categories were compiled and coded in a matrix for implicative analysis performed using CHIC (Bodin et al. 2000) software (see item 3.3).

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[1] The three requirements to teach mathematics in Spanish secondary schools are: (a) a B.Sc. in mathematics or science, (b) an M.Sc. in education for secondary school, and (c) passing a series of public exams.

The subjects of this study are working toward a B.Sc. in mathematics. Spanish Faculties of Mathematics offer specialised training in secondary school mathematics education as part of the undergraduate mathematics curriculum. The subjects presently available include: "Mathematics Education in Secondary Schools", "Mathematics for Teaching" and Practicum (practice teaching in secondary schools).

Most future teachers participating in this program believe that they have sound mathematics training, after having taken advanced courses in several areas of geometry, algebra and analysis. Two-thirds of these students acquire teaching experience prior to the training plan (Practicum) as private tutors or in tutoring schemes for secondary school students.

The M.Sc. in Education (Secondary) is a post-graduate course that builds on prior learning and develops advanced professional knowledge, practice and relationship skills relevant to teaching.

The data was coded into categories:

Emotions regarding computer (GeoGebra) use: Positive (EmoP), Negative (EmoN), depend on the task and activity (Emodep).

Preference for visual reasoning: VisualA (like); VisualN (indifferent); VisualD (dislike)

Attitudes toward mathematics and technology: self-confidence in mathematics (mathconf), mathematical motivation (mathmot), mathematics engagement (matheng), computer motivation (compmot) and interaction between mathematics and computers (mathcompuint).

Cognitive learning difficulties in technology-assisted mathematic work: Diff_Proc (understanding and interpreting the problem in the initial phase of problem solving); Diff_Ins (instrumental genesis (software commands and mathematical meaning or the dependencies between objects in geometry dynamics); Diff_Block (blockage in overall control of the geneses involved in geometric work (blockage in the switch from discursive to instrumental and from discursive to visual)).

In Study I, survey-based assessments of attitudes and emotions were supplemented by observation during teaching experiments (Gómez-Chacón and Kuzniak 2011) designed to explore the interaction between cognition and affect in mathematics learning. The teaching experiment findings supported the survey results, revealed the complexity of interpreting the data (contextual nature of emotions) and contributed to defining better instruments. In these experiments, the causal relationships among emotions, cognitive processes expressed as cognitive difficulties and attitudes toward technology are highly context-dependent. The results also highlighted the importance of visualization in understanding and solving problems and showed that visualization may be associated with different emotions and beliefs. Based on these analyses and results, and with the classification of the data into the aforementioned categories, new conjectures and hypotheses were defined and tested in Study II (section "Methodology of Study II") and better instruments were developed.

Methodology of Study II

The subjects for this study were 32 mathematics undergraduates (future secondary school math teachers) classed in groups whose attitudes and belief systems were representative of the profiles identified in the first study (see Table 5 for the individual characteristics).

The study was mainly based on Design Experiment methodology (Cobb et al. 2003). Qualitative research was conducted by observing subjects during training, performance analysis sessions, video-recorded, semi-structured interviews and two questionnaires.

Geometric locus training was conducted in three 2-h sessions. In the first two sessions, subjects were asked to individually solve six non-routine geometric locus problems using GeoGebra in accordance with a proposed problem-solving procedure. A description of the problems, their graphic solutions and design are given in Gómez-

Chacón and Escribano (2011). That paper discusses the results of the following problem (P4) *the top of a 5-m ladder rests against a vertical wall, and the bottom on the ground. Define the locus generated by midpoint M of the ladder when it slips and falls to the ground. Define the locus for any other point on the ladder.*

The wording provides no explicit instructions for constructing the *geometric locus*. The situation is realistic and easy to understand, but its translation to a GeoGebra construct is not straightforward. Once that initial difficulty is overcome, *visual reasoning* is deployed, and the ladder is drawn with the aid of an auxiliary object, GeoGebra accurately represents the locus. Finding the algebraic answer entails defining five points on the locus and then the conic passing through all five. The result is an algebraic equation. Using *instrumental reasoning*, the two keys to the problem are: (1) construction of the ladder with an auxiliary circle, and (2) the precise definition of the point (mid-point, ¼ point) to study the locus of the positions occupied by the ladder.

The subjects were also asked to describe and record their emotions, feelings and mental blocks when solving problems on protocols designed for that purpose. The third session was devoted to discussing joint approaches and the difficulties experienced in problem solving.

Data were collected from the subjects' problem-solving protocols mentioned above, as well as with two questionnaires, one on beliefs and emotions about visual reasoning completed at the outset and the other on the interaction between cognition and affect in a technological context filled in after each problem was solved.

A first questionnaire focused on identifying subjects' beliefs about visualization and computers to study their global affect and determine whether a given belief can elicit different emotions from different individuals. A second questionnaire was completed at the end of each problem. The main questions are listed in Table 1.

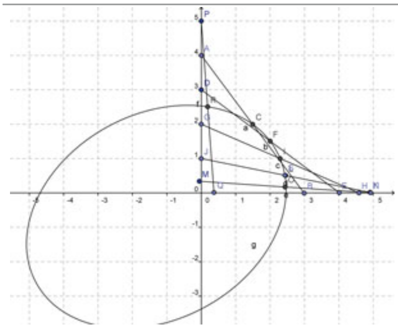
Two types of analyses were conducted in this study. The first was exploratory, descriptive and interpretational, involving mainly inductive data analysis, with categories and interpretation building on the information collected (section "Theoretical Considerations"). This analysis used a qualitative approach based on cross-checking the solutions by three researchers. The first step in data analysis was to classify the methods used by the students and identify classes of answers in accordance with a set of visualization variables and emotions. For instance, subjects' visualization, use of imaging and emotions as reported on their solution protocols were identified for each problem. This information was summarized in schemes such as outlined in Table 7. The second was based on implication analysis, for which the *following categories* were defined.

1. *Emotion associated with visual reasoning* in the ladder exercise: P4EviP (like), P4EviN (dislike), P4EviM (mixed emotions), and P4viInd (indifferent).
2. *Instrumental difficulties*: the focus in this category was on two types of difficulties arising around the six problems (Table 2 and Gómez-Chacón and Escribano 2011). *Typology 1: Static constructions (discrete)* (DT1P4). In this typology, subjects used GeoGebra as an advanced blackboard, but they did not use dynamic properties. They repeated the constructions for a number of points. To draw the

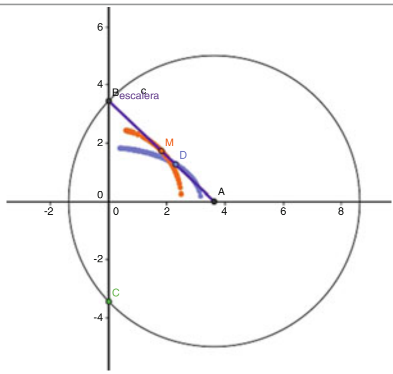
Table 1 Student questionnaire on the interaction between cognition and affect

Please answer the following questions after solving the problem:	
1.	Was this problem easy or difficult? Why?
2.	What did you find most difficult?
3.	Do you usually use drawings when you solve problems? When?
4.	Were you able to visualize the problem without a drawing?
5.	Describe your emotional reactions and specify whether you hit a mental block when doing the problem with a pencil and paper or with a computer.
6.	Which of the following routes best describes your emotional pathway when solving the problem? If you identify with neither, please describe your own pathway.
	Affective pathway 1 (enabling problem solving): curiosity → puzzlement → bewilderment → encouragement → pleasure → elation → satisfaction → global structures of affect (specific representation, general self
	Affective pathway 2 (constraining or hindering problem solving): curiosity → puzzlement → bewilderment → frustration → anxiety → fear/distress → global structures of affect (general self
7.	Specify whether any of the aforementioned emotions was related to problem visualization or representation and the exact part of the problem concerned.

Table 2 Examples of subjects' difficulties with the "ladder" exercise



DT1 difficulty



DT2 difficulty

geometric locus, they used the "5-point conic" tool. *Typology 2: Incorrect definition of the construction* (DT2P4). The subjects solved the problem (albeit inaccurately), but the GeoGebra tools are unusable in such an approach. To use the "locus" tool, the points must be precisely defined (they may not be free points). The best subjects could do when broaching the exercise from this perspective was to obtain a partially valid construction, but, without the GeoGebra tools, the algebraic answer could not be found. In this exercise, the difficulty consisted of precisely defining a point other than the mid-point. Using an undetermined point on the ladder would preclude using the locus tool.

3. Initial problem visualization: VisiP4

4. Beliefs about visual reasoning: BeviP (positive), BeviN (negative)
5. Preferences and emotions around the use of visualization: EviP (like), EviN (dislike), EviInd (indifference)
6. Beliefs about computer-aided learning: BeGeoP (positive), BeGeoN (negative)
7. Emotions around computer use: EGeoP (like), EGeoN (dislike), EGeInd (indifference)
8. Affective-cognitive pathways R1 and R2 (explained in the questionnaire in Table 1) and R3 (subject-formulated, as described in Table 7).

Each researcher conducted a separate analysis, using two dimensions of the genesis of mathematical work as a guide. The use and role of instruments and techniques were used for instrumental genesis, while the basis for visual-figurative genesis was the use and role of figures and images. The researchers' findings were compared and discussed where disagreements were detected. The identification of possible links among affective-cognitive pathways, emotions and meta-emotion was the object of joint analysis.

Implicative Data Analysis

The implicative data analysis method (Gras et al. 1997) used in both Study I and Study II to supplement qualitative analysis is described briefly below. This procedure begins with a group of individuals (the 98 subjects in the first study, for instance) described by a finite set of binary variables (computer attitudes and emotions, preference for visual reasoning, cognitive learning difficulties). The question posed is: to what extent is variable b true when variable a is true? In other words, do subjects known to be characterized by a tend also to exhibit b? In real-life situations deductive theorems of the logical form $a \rightarrow b$ are often difficult to establish due to the existence of exceptions. The dataset must consequently be mined to extract rules reliable enough to conjecture causal relationships around which population is structured. At the descriptive level, they can be used to detect a certain degree of stability in the structuring, while for predictive purposes, they provide the grounds for assumptions.

This statistical analysis was then used to establish rules of association for data series in which variables and individuals were matched to define trends in sets of properties on the grounds of inferential, non-linear measurement. This non-symmetrical statistical approach draws from the notion of implication, borrowed from Boolean algebra and artificial intelligence. Knowledge is formed inductively after a number of successful attempts ensure a certain level of confidence in a given rule. As soon as this (subjective) level is reached, the rule is accepted and implemented.

According to Gras (Gras et al. 1997), learning begins with inter-related facts and rules that progressively form learning structures. That is precisely the aim of the present study, to find rules that reduce the number of categories (listed in items 3.21 and 3.1) while furnishing information on the factors involved in the cognition-affect structure. Gras defines three important rules that can be described in learning processes: (1) $a \rightarrow b$, where a and b may be categories or rules; (2) $a \rightarrow (b \rightarrow c)$; and (3) $(a \rightarrow b) \rightarrow (c \rightarrow d)$. These rules describe a hierarchical, oriented and non-symmetrical learning structure, that can be obtained with cohesive hierarchical implicative classification (CHIC) software (Bodin et al. 2000). The result is three types of diagrams that contain different types of information. (a) Similarity trees group variables on the grounds of their uniformity, allowing for interpretation of the groupings with which the variables are handled. Each level on the resulting graph contains groups arranged in descending order of similarity. (b) Hierarchy trees are used to interpret classes of variables defined in terms of significant levels along the lines of similarity, identifying association rules and levels of cohesion among variables or classes. (c) Implication graphs are constructed around both an intensity index and a validity index to show associations among implications that are significant at specific levels.

Results

Study I: Identifying Cognitive-Affective Interaction Phenomena in a Technological Context

The phenomena identified to characterize cognition-affect interaction in technology-mediated learning were: (a) initial medium-high acceptance of computer-aided mathematical learning; (b) the situational and contextual nature of emotions and cognitive processes.

Initial Attitude Towards Learning Mathematics in a Technological Context

This group showed an initially satisfactory (medium-high) attitude, i.e., appropriate disposition from the outset, characterized by dimensions such as self-confidence in mathematics (mathconf), mathematical motivation (mathmot), mathematics engagement (matheng) and positive beliefs about mathematics and computer-enhanced mathematics learning. The mean values for these dimensions were similar in this group, although statistically significant differences were found among the standard deviations at a 95.0 % confidence level (Table 3).

Many subjects provided details on their initial attitude, noting that they felt joy when they were able to solve a problem with the computer and let go of their apprehension. The reasons given were as follows. (a) Working with a computer enhanced their pleasurable classroom experience and made mathematics more interesting, less abstract, by helping them to find a "meaning". (b) Computers favored learning and success in mathematics, strengthening visualization skills and the accuracy of calculations. (c) Computers helped them establish connections between the

Table 3 Mean, standard deviation and internal consistency (Cronbach's α) for each sub-scale

	Mathconf	Mathmot	Matheng	Compmot	Mathcompuint
Mean	3.32	3.53	3.41	3.41	3.41
Median	3.25	3.63	3.38	3.25	3.45
Standard deviation	0.48	0.69	0.65	0.65	0.50
Coefficient of variation	14.55 %	19.61 %	19.28 %	18.97 %	14.79 %
(Cronbach's α)	0.77	0.80	0.57	0.82	0.69

algebraic and analytical dimensions. (d) And a final category was associated with their goals as future teachers.

Cognitive Processes and Emotions

Subjects' emotions when solving mathematical problems with a computer were analyzed to attempt to answer the question: what cognitive-emotional processes govern subjects' positive or negative appraisal of the use of GeoGebra to learn mathematics? Positive emotions were described by 52.6 %, negative emotions by 19.6 % and 27.8 % replied that they could not give a general answer, for their emotions were task-specific.

The frequency of emotional categories varied, partly depending on task typology. On the whole, the emotions that subjects reported as obstructions to learning were lack of self-confidence and apprehension, which respectively accounted for 15 and 25 % of all the responses. The reasons were mental blockage around the use of the tool and the application of mathematical knowledge with the software: "*At first you feel overwhelmed when you don't know how to apply your mathematical knowledge with the tools that perform these operations*" (A18-GA-E2). Frustration and disappointment were other negative emotions frequently reported in retrospect, attributed to an uncertain command of the technical language or the time that had to be devoted to solving the problem.

The characteristics of the cognitive-affective structure were explored by applying Statistical Implicative Analysis. Figure 1 shows the similarity analysis findings. The variables (listed in item 3.1.) grouped in two classes are underlined in red.

Group 1: this group was characterized by variables specifying a clear preference for visual reasoning and working on computers (EmoP VisualA) and positive attitudinal dimensions (MathconfA ((MathmotA MathengA) (CompmotA MathcompuintA))). One of the most prominent elements was cognitive difficulties and variables relating to the global control of geometric tasks that affect discursive and instrumental processes in technology-assisted work.

Group 2: this group was associated with the variable describing instrumental genesis cognition and problem visualization difficulties in the initial phase of problem solving and negative feelings toward computer-aided mathematics. Not significant.

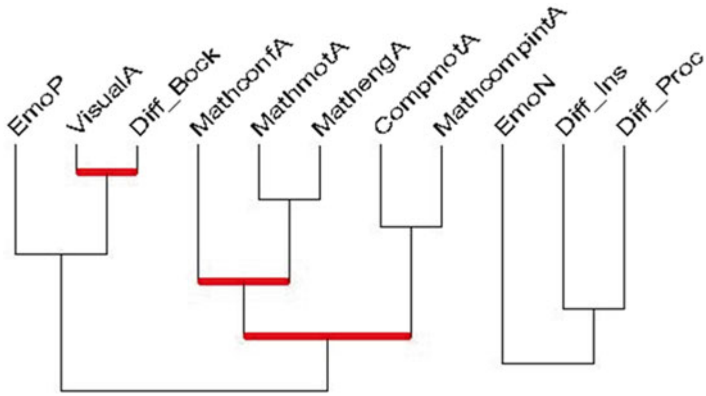


Fig. 1 Similarity tree

The implicative graph revealed causal relationships with a reliability index of at least 84 % between: computer motivation and attitude toward the interaction between mathematics and computers (CompmotA$\rightarrow^{0.85}$ MathCompuint); non-preference for visual reasoning and instrumental difficulties (VisualD$\rightarrow^{0.84}$ Diff_Ins); and a positive preference for visual reasoning and variables relating to the global control of geometric tasks that affect visual, discursive and instrumental processes in technology-assisted work (VisualA$\rightarrow^{0.95}$ Diff_Block).

In a nutshell, motivation to use computers is the variable observed to have the heaviest impact on mathematics-technology interaction. The results also revealed the importance of a preference for visual reasoning in understanding and solving problems.

Study II: The Role of Emotion and Meta-emotion in Interactive Visualization

The objective here was a more thorough review of the conceptual structure underlying visualization, along with productive pathways and meta-emotional intervention in locus problems.

Beliefs and Emotions

In this study a similar response was received when the beliefs explored related to the use of dynamic geometry software as an aid to understanding and visualizing the geometric locus idea. All the subjects claimed to find it useful and 80 % expressed positive emotions based on its reliability, speedy execution and potential to develop their intuition and spatial vision. They added that the tool helped them surmount mental blocks and enhanced their confidence and motivation. As future teachers

Table 4 Frequency of affective-cognitive and emotional pathways associated with visualization in the ladder exercise (N = 32)

	R1	R2	R3	EviP	EviN	EviM	EviInd
Problem 4	15	4	13	6	8	17	1

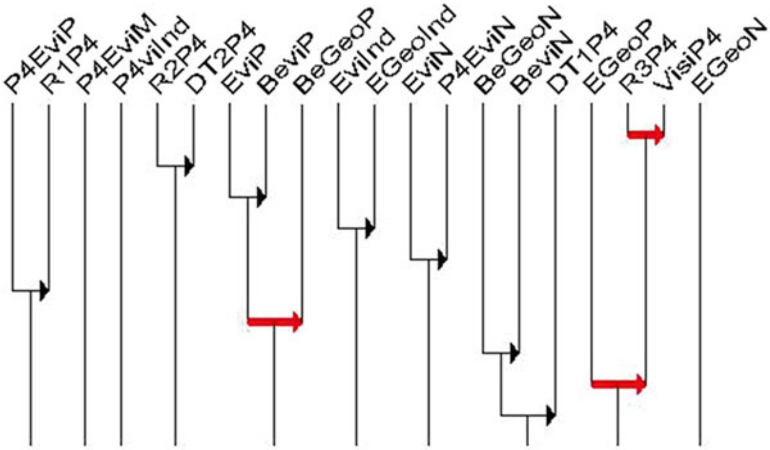


Fig. 2 Hierarchy tree

they stressed that GeoGebra could favor not only visual thinking, but help maintain a productive affective pathway. They indicated that working with the tool induced positive beliefs towards mathematics itself and their own capacity and willingness to engage in mathematics learning (self-concept as a mathematics learner).

Table 4 summarizes the frequencies of pathways and emotions associated with visualization in the ladder problem. Mixed affective pathways were identified, with alternating negative and positive emotions and optimized self-control of emotions.

The question posed to study the mix of emotions and meta-emotion in greater detail was: what are the differences in a subject's choice of these three pathways? A preliminary analysis showed that pathway R3 was largely self-formulated and contained a much greater mix of emotions. In most cases, moreover, the trend was not as explicit as in R1 (positive) or R2 (negative). Rather, negative feelings (which were controlled) were attributed to certain stages of the visualization process, and positive feelings to success in representing the desired images. A hierarchy study of R3 yielded some significant affective-cognitive implications respecting visual processes, such as: R3P4$\rightarrow^{0.99}$ VisiP4 and R2P4$\rightarrow^{0.90}$ DT2P4 (Fig. 2).

Three of the nine nodes obtained in the hierarchy tree were significant and identified the following groups.

Group 1 (N (level 1, cohesion: 0.998) = (R3P4 VisiP4)), comprising over 40 % of the initially visualizing subjects (in problem 4) who indicated pathway R3 as the expression of their cognition-affect interaction. The most significant characteristic of these individuals was their positive feelings towards computers (use of GeoGebra (EGeoP) software).

Group 2 (N (level 7, cohesion: 0.276) = ((EviP BeviP) BeGeoP))), where the most prominent finding was that a belief in the use of GeoGebra was attendant upon a belief in and a preference for visual reasoning.

Meta-emotion and Visualization: Maintaining Productive Affective Pathways

The study subsequently focused on the Group 1 subjects to ascertain the characteristics of their R3 pathways and explore the implied relationships. The affective pathways they reported were compared to glean information on meta-emotion and visualization. The comparison revealed: (a) the use of visualization and associated emotion; and (b) their emotional self-control depended on their individual perception, which was influenced by style, disposition, type of activity or skill, instrumental command and belief systems around technology-aided mathematical learning.

By way of illustration of the foregoing, three case studies with varying characteristics are discussed below. The key characteristics of the case studies are given in Table 5.

The ladder problem was chosen for this analysis. According to the subjects' affective pathways for this problem given in Table 6, two identified with R3 and one with R2.

The first case was *S-19*, a visualizer. In the interview he said that the pleasure he derived from visualization was closely associated with his view of mathematics. He regarded visual reasoning as essential to problem solving to monitor, contribute to the intuitive dimension of knowledge and form mental images.

When he was asked whether his feelings were related to visualization and to specify the parts of the problem where they were, he replied: "*curiosity predominated in visualization. Since the problem was interesting. I was keen on finding the solution. I had a major mental block when it came to representing the problem and later, when I sought a strategy, I was unable to define a good strategy to find the answer. I was puzzled long enough to leave the problem unsolved and try again later. When I visualized the problem in a different way, I found a strategy: construct a circle with radius 5 to represent the ladder and another smaller circle to represent the point in question. When I reached that stage, I felt confident, happy and satisfied*" (S-19*).*

An in-depth analysis of the problem-solving protocol for this exercise and the affective-cognitive pathway reported showed that this subject was able to describe and control emotions and identify causes. First he tried to find the mathematical object, but reached a block (Table 7 (1)). He even attempted construction with physical objects. That led him to iconic visualization and from there to visual and semiotic exploitation within a dynamic geometry environment. For instance, in Table 7 (1), the subject's search for a "mental image", involved "implemented discovery" including empirical pursuit, as if his figures were objects of experimentation, anticipating

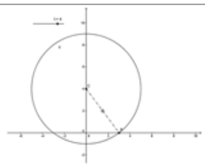
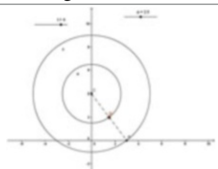
Table 5 Three case studies: characteristics

Case	Gender	Mathematical achievement	Visual style	Beliefs about computer learning	Feelings about computers	Beliefs about visual thinking	Feelings about visualization	Pathway	Global affect
Student 19	Male	High	Visualizing subject	Positive	Likes	Positive	Likes	R3	Positive self-concept
Student 20	Female	Average	Non-visualizing subject	Positive	Dislikes	Positive	Dislikes	R3	Positive self-concept
Student 6	Female	Low	Style not clear	Positive	Dislikes	Positive	Likes	R2	Negative self-concept

Table 6 Affective pathways and visual cognitive processes reported for this problem by three subjects

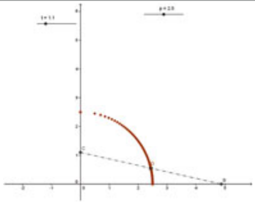
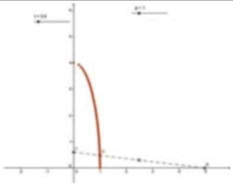
Ladder exercise	Cognitive-emotional process	
Student 19	Curiosity	Reading and understanding problem
Pathway R3	Confusion	Drawing (patterns and lines/figure)
		Analytical
	Puzzlement	(Search for mental image) (specific illustration and dynamic image)
	Mental block	
	Confidence	Search for mental image
	Perseverance-motivation	Search for mental image
	Exhilaration	Physical manipulation – kinetics
		Kinesthetic learning
		Mental image Identification mathematical object
	Confidence	Technological manipulation with the computer
		Representing circle radius (specific illustrations)
	Confidence, joy	Interactive image generation, slider (analogical)
	Joy	Interactive image generation, slider (analogical)
	Perceived beauty	Specific illustration with interactivity (analogical)
	Satisfaction	Analytical-visual
		Memorized formulaic typology
	Global affect	Positive self-concept
Student 20	Curiosity	Reading problem
Pathway R3	Frustration	Global visualization of problem
		Pictorial image
	Confusion	Search for mental image
		Inability to visualize the ladder as the radius of a circle
	Puzzlement	Search for mental image
		Dynamic and interactive image with GeoGebra
	Stimulus, motivation	Technological manipulation with the computer
		Pictorial representation with GeoGebra
	Satisfaction	Pictorial representation with the GeoGebra "trace on" function
		Full construction from scratch
		Obtaining a final solution
	Global affect	Positive self-concept
Student 6	Curiosity	Reading problem
Pathway R2	Puzzlement	Global visualization of problem
		Pictorial image
	Bewilderment	Search for an instrumental image with GeoGebra
	Frustration	Computer skills
	Apprehension	Inability to visualize the ladder as the radius of a circle or to use the "trace on" function
	Fear/despair	Need for help to find the solution
	Global affect	Negative self-concept

Table 7 Analysis of S-19's solution to the ladder exercise and use of images as reported by the subject in his protocol

Description of method, including visualization	Use of image	Emotions/affect
(1) First I sketched the problem on paper. I tried to find a way to solve it analytically but couldn't. I imagined the possible relationships between the triangles that the ladder would gradually generate as it slides downward against the wall to the ground, but that got me nowhere.	Drawing (patterns and lines/figure)	Curiosity
	Analytical (search for mental image (specific figure/illustration and dynamic image)	
(2) Then I tried to envisage the answer: would it be a straight line, an ellipse or a circle?		Confusion
(3) I decided to come back to the problem another day, but I kept thinking about it meanwhile. I trusted my subconscious to help me.	Search for mental image	Puzzlement
		Mental block
		Confidence
		Perseverance-motivation
(4) I came back to the problem with new energy. I experimented with a biro attached in the middle to a rubber band. It seemed to form the arc of a circle. At least I had something to go on.	Physical manipulation - kinetics (Kinesthetic manipulation)	Exhilaration
	Mental image – identification	
	Mathematical object	
(5) I started to work with GeoGebra. After trying a few straight line constructs, I noticed that the ladder could be viewed as a radius of a circle of length 5 running along the y-axis.	Technological manipulation with the computer	Confidence
	Representation of the radius of the circle (Specific illustrations)	
(6) I generated a slider that I called t and defined the center of the circle C = (0,t). The slider would shrink from 5 to 0 when the ladder is lying on the ground. Point B represented the intersection of the circle with the x-axis.		Confidence, joy
	Interactive image generation, Slider (analogical)	
(7) After drawing the ladder, I constructed another circle with a variable radius (using a second slider, p). This circle would be the path of the point constituting the object of the study.		Joy
In the first case, it is the midpoint of the line representing the ladder.	Interactive image generation Slider (analogical)	

(continued)

Table 7 (continued)

Description of method, including visualization	Use of image	Emotions/affect
(8) I observed the path of the midpoint of the ladder when I activated the drawing.	 Specific illustration with interactivity (analogical)	Perceived beauty
(9) I found that I had an arc of a circle with a center C = (0,0) and radius r = 2.5. Then, I tried the same procedure with a point located 4 m from the bottom of the ladder in the initial position (on the y-axis). The result was an arc of an ellipse whose major vertical semi-axis was 4 and minor semi-axis 1.	 Specific illustration with interactivity (analogical)	Perceived beauty
(10) I calculated the geometric locus generated by point D to be an ellipse whose major semi-axis was max(h, 5-h), positioned vertically if h > (5-h) and horizontally if h < (5-h)); and whose minor semi-axis was min (h, 5-h) positioned horizontally if h > (5-h) and vertically if h < (5-h)), where h is the distance of the point from the base of the ladder when set vertically.	Analytical-visual Formulation from knowledge of mathematics	Satisfaction
Where h = 2.5, the two semi-axes are equal, confirming that it is a circle.		

the right loci. Here, inductive reasoning could be seen to be supplemented with analytical reasoning in certain steps. In others, analytical reasoning was explicit, such as in Table 7 (10).

Three types of affective perspective were identified. First, S-19, always tried to find an answer even when in doubt or blocked. S-19 was continuously active, which is one way that many students cope with stress. Secondly, he was able to walk away from the problem, aware of the role of the subconscious in mathematics (Table 7 (3)). Thirdly, he struck a balance between the combination of graphic geometric thought and analytical task solving. These three behaviors were indicative of interaction between the cognitive-affective system and self-control. The description of emotions revealed that self-confidence, stimulus and joy were associated with the reproduction of physical forms and the visual/perceptive control implicit in a command of ancillary mathematical objects, from both a mathematical and a technical-instrumental perspective.

S-20 was a non-visualizing thinker with positive beliefs about the importance of visual reasoning. However, she claimed that her preference for visualization depended on the problem and that she normally found visualization difficult. It was easier for her to visualize "real life" (such as in the ladder exercise) than more theoretical problems.

Her motivation and emotional reactions to the use of computers were not positive, although she claimed to have discovered the advantages of GeoGebra and found its environment friendly. She also found that working with GeoGebra afforded greater assurance than manual problem solving because the solution is dynamically visible. Convincing trainees such as S-20 that mathematical learning is important to teaching their future high school students helps them keep a positive self-concept, even if they don't always feel confident in problem-solving situations (Table 6).

S-6's visual thinking style could not be clearly identified. She expressed a belief in the importance of positive visual reasoning ("because visual reasoning helps gain a better understanding of the problem and consequently the solution"). This confirmed a liking for visualization and representation because it made it easier to understand the problem and she found formalization helpful. She added, however, that she felt insecure applying technological software to mathematics, although she believed GeoGebra, specifically, to be useful. In her own words, "*I don't like it and never will. I feel a little nervous and unsure of myself, not because of GeoGebra but because computers intimidate me because I don't understand them completely. But when I managed to represent the problem with GeoGebra, I felt more satisfied with the result than when I solved it with paper and pencil*". Although S-6's pathway was essentially negative, she persisted until she found the solution. In some cases subjects were unaware of their mistakes and misunderstandings, however.

Comparing these three cases in terms of meta-emotion (self-awareness of the nature, cause and control of emotions) showed that the subjects exhibited meta-cognitive and meta-affective understanding in connection with the knowledge acquired and their own beliefs and cognitive processes. And further to Demetriou and Kazi (2001), this self-image makes a fairly effective contribution to learning, for it forms an integral part of mental control.

The analysis of the relationship between these three subjects' affective pathways (Table 6) and their cognitive visualization showed that negative feelings and interactions around visualization stemmed essentially from subjects' lack of familiarity with the tools. They were intimidated by their want of resources in their search for computer-transferable analogue images and their conversion from a paper and pencil to a computer environment in their interpretation of the mathematical object. The results nonetheless showed that although the construction of interactive images may be an obstacle, interactivity may be beneficial. Here, interactive computer programs provided feedback and clues to raise subjects' awareness of their cognitive and metacognitive processes. A review of S-19's pathways in the six problems revealed that the interaction between visual reasoning and negative feelings arose around the identification of interactive representation strategies and the formulation of certain images involving the identification of parametric variations. This subject's command of the use of concrete, kinesthetic and analogical images was very helpful and contributed to his global affect and his positive overall self-concept when engaging in computer-aided mathematics.

The data also revealed the relationship between beliefs, goals and emotional pathways. The analysis of S-20's responses showed that while she had no inclination to use computers, the importance she attached to mathematics and IT in specific objectives and the structuring of her overall objective kept her on a productive affective pathway (McCulloch 2011).

S-20's solution to the ladder exercise (Table 6), for instance, constitutes a good example of a productive pathway: despite negative feelings, she maintained a positive mathematical self-concept, which she reported when she explained her global affect. Questions designed to elicit the reasons for her positive mathematical self-concept in terms of technology showed that objectives, purposes and beliefs were clearly interrelated. Her own words were: "*I think that computers, not only the GeoGebra program, are an excellent tool for anyone studying mathematics. Nowadays, the two are closely linked: everyone who studies mathematics needs a computer at some point… mathematics is linked to computers and specifically to software like GeoGebra (if you want to teach high school mathematics, for instance. I at least am trying to learn more to be a math teacher)*" (S-20).

Conclusions

Like the results of related research, the present findings revealed that in the usage schemes for technology-assisted mathematics learning, emotions interact with cognition, conation and instrumental dimensions that determine problem solving procedures and practice. The two studies described identified several emotional phenomena associated with technology-assisted learning: (a) an initially positive attitude toward computer-aided mathematics learning and a preference for visual reasoning; (b) instrumental genesis associated with social and contextual dimensions of emotion and cognition; and (c) the effect of meta-emotion on task performance and the development of visual processes.

Contrary to the evidence reported in earlier studies (Galbraith and Haines 2000), mathematics-computer interaction is determined not by these two dimensions separately: the relationship is more complex. While individuals' positive belief systems about visualization constitute a core value in mathematics-computer interaction, this does not confirm prior studies on preferences for visualization (e.g., Eisenberg 1994). Rather, different emotions are associated with such beliefs. This emotional plurality and the individual and social elements in visualization-related instrumental genesis were identified by analysis of artifact use from an instrumental and cognitive approach, focusing on instrumentalization and instrumentation processes.

For the group studied, instrumental mediation (DGS) favored visualization and revealed the existence of social usage processes in their beliefs. Emotions arose in connection with the appraisal of success. The value accorded the situation (importance of the goal, whether it be "to build an interactive image" or the longer term "to become a teacher") and its appraisal determined the emotional response (Pekrun 2006; Weiner 1985; Hannula 2002) and meta-emotion.

Evidence of positive instrumental mediation was observed in situation "controllabilty" (Weiner 1985) expressed as interaction with software that affords an immediate answer and lowers apprehension thanks, for instance, to the algebraic and geometric windows that contributed to analytical and geometric comprehension. Interaction with the medium was found to consist of both intrinsic and extrinsic components that favored emotional self-control.

Students exhibited a variety of cognitive-affective pathways in which emotional self-control was not always optimized. Pathway R3 was characteristic of subjects who optimized their emotions and learned how emotional activation could turn perplexity and confusion into success (or de-fuse apprehension, frustration or anger).

Other factors present in visual reasoning schemes included cognitive styles and beliefs about the technological context. The use of different theoretical frameworks favored the understanding of a preference for visualization. According to the data, subjects experienced positive feelings in the initial stage of problem solving, when seeking an image that would describe the structure sought; or of satisfaction and happiness when they were able to construct an interactive image. However, the attempted generation of interactive images, the use of analogical visualization and progressive schematization based on the analytical-algebraic analysis and geometric figures prompted confusion, mental blocks and frustration.

Finally, another source of information on performance in technological contexts was social persuasion (portrayal of computer use in education) and the individual's psychological and emotional appraisal. Summing up, in this group the two crucial elements that contributed to sustaining effective interaction between technology and mathematics learning were the subjects' self-perception or self-image and the meta-affective experiences involved in task performance, to which the instrument contributed.

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