Chapter 1
Power System Dynamic Equilibrium, Power Flow, and Steady-State Stability

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Abstract The material in this chapter focuses on the relationship between power system dynamic equilibrium, power flow, and operating point stability. It addresses issues relating steady-state equilibrium in electric power systems with possible implications about stability of the associated operating point. It presents various connections between dynamic models, dynamic equilibrium, power-flow analysis, and the significance of singularities of Jacobian matrices involved in various computations. It includes advances on earlier work on this subject and provides recent results on computing the equilibrium of "post-contingency" models. These post-contingency models are created to enforce the concept of "constant control inputs" in the steady-state analysis. This impacts subtle things such as post-contingency speed (frequency) and remote voltage regulation. The concepts are illustrated on small system models. Different methods of computation are presented to provide alternatives for possible practical implementation.

Keywords Steady-state stability · Power-flow Jacobian · Power system dynamics · Contingency analysis · Equilibrium · Small-signal stability

1.1 Introduction[1]

There are many subtle issues associated with power systems and the viability of mathematical models associated with the physical system. A number of these issues have to do with voltage collapse, voltage instability, and equilibrium conditions. These include the concept of maximum loadability. The issue of load modeling alone raises many questions about the proper modeling techniques and assumptions

[1] Portions of this chapter were previously published with IEEE copyright in the paper by P. W. Sauer and M. A. Pai, "Power System Steady-State Stability and the Load flow Jacobian," IEEE Transactions on Power Systems, Vol. 5, No. 4, November 1990, pp. 1374–1383, reprinted here with permission from IEEE.

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S. C. Savulescu (ed.), *Real-Time Stability in Power Systems,*
Power Electronics and Power Systems, DOI 10.1007/978-3-319-06680-6_1,

about aggregate behavior. This chapter focuses primarily on equilibrium analysis and information that can be obtained from various types of steady-state analysis. The subjects of voltage collapse and voltage instability have created interest in power-flow Jacobian singularities and their relationship with steady-state stability and voltage collapse.

In 1975, Venikov et al. published a paper that proposed that, under certain conditions, a direct relationship exists between the singularity of the standard power-flow Jacobian and the singularity of the system dynamic state Jacobian (Venikov et al. 1975). This paper has been cited as the primary justification for studying the power-flow Jacobian matrix to determine critical load levels.

In this chapter, we clarify this result in the context of a fairly general dynamic model and show that the result should be considered optimistic for any type of steady-state stability analysis. This chapter includes a tutorial on the role of power flow in dynamic analysis.

1.2 Detailed Dynamic Model Without Stator/Network Transients

This section presents a basic nonlinear multi-machine dynamic model that includes the fundamental features of voltage and frequency control, but assumes that all stator/network transients have been eliminated. The elimination of the stator/network transients leads to algebraic equations that accompany the multi-machine dynamic model as follows:

$$\frac{d\delta_i}{dt} = (\omega_i - \omega_s) \quad i = 1, \ldots, m \tag{1.1}$$

$$M_i \frac{d\omega_i}{dt} = T_{Mi} - \left[E'_{qi} - X'_{di} I_{di}\right] I_{qi} - \left[E'_{di} + X'_{qi} I_{qi}\right] I_{di} - D_i\left(\omega_i - \omega_s\right) \quad i = 1, \ldots, m \tag{1.2}$$

$$T'_{doi} \frac{dE'_{qi}}{dt} = -E'_{qi} - \left(X_{di} - X'_{di}\right) I_{di} + E_{fdi} \quad i = 1, \ldots, m \tag{1.3}$$

$$T'_{qoi} \frac{dE'_{di}}{dt} = -E'_{di} + \left(X_{qi} - X'_{qi}\right) I_{qi} \quad i = 1, \ldots, m \tag{1.4}$$

$$T_{Ei} \frac{dE_{fdi}}{dt} = -\left(K_{Ei} + S_{Ei}\left(E_{fdi}\right)\right) E_{fdi} + V_{Ri} \quad i = 1, \ldots, m \tag{1.5}$$

$$T_{Ai} \frac{dV_{Ri}}{dt} = -V_{Ri} + K_{Ai} R_{fi} - \frac{K_{Ai} K_{Fi}}{T_{Fi}} E_{fdi} + K_{Ai}(V_{refi} - V_i) \quad i = 1, \ldots, m \tag{1.6}$$

$$T_{Fi} \frac{dR_{fi}}{dt} = -R_{fi} + \frac{K_{Fi}}{T_{Fi}} E_{fdi} \quad i = 1, \ldots, m \tag{1.7}$$

$$T_{RHi}\frac{dT_{Mi}}{dt} = -T_{Mi} + \left(1 - \frac{K_{HPi}\,T_{RHi}}{T_{CHi}}\right)P_{CHi} + \frac{K_{HPi}\,T_{RHi}}{T_{CHi}}P_{SVi} \quad i = 1,\ldots,m \tag{1.8}$$

$$T_{CHi}\frac{dP_{CHi}}{dt} = -P_{CHi} + P_{SVi} \quad i = 1,\ldots,m \tag{1.9}$$

$$T_{SVi}\frac{dP_{SVi}}{dt} = -P_{SVi} + P_{ci} - \frac{1}{R_i}\left(\frac{\omega_i}{\omega_s}\right) \quad i = 1,\ldots,m \tag{1.10}$$

$$\begin{aligned}0 = V_i e^{j\theta_i} + \left(R_{si} + jX'_{di}\right)\left(I_{di} + jI_{qi}\right)e^{j(\delta_i - \pi/2)} \\ -\left[E'_{di} + \left(X'_{qi} - X'_{di}\right)I_{qi} + jE'_{qi}\right]e^{j(\delta_i - \pi/2)} \quad i = 1,\ldots,m\end{aligned} \tag{1.11}$$

$$0 = -P_i - jQ_i + V_i e^{j\theta_i}\left(I_{di} - jI_{qi}\right)e^{-j(\delta_i - \pi/2)} + P_{Li}\left(V_i\right) + jQ_{Li}\left(V_i\right) \quad i = 1,\ldots,m \tag{1.12}$$

$$0 = -P_i - jQ_i + P_{Li}\left(V_i\right) + jQ_{Li}\left(V_i\right) \quad i = m+1,\ldots,n \tag{1.13}$$

$$0 = -P_i - jQ_i + \sum_{k=1}^{n} V_i V_k Y_{ik}^{\,j(\theta_i - \theta_k - \alpha_{ik})} \quad i = 1,\ldots,n, \tag{1.14}$$

where the notation is standard for a machine with one damper winding plus field (two-axis model), IEEE type I excitation system, and simplified turbine/governor model (Sauer and Pai 1998). The notation for an m-machine, n-bus system is,

$$V_i e^{j\theta_i} = \text{voltage at bus} \quad i = 1,\ldots,n \tag{1.15}$$

$$V_i e^{j\theta_i} = \left(V_{di} + jV_{qi}\right)e^{j(\delta_i - \pi/2)} \quad i = 1,\ldots,m \tag{1.16}$$

$$I_{Gi}e^{j\gamma_i} = \left(I_{di} + jI_{qi}\right)e^{j(\delta_i - \pi/2)} \quad i = 1,\ldots,m \tag{1.17}$$

$$Y_{ik}e^{j\alpha_{ik}} = \text{standard power-flow bus admittance matrix entry} \quad i,k = 1,\ldots,n \tag{1.18}$$

$$\begin{aligned}P_i + jQ_i = \text{net injected power into bus}\,i\,\text{from a path not} \\ \text{included in the bus admittance matrix} \quad i = 1,\ldots,n.\end{aligned} \tag{1.19}$$

The algebraic variables P_i and Q_i are introduced so that the standard power-flow equation (1.14) is preserved for any load models (1.12) and (1.13). Note that these equations can be supplemented with algebraic or dynamic models that have an output of $P_{Li}(V_i) + jQ_{Li}(V_i)$. This full dynamic model contains $10m$ dynamic states $(\delta, \omega, E'_q, E'_d, E_{fd}, V_R, R_f, T_M, P_{\mathrm{CH}}, P_{\mathrm{SV}})$, $2m + 4n$ real algebraic states

$(I_d, I_q, P, Q, \theta, V)$, *and* $2m$ inputs (V_{ref}, P_c). Equation (1.11) is the stator algebraic equation, which is normally expressed either as a phasor diagram or as a quasi-static phasor circuit in the literature (Sauer and Pai 1990). There are $m+2n$ complex algebraic equations, which should in principle be solved for the $2m+4n$ real algebraic states as functions of the $10m$ dynamic states.

The machine currents I_d and I_q can easily be eliminated by solving (1.11) and substituting into (1.2)–(1.12). The P and Q algebraic states can easily be eliminated by substituting (1.12) and (1.13) into (1.14), leaving only n complex algebraic equation (1.14) to be solved for the $2n$ real algebraic states θ and V. These remaining algebraic equations cannot normally be solved explicitly.

In the special case of constant impedance loads, it is customary to use an internal generator bus model and include all loads and the machine impedance $R_s + jX'_d$ in the bus admittance matrix (enlarged to $m+n$ buses).

With the additional assumption of $X'_q = X'_d$, all algebraic states can be explicitly eliminated with a reduced $(m \times m)$ admittance matrix. For nonlinear load models, the algebraic equations must be retained. This chapter does not introduce internal machine buses.

Wind turbine generators can be included in this model using the basic model of Pulgar and Sauer (2011), where the wind turbine controls add additional dynamic states to the traditional blade rotation and generator electrical transients.

1.3 Standard Power Flow

Standard load flow (or power flow) has been the traditional mechanism for computing a proposed steady-state operating point. For this chapter, we define standard power flow as the following algorithm (Pai 2004):

- Specify bus voltage magnitudes numbered 1 to m
- Specify bus voltage angle number 1 (slack bus)
- Specify net injected real power P_i at buses numbered 2 to m
- Specify load powers P_{Li} and Q_{Li} at all buses numbered 1 to n
- Solve the following equations ((1.13) and (1.14) rewritten) for $\theta_2, \ldots, \theta_n, V_{m+1}, \ldots, V_n$

$$0 = -P_i + \sum_{k=1}^{n} V_i V_k Y_{ik} \cos\left(\theta_i - \theta_k - \alpha_{ik}\right) \quad i = 2, \ldots, m \tag{1.20}$$

(PV buses)

$$0 = -P_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik} \cos\left(\theta_i - \theta_k - \alpha_{ik}\right) \quad i = m+1, \ldots, n \tag{1.21}$$

(PQ buses)

$$0 = -Q_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik} \sin\left(\theta_i - \theta_k - \alpha_{ik}\right) \;\; i = m+1,...,n \tag{1.22}$$

(PQ buses)

where

$$P_i(i = 2,...,m), V_i(i = 1,...,m), P_{Li}(i = m+1,...,n), Q_{Li}(i = m+1,...,n)$$

and θ_1 are the specified numbers. The standard power-flow Jacobian matrix is the linearization of (1.20)–(1.22) with respect to $\theta_2,...,\theta_n$ and $V_{m+1},...,V_n$. After the power-flow solution, compute

$$P_1 + jQ_1 = \sum_{k=1}^{n} V_1 V_k Y_{1k} e^{j\left(\theta_1 - \theta_k - \alpha_{1k}\right)} \tag{1.23}$$

$$Q_i = \sum_{k=1}^{n} V_i V_k Y_{ik} \sin\left(\theta_i - \theta_k - \alpha_{ik}\right) \;\; i = 2,...,m. \tag{1.24}$$

This standard power flow has many variations, including the addition of other devices such as Tap Changing Under Load (TCUL) transformers, switching Volt–Ampere Reactive (VAr) sources, and High-Voltage Direct Current (HVDC) converters. It can also include inequality constraints on quantities such as Q_i, and can be revised to distribute the slack power between all generators.

We would like to make one important point about power flow. Power flow is normally used to evaluate operation at a specific load level (specified by a given set of powers). For a specified load and generation schedule, the solution is independent of the actual load model. That is, it is certainly possible to evaluate the voltage at a constant impedance load for a specific case where that impedance load consumes a specific amount of power.

Thus, the use of "constant power" in power-flow analysis does not require or even imply that the load is truly a constant power device. It merely gives the voltage at the buses when the loads (any type) consume a specific amount of power. The load characteristic is important when the analyst wants to study the system in response to a change such as contingency analysis or dynamic analysis. For these purposes, standard power flow usually provided the "initial conditions."

1.4 Initial Conditions for Dynamic Analysis

For any dynamic analysis using (1.1)–(1.14), it is necessary to compute the initial values of all dynamic states and to specify the fixed inputs (V_{ref}, P_c). In the power system dynamic analysis, the fixed inputs and initial conditions are normally found from a base case power-flow solution. That is, the values of V_{ref} are computed such that the *m* generator voltages are as specified in the power flow.

The values of P_c are computed such that the *m* generator real power outputs are as specified and computed in the power flow for rated speed (ω_s). To see how this is done, we assume that a power-flow solution (as defined in Sect. 1.3) has been found. The first step in computing initial conditions is normally the calculation of generator currents from (1.12) and (1.17) as:

$$I_{Gi}e^{j\gamma_i} = \left[\left(P_i - P_{Li}\left(V_i\right)\right) - j\left(Q_i - Q_{Li}\left(V_i\right)\right)\right] \Big/ \left(V_i e^{-j\theta_i}\right) \quad i = 1,\dots,m \tag{1.25}$$

and machine relative rotor angles from the manipulation of (1.11) and the algebraic equation from (1.4)

$$\delta_i = \text{angle of}\left[V_i e^{j\theta_i} + \left(R_{si} + jX_{qi}\right) I_{Gi} e^{j\gamma_i}\right] \quad i = 1,\dots,m. \tag{1.26}$$

With these quantities, the remaining dynamic and algebraic states can be found by

$$I_{di} + jI_{qi} = I_{Gi} e^{j(\gamma_i - \delta_i + \pi/2)} \quad i = 1,\dots,m \tag{1.27}$$

$$V_{di} + jV_{qi} = V_i e^{j(\theta_i - \delta_i + \pi/2)} \quad i = 1,\dots,m \tag{1.28}$$

followed by E_{fd} from (1.3), (1.4), (1.11), and (1.16)

$$E_{fdi} = X_{di} I_{di} + V_{qi} + R_{si} I_{qi} \quad i = 1,\dots,m. \tag{1.29}$$

With this field voltage, R_{fi}, V_{Ri}, and V_{refi} can be found from (1.5)–(1.7) as

$$R_{fi} = \frac{K_{Fi}}{T_{Fi}} E_{fdi} \quad i = 1,\dots,m \tag{1.30}$$

$$V_{Ri} = \left(K_{Ei} + S_{Ei}\left(E_{fdi}\right)\right) E_{fdi} \quad i = 1,\dots,m \tag{1.31}$$

$$V_{refi} = V_i + \left(V_{Ri} / K_{Ai}\right) \quad i = 1,\dots,m. \tag{1.32}$$

The initial values of E'_{qi} and E'_{di} are then found from (1.3) and (1.4):

$$E'_{qi} = -\left(X_{di} - X'_{di}\right) I_{di} + E_{fdi} \quad i = 1,\dots,m \tag{1.33}$$

$$E'_{di} = -\left(X_{qi} - X'_{qi}\right) I_{qi} \quad i = 1,\dots,m. \tag{1.34}$$

Note that, if the machine saturation is included, this calculation for E'_{qi} and E'_{di} may be iterative. The mechanical states and P_c are found from (1.1), (1.2), and (1.8)–(1.10) as:

$$\omega_i = \omega_s \quad i = 1,\ldots,m \tag{1.35}$$

$$T_{Mi} = \left[E'_{di} + X'_{qi} I_{qi}\right] I_{qi} + \left[E'_{di} + X'_{qi} I_{qi}\right] I_{di} \quad i = 1,\ldots,m \tag{1.36}$$

$$P_{CHi} = T_{Mi} \quad i = 1,\ldots,m \tag{1.37}$$

$$P_{SVi} = P_{CHi} \quad i = 1,\ldots,m \tag{1.38}$$

$$P_{ci} = P_{SVi} + \left(1/R_i\right) \quad i = 1,\ldots,m. \tag{1.39}$$

This completes the computation of all dynamic state initial conditions and fixed inputs.

For a given disturbance, the inputs remain fixed throughout the simulation. If the disturbance occurs in the algebraic equations, the algebraic states must change instantaneously to satisfy the initial condition of the dynamic states and the new algebraic equations. Thus, it may be necessary to re-solve the algebraic equations with the dynamic states specified at their initial conditions to determine the new initial values of the algebraic states.

From the above description, it is clear that once a standard power-flow solution is found, the remaining dynamic states and inputs can be found in a straightforward way. The machine relative rotor angles δ_i can always be found provided

$$V_i e^{j\theta_i} + \left(R_{si} + jX_{qi}\right) I_{Gi} e^{j\gamma_i} \neq 0 \quad i = 1,\ldots,m. \tag{1.40}$$

If control limits are enforced, a solution satisfying these limits may not exist. In this case, the state that is limited would need to be fixed at its limiting value, and a corresponding new steady-state solution would have to be found.

This would require a new power-flow solution specifying either different values of generator voltages, different generator real powers, or possibly generator reactive power injections, thus allowing generator voltage to be a part of the power-flow solution. In fact, the use of reactive power limits in power flow can usually be traced back to an attempt to consider excitation system limits or generator capability limits.

1.5 Angle Reference

In any rotational system, the reference for angles is arbitrary. Examination of (1.1)–(1.14) clearly shows that the order of this dynamic system can be reduced from $10m$ to $10m - 1$ by introducing the new relative angles (arbitrarily selecting δ_1 as reference)

$$\delta_i' = \delta_i - \delta_1 \quad i = 1, \ldots, m \tag{1.41}$$

$$\theta_i' = \theta_i - \delta_1 \quad i = 1, \ldots, m. \tag{1.42}$$

The full system remains exactly the same as (1.1)–(1.14) with each δ_i replaced by δ_i', each θ_i replaced by θ_i', and ω_s replaced by δ_i' in (1.1). During a transient, the angle δ_1 still changes from its initial condition (as found in the last section) as ω_1 changes, so that each original δ_i and θ_i can be easily recovered if needed.

The angle δ_1' remains at zero for all time. Thus, for dynamic simulation, the differential equation for δ_1 is normally replaced by the algebraic equation which simply states $\delta_1' = 0$. Notice that θ_1 is normally arbitrarily selected as zero for the power-flow analysis.

This means that the initial value of δ_1 is normally not zero. During a transient, θ_1' and θ_1 change as all angles except δ_1' change. If the inertia of machine 1 is set to infinity, ω_1 and δ_1 remain constant for all time.

1.6 Steady-State Stability

The steady-state stability of multi-machine systems is usually evaluated by computing the eigenvalues of the system dynamic state Jacobian matrix that is the linearized version of (1.1)–(1.14) with all algebraic equations eliminated. This dynamic model has one zero eigenvalue corresponding to the angle reference discussed above.

Elimination of δ_1 through the use of (1.41) and (1.42) would eliminate this zero eigenvalue. The system is linearized around a steady-state operating point found using standard power flow. The Jacobian matrix for this standard power flow appears as a sub-matrix in the lower right block and is denoted as J_{LF} below:

$$\begin{bmatrix} \frac{d\Delta y}{dt} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & J_{LF} \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta z \\ \Delta v \end{bmatrix} \tag{1.43}$$

where v contains the power-flow variables $\theta_2, \ldots, \theta_n, V_{m+1}, \ldots, V_n$. In order to evaluate the stability of the dynamic system, the algebraic equations must be eliminated. This requires the nonsingularity of the algebraic equation Jacobian (J_{AE}):

$$J_{\mathrm{AE}} = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & J_{\mathrm{LF}} \end{pmatrix} \tag{1.44}$$

For B equal to the submatrices B_1 and B_2 and C equal to the submatrices C_1 and C_2, the stability of the steady-state equilibrium point is then determined by the system dynamic state Jacobian:

$$J_{\text{sys}} = A - BJ_{\text{AE}}^{-1}C \tag{1.45}$$

Special cases where the three Jacobians $J_{\text{sys}}, J_{\text{AE}}$, and J_{LF} can be more explicitly related are given in the following section.

1.7 Special Cases

There are two special cases where the standard power-flow Jacobian can be directly related to the system dynamic state Jacobian. We do not claim that these are necessarily realistic cases, only that they lead to cases where the three Jacobians can be explicitly related.

a. The first special case makes the following assumptions:
   (a1) Stator resistance of every machine is negligible $(R_{si} = 0,\ i = 1,\dots,m)$.
   (a2) Transient reactances of every machine are negligible $(X'_{di} = 0, X'_{qi} = 0,\ \ i = 1,\dots,m)$.
   (a3) Field and damper winding time constants for every machine are infinitely large $(E'_{qi} = \text{constant}, E'_{di} = \text{constant},\ \ i = 1,\dots,m)$.
   (a4) Constant mechanical torque to the shaft of each generator $(T_{Mi} = \text{constant},\ \ i = 1,\dots,m)$.
   (a5) Generator number one has infinite inertia. This together with (a1)–(a3) makes $V_1 = \text{constant}, \theta_1 = \text{constant}$ (infinite bus).
   (a6) All loads are constant power $(P_{Li}(V_i) = \text{constant}, Q_{Li}(V_i) = \text{constant}, i = 1,\dots,n)$.

With these assumptions, each δ_i is equal to its corresponding θ_i plus a constant, and each V_i is constant. Choosing ω_1 as ω_s and θ_1 as zero, the dynamic model for this special case (after eliminating P_i and Q_i) is

$$\frac{d\theta_i}{dt} = \omega_i - \omega_s \quad i = 2,\dots,m \tag{1.46}$$

$$M_i \frac{d\omega_i}{dt} = T_{Mi} + P_{Li} - \sum_{k=1}^{n} V_i V_k Y_{ik} \cos\left(\theta_i - \theta_k - \alpha_{ik}\right) - D_i\left(\omega_i - \omega_s\right) \quad i = 2,\dots,m \tag{1.47}$$

$$0 = -P_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik} \cos\left(\theta_i - \theta_k - \alpha_{ik}\right) \quad i = m+1,\dots,n \tag{1.48}$$

$$0 = -Q_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik} \sin\left(\theta_i - \theta_k - \alpha_{ik}\right) \quad i = m+1,\dots,n \tag{1.49}$$

with

$$T_{Mi} = \text{constant} \quad i = 1,\ldots,m \tag{1.50}$$

$$V_i = \text{constant} \quad i = 1,\ldots,m \tag{1.51}$$

$$P_{Li} = \text{constant} \quad i = 1,\ldots,n \tag{1.52}$$

$$Q_{Li} = \text{constant} \quad i = 1,\ldots,n \tag{1.53}$$

$$\theta_1 = 0 \tag{1.54}$$

The linearized form of this model is,

$$\begin{bmatrix} \dfrac{d\Delta\theta_g}{dt} \\ M\dfrac{d\Delta\omega_g}{dt} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & \mathrm{I} & 0 \\ K_1 & -D & K_2 \\ K_3 & 0 & K_4 \end{bmatrix} \begin{bmatrix} \Delta\theta_g \\ \Delta\omega_g \\ \Delta\theta_L V_L \end{bmatrix} \tag{1.55}$$

where $\Delta\theta_L V_L$ is the vector $\left[\Delta\theta_L^t \quad \Delta \mathrm{V}_L^t\right]^t$ of incremental load angles and voltages.

For this case (a), the algebraic equation Jacobian $(J_{\mathrm{AE}}^{(\mathrm{a})})$ is K_4. For nonsingular K_4, the system dynamic state Jacobian for this case (a) is

$$J_{\text{sys}}^{(\mathrm{a})} = \begin{bmatrix} 0 & I \\ M^{-1}\left(K_1 - K_2 K_4^{-1} K_3\right) & -M^{-1}D \end{bmatrix} \tag{1.56}$$

The determinant of $J_{\text{sys}}^{(a)}$ is (Sauer and Pai 1990)

$$\det J_{\text{sys}}^{(\mathrm{a})} = \frac{\det\left(K_1 - K_2 K_4^{-1} K_3\right)}{\det M}(-1)^{m-1} \tag{1.57}$$

The standard power-flow Jacobian as previously defined can be written in terms of these submatrices as

$$J_{\mathrm{LF}} = \begin{bmatrix} -K_1 & -K2 \\ K3 & K4 \end{bmatrix} \tag{1.58}$$

Again, for nonsingular K_4, the determinant of the power-flow Jacobian is (Sauer and Pai 1990)

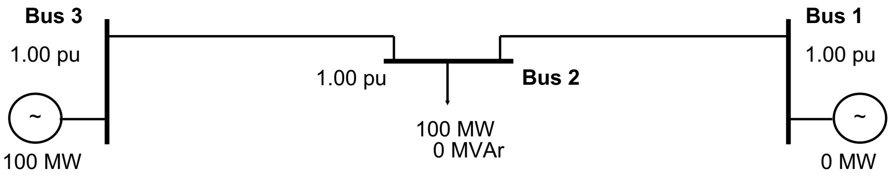

Fig. 1.1 Three-bus power system

$$\det J_{\mathrm{LF}} = \det K_4 \ \det\left(K_1 - K_2 K_4^{-1} K_3\right)\left(-1\right)^{m-1} \tag{1.59}$$

For this case, a clear relationship between the determinant of the standard power-flow Jacobian and the determinant of the system dynamic state Jacobian exists as

$$\det J_{\mathrm{sys}}^{(\mathrm{a})} = \frac{\det J_{\mathrm{LF}}}{\det K_4 \det M} \tag{1.60}$$

This means that under these assumptions, monitoring the power-flow Jacobian determinant can detect a possible dynamic instability. The basic structure of this case (a) is used frequently, but assumptions (a1)–(a3) are slightly different. The same structure of (1.46)–(1.49) can be obtained by assuming a constant voltage behind transient reactance model with the terminal buses eliminated. This leads to a non-standard power-flow Jacobian matrix that includes machine parameters in the bus admittance matrix.

The results of (1.58)–(1.60) above, which were previously published in Sauer and Pai, (Sauer and Pai 1990), were questioned by M. K. Pal, a discusser of Rajagopalan et al. (1992). The question had to do with the assumption of a nonsingular K_4. A single-machine example was presented by Pal in his discussion to Yorino et al. (1992).

This example had a special structure where the determinant of K_4 was in fact zero at the same condition as $\det J_{\mathrm{LF}}$ and $\det J_{\mathrm{sys}}^{(\mathrm{a})}$. However, additional analysis was not done to determine the general validity of the test. Additional testing by Verastegui (Verastegui 2000) indicated that, for more general systems, the power-flow Jacobian and system Jacobian can become zero while maintaining nonsingular K_4. Figure 1.1 shows a three-bus power system that was used to illustrate the result.

The impedance of the two lines is purely reactive with an impedance of 0.1 pu. The load at bus 3 starts out at 1.0 pu (power base is 100 MW). The two-generator bus voltages are maintained at 1.0 pu. The load at bus 3 remains at unity power factor for the entire example.

For this example, the load at bus 3 is increased from 1 pu while monitoring the values of $\det K_4, \det J_{\mathrm{LF}},$ and $\det J_{\mathrm{sys}}^{(\mathrm{a})}$. The values of these three quantities as the load is increased are shown in Figs. 1.2, 1.3, 1.4.

b. A second case where such a relationship can be firmly established was proposed in principle by Venikov et al. (1975). This special case makes the following assumptions:

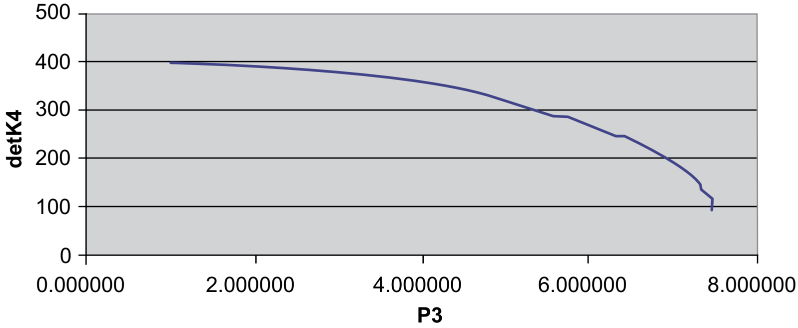

Fig. 1.2 Plot of det$\boldsymbol{K}_4$ versus load $\boldsymbol{P}_3$

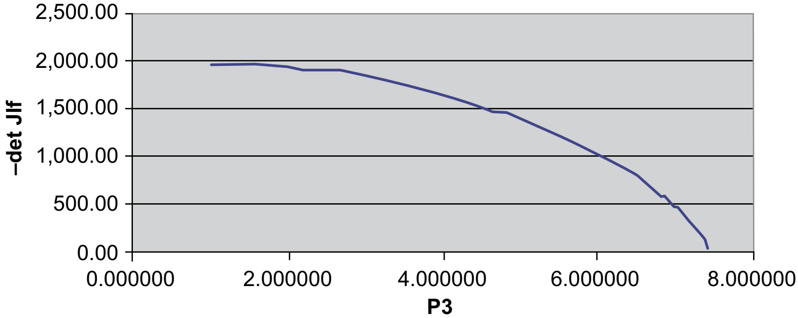

Fig. 1.3 Plot of $-$det$\boldsymbol{J}_{LF}$ versus load $\boldsymbol{P}_3$

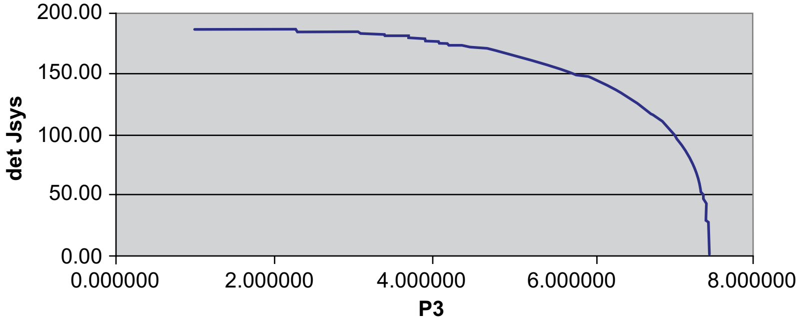

Fig. 1.4 Plot of det$\boldsymbol{J}_{\mathrm{sys}}^{(a)}$ versus load $\boldsymbol{P}_3$

(b1) Stator resistance is negligible $\left(R_{si} = 0, \quad i = 1, \ldots, m\right)$
(b2) No damper windings or speed damping

$$\left(T'_{qoi} = 0, D_i = 0, \;\; i = 1, \ldots, m\right)$$

(b3) High gain and fast excitation systems so that all generator terminal voltages are constant $(V_i = \text{constant}, \ \ i = 1,\ldots,m)$
(b4) Constant mechanical torque to the shaft of each generator

$$(T_{Mi} = \text{constant}, \ \ i = 1,\ldots,m)$$

(b5) Generator number one has infinite inertia and negligible reactances. This together with (b1)–(b3) makes $V_1 = \text{constant}$ and $\theta_1 = \text{constant}$ (infinite bus)
(b6) All loads are constant power

$$\left(P_{Li}(V_i) = \text{constant}, Q_{Li}(V_i) = \text{constant}, i = 1,\ldots,n\right).$$

With these assumptions, the special case dynamic model (after eliminating P_i and Q_i) is

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s \quad i = 2,\ldots,m \tag{1.61}$$

$$M_i \frac{d\omega_i}{dt} = T_{Mi} - \left[E'_{qi} + (X_{qi} - X'_{di}) I_{di}\right] I_{qi} \quad i = 2,\ldots,m \tag{1.62}$$

$$\begin{aligned} 0 &= V_i e^{j\theta_i} + jX'_{di}\left(I_{di} + jI_{qi}\right)e^{j(\delta_i - \pi/2)} \\ &\quad -\left[\left(X_{qi} - X'_{di}\right)I_{qi} + jE'_{qi}\right]e^{j(\delta_i - \pi/2)} \quad i = 2,\ldots,m \end{aligned} \tag{1.63}$$

$$\begin{aligned} 0 &= -\sum_{k=1}^{n} V_i V_k Y_{ik} e^{j(\theta_i - \theta_k - \alpha_{ik})} + V_i e^{j\theta_i}\left(I_{di} - jI_{qi}\right)e^{-j(\delta_i - \pi/2)} \\ &\quad + P_{Li} + jQ_{Li} \quad i = 2,\ldots,m \end{aligned} \tag{1.64}$$

$$0 = -\sum_{k=1}^{n} V_i V_k Y_{ik} e^{j(\theta_i - \theta_k - \alpha_{ik})} + P_{Li} + jQ_{Li} \quad i = m+1,\ldots,n, \tag{1.65}$$

with

$$T_{Mi} = \text{constant} \quad i = 1,\ldots,m \tag{1.66}$$

$$V_i = \text{constant} \quad i = 1,\ldots,m \tag{1.67}$$

$$P_{Li} = \text{constant} \quad i = 1,\ldots,n \tag{1.68}$$

$$Q_{Li} = \text{constant} \quad i = 1,\ldots,n \tag{1.69}$$

$$\theta_1 = 0. \tag{1.70}$$

The $m+n-2$ complex algebraic equations must be used to eliminate the $2m+2n-4$ real algebraic variables $E'_{qi}, I_{di}, I_{qi} (i=2,...,m), \theta_i (i=2,...,n), and V_i (i=m+1,...,n)$. We begin by first noting that from (1.63) and (1.64),

$$\left[E'_{qi}+\left(X_{qi}-X'_{di}\right)\right] I_{di} I_{qi}=-P_{Li}+\sum_{k=1}^{n} V_i V_k Y_{ik} \cos\left(\theta_i-\theta_k-\alpha_{ik}\right) \quad i=2,...,m. \tag{1.71}$$

This can be substituted into (1.62). Second, we note that (1.63) and (1.64) can be rewritten as

$$X_{qi} I_{qi}=-V_i \sin\left(\theta_i-\delta_i\right) \quad i=2,...,m \tag{1.72}$$

$$E'_{qi}-X'_{di} I_{di}=V_i \cos\left(\theta_i-\delta_i\right) \quad i=2,...,m \tag{1.73}$$

$$\begin{aligned} V_i I_{di}= & \sum_{k=1}^{n} V_i V_k Y_{ik} \sin\left(\delta_i-\theta_k-\alpha_{ik}\right)+P_{Li} \sin\left(\theta_i-\delta_i\right) \\ & -Q_{Li} \cos\left(\theta_i-\delta_i\right) \quad i=2,...,m \end{aligned} \tag{1.74}$$

$$\begin{aligned} V_i I_{qi}= & \sum_{k=1}^{n} V_i V_k Y_{ik} \cos\left(\delta_i-\theta_k-\alpha_{ik}\right)-P_{Li} \cos\left(\theta_i-\delta_i\right) \\ & -Q_{Li} \sin\left(\theta_i-\delta_i\right) \quad i=2,...,m. \end{aligned} \tag{1.75}$$

Eliminating E'_{qi}, I_{di}, and I_{qi} (simply equating I_{qi} in (1.72) and (1.75)) gives

$$\begin{aligned} -V_i^2 \sin\left(\theta_i-\delta_i\right)= & X_{qi} \sum_{k=1}^{n} V_i V_k Y_{ik} \cos\left(\delta_i-\theta_k-\alpha_{ik}\right) \\ -X_{qi} P_{Li} & \cos\left(\theta_i-\delta_i\right)-X_{qi} Q_{Li} \sin\left(\theta_i-\delta_i\right) \quad i=2,...,m. \end{aligned} \tag{1.76}$$

Using (1.71), (1.76), and (1.65), this special case dynamic model with E'_{qi}, I_{di}, and $I_{qi} (i=2,...,m)$ eliminated is

$$\frac{d\delta_i}{dt}=\omega_i-\omega_s \quad i=2,...,m \tag{1.77}$$

$$M_i \frac{\mathrm{d}\omega_i}{\mathrm{d}t}=T_{Mi}+P_{Li}-\sum_{k=1}^{n} V_i V_k Y_{ik} \cos\left(\theta_i-\theta_k-\alpha_{ik}\right) \quad i=2,...,m \tag{1.78}$$

$$\begin{aligned} 0= & V_i^2 \sin\left(\theta_i-\delta_i\right)-X_{qi} Q_{Li} \sin\left(\theta_i-\delta_i\right)-X_{qi} P_{Li} \cos\left(\theta_i-\delta_i\right) \\ & +X_{qi} \sum_{k=1}^{n} V_i V_k Y_{ik} \cos\left(\delta_i-\theta_k-\alpha_{ik}\right) \quad i=2,...,m \end{aligned} \tag{1.79}$$

$$0 = -P_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik} \cos\left(\theta_i - \theta_k - \alpha_{ik}\right) \;\; i = m+1,\ldots,n \tag{1.80}$$

$$0 = -Q_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik} \sin\left(\theta_i - \theta_k - \alpha_{ik}\right) \;\; i = m+1,\ldots,n, \tag{1.81}$$

with

$$T_{Mi} = \text{constant} \quad i = 1,\ldots,m \tag{1.82}$$

$$V_i = \text{constant} \quad i = 1,\ldots,m \tag{1.83}$$

$$P_{Li} = \text{constant} \quad i = 1,\ldots,n \tag{1.84}$$

$$Q_{Li} = \text{constant} \quad i = 1,\ldots,n \tag{1.85}$$

The linearized form of this dynamic model is

$$\begin{bmatrix} \frac{d\Delta\delta_g}{dt} \\ M\frac{d\Delta\omega_g}{dt} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & \mathrm{I} & 0 & 0 \\ 0 & 0 & K_1 & K_2 \\ K_5 & 0 & K_6 & K_7 \\ 0 & 0 & K_3 & K_4 \end{bmatrix} \begin{bmatrix} \Delta\delta_g \\ \Delta\omega_g \\ \Delta\theta_g \\ \Delta\theta_L V_L \end{bmatrix} \tag{1.86}$$

For the case (b), the algebraic equation Jacobian $J_{AE}^{(\mathrm{b})}$ is

$$J_{\mathrm{AE}}^{(\mathrm{b})} = \begin{bmatrix} K_6 & K_7 \\ K_3 & K_4 \end{bmatrix} \tag{1.87}$$

Defining B' and C' as

$$B' = \begin{bmatrix} K_1 & K_2 \end{bmatrix}, C' = \begin{bmatrix} K_5 & 0 \\ 0 & 0 \end{bmatrix} \tag{1.88}$$

for nonsingular $J_{AE}^{(b)}$, the system dynamic state Jacobian for case (b) is

$$J_{sys}^{(b)} = \begin{bmatrix} 0 & I \\ M^{-1}\left(-B' J_{AE}^{(b)-1} C'\right) & 0 \end{bmatrix} \tag{1.89}$$

and (Sauer and Pai 1990)

$$\det J_{sys}^{(b)} = \frac{\det\left(-B' J_{AE}^{(b)-1} C'\right)}{\det M} \left(-1\right)^{m-1} \tag{1.90}$$

Note that the eigenvalues of $J_{\text{sys}}^{(\text{b})}$ will all be either pure imaginary or will include one or more values which are positive real. We will consider the dynamic system case (b) to be stable as long as no eigenvalues are positive real. By rearranging rows and columns of the matrix in (1.86) (Sauer and Pai 1990),

$$\det J_{AE}^{(b)} \det\left(-B' J_{AE}^{(b)-1} C'\right) = \det K_5 \det J_{LF} \tag{1.91}$$

where J_{LF} is as in (1.58). This gives the following relationship between the determinants of the standard power-flow Jacobian, the algebraic equation Jacobian, and the system dynamic state Jacobian:

$$\det J_{sys}^{(b)} = \frac{\det K_5 \det J_{LF}}{\det M \det J_{AE}^{(b)}} \tag{1.92}$$

This means that under these assumptions, monitoring the power-flow Jacobian determinant can detect a possible dynamic instability. This is discussed in the following section.

1.8 Instability and Maximum Loadability

When studying a proposed load or interchange level, a power-flow solution is required before steady-state stability can be analyzed. If a power-flow solution cannot be found, then it is normally assumed that the proposed loading exceeded the "maximum power transfer" capability of the system. This maximum power transfer point is normally assumed to coincide with a zero determinant for the standard power-flow Jacobian.

Using this as a criterion, any load level that produces a zero determinant for the standard power-flow Jacobian is an upper bound and hence an optimistic value of the maximum loadability. It is also important to note that non-convergence of power-flows is also a matter of solution technique. Cases have been cited where Gauss–Seidel routines converge when Newton–Raphson routines do not.

If a standard power-flow solution and associated dynamic system equilibrium point are found (as described in Sects. 1.3 and 1.4), the stability of the point must be determined. In order to do this, the algebraic equation Jacobian must be nonsingular. This matrix is given by (1.44) in general, by K_4 for case (a), and by (1.87) for case (b). Assuming these algebraic equation Jacobians are nonsingular for a given case, steady-state stability must be evaluated from the eigenvalues of the system dynamic state Jacobian. This matrix is given by (1.45) in general, by (1.56) for case (a), and by (1.89) for case (b).

A system is at a critical point when the real part of one of its eigenvalues is zero. If a real eigenvalue is zero, then the determinant is zero. In the general case of (1.45), the zero eigenvalue due to the angle reference can easily be removed by using a dynamic model reduced in order by 1 (see Sect. 1.5). Clearly, many cases can be found where an equilibrium point can be critically unstable (at least one eigenvalue has a zero real part) and the power-flow Jacobian is nonsingular.

In cases (a) and (b), all detailed model dynamic states have been eliminated by making rather drastic assumptions. In special case (a), as long as det$M$ and det K_4 are nonzero and bounded, a dynamic equilibrium point exists and has a system dynamic state Jacobian that is singular if and only if the power-flow Jacobian is singular. In special case (b), we need to look at the matrix K_5. Examination of (1.79) shows that K_5 is diagonal with the ith diagonal entry equal to

$$K_{5i} = -V_i^2 \cos(\theta_i - \delta_i) + X_{qi} Q_{Li} \cos(\theta_i - \delta_i) \\ -X_{qi} P_{Li} \sin(\theta_i - \delta_i) - X_{qi} \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\delta_i - \theta_k - \alpha_{ik}) \quad i = 2,\dots,m \tag{1.93}$$

From (1.74),

$$K_{5i} = -X_{qi} V_i I_{di} - V_i^2 \cos(\theta_i - \delta_i) \quad i = 2,\dots,m \tag{1.94}$$

and from (1.73),

$$K_{5i} = -V_i \left(E'_{qi} + \left(X_{qi} - E'_{di} \right) I_{di} \right) \quad i = 2,\dots,m \tag{1.95}$$

In steady state, (1.4) and (1.11) give (with $R_{si} = 0$)

$$0 = V_i e^{j\theta_i} + jX_{qi} \left(I_{di} + jI_{qi} \right) e^{j(\delta_i - \pi/2)} \\ - j\left[E'_{qi} + \left(X_{qi} - X'_{di} \right) I_{di} \right] e^{j(\delta_i - \pi/2)} \quad i = 2,\dots,m \tag{1.96}$$

This means that K_{5i} can be zero (for nonzero V_i) only if (see (1.17))

$$V_i e^{j\theta_i} + jX_{qi} I_{Gi} e^{j\gamma_i} \neq 0 \quad i = 2,\dots,m \tag{1.97}$$

This also shows that K_{5i} is proportional to the magnitude of the voltage behind X_{qi} in steady state. This was discussed in Sect. 1.4 as a condition for the existence of a dynamic equilibrium from a power-flow solution. Thus, if a dynamic equilibrium point exists (Eq. (1.40) is satisfied), then K_5 cannot be singular. Thus, if det$M$ and det $J_{\mathrm{AE}}^{(\mathrm{b})}$ are nonzero and bounded, then the system dynamic state Jacobian of case (b) is singular if and only if the power-flow Jacobian is singular.

Since $J_{\text{sys}}^{(\text{b})}$ must have all pure imaginary eigenvalues to be stable, $\det J_{\text{sys}}^{(\text{b})}$ must be positive to be stable. Venikov et al. (1975) originally proposed the monitoring of the power-flow Jacobian determinant during power-flow iterations to see if it changed sign between the initial guess and the converged solution. The implication was that if it did, then the case (b) dynamic model would be unstable at that solution, and if it did not then the case (b) dynamic model would be stable (all pure imaginary eigenvalues).

Our interpretation indicates that they did not account for possible values of $\det K_5$ and $\det J_{\text{AE}}^{(\text{b})}$. A change in sign of either of these would affect stability issues. We have shown that $\det K_5$ would probably never change sign, but whether the $\det J_{\text{AE}}^{(\text{b})}$ changes sign or not remains an open question.

1.9 Post-Contingency Equilibrium Analysis

The earlier sections of this chapter presented a method to compute the equilibrium condition based on standard Power-Flow Methods (PFMs) assuming generator voltages and rated frequency. When contingencies occur, the equilibrium immediately following the contingency will be based on the constant control inputs. This section proposes and analyzes several techniques for computing this "post-contingency equilibrium condition."

Traditional contingency analysis of power systems uses either standard power-flow analysis or full transient simulation. The full transient simulation is normally only used to assess stability information following a hypothetical disturbance. It is rarely used to compute the steady-state equilibrium condition. The reference values fed into the control systems of a generator eventually determine the real power output and some desired bus voltage magnitude (possibly modified by a compensation circuit).

In a dynamic simulation, these control inputs are normally computed from a base case power-flow solution as discussed in Sect. 1.4. If the base case is subjected to a contingency (loss of line), the dynamic model (and therefore presumably also the real system) would respond according to these fixed control inputs until they are changed by an operator or other higher-level control. This means that the post-contingency equilibrium is determined by the fixed controls. This may not produce the same result as a simple power-flow solution modified to reflect a line outage.

This section reports on various techniques to compute the post-contingency equilibrium with fixed control inputs, and compares the results with simple power-flow results. Clearly, running a dynamic solution until steady state is reached (for stable systems) would give the nearly exact post-equilibrium condition and would be considered the benchmark for all other approximate or alternative solution techniques.

The nearly exact post-equilibrium condition should also be computable using analytical methods. However, commercial power-flow programs already solve a subset of the equilibrium equations and can be utilized to solve for part of the post-contingency equilibrium state. So, rather than creating a completely new Newton-based program to solve the entire problem, an alternative would be to create a new program only for the machine dynamic equilibrium equations and couple this in an iterative fashion with the standard power-flow program.

The potential for decoupling each of the machine equations is also a motivation for pursing this technique. The remainder of this chapter focuses on these techniques and compares them with the "Time Domain Method" (TDM) and "PFM".

The basic dynamic model that describes a power system has been given in Sect. 1.1 above. It includes the dynamics of the generator, exciter, and turbine governor along with the algebraic network constraints. The model was changed slightly from above to coincide with the model used in the commercial software known as PSS/E (Power System Simulator for Engineering).

The change included the use of power over per unit speed for the generator mechanical torque, and $-D_i((\omega_i - \omega_s)/\omega_i)$ rather than $-D_i(\omega_i - \omega_s)$ for the damping torque. This was important because the post-contingency generator speeds may not be equal to the nominal rated speed.

The power system equilibrium equations were obtained by setting the time derivatives of the dynamic model to zero. The equations are for an n bus system where the first m buses are connected to a generator. Also, all the machine and bus voltage angles were referenced to the machine angle (δ_1) of the generator at bus number 1 (also the slack bus for the power-flow portion of the analysis). This is denoted by the prime above all the angle variables.

These post-contingency equilibrium equations include the power-flow equations and the steady-state machine equations. The post-contingency equilibrium conditions contain the voltages (magnitude and angle) of a contingency analysis ($2n$ states) plus the states introduced by the machine, referred to as the *reduced machine states* ($5m$ states).

$$\underline{\mathbf{x}}_m^T = \left[\omega, \delta'_j, I_{di}, I_{qi}, E_{fdi}, P_{Mi}\right] \quad i = 1, \dots, m \quad j = 2, \dots, m \tag{1.98}$$

These are referred to as the *reduced machine states* because the other machine dynamic states have been eliminated by substitution. Their equilibrium values can be recovered by simple substitution at the end if desired. Collectively, there are $5m + 2n$ equations to be solved for the $5m + 2n$ states.

The challenge of post-contingency analysis is to compute the solution of the equilibrium algebraic equations. A solution of these equations by a "Full Newton Method" (FNM) should produce nearly the same post-contingency solution as the full dynamic simulation run to steady-state TDM.

1.9.1 Partitioned Newton Method

In the "partitioned Newton method" (PNM), the equilibrium equations are divided into two sets. The power-flow program is used to solve the power-flow equations. The dependent and independent variables for the power-flow equations are

$$\underline{\mathbf{x}}_{pf}^{T} = \left[V_i, \theta_j'\right] \quad i = m+1,\ldots,n \quad j = 2,\ldots,n \tag{1.99}$$

$$\underline{\boldsymbol{u}}_{pf}^{T} = \left[V_i, \theta_1', P_{Gj}, P_{Lk}, Q_{Lk}\right] \quad i = 1,\ldots,m \quad j = 2,\ldots,m \quad k = 1,\ldots,n \tag{1.100}$$

where P_{Gj} is defined as $P_j - P_{Lj}$. This power-flow step includes the solution of $2n-m-1$ equations for $2n-m-1$ states. The remaining $6m+1$ equations are solved using Newton's method for the $6m+1$ states. The dependent and independent variables for this set of machine equations are:

$$\underline{\mathbf{x}}_{mp}^{T} = \left[\theta_1', V_i, \omega, \delta_j', I_{di}, I_{qi}, E_{fdi}, P_{Mi}\right] \quad i = 1,\ldots,m \quad j = 2,\ldots,m \tag{1.101}$$

$$\underline{\boldsymbol{u}}_{mp}^{T} = \left[V_{refi}, P_{ci}, V_l, \theta_j', \delta_1', P_{Lk}, Q_{Lk}\right] \quad \begin{matrix} i = 1,\ldots,m & j = 2,\ldots,n \\ k = 1,\ldots,n & l = m+1,\ldots,n \end{matrix} \tag{1.102}$$

The steps for computing the equilibrium condition by the PNM are shown in Fig. 1.5.

Starting from the base case power-flow solution and the reference values of the generators that satisfy this condition, a disturbance in the network is introduced that changes the network topology. The machine equations are then solved using Newton's method to give a new voltage value of the slack bus generator and a new voltage magnitude and real power output of the Power–Voltage (PV) bus generators.

These are the new independent variable values for the power-flow equations. Then the power-flow is solved to provide the new network voltages that are used to solve the machine equations. This loop of updating the independent variables in each equation set is made until the change in the power and voltage values of the generators becomes negligible.

In this technique, the calculation of the power and voltage values of the generators was done by solving all of the machine equations together using Newton's method. This method alternates between this Newton solution and a standard power-flow solution. It does not exploit the natural decoupling between generator dynamics.

1.9.2 Decoupled Newton Method

All the machines are coupled together by the synchronism of the machine speed at equilibrium. In fact, the speed term ω is the only term that couples the different machines together among the machine states. In the "decoupled Newton method"

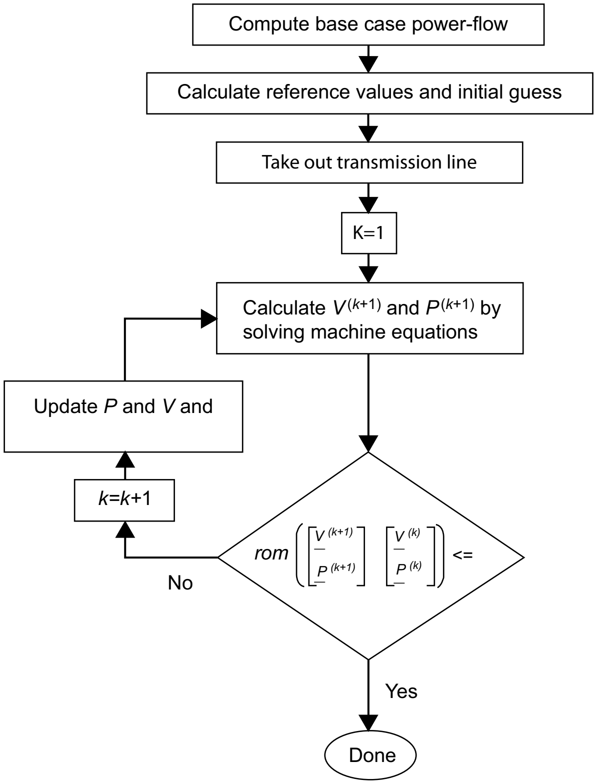

Fig. 1.5 Post-contingency equilibrium calculation. (Yeu 2005)

(DNM), the speed term is solved for by only using the power-flow solution and the machine states of the slack bus generator. This decouples the calculation of the power and voltage values of the generators from other generator machine states.

As before in the PNM, the power-flow program is utilized to solve the power-flow equations. Then the slack bus voltage magnitude and angle are calculated by solving the machine equilibrium equations for $i = 1$ using Newton's method. The dependent and independent variables for this calculation are shown in (1.103) and (1.104). The decoupling of the machines allows the voltage magnitudes and angles of all the other buses, including generator buses, to be independent variables. These voltage values are obtained from the power-flow solution.

$$\underline{\boldsymbol{x}}_{md_1}^T = \left[\theta_1', V_1, \omega, I_{d1}, I_{q1}, E_{fd1}, P_{M1}\right] \tag{1.103}$$

$$\underline{\boldsymbol{u}}_{md_1}^T = \left[V_{ref1}, P_{c1}, V_k, \theta_k', \delta_1', P_{Li}, Q_{Li}\right] \quad k = 2,\ldots,n \quad i = 1,\ldots,n \tag{1.104}$$

The slack bus calculation must be made first in the DNM. This is because, while all the voltage magnitude, real power output, and machine speed values of each generator are updated after the calculation is made for all the generators, the slack bus angle must be updated before the calculation of the next set of power and voltage values of the PV buses.

An update in the slack bus angle along with the other updates would just change the angle reference value of the power flow and would neglect the control of the power output of the slack bus generator driven by the power reference, P_{c1}. The update in the slack bus angle is made before the other updates so that the power and voltage updates of the PV bus generators take into account not only its own control but also the slack bus generator power requirements through the network constraints.

The calculation of the remaining generator voltage and power values can be made in an arbitrary order using Newton's method. The dependent and independent variables for this calculation are shown in (1.105) and (1.106) for $j = 2,\ldots,m$.

$$\underline{\boldsymbol{x}}_{mj}^{T} = \left[\delta'_j, V_j, I_{dj}, I_{qj}, E_{fdj}, P_{Mj} \right] \tag{1.105}$$

$$\underline{\boldsymbol{u}}_{mj}^{T} = \left[V_{refj}, P_{cj}, \omega, V_k, \theta'_i, P_{Li}, Q_{Li} \right] \quad \underset{k \neq j}{k} = 1,\ldots,n \quad i = 1,\ldots,n. \tag{1.106}$$

The steps of the DNM are similar to those of the PNM. The starting point is the base case power flow. Then power and voltage reference values of the generators are calculated to satisfy the base case condition. After a line is opened, the slack bus voltage magnitude and angle are calculated and the slack bus angle is updated. Then the PV bus generator real power and voltage magnitude values are calculated in arbitrary order or in parallel as previously described.

If the change in the power and voltage values of the generators is within the tolerance range, then the equilibrium solution is reached. Otherwise, the power and voltage values of the generators are updated and used to calculate a new power-flow solution. Another set of power and voltage values of the generators are then calculated using the new power-flow solution and this loop of calculating the updates and calculating the power flow is repeated until the change in the updates is within the tolerance range.

Figure 1.5 with one modification describes these steps. The block for calculating the voltage and power values is replaced by *m* blocks. The first block represents the calculation of the slack bus voltage values and the update of the slack bus angle, and the remaining blocks represent the voltage and power update of each PV bus generators.

Since the machines are decoupled, the size of the Jacobian used in each of the Newton's methods to calculate the machine states is 7×7 for the slack bus calculation and 6×6 for each of the $m-1$ PV bus calculations.

This is a significant reduction in the size of the Jacobian from the one used in the PNM, which is $(6m+1) \times (6m+1)$. Another advantage of the DNM is that different machine models can be incorporated for the equilibrium analysis easily. This is because of the fact that independent variables for a given machine stay the same for different machine models while the dependent variables are independent from other machine states.

Table 1.1 Maximum pu voltage magnitude difference from TDM (nine-bus case)

Bus	FNM	PNM	DNM	PFM
1	0.0001	0.0001	0.0001	0.0031
2	0.0004	0.0004	0.0004	0.0088
3	0.0002	0.0002	0.0002	0.0131

Table 1.2 Maximum pu real power difference from TDM (nine-bus case)

Bus	FNM	PNM	DNM	PFM
1	0.0052	0.0052	0.0052	0.0311
2	0.0036	0.0036	0.0036	0.0184
3	0.0014	0.0014	0.0014	0.0177

1.9.3 Test Cases

The post-contingency equilibrium analysis was done on two power-system cases using five methods:

- TDM
- FNM
- PNM
- DNM
- PFM

As a benchmark reference, a dynamic simulation using the PSS/E-30 software was run to equilibrium to calculate the post-contingency equilibrium state for the TDM. The next three methods were programmed in Matlab 7.0.1. The Power System Toolbox 2.0 power-flow program was used to solve the power-flow equations in the PNM, DNM, and PFM. The first test case was the so-called Western Electric Coordinating Council (WECC) nine-bus power system (Sauer and Pai 1998). This system contains three machines and nine transmission lines. Only three single line outage cases were stable according to the PSS/E simulations.

Out of these three cases, the maximum per unit differences of the voltage magnitude and real power output from the four methods were compared to the results from the TDM equilibrium analyses with the results shown in Tables 1.1 and 1.2. The maximum differences are shown only for the generator buses.

The second case was the 57-bus test case (Power System Test Case Archive 2014). This system contains 6 machines and 80 transmission lines. All but two single line outage cases were stable.

The maximum per unit differences of the voltage magnitude and real power output from the four methods were compared to the results from the TDM equilibrium analyses with the results shown in Tables 1.3 and 1.4. As before, the maximum differences are only shown for the generator buses.

The results of the post-contingency equilibrium analysis using the FNM, PNM, and DNM and the results from the TDM are basically the same as would be expected. The only explanation we have for the errors shown is the truncation error of

Table 1.3 Maximum pu voltage magnitude difference from TDM (57-bus case)

Bus	FNM	PNM	DNM	PFM
1	0.0001	0.0001	0.0001	0.0054
2	0.0003	0.0003	0.0003	0.0653
3	0.0004	0.0004	0.0004	0.0141
6	0.0002	0.0002	0.0002	0.0162
8	0.0001	0.0001	0.0001	0.0015
9	0.0004	0.0004	0.0004	0.0372
12	0.0002	0.0002	0.0002	0.0058

Table 1.4 Maximum pu real power difference from TDM (57-bus case)

Bus	FNM	PNM	DNM	PFM
1	0.0058	0.0058	0.0058	0.2793
2	0.0027	0.0027	0.0027	0.0442
3	0.0076	0.0076	0.0076	0.0453
6	0.0021	0.0021	0.0021	0.0440
8	0.0045	0.0045	0.0045	0.0532
9	0.0027	0.0027	0.0027	0.0440
12	0.0054	0.0054	0.0054	0.0501

numerical integration to steady state and convergence mismatch of iterative solvers. This shows that the different equilibrium analyses with fixed control inputs accurately characterized the post-contingency equilibrium states.

On the other hand, there are some significant differences between the PSS/E simulation (TDM) and the traditional contingency analysis (PFM). This is also expected since the voltage magnitude and the real power output of a generator do not necessarily stay constant from the base case to the post-contingency equilibrium state as is generally assumed in standard power-flow contingency analysis.

Generally, the voltage and power values did not deviate by a great amount from the base case to the post-contingency state. However, significant deviations such as the 0.06 pu voltage difference on bus 2 for the 57-bus case can occur. Deviations of this magnitude can affect the reliability of a power system considerably and can introduce errors in a contingency analysis.

1.10 Conclusions

Standard power flow is used to find system voltages for a specified level of loading or interchange (regardless of the dynamic load model). It is also the starting point for determining the initial conditions for dynamic analysis. The standard power-flow Jacobian can provide information about the existence of a steady-state equilibrium point for a specified level of loading or interchange. There are two very special cases when the determinant of the standard power-flow Jacobian implies something about the steady-state stability of a dynamic model.

While these two cases have been presented previously, the extensions here illustrate the validity of the basic results. Both of these cases involve very drastic assumptions about the synchronous machines and their control systems. The load level, which produces a singular power-flow Jacobian, should be considered an optimistic upper bound on maximum loadability. The actual upper bound would be either the same or lower since it requires both the existence of a solution *and* stable dynamics.

For voltage collapse and voltage instability analysis, any conclusions based on the singularity of the standard power-flow Jacobian would apply only to the phenomena of voltage behavior near maximum power transfer. Such analysis would not detect any voltage instabilities associated with synchronous machine characteristics or their controls.

This chapter also addressed the issue of post-contingency steady state conditions. New analytical methods of computing the post-contingency equilibrium state of a power system have been described and illustrated. The main observation is that all of the methods provide a solution, which agrees reasonably well with the so-called exact solution of a dynamic simulation run to steady state. The particular illustrations also indicated that there can be substantial error between this true post-contingency equilibrium and standard power-flow contingency analysis.

The FNM is a brute force analytical method requiring an entirely new software program to solve all of the machine and network equations simultaneously. The PNM is an alternative method that divides the solution into two parts—one utilizing a standard power-flow program and another new one for the machine equations—alternating solution updates. The DNM is the most promising method because it exploits the utilization of a standard power-flow program and also decouples the machine equations so that they can be solved by a very low-order iterative program. This also allows the extension to more detailed generator dynamic models that include more complex local dynamics and control actions.

Recent activity in this area has focused on using phasor measurement unit data to detect steady-state stability criteria. This activity is summarized in Reinhard et al. (2013) where Thévenin equivalents are used to represent complete systems on either side of each transmission line. The results propose that as the system approaches a steady-state stability limit, the angle across the entire system approaches 90° (between the two Thévenin equivalents) for at least one line in the system.

Acknowledgments The authors thank Mr. Rodney Yeu for contributing the results from his graduate thesis on computing the post-contingency equilibrium point. Special thanks also to Mr. Tom Moone for providing editorial assistance. This work was supported in part by National Science Foundation grants NSF ECS 87-19055 and NSF EEC 0120153, and in part by the Grainger endowments to the University of Illinois.

References

Pai MA (2004) Computer techniques in power system analysis, 2nd ed, McGraw-Hill, New Delhi
Power System Test Case Archive (2014) University of Washington College of Engineering, The IEEE 57 bus network model, http://www.ee.washington.edu/research/pstca/. Accessed 5 May 2004
Pulgar-Painemal HA, Sauer Peter W (2011) Reduced-order model of type C wind turbine generators. Electr Power Syst Res 81(4):840–845
Rajagopalan C, Lesieutre B, Sauer PW, Pai MA (1992) Dynamic aspects of voltage/power characteristics. IEEE Trans Power Syst 7(3):990–1000
Reinhard KE, Sauer Peter W, Dominguez-Garcia Alejandro D (2013) On computing power system steady-state stability using synchrophasor data, Proceedings, 46th Hawaiian International Conference on System Science (HICSS), 7–10 Jan 2013, Maui, Hawaii.
Sauer PW, Pai MA (1990) Power system steady-state stability and the power-flow Jacobian. IEEE Trans Power Syst 5(4):1374–1383
Sauer PW, Pai MA (1998) Power system dynamics and stability. Prentice Hall, Upper Saddle River
Venikov VA, Stroev VA, Idelchick VI, Tarasov VI (1975) Estimation of electrical power system steady-state stability. IEEE Trans Power Appar Syst PAS-94(3):1034–1040
Verastegui P (2000) Examination of the relationship between power-flow Jacobian singularities and system dynamics Jacobian singularities, special ECE 272 project report, Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign
Yeu R (2005) Postcontingency equilibrium analysis of power systems, MSEE Thesis, Department of Electrical and Computer Engineering. University of Illinois, at Urbana-Champaign, May 2005, pp 1–63
Yorino N, Sasaki H, Masuda Y, Tamura Y, Kitagawa M, Oshimo A (1992) An investigation of voltage instability problems. IEEE Trans Power Syst 7(2):600–611