Towards a Participatory Approach to 'Beliefs' in Mathematics Education

Jeppe Skott

Abstract Over the last three decades research in beliefs, and affect more generally, has developed into a significant field of study. It attempts to make sense of teachers' and students' understandings of mathematics, of its teaching and learning, and of themselves as doers, teachers, and learners of mathematics and of how these understandings relate to classroom practice. Studies of these issues have been published widely and in the most prestigious journals and book series. However, belief research is still confronted with significant conceptual and methodological problems. I suggest that this is at least in part due to the dominant conceptualization of individual functioning in belief research, one that is based on acquisitionism with its emphasis on human action as an enactment of previously reified mental entities. In the present chapter I build on social practice theory and symbolic interactionism to rephrase key issues of belief research, especially that of the relationship between beliefs and practice, in more participatory terms. The suggestion is to shift the focus from beliefs to the pre-reified processes that are said to give rise to them. This leads to more dynamic understandings of learning and lives in mathematics classrooms and serves to overcome some of the conceptual and methodological problems of the field.

Keywords Belief research • Mathematics teachers • Dynamic views of beliefs • Acquisition • Patterns of Participation (PoP)

Over the last three decades large numbers of studies have investigated the character of students' and teachers' beliefs about mathematics, about mathematics as taught in school, about the teaching and learning of the mathematics, and about themselves as learners, teachers, and doers of mathematics. Some studies focus on the development or relative stability of beliefs, for instance as they relate to the education of prospective or practising teachers, while others address the question of a possible correlation between students' and teachers' beliefs and the teaching-learning

J. Skott (✉)
Department of Mathematics Education, The Linnaeus University, Växjö, Sweden

Department of Education, Aarhus University, Copenhagen, Denmark
e-mail: jeppe.skott@edu.au.dk

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practices that unfold in mathematics classrooms. As it relates to the students, the field is in the latter case concerned with how beliefs structure the students' approach to mathematics, sometimes in more domain-specific fields such as statistics or proof, and even "determine the way they engage in mathematical learning and problem solving" (De Corte et al. 2002, p. 298). In relation to teachers, the research interest in beliefs is part of an effort to supplement the focus on their knowledge of the contents and of the related educational issues with a more meta-cognitive and affective perspective. It is often assumed that the structuring effect of their beliefs on behaviour is as significant as suggested by de Corte et al. for the students (Schoenfeld 1992). The premise of the field of beliefs, then, was – and to some extent still is – that beliefs, understood as relatively stable, reified mental constructs, significantly influence students' and teachers' behaviour, also if they run counter to curricular intentions developed for instance in the research community. The promise of the field was – and still is – to solve, or at least alleviate the 'problems of implementation', i.e. the lack of congruity between such intentions and instructional practice, by changing the beliefs of prospective and practising teachers.

This research effort has contributed with more profound understandings of what Goldin (2002) calls affective/cognitive configurations and of the role they play for students and teachers engaged in classroom interaction. However, belief research is not an unproblematic endeavour. The key concept of the field, the one of beliefs, is ill-defined, and the methodological problem of getting access these elusive constructs is unresolved. Further, it has turned out to be difficult to fulfil the promise of making significant contributions to the current reform agenda, in spite of comprehensive development and research efforts to do so. There are two sides to this, as it is no easy task to facilitate belief change, and even when beliefs attributed to the teacher on the basis of questionnaires and interviews are in line with reform intentions, classroom practices do not always comply. The latter of these problems is the background to the development of interpretations of the beliefs-practice quandary that are less causal and more dynamic than the ones that dominate traditional belief research (Op't Eynde et al. 2006; Schoenfeld 2011a, b). They suggest that the impact of mathematics related beliefs may be moulded by other mental constructs the role and significance of which are modified by contextual constraints. These interpretations, then, emphasise the dynamic relationships among mental reifications in the form of beliefs and knowledge, and between such reifications and classroom processes.

In line with more traditional approaches to beliefs most of these dynamic interpretations rely on acquisitionist, especially constructivist, interpretations of human functioning. Students and teachers are expected to come to hold or possess reified mental entities, beliefs, through processes of assimilation and accommodation as they engage in social interaction. Subsequently they are to enact these reifications, though possibly in modified form due to contextual constraints. Acquisitionism, however, has been challenged in recent years, for instance by the more participatory approach adopted in most studies of identity (Hodgen and Askew 2007; Horn et al. 2008; Ma and Singer-Gabella 2011). In line with this, I suggest interpreting students' and teachers' affectively laden action and meaning-making as

shifting modes of participation in different social practices, rather than as contextually constrained release of mental reifications. The argument is that the general lack of confirmation of the congruity thesis in belief research, i.e. the thesis of close correspondence between beliefs and practice, may be addressed not by suggesting that the role and significance of beliefs is contextually constrained, but by shifting the emphasis from mental reifications to the social processes on which they are assumed to be based. The use of 'beliefs' (with inverted commas) in the title of this chapter is to indicate that I address affective issues normally dealt with in the field of beliefs, but that I suggest minimizing the emphasis on mental reifications and conceptualising affective issues in more processual terms.

To make my argument I focus primarily, but not exclusively, on teachers' beliefs. I begin by discussing the concept of beliefs and argue that there seems to be some agreement about a core of the concept in spite of the lack of an agreed-upon definition. Next, I outline and categorise some of the dynamic approaches to beliefs and link them to aspects of this core (section "Dynamic views of teachers' beliefs"). Following from that, I discuss the acquisitionist underpinnings that orient the larger part of the beliefs literature, including at least some the more dynamic approaches, and argue that the concept of beliefs is used about objectifications, i.e. about reified mental entities assumed to have explanatory power for practice (section "Belief research and acquisitionism – or why believe in beliefs?"). I build on Sfard (2008) to outline some of the drawbacks of such an approach and argue that an over-reliance on objectifications is somewhat ironic in relation to beliefs. This is the backdrop for the suggestion to adopt a more participatory stance in the form of a conceptual framework in the making that I call *Patterns of Participation* (sections "PoP – towards a participatory account" and "Using PoP for empirical purposes"). My colleagues and I have argued elsewhere that PoP addresses some of the conceptual and methodological problems of belief research (Palmér 2013; Skott 2013; Skott et al. 2011); in the present context I highlight how it differs from dynamic approaches developed within the field.

The Concept and Expected Functions of Beliefs

One of the challenges of belief research is that the key concept of the field, the one of beliefs, is not easily defined. Some scholars, both in mathematics education and beyond, engage in lengthy discussions of the concept, while others define it only implicitly and in use. The latter approach may be based on one or more rationales. It may be implied that there is a core or an essence to the notion of beliefs that is generally accepted, even though it is difficult to phrase a definition that captures all aspects of the concept and delineates its borders sufficiently clearly vis-à-vis related ones. It may also be based on the recognition that empirical work on a concept that is initially ill-defined may invite dialogue, which in turn serves to specify the concept in question in greater detail. And it may simply acknowledge that explicit definitions do not carry unequivocal meanings and may be interpreted in a multitude of

ways, even if agreement is reached. Each of these rationales suggests that a further search for an agreed-upon definition is not worth the effort, at least at present, but that continued research in the field is, even though we may not be able to specify the contents of the concept of beliefs as clearly as we would like.

I have argued elsewhere that whether beliefs are defined explicitly or not, there seem to be four key aspects to the concept (Skott 2014). First, beliefs are used about mental constructs that are subjectively true for the person in question. This implies that beliefs are characterised by individual conviction, but also that the individual holding them may accept alternatives as reasonable and justifiable. Second, there is an element of affect to beliefs. Beliefs, then, are value-laden and characterised by a certain degree of commitment. Third, beliefs are considered relatively stable. The individual is expected to carry his or her beliefs in and out of different settings without changing them significantly, and belief change is expected to occur only as a result of substantial, new personal experiences. Fourth, and as argued above, beliefs are expected to significantly influence individuals' perceptions and interpretations of experiential encounters as well as their contributions to the practices in which they engage. In fact, it is unlikely that research on teachers' beliefs would have attracted more than minimal attention, if they were not believed (!) to impact practice.

To sum up, the notion of beliefs is used in the literature about mental reifications that are acquired on the basis of comprehensive, previous social experiences and that are characterised by considerable degrees of conviction, commitment, stability, and impact. The core of the beliefs concept may, then, be defined as subjectively true, value-laden mental constructs that are the relatively stable results of substantial prior experiences and that have significant impact on practice. I do not mean to imply that this definition is helpful when describing an everyday use of the notion (e.g. *I believe it is going to rain tomorrow*). However, the four key aspects may be said to constitute the core of the concept as understood in mainstream belief research. In fact, I use the phrase of *mainstream belief research* to designate approaches that focus on beliefs about mathematics and its teaching and learning and explicitly or implicitly endorse all four.

The last of the four characteristics of the core of beliefs suggests that once established these reified mental constructs serve two functions. The flow downwards in Fig. 1 emphasises the reification process itself and the subsequent, (semi-)causal relationship between beliefs on the one hand and instructional behaviour and classroom practice on the other. However, beliefs also serve a function for the reverse movement, i.e. for guiding perception and interpretation and turning immediate social encounters into more coherent life experiences. In this sense, they are an assimilatory filter that shapes the flow backwards in Fig. 1.

In the literature, however, the thesis that there is congruity between beliefs and behaviour has been challenged as much as confirmed (Fives and Buehl 2012). This obviously calls for an explanation in view of the premise of the field. One response is to capitalise on the conceptual and methodological problems of belief research and argue that the methods used in the field do not provide access to what people really believe or at least not to beliefs that matter for the situation at hand.

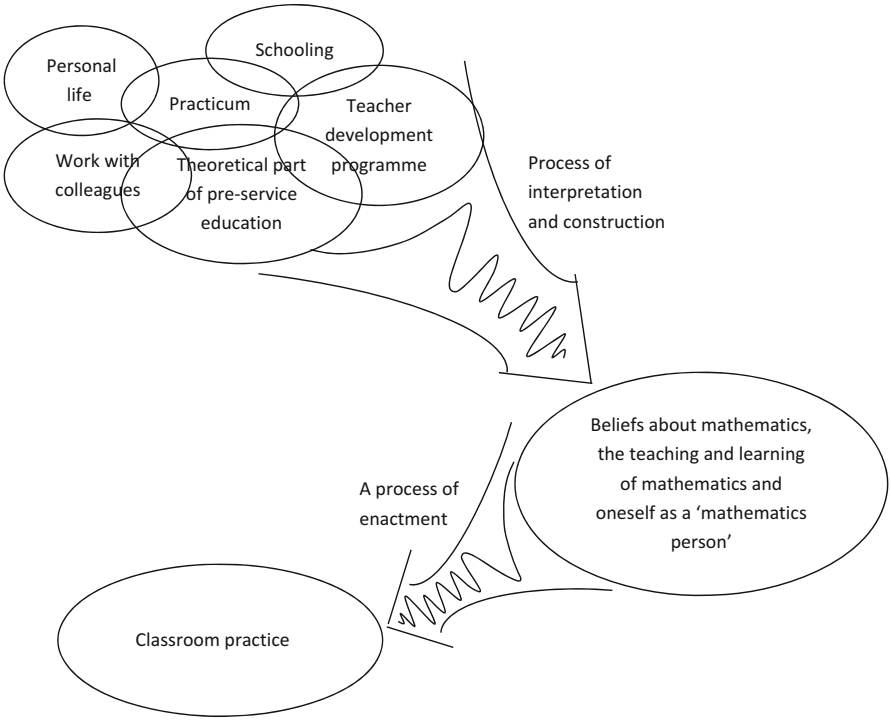

Fig. 1 Beliefs in mainstream belief research

Consequently it is suggested that other types of analyses are needed (Speer 2008), or that beliefs are held in "clusters" (Green 1971) or "bundles" (Aguirre and Speer 2000) that have different relations to instruction and are structured so that the beliefs one does get access to for instance in interviews are different from the ones that manifest themselves in the classroom. These explanations are fully compatible with the claim that beliefs explain behaviour. Another set of responses, the ones discussed in the present context, modify this claim and suggest that 'context', in one or other interpretation of the term, may be a constraint on the opportunities for 'belief enactment', and that a more dynamic approach is needed to understanding the functioning of the individual in that 'context'.

In relation to the students, such dynamics is apparent for instance in the work of Malmivuori (2006). She discusses *self-systems*, i.e. relatively stable mental structures encompassing knowledge of mathematics, beliefs about the subject and about self in mathematics, affective schemata, and habitual behavioural patterns in mathematical situations. In Malmivuori's analysis, self-systems are "the basis for the functioning of students' […] metacognitive, cognitive, and affective capacity used in mathematical thinking", but their role is conditioned by situation-specific factors (p. 151). Also working with students, Goldin et al. (2011) introduce *engagement structures* that have beliefs and values as one of ten inherent components or strands,

and which they describe as "*behavioural/affective/social constellation[s]* situated in the person" (p. 548; emphasis in original). Examples of engagement structures include "Get the work done", i.e. completing assignments by following instructions; "Look how smart I am", i.e. impressing others with one's mathematical performance; and "Stay out of trouble", i.e. avoiding interactions that may cause conflict or distress. It is a main aim for Goldin et al. to describe the "*particulars* of *how* beliefs, values, emotional feeling, and social situations interact in a structured way to influence in-the-moment engagement with mathematics" (p. 552; emphasis in original). Engagement structures are embedded in people, but activated in particular situations and as such descriptive of a person's state. Beliefs, in contrast, are taken as traits that in the particular situation may motivate involvement in certain engagement structures, but inhibit the activation of others.

There is dual dynamic involved in the view of beliefs in these studies. First, they both acknowledge that immediate social interaction and the related contingencies play a role for the extent to which beliefs inform students' participation in the classroom. This dynamic relates to the person-context interface. Second, the studies consider an internal dynamic in the form of shifting relationships between the elements of self-systems (Malmivuori 2006) or engagement structures (Goldin et al. 2011). While these studies modify the assumption of a direct causality between beliefs and behaviour, they still expect relatively high levels of stability and impact of students' and teachers' mathematics related beliefs, and in this sense they are in line with mainstream belief research. Similar conceptualisations may be found in studies of teachers' beliefs.

Dynamic Views of Teachers' Beliefs

One may differentiate at least four possible contextual and dynamic categories of perspectives on the role of teachers' beliefs for practice, which I label *enactment*, *activation*, *situatedness*, and *emergence*. They relate differently to the last two of the four characteristics of the core of the beliefs concept, the ones of relative stability and of expected impact. I build on different studies to elaborate on the distinctions between the categories. However, my intention is not primarily to 'locate' these studies in particular categories, but to use them as starting points for specifying the character of the categories themselves.

One dynamic interpretation, labelled *enactment*, is compatible with mainstream belief research in the sense defined above. Schoenfeld's recent work may be taken as a starting point for a description of this category (Schoenfeld 2011a, b). He subsumes beliefs under a broader concept of 'orientations' and takes what he calls resources (most notably knowledge) and goals into account.

Schoenfeld emphasises the role of planned behaviour in instruction, as "the vast majority of a teacher's actions in the classroom are shaped by the teacher's agenda", for instance as it materialises in the lesson plan (Schoenfeld 2011a, p. 9). Further he suggests that teachers base their behaviour on combinations of their goals, resources,

and orientations both when teaching goes according to plan and when planned action is disrupted by contingencies, such as an unexpected response or suggestion from a student. There is in Schoenfeld's interpretation a dynamic relationship between the orientations, resources, and goals brought to the situation by the teacher and the goals that are pursued at the instant. Schoenfeld, then, assigns a significant role to orientations, most notably beliefs, as the individual teacher's choice of action among a range of possible options "depends on that teacher's orientation […] and what resources the teacher can bring to bear in support of the option he or she has chosen" (Schoenfeld 2011a, p. 13).

The dynamics between beliefs and behaviour suggested by Schoenfeld resemble the ones described by Malmivuori (2006) and Goldin et al. (2011). He acknowledges the significance of social interaction for the degree and character of belief enactment, and points to an internal dynamic that links emerging goals to shifting relationships between the goals, orientations, and resources brought to the classroom (Schoenfeld 2011a, b). This is all in line with the constructivist tenet that social interaction is a significant source of perturbations to what is conceived as the relatively autonomous functioning of the individual. In this interpretation, Schoenfeld assigns considerable stability and impact to teachers' beliefs. The position may be described as one of contextually constrained enactment of mathematics related beliefs.

In the cases Schoenfeld works with, the orientations that are enacted are closely related to the contents. This may be because of his emphasis on planned behaviour and because the examples he presents are from high school or college education and/or conducted by very experienced mathematics educators. A second dynamic perspective, the one of belief *activation*, is developed by others, who work with teachers with weaker backgrounds in mathematics and/or mathematics education. They have found that content-related beliefs may lose part of their significance or may be transformed in the educational process. Sztajn (2003), for instance, compares two elementary school teachers, Teresa and Julie. They both consider themselves in line with current reform initiatives, although their interpretations of the reform differ, and they are both convinced that their instructional approaches comply with these recommendations. However, these approaches are very different, and in Sztajn's interpretation the differences are not adequately accounted for by differences in the teachers' beliefs about the reform. Instead, she suggests that the teachers base instructional decisions on broader aspects of their students' lives than those related to their mathematical learning, and in particular it seems significant that Teresa and Julie teach children from very different socioeconomic backgrounds. Teresa teaches in a relatively poor neighbourhood, and in Sztajn's interpretation she emphasises rules and drill, as she seeks to "transform lower socioeconomic students into good citizens" (p. 69). In contrast, Julie works in a well-to-do area and teaches "higher-order thinking through educationally rich projects" in order to make schooling a good experience for the students (p. 69). Sztajn's point, then, is that beliefs beyond mathematics, especially a broad view of students' needs, play prominently in instruction, as teachers make ideological decisions "about what within the reform rhetoric fits particular children" (p. 70).

Also, my own previous work has questioned the extent to which teachers base instructional decisions on their beliefs about mathematics (Skott 2001, 2009). In one study, the teacher, Christopher, presented *school mathematics images* (SMIs) in interviews and questionnaires that were deemed highly compatible with aspects of the reform discourse (Skott 2001). However, in specific classroom episodes Christopher reacted to different groups of students in ways that appeared to be in mutual conflict with one another, and often also in conflict with his SMIs. Rather than interpreting these conflicts as expressions of teacher inconsistency, I saw them as cases in which Christopher's reformist intentions were to different degrees dominated by other concerns such as supporting the students' self-confidence and ensuring their position in the classroom community. In another study, a novice teacher, Larry, presents beliefs that are highly inspired by the reform, but he gets his first teaching position at a very conservative private school (Skott 2009). The tension between Larry's intention of supporting the students' own investigations and the school's emphasis on testing their command over standard procedures highlights the need for a contextualised views of beliefs that relate Larry's contributions to classroom interaction to three actual and virtual communities of practice, the ones of immediate classroom interaction, of the teachers at his school, and of his college education.

There is nothing in Schoenfeld's model of the moment-to-moment decision-making in teaching that is in principle at odds with the emphasis on broader educational issues in the studies by Sztajn and Skott. Both studies share Schoenfeld's view of teaching as goal-oriented, and the teachers may be seen as just bringing a broader set of orientations to the classroom, a set that encompasses educational issues beyond mathematics.

However, there is a difference in degree, if not in kind, between the types of dynamics involved in Schoenfeld's model (Schoenfeld 2011a) and my own previous work (Skott 2001, 2009). For Schoenfeld teaching is primarily a matter of planned enactment of orientations and resources, even though he does allow for contingencies. There is a stronger emphasis on the emergence of goals in the locally social in my own work. The two approaches share the view of beliefs as relatively stable constructs; the difference concerns the expected impact of those related to mathematics. Schoenfeld's model suggests that pre-existing, mathematics related beliefs are highly influential as teaching is basically a matter of enacting them (with due consideration of contingencies); in comparison I (in the studies above) suggest a stronger contextual dynamic that leaves it as a more open question if, what, and how beliefs are activated in classroom interaction.

Another interpretation, the *situated* perspective, suggests that mainstream belief research should not primarily be questioned on the expectation of impact of mathematics related beliefs, but on the one of their contextual stability. Beliefs, the argument goes, are situated or distributed, and there is little reason to expect that beliefs espoused in questionnaires and research interviews resemble the ones that are observed in classroom interaction. Hoyles (1992), for instance, suggests that once the situated character of beliefs is recognised, it becomes "self-evident that any individual can hold multiple (even contradictory) beliefs, and 'mismatch', 'transfer',

and 'inconsistency' are irrelevant considerations" (p. 40). Looking back on her own previous study of a female teacher, Ms. X, who teaches a group of high ability girls, Hoyles raises the following questions:

> How far was Ms X's mathematical perspective constructed by the 'high ability' of the group? How far was her emphasis on effort related to her sex and the sex of the students? Was her particular blend of exposition/interaction partly a function of the age and specialism of the students? I would of course now answer these questions in the affirmative! (p. 40)

Somewhat in line with Hoyles, Lerman (2001) argues that although there is "a family resemblance between concepts, beliefs, and actions in one context and those in another, they are qualitatively different by virtue of those contexts" (p. 36). The beliefs observed in the classroom, then, are not unrelated to the ones that may dominate the interview situation, "but the classroom is its own setting" (p. 36).

The notion of situatedness clearly questions the assumption of belief stability across contexts, but not necessarily the one of belief impact. Indeed, if the situatedness of teachers' beliefs is used to explain why observed classroom practices differ when the same teacher works with different groups of students and why classroom practices differ from beliefs as inferred from research interviews or questionnaires, it is still implied that beliefs are an explanatory principle for practice. The differences are accounted for, not by suggesting that beliefs, still understood as reified prior experiences, do not matter for practice, but that the experiences gained in different settings are sufficiently different for the beliefs, to differ as well (Skott, 2014).

A fourth dynamic and contextual perspective on belief-practice relationships, the one of *emergence*, also involves a view of beliefs as in some sense situated. In this interpretation, however, beliefs are reflexively related to the classroom processes that evolve at the instant, and consequently less reified. This is the case for instance in work of Cobb and Yackel (1996), whose emphasis is on students rather than on teachers. Their framework includes the well-known concept of socio-mathematical norms, i.e. "the normative aspects of whole-class discussions that are specific to students' mathematical activity" (p. 178). Socio-mathematical norms are seen as collective counterparts to "mathematical beliefs and values". At a more general level of analysis, "classroom social norms" correspond to "beliefs about own role, others' role, and the general nature of mathematical activity in school", while at a more specific level "classroom mathematical practices" are seen as social correlates of individuals' "mathematical conceptions and activity".

Cobb and Yackel are explicit that they developed the framework for the purpose of accounting for and supporting the students' mathematical development, not to analyse the norms in their own right. The same is the case in Cobb's and his colleagues' subsequent use of the framework in development activities concerned with elementary students' learning of measurement (Stephan et al. 2003) and lower secondary students' work on data handling (Cobb et al. 2001, 2003). However, in order to make such accounts, presumably including accounts of students' beliefs, they found it necessary to include a social perspective and conduct analyses of classroom social norms, of socio-mathematical norms, and of classroom mathematical practices.

Two elements of the argument made by Cobb and his colleagues are particularly relevant in the present context. First, they argue that the relationship between beliefs and norms is reflexive rather than causal. This means for instance that "neither the social norms nor individual students' belief are given primacy over the other […] social norms and beliefs are seen to be reflexively related such that neither exists independently of the other" (Cobb and Yackel 1996, p. 178). Supposedly, the same holds also for teacher's beliefs. Second, all norms are seen as established jointly by the students and the teacher. Classroom practices, then, are not interpreted as the teacher's practices, but as the result of continuous renegotiation among all participants in the classroom community, in spite of the special role of the teacher in it.

As indicated above, Cobb and his colleagues were not primarily interested in teachers' beliefs. However, the reflexivity between the social and the individual and the view of the classroom as a jointly emerging reality suggest a fourth possible category of perspectives on teachers' beliefs, the one of emergence, according to which they are neither necessarily stable nor determiners of the practices that evolve in the classroom.

The point in this section is that it makes sense to discuss the dynamic character of beliefs in relation to the two dimensions of stability and impact. To make the point, I have used particular studies to elaborate on the meaning of each of the four cells in Table 1. I should reiterate, however, that this does not mean that these studies are necessarily 'located' in those cells. For instance, both Lerman and Hoyles argue for a 'low-stability' view of beliefs, but neither of them is explicit that situated beliefs are 'high-impact'. However, I have used their studies to suggest what may characterise a 'low-stability, high-impact' approach to belief research. The studies mentioned in the cells in Table 1, then, are meant as reminders of the studies used to elaborate on the meaning of the particular cells, not necessarily as studies deemed exemplary for the cell in question.

The four cells in Table 1 all represent relatively dynamic interpretations of the belief-practice quandary. Possibly with the exception of the emergent perspective, however, they all interpret beliefs as individual reifications that significantly influence practice, although the beliefs in question are not necessarily related to mathematics (*activation*) and not necessarily stable across contexts (*situatedness*). This indicates that the acquisitionist underpinnings of mainstream belief research continue to orient the field, also when more dynamic interpretations are developed.

Table 1 Dynamic perspectives on teachers' beliefs

		Impact of teachers' mathematics related beliefs on classroom practice	
		High	Possibly low
Stability of teachers' mathematics related beliefs across contexts	High	Enactment (Schoenfeld 2011a, b)	Activation (Sztajn 2003; Skott 2001)
	Possibly low	Situatedness (Hoyles 1992; Lerman 2001)	Emergence (Cobb and Yackel 1996)

Belief Research and Acquisitionism – Or Why Believe in Beliefs?

The notions of reification and objectification have been discussed by Sfard as part of her challenge to acquisition as a metaphor for learning and knowing (Sfard 2007, 2008). In her terminology, objectification is a two-stage process that transforms human engagement in discursive practices into apparently self-sustained, mental entities. The first stage is a reification, in which "sentences about processes and actions [are replaced by] propositions about states and objects" (Sfard 2008, p. 44). The second stage is an alienation in which reified objects get a life of their own, independently of the processes that initially gave rise to them.

Sfard's examples of objectification include the notions of number and of thinking. Number, she says, is a reification of a counting procedure (1, 2, 3, …) that is transformed into an adjective (three apples) and then into a noun-like entity that may be operated on (3 + 5 =). Finally, number becomes a mind independent, alienated object that has its own characteristics independently of any mental activity (whether 3517211 is a prime is independent of whether anybody cares to find out). As far as the other example, thinking, is concerned, Sfard argues that a discourse dominated by the acquisition metaphor makes us think "of knowledge as a kind of material, of human mind as a container, and of the learner as becoming an owner of the material stored in the container." (p. 49). The acts of knowing or coming to know in a particular situation are decontextualized and the content of knowing is considered an entity with a strong element of permanence. As a final example, one may use the concept of objectification on itself. Describing the stages of objectification, Sfard objectifies the process, and uses the term of objectification to point to an independent entity as well as to the process.

The core of the beliefs concept (cf. section "The concept and expected functions of beliefs") implies that beliefs are generally regarded as objectifications. Abelson (1986), for instance, suggests that beliefs resemble possessions that are acquired, kept, valued, and sometimes lost, and although people do not buy or sell beliefs, they often accept that beliefs come at a cost. Referring to Abelson's earlier work (Abelson 1979), Nespor (1987) develops a conceptualisation of beliefs on the basis of a qualitative study of teachers of different subjects. He suggests that beliefs are characterised among others by what he calls an "existential presumption". This is the tendency to phrase beliefs in terms of reifications that point to the existence or non-existence of the involved entities. The two mathematics teachers in Nespor's study, for example, explained students' (lack of) learning with reference to their "ability", "maturity", and "laziness". In Nespor's interpretation these terms not only describe the students' participation in the classroom, but function as "labels for entities thought to be embodied by the students" (p. 318). Nespor's point is that the teachers have acquired and now possess reified mental constructs that allow them to bring order to the poorly structured problems of practice by interpreting and explaining student action and subsequently to define their own tasks in the classroom.

Nespor's examples suggest that there are advantages and disadvantages to objectification. Similarly Sfard (2008) says that it increases the effectiveness of communication and constitutes the basis for accumulation of experience. However, objectified entities are the result of an ontological collapse, as the discursive construction of the object is disregarded, and the object itself is mistakenly conceived as belonging to a mind-independent, perceptually accessible reality. Consequently, objectified entities carry connotations of permanence and repetitiveness that may be unfounded, and they invite interpretations of the future in the image of the past. Sfard mentions mathematical inability and giftedness as examples to make her point.

In belief research, teachers' beliefs are viewed as the result of a two-stage objectification process on the part of the teachers themselves. However, the beliefs attributed to, or symbolically imposed upon, teachers are a result of a similar, but second-order objectification process on the part of an observer. The researcher working with beliefs attributes sets of reified priorities, beliefs, to teachers and uses them to explain instructional decisions. For example, Nespor's argument that the teachers in his study impose personality traits (ability; maturity; laziness) on the students to account for their behaviour may be turned on the field of beliefs itself to the extent that researchers in the field impose trait-like beliefs on the teachers (traditionalist; reform oriented; inconsistent) so as to explain *their* classroom behaviour.

The drawbacks of objectification that Sfard points to, then, are apparent in the dominant use of beliefs. The very idea that beliefs are a priori expected to influence classroom practice is a paradigm case of how the experiential base of an objectified entity is disregarded and of how the reifications themselves are expected to mould future activities. One may object that this is no more and no less than yet another example that people understand and act in and towards the world in ways that reflect the meaning they attribute to that world. However, mainstream belief research is based on the premise that pre-existing, de-contextualized, and temporally stable beliefs about mathematics and its teaching and learning are the main, if not the sole determiners of such meaning. The more participatory approach outlined below allows for the possibility that such meaning-making is more or at least differently dynamic than usually assumed in belief research, also in approaches that may be 'located' in cells [11], [12] and [21] in Table 1.

PoP – Towards a Participatory Account

Recently, attempts have been made to challenge acquisitionism and develop or use more participatory accounts of human functioning. Sfard's work, referred to previously, is an ambitious attempt to develop such an account; Barwell (2013) draws on discursive psychology to make more locally social and dynamic analyses of what is normally discussed in terms of knowledge; and Wagner and Herbel-Eisenmann (2009) build on positioning theory to do so in the field of identity. In the field of affect few such attempts have been made, but Evans et al. (2006) and Horn (2007) are notable exceptions. In what follows I recapitulate my own attempt to build on

social practice theory and symbolic interactionism to reconceptualise what is normally phrased in terms of beliefs, and develop a conceptual framework, called Patterns of Participation (PoP), that as far as possible avoids relying on objectifications when analysing classroom practices. In a more positive wording, PoP may be described as an approach to classroom interaction that views individuals' contributions in processual and participatory terms and interprets them as meaningful re-engagement in other past and present practices in view of the ones that unfold at the instant.

I suggested previously that Schoenfeld's model of teaching implies that belief enactment may be modified by two distinct types of dynamics, one that concerns the person-context interface and another – depending on the first – that refers to shifting relationships among different orientations, goals and resources, i.e. among different reifications. Similarly, there are dual dynamics involved also in 'belief activation', i.e. when belief selection is based primarily on immediate social interaction, and in what I described as the situated perspective (cf. Table 1). Although the three perspectives differ in their views of the character and functioning of the dynamic relationships, they all locate these in the bottom half of Fig. 1, i.e. between beliefs and classroom practice or among the beliefs themselves, possibly supplemented with other reifications in the form of knowledge and goals.

Reducing the emphasis on objectifications, PoP assigns greater significance to the relationship between the experiences at the top of Fig. 1 and classroom practice without relying on beliefs as an intermediary reification. However, merely transforming Fig. 1 by turning the top arrow clockwise and erasing beliefs would indicate an immediate, causal connection between prior experiences and classroom practice. This loses the potential of the more interactive interpretations of classroom processes that have been developed recently, including the ones outlined in section "Dynamic views of teachers' beliefs", and it may even be read as disregard for the attempts to understand individual meaning-making that have always fuelled belief research. The intention of PoP is exactly to focus on such meaning-making, but in contrast to belief research to suggest that it is to a lesser extent based on reifications (beliefs) than on dynamic re-engagement in the practices that in belief research are assumed to be the basis for them.

There is also a dual dynamic involved in PoP interpretations of classroom interaction, although it is somewhat different from the ones described previously. First, and somewhat in line with Cobb and Yackel (1996), classroom practices are viewed as social phenomena, not as an outcome of any individual's actions. In PoP they are seen as constituted in a process during which each individual continuously makes symbolic interpretations of others' actions as well as of others' (possible) reactions to one's own behaviour. This is inspired by symbolic interactionism, especially of its view of the self as consisting of two phases, an *I* and a *me* (Blumer 1969; Mead 1934). The *I* acts, but in the process the individual becomes an object to him- or herself, i.e. becomes the *me*. In the action, then, the individual takes the attitude of individual or generalised others and adjusts his or her actions accordingly. This is significant not least in relation to affective issues (Shott 1979). Second, behaviour is not seen as a release of reified mental entities, whether in the form of beliefs,

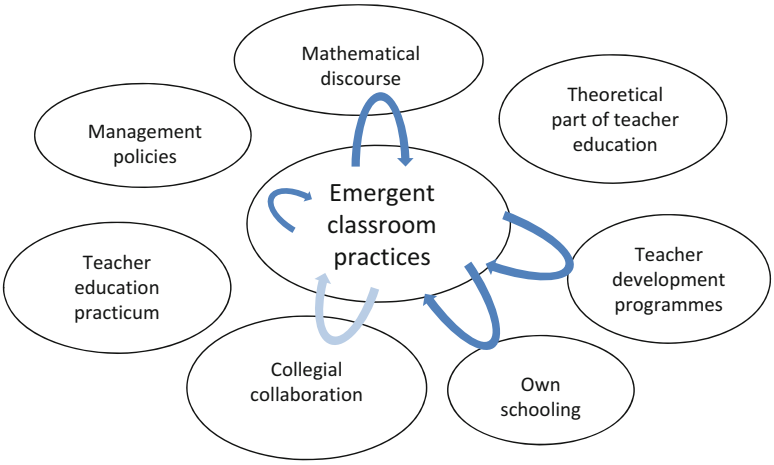

Fig. 2 Drawing on other practices and figured worlds in classroom interaction

knowledge, or any combination of the two, but as an outcome of the teacher reengaging in a range of other social practices stemming for instance from team or department meetings, theoretical discussions in teacher education or development programmes, experiences from their own schooling, and many more.

Consider for example a teacher working with a group of students, who are trying to substantiate a mathematical conjecture, but finds it difficult to do so. The teacher's contributions to the interaction may change, if she, while engaging in a mathematical discourse in order to assist the students, also orients herself towards the reform, possibly as propagated in a recent teacher development programme; positions herself within a team of teachers, whose cooperation focuses on the well-being of individual students rather than on their subject matter learning; and manifests her own professional authority, as her mathematical competence was recently questioned in the class. In the interaction, then, i.e. as classroom practices emerge, the teacher draws upon and renegotiates the meaning of prior social practices (Fig. 2). This exemplifies that in PoP we look at classroom processes in an attempt to link the teacher's contributions to the interaction to other significant practices or to what Holland and her colleagues call figured worlds (Holland et al. 1998), i.e. collective as-if worlds in which "particular characters and actors are recognised, significance is assigned to certain acts, and particular outcomes are valued over others" (p. 52). The imaginary example above indicates that beyond mathematics such practices and figured worlds may include – among many more – a reform agenda, a team of cooperating teachers, and even schooling in a general sense. Schooling, for instance, qualifies as a significant figured world in relation to the challenge to the teacher's professional authority. Students and teachers are recognised as such due to their different positions in the interaction; significance is assigned to acts of teaching that position the teacher as knowledgeable and able to support students in solving the tasks at hand; and the valued outcomes include identifiable and recognisable shifts

in understanding or proficiency on the part of the students. In the example the degree to which this sense of 'schooling' plays a role for the teacher may inform how she seeks to contribute to the students' reasoning about the task. I refer to two recent studies to indicate how PoP may be used for empirical purposes.

Using PoP for Empirical Purposes

I indicated previously that belief research is faced with considerable methodological problems, as there is no easy access to the mental reifications assumed to reside entirely in people's minds. Consequently, a combination of for instance observations, interviews, and surveys is often used, based on the expectation that between them they shed sufficient light on what teachers and students 'really believe'. As I have argued elsewhere, however, methodical triangulation is itself problematic, as it takes contextual and temporal stability of beliefs for granted (Skott 2014).

Although teachers' participation in discourses and non-verbal practices is more readily observable than their beliefs, PoP-research is clearly confronted with its own methodological challenges (Skott 2013). As Fig. 2 indicates, the task is to interpret classroom action and meaning-making as they relate to the teachers' participation in other past and present practices that are not all equally accessible and the character and significance of which cannot be specified beforehand. To meet these challenges we use an approach inspired by recent developments of grounded theory (GT) in combination with methods that are often associated with educational ethnography (Charmaz 2006; Charmaz and Mitchell 2001). Initially data are generated from video recordings of considerable amounts of classroom teaching and from relatively open qualitative interviews, sometimes using stimulated recall. These methods are combined with informal observations of staffroom communication. The data are continuously coded, compared, and theorised in line with GT guidelines, but without the naturalistic connotations often associated with them. The analyses of the data lead to suggestions for practices and figured worlds beyond the classroom that are significant for the teacher's contributions to the ones that unfold within it. Subsequently new data are generated on these other practices and compared to those stemming from the classroom. In the case of Anna, discussed briefly below, it became apparent from the initial interviews that she relates closely to the functioning of "her team", i.e. the group of four teachers, who teach all subjects in grade 7 at her school. Consequently, I observed team meetings and conducted a group interview with the other team members, so as to get a sense of how the team negotiates the task of taking on the full responsibility for the year-group; of how Anna contributes to that negotiation; and of how the negotiation relates to Anna's shifting tales of herself as a professional at Northgate, including how she positions herself in the classroom.

It is apparent, then, that PoP makes use combinations of methods that are somewhat similar to the ones used in belief research, but does so for other reasons. In research on beliefs, multiple methods are used in attempts to gain access to the

same mental constructs, a teacher's or student's beliefs. In PoP the intention is in some sense the opposite, i.e. as far as possible to get access to *different* practices and figured worlds. An open, qualitative interview may, for instance, shed light on aspects of how a teacher engages discursively with mathematics education, including if and how she relates to elements of the current reform; a set of classroom observations may suggest how she copes with the multiple challenges involved in classroom interaction in a particular context. Although her discursive engagement with the reform may matter for her contributions to classroom practice, the reform discourse and classroom interaction are viewed as decidedly different practices or figured worlds. The methodological decisions in PoP, then, are concerned with how best to develop an understanding of the range of different practices and figured worlds that are currently significant for the teacher in question, and of how they may inform one another, in particular how they relate to her contributions to classroom interaction. It follows that interviews, let alone surveys, are of limited value in PoP as the sole sources of data generation, as they provide little access to the range of practices beyond teachers' rhetorical commitment to specific discourses such as the reform.

To exemplify the methods as well as some results, I refer to the study of Anna, mentioned above (Skott 2013). Anna is a young, novice teacher, who teaches mathematics at Northgate Primary and Lower Secondary School in Denmark. She is selected for the study because of her mathematical and professional self-confidence and commitment at the time of her graduation. She explicitly considers herself a *mathematics* teacher, not just someone who teaches mathematics, and the initial interviews and observations suggest that one aspect of her dedication is linked to the current reform agenda, not least to investigations and student communication, which, she says, was promoted by her teacher education programme. The initial data also indicate that there are two other, but partly overlapping, practices and figured worlds beyond *mathematics* and *the reform* that are less immediately related to the contents of instruction, but that play prominently for Anna's tales of herself as young professional and for how she positions herself in the classroom. One of these is *teaming*. The initial interviews with Anna as well as the subsequent data from observations and interviews with the team suggest that the team does not plan instruction or teach together, at least not in the first 2 years after Anna's graduation. The team focuses on less content specific aspects, such as the social functioning of the classes and individual students' social and personal problems. This relates closely to the fourth figured world discerned from the initial data, one concerned with Anna's attempts to build trusting relationships with the students. The valued outcome of this world of *relationing* positions Anna as what she half-jokingly describes as being "somewhere between a mother and a friend" for the students. Between them the data generated with the different methods invite interpretations of the meanings these four practices and figured worlds have for Anna in different situations, including classroom contexts.

In the above analysis, then, the most prominent practices and figured worlds for Anna's meaning-making and instructional decisions in her classroom are *mathematics*, *the reform*, *teaming*, and *relationing*. However, the character and relative significance of each of them change as the interactions unfold. It is noticeable, for instance, that the aspect of the reform concerned with students' mathematical

communication sometimes loses the qualifier of mathematical and becomes merely a matter of verbal exchanges. This may happen when Anna takes the attitude of what she at the instant considers vulnerable students, as the emphasis on communication in *the reform* is submerged by or embedded in *relationing* in her attempts to avoid jeopardising the students' self-confidence or her own relationship with them.

My colleagues and I also worked with another novice teacher, Susanne, who teaches mathematics in grade 5 at a school called Southern Heights (Skott et al. 2011). The students at the school have mixed social backgrounds, and as Susanne points out, a significant number of them come to school every morning "without breakfast and without a kiss and a hug and without all the other things that the rest of us consider matters of course" (p. 41).

Susanne draws heavily on what she describes as traditional teaching: "teaching-from-the-board and exercises" (p. 38). However, in a PoP interpretation, elements of other practices and figured worlds are renegotiated and inserted in isolated instances and on the fringes of the dominant instructional approaches. This is so for instance with the reform discourse on student understanding, i.e. "that doctrine that they need to understand and not just follow the rules" (p. 40). This discourse plays small but significantly different roles, when she introduces procedures for the students to copy and routinize, and when students, in spite of Susanne's emphasis on standard procedures, come up with unexpected suggestions for how to solve the tasks. Also, Susanne reinterprets what she considers successful initiatives on the part of the school to support children with social or personal problems. One such initiative is a special needs department for students with learning problems in particular subjects; another is the 'observation class', 'the obs', where students are sent, if they are unruly, but which Susanne, at least when she talks about the students in general, also sees as an opportunity for them to calm down, if they are under pressure. Susanne makes frequent use of both the special needs department and 'the obs'. Further, she seems inspired by these organisational measures at Southern Heights also in relation to other students in her class and asks groups of students to work on their own elsewhere or to work independently in the classroom, apparently in an attempt to create a sufficiently homogeneous group of students to work with herself. We suggest, however, that in the process this partial imitation of the school's segregation policies shifts its meaning for Susanne from taking care of students with problems to handling problematic students.

The practices and figured worlds that are significant differ in the two cases above and so do the ways in which their mutual relationships develop as classroom practices unfold. In spite of the differences, however, they both lend themselves to PoP interpretations.

Concluding Remarks

Irrespective of the problems of defining and accessing mathematics related beliefs, empirical findings in belief research more than suggest that the expectation of a causal relationship between such reified constructs and behaviour needs to be

modified. The section "Dynamic views of teachers' beliefs" outlines four such modifications that differ in their interpretations of the contextual and temporal stability of beliefs and of what and if beliefs are influential, even if stable. These responses, then, relate differently to the last two of the core aspects of beliefs as outlined in section "The concept and expected functions of beliefs", the ones of stability and impact.

In spite of the differences among these more dynamic interpretations, at least three of them conceptualise beliefs as objectifications. Section "Belief research and acquisitionism – or why believe in beliefs?" discusses some of the problems with this, including the somewhat ironic observation that the use of objectifications in the beliefs literature is characterised by a similar existential presumption to the one Nespor assigns to the beliefs of the teachers in his study. It is implicit in Nespor's discussion that teachers' beliefs that their students are able, mature or lazy do not qualify as reasonable explanations for the students' actions. One may wonder why belief research attributes similar explanatory power to the trait-like beliefs attributed for instance to teachers.

This is the backdrop to the presentation of PoP, a framework that suggests a negative answer to the question of whether we need to rely on objectified beliefs, when attempting to understand what roles the teacher plays in the practices that emerge in the classroom. This, however, needs an elaboration.

As Sfard points out, there are certainly advantages to the use of reifications, and we tend to make and use them in order to make sense of and function in complex situations. However, there are at least two reasons why belief research seems to rely too heavily on beliefs as reifications. First, researchers attribute beliefs to teachers and students, and in this sense the beliefs described in the literature are second-order reifications. More often than not the research participants do not explicate these reifications themselves. In spite of that they are expected to make sense of the world by using them. Second, even when research participants do describe their relationship to mathematics and its teaching and learning in reified terms, it is an empirical question what role the reifications play, if any. In the interviews with Anna and Susanne, they are both close to using reifications to describe themselves as teachers of mathematics, Susanne explicitly calling herself a traditionalist, and Anna emphasising that she is a *mathematics* teacher, who prioritises certain aspects of the reform. Doing so, they engage in a discursive construction of themselves as professionals, which may resemble or inform their contributions to unfolding classrooms events. Anna, for instance, time and again requires the students to find their own solution strategies and discuss their methods and results with one another. However, Anna's reengagement in the reform discourse in the classroom is often transformed as she appears to position herself in her team or among her colleagues in general or by her attempts to be "somewhere between a mother and a friend" for the students. As she makes sense of and contributes to emerging classroom practices, then, Anna takes the attitude of different individual and generalised others (e.g. students), including some that are not physically present (e.g. other team members) and others that are only established discursively (e.g. *the reform*).

PoP implies taking the dynamic perspective beyond an approach that looks at shifting relationships between pre-established reifications. Instead, it suggests attempting to understand how teachers draw on and renegotiate their participation in a range of other past and present practices and figured worlds as they engage in classroom interaction. This provides a differently dynamic perspective on what in beliefs terminology may be phrased as the beliefs-practice quandary.

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