A Novel Approach to Robust Weighted Averaging of Auditory Evoked Potentials

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Abstract. The paper describes the robust weighted averaging method applied to averaging of auditory brainstem responses. This type of signals is characterized with extremely low signal-to-noise ratio. Suppression of noise that contaminates this type of signals can be achieved with the use of the averaging technique. The auditory evoked potentials are time-aligned and then the average template is determined. The weighted averaging operation can be regarded as special case of clustering. In this work the averaging process is formulated as the problem of certain criterion function minimization. The maximum likelihood estimator of location based on the generalized Cauchy distribution is used as the measure of dissimilarity function. The proposed methods performance is experimentally evaluated and compared to the reference methods in the presence of the artificial noise and in the case of real signals. The experiments show usefulness of the proposed method for robust weighted averaging of periodic signals, for instance the evoked potentials.

Keywords: auditory brainstem response, robust weighted averaging, General Cauchy distribution.

1 Introduction

Sensory evoked potentials (EPs) are time-aligned changes of the electrical activity of the brain recorded from the human scalp. These potentials represent aggregated electrical activity from a large number of temporally and spatially aligned neurons of the brain and arise as the response to a variety of controlled external stimuli [3,9]. The clinical utility of EPs is based on their ability to show abnormal sensory system conduction, to discover asymptomatic engagement of a sensory system, to help define the anatomic distribution of a disease process and monitor changes in a patient's neurological status [19]. In clinical practice the auditory (AEPs), visual (VEPs) and somatosensory (SEPs) are tested most frequently and they can be recorded in patients who are anaesthetised or comatose [3,19]. In general, the electroencephalogram (EEG) is extensively used within neurophysiology, cognitive neuroscience, cognitive psychology, and brain-computer interfacing [9].

The peak amplitudes and the latencies are commonly used for quantitative evaluation of EPs which last for a few hundreds of milliseconds, with it various

E. Pietka, J. Kawa, and W. Wieclawek (eds.), *Information Technologies in Biomedicine, Volume 4,* Advances in Intelligent Systems and Computing 284,
DOI: 10.1007/978-3-319-06596-0_30,

features categorized into early, middle and late components [3,19]. Early components of AEPs are auditory brainstem responses (ABRs) and auditory steady state response arising within 0-10 ms [3]. AEPs are much smaller than the EEG and early and middle components typically have a signal-to-noise ratio (SNR) in the range from -30 to -20 dB [3]. The most common method to isolate the AEP from background EEG is averaging the EEG responses to multiple identical auditory stimulations. The ABR can be regarded as the deterministic component for all stimuli (because it is time-locked) while the background noise (including spontaneous EEG) will vary and thus be reduced by averaging [17]. The assumptions for realize the averaging are the following: (i) the signal remains constant from trial to trial, (ii) the noise on any trial is uncorrelated with the noise on other trials, and (iii) the noise statistics remain stationary from trial to trial [5,7]. But if the noise varies from one trial to the next, averaging is less effective. The random phenomena, sudden transients as well as non-stationarities in the signals (caused both by a physiological adaptation process to the series of stimuli or by possible pathological evidences), do cause that the obtained average, in most of the cases, is only a very general information about a mean behaviour of the system under study.

From another point of view, the averaging methods are similar to the fuzzy robust clustering methods [2,6]. The averaging process can be regarded as a special case of clustering with only one prototype. The clustering method should be robust for data corrupted by outliers or heavy-tailed distributed noise. The influence of outliers on the averaged signal can be reduced by choosing a median as the aggregation operation. However the median averaging does not only remove the outliers but also the rest of data [11,13]. The alternative approach is based on the trimmed mean method [13]. The possibility of using the myriad cost function to develop a procedure for robust weighted averaging is presented in [15]. The approach based on L_p-norm is presented in [16]. An application of Vapnik ε-insensitive function allows to increase the robustness of the weighted averaging is presented in [11].

The goal of this paper is to show the novel approach to robust weighted averaging applied to processing of ABRs. This paper presents a robust cost function based on Generalized Cauchy probability distribution which plays role of the dissimilarity function in the weighted averaging method based on the minimizing of certain criterion function. The paper is divided into four sections. Section 2 presents the idea of the weighted averaging method based on the minimization of the scalar criterion function and introduces the proposed method. Section 3 describes the proposed experiments. Section 4 presents the obtained results and discussion. Finally, the conclusions are given in Section 5.

2 Methods

2.1 Weighted Averaging

Let us consider N sweeps of ABR potentials where $\mathbf{x}_i = [x_{i1}, x_{i2}, \ldots, x_{iM}]^T$ is the ith sweep which consists of M samples and $1 \leq i \leq N$. Let $\mathbf{y} = [y_1, y_2, \ldots, y_M]^T$

is the averaged signal and $\mathbf{w} = [w_1, w_2, \ldots, w_N]^T$ is the weight vector that satisfies the following conditions:

$$\underset{1 \le i \le N}{\forall} \quad w_i \in [0,1], \quad \sum_{i=1}^{N} w_i = 1. \tag{1}$$

Each of the $\mathbf{x}_i$ sweep is the sum of a deterministic component and the noise $\mathbf{x}_i = \mathbf{s} + \nu_i$. The noise ν_i is a random component with zero mean and variance σ_i^2. The weighted average of N sweep waveforms is given by:

$$\mathbf{y} = \sum_{i=1}^{N} w_i \mathbf{x}_i. \tag{2}$$

The general idea of weighted averaging rests on the assumption that each signal $\mathbf{x}_i$ affects the resulting averaged signal $\mathbf{y}$ in the manner specified by the value of weight w_i. The simplest case of weighted averaging is the arithmetic averaging, where all weights are the same, equal to N^{-1}. Estimation of the weights values is a crucial for the process of averaging. The easy mathematical consideration shows that, given the variance σ_i^2 of each individual sweep, the weights are the following [10]:

$$w_i = \sigma_i^{-2} \left(\sum_{j=1}^{N} \sigma_j^{-2} \right)^{-1}. \tag{3}$$

This method is one of the reference methods and is denoted as the WA (the weighted averaging) method. There are different methods to estimate the noise variances or to estimate the optimal weights without direct knowledge of the noise variance. One of such approach is presented below.

2.2 Criterion Function Minimization

The WACFM (weighted averaging method based on criterion function minimization) is based on minimization the following scalar function [2,11]:

$$I_m(\mathbf{w}, \mathbf{y}) = \sum_{i=1}^{N} \sum_{j=1}^{M} (w_i)^m \rho(z_{ij}), \tag{4}$$

where $z_{ij} = x_{ij} - y_j$, $m \in (1, \infty)$ is the assumed weighting exponent and $\rho(\cdot)$ is a measure of dissimilarity for the vector argument. The eq. (4) can be rewritten as:

$$I_m(\mathbf{w}, \mathbf{y}) = \sum_{i=1}^{N} (w_i)^m \left(\sum_{j=1}^{M} \rho(z_{ij}) \right), \tag{5}$$

where $\sum_{i=1}^{M} \rho(z_{ij}) = \rho(\mathbf{z}_i)$ and then $\mathbf{z}_i = \mathbf{x}_i - \mathbf{y}$. The $\rho(\cdot)$ function is a measure of dissimilarity for a vector argument [11]. It plays a similar role to the cost function in the robust M-estimator [2]. The task of searching for an optimal an optimal weight vector $\mathbf{w}^*$ and optimal averaged signal $\mathbf{y}^*$, can be formulated as follows:

$$I_m(\mathbf{w}^*, \mathbf{y}^*) = \min_{\mathbf{w},\mathbf{y}} I_m(\mathbf{w}, \mathbf{y}). \tag{6}$$

The problem of I_m minimization is considered as the constrained optimization problem and the method of Lagrange multipliers is applied [11] to find the optimal weights vector $\mathbf{w}$ which are given in the following way:

$$\underset{1\le i\le N}{\forall} \qquad w_i = \frac{\rho\left(\mathbf{z}_i\right)^{1/(1-m)}}{\sum_{j=1}^{N}\left[\rho(\mathbf{z}_j)\right]^{1/(1-m)}}. \tag{7}$$

The robustness of the weighted averaging depends strictly on the function $\rho(\mathbf{z})$ which should be symmetric positive-definite function [2,11]. The weights w_i corresponds to membership (belonging) of $\mathbf{x}_i$ to the prototype (the averaged signal) [2].

If we assume that $\mathbf{y}$ is fixed, the next step of the algorithm consists in estimation of the $\mathbf{y}$ averaged signal. From the theory of M-estimators, the derivative of $\rho(z)$ is called the influence function $\psi(z) = \frac{\mathrm{d}\rho(z)}{\mathrm{d}z}$, while the weighted function is defined as $\varpi(z) = \frac{\psi(z)}{z}$ [12]. Then, for given data set $\mathbf{x} = [x_1, x_2, ..., x_N]$, using the weighted function of the dissimilarity function, the averaged signal $\mathbf{y}$ can be found by applying the fixed-point search algorithm which can be written as [16]:

$$\mathbf{y}^{(k+1)} = \frac{\sum_{i=1}^{N}(w_i)^m \cdot \varpi(\mathbf{x}_i - \mathbf{y}^{(k)}) \cdot \mathbf{x}_i}{\sum_{i=1}^{N}(w_i)^m \cdot \varpi(\mathbf{x}_i - \mathbf{y}^{(k)})}, \tag{8}$$

where the superscript (k) denotes the iteration number. The algorithm is regarded as convergent when $||\mathbf{w}^{(k+1)} - \mathbf{w}^{(k)}|| < \epsilon$, ϵ is a small positive value (e.g. $\epsilon = 10^{-6}$).

2.3 Generalized Cauchy Distribution-Based Cost Function

The family of Generalized Cauchy Distribution (GCD) has the feature that its pdf has the closed form for the whole family and also has algebraic tails which makes it suitable to model many impulsive process in real world of signals (the Cauchy distribution is the special case of the α-stable distribution for $\alpha = 1$ [8]). The pdf of the GCD has the following form [1]:

$$f(z) = a\sigma(\sigma^p + |z|^p)^{(-2/p)} \tag{9}$$

with $a = p\Gamma(2/p)/2(\Gamma(1/p))^2$; σ is the scale parameter and p is the tail constant. The family of GCD contains the Meridian [1] ($p = 1$) and Cauchy ($p = 2$) distributions as special cases. The adjustable and closed-form feature of the GCD makes it useful for robust estimation and filtering techniques [1]. Equation (9) is the basis for determination of the maximum likelihood estimators of location; the cost function has the following form:

$$\rho(z) = \log\left(\sigma^p + |z|^p\right), \tag{10}$$

where $\sigma > 0$ and $0 < p \leq 2$. Using (5) and (10), the scalar criterion function can be rewritten as:

$$I_m(\mathbf{w}, \mathbf{y}) = \sum_{i=1}^{N} (w_i)^m \sum_{l=1}^{M} \log\left(1 + \left(\frac{|x_{il} - y_l|}{\sigma}\right)^p\right). \tag{11}$$

According to (7) the weights w_i are given by:

$$\underset{1\leq i\leq N}{\forall} \; w_i = \frac{\left[\sum_{l=1}^{M} \log\left(1 + \left(\frac{|x_{il}-y_l|}{\sigma}\right)^p\right)\right]^{1/(1-m)}}{\sum_{j=1}^{N}\left[\sum_{l=1}^{M} \log\left(1 + \left(\frac{|x_{jl}-y_l|}{\sigma}\right)^p\right)\right]^{1/(1-m)}}. \tag{12}$$

The influence function of (10) has the form $\psi(z) = \frac{p|z|^{p-1}\mathrm{sgn}(z)}{\sigma^p + |z|^p}$ and the corresponding weighted function is given by:

$$\varpi(z) = \left(\frac{1}{p}|z|^{2-p}\left(\sigma^p + |z|^p\right)\right)^{-1}. \tag{13}$$

The averaged signal $\mathbf{y}$ is obtained with using formulas (8) and (13) in the following form:

$$\mathbf{y}^{(k+1)} = \frac{\sum_{i=1}^{N} (w_i)^m \mathbf{x}_i \left[\sum_{l=1}^{M} \frac{|x_{il}-y_l^{(k)}|^{2-p}}{p}\left(\sigma^p + |x_{il} - y_l^{(k)}|^p\right)\right]^{-1}}{\sum_{i=1}^{N} (w_i)^m \left[\sum_{l=1}^{M} \frac{|x_{il}-y_l^{(k)}|^{2-p}}{p}\left(\sigma^p + |x_{il} - y_l^{(k)}|^p\right)\right]^{-1}}. \tag{14}$$

Finally, using (12) and (14) the algorithm of the weighted averaging with criterion function minimization with the GCD cost function (WACFMGC) can be described as follows, where ϵ is a preset parameter:

1^o fix $m = 2$, σ and p, initialize $\mathbf{y}^{(0)} = \mathbf{0}$, set the iteration index $k = 1$,
2^o calculate $w_i^{(k)}$ for the kth iteration using the formula (12),
3^o update the averaged signal for the kth iteration $\mathbf{y}^{(k)}$ using the formula (14), and $\mathbf{w}^{(k)}$
4^o if $||\mathbf{w}^{(k+1)} - \mathbf{w}^{(k)}|| > \epsilon$ then $k \leftarrow k + 1$ and go to 2^o.

3 Experiments

The real ABR data was recorded with the participation of a young healthy man (24 years old) with a properly functioning sense of hearing. He sat in a chair in a acoustically shielded room. Recordings were obtained using standard acquisition and stimulus parameters setup. The auditory stimulus was delivered as a rarefaction 'click stimulus' with a 0.1 ms pulsewidth from the standard TDH-49p supra-aural Telephonics headphones. The evoked ABRs were recorded with a bioamplifier at sampling frequency 48 kHz and bandpass filtered within 100-3000 Hz. Amplifier gain was 1000, A/D converter had 12 bits (providing the resolution of 0.084 μV/bit), amplitude range 5.5 mV. The example of the noisy ABR sweep and the averaged ABR are presented in Figure 1.

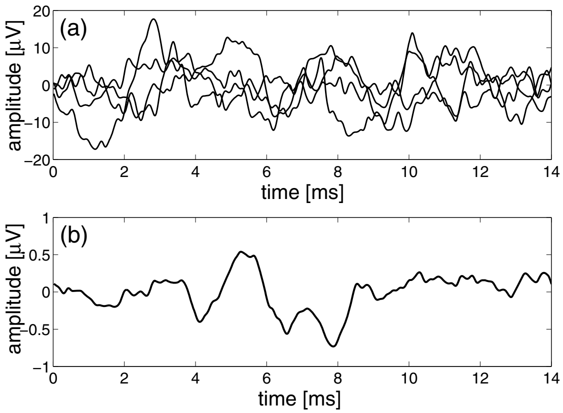


Fig. 1. (a) Examples of four raw ABRs waves, (b) the ABR wave after arithmetic averaging of 1800 sweeps at 40 dB normal hearing level. Stimulus is presented at zero time point

The simulated reference signal used in this study is based on ABR and consists of typical waves. The three sinc functions represent prominent ABR waves I, III and V [3]:

$$s(t) = 0.4\mathrm{sinc}\left[0.13\pi(4t - 6 + l)\right] + 0.4\mathrm{sinc}\left[0.13\pi(4t - 14 + l)\right] \\ -\mathrm{sinc}\left[0.13\pi(4t - 24 + l)\right], \tag{15}$$

where l determines the latency variation and in this work $l = 0$. The artificial reference signal is shown in Figure 2.

In order to simulate the real conditions of data recordings the symmetric α-stable (SαS) distribution [8] is applied to model the noise. The level of impulsiveness is controlled with the characteristic exponent α. The case of $\alpha = 2.0$ corresponds to the Gaussian probability distribution and $\alpha = 1.0$ corresponds to the Cauchy distribution. Figure 3 presents histogram of the characteristic

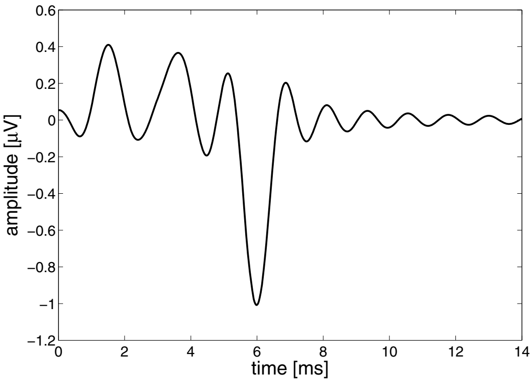


Fig. 2. Ideal artificial reference ABR wave

exponent α over 1000 sweeps of ABR. In this study α is calculated applying method presented in [14]. As can be seen, a certain part of sweeps are disturbed noise that has an impulsive nature. For that reason in this work α varies from 1.8 to 2.0 with the step 0.05.

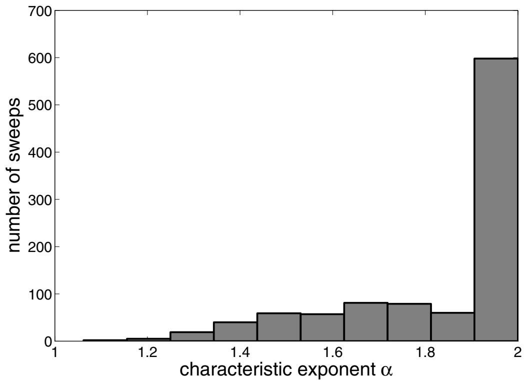


Fig. 3. Histogram of the characteristic exponent α calculated for 1000 ABRs sweeps

The class of the symmetric α-stable distributions does not have finite moments of the second or higher order. It means that the use of variance in the standard definition of the signal-to-noise ratio is meaningless [18]. For that reason the Geometric-SNR (GSNR) is applied [4]:

$$\mathrm{GSNR} = \frac{1}{2C_g}\left(\frac{A}{S_0}\right)^2, \tag{16}$$

where $C_g = e^{C_e} \approx 1.78$ is the exponential of the Euler constant, A is the amplitude of a modulated signal in an additive-noise channel with noise geometric power S_0. The normalization constant $2C_g$ is used to ensure that the definition of the GSNR corresponds to that of the standard SNR if the channel noise is Gaussian. The geometric power definition is the following $S_0 = \exp\left(\frac{1}{N}\sum_{i=1}^{N}|x_i|\right)$ [4]. A series of 1000 ABRs sweeps are generated with the same deterministic component (given with eq. (15)) disturbed by the artificial noise, according to the two predefined simulation patterns — for the 1st, the 2nd, the 3rd, the 4th and the 5th part of ABR sweeps (each part containing 200 cycles), the GSNR values are equal to:

P1: -30, -20, -10, -15, -25 dB, respectively.
P2: -30, -30, -15, -30, -30 dB, respectively.

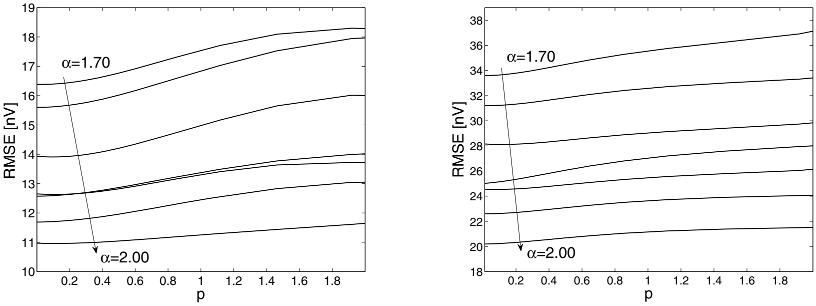
These two simulation patterns allow to test the efficiency and quality of the proposed methods of weighted averaging in the presence of artificial impulsive noise as well as real noise. The P1 and P2 patterns represent the case of lower and greater power noise variations.

4 Results and Discussion

The averaging process should not deform the signal. For that reason, the presented methods are evaluated using the root mean-square error (RMSE) between the deterministic component and the averaged signal. Subtraction of the deterministic component from the averaged signal gives the residual noise. The second index used to evaluate the quality of the tested methods, is the maximal absolute difference between the deterministic component and the averaged signal (MAX). The methods based on minimization of scalar criterion function are initialized with zero vector $\mathbf{y}$ and $m = 2$. The proposed methods performance is compared to that of the arithmetic mean (AM), the trimmed mean (TM) and the classical weighted averaging (WA) method. All experiments were done in MATLAB environment.

4.1 Selection of *p*

Selection of the optimal values for the p and σ parameters of the WACFMGC method is important but there is a concern that it will be attuned to the analysed signal only. In [1] the multiparameter estimation algorithm is presented that tries to solve this problem. This solution is suitable in the case of robust filtering [1], but in the case of cycles averaging it would require additional optimization of p and σ, leading to an increase of the computational complexity without the warranty to achieve the parameters optimal values. The influence of p on RMSE value obtained with the WACFMGC method is presented in Figure 4. The smallest values of RMSE are obtained for small p for both experiments and for further research will be used $p = 0.01$ as well as $\sigma = 1.0$.



(a) P1 experiment (b) P2 experiment

Fig. 4. Comparison of RMSE of the averaged 1000 ABRs using different p value

4.2 Efficacy of Averaging

The results of averaging are presented in Tables 1 and 2. The purpose of these experiments are to investigate the proposed method in the presence of the Gaussian noise ($\alpha = 2$) and nearly-Gaussian noise ($1.8 \leq \alpha < 2$). The smallest value of RMSE allows to achieve the highest signal-to-noise improvement.

Table 1. RMSE for averaged signals with artificial noise (P1 experiment)

α	AM	TM 25%	WA	WACFMGC $\sigma = 1$, $p = 0.01$
		RMSE [nV]		
1.80	47.87	20.57	14.60	**13.92**
1.85	56.95	22.54	13.57	**12.64**
1.90	37.38	21.79	13.08	**12.57**
1.95	51.44	21.33	12.42	**11.68**
2.00	31.53	21.20	11.69	**10.96**
		MAX [nV]		
1.80	241.54	74.27	**41.72**	41.81
1.85	462.99	75.30	40.34	**34.22**
1.90	139.02	63.53	**35.81**	36.38
1.95	1034.48	70.96	46.48	**40.13**
2.00	92.53	78.26	33.03	**30.14**

In the case of the P1 experiment, the RMSE value of the residual noise for the investigated methods are presented in first part of Table 1. The worst results (the highest value of RMSE) are achieved by a method of arithmetic averaging. The performance of AM improves a little for higher value of the characteristic exponent α. To the slightly better results leads the trimmed mean method; RMSE values for all α remain more or less stable. The WA method allows to achieve significantly better performance then the methods of equal weights (AM and TM methods). But the best results are obtained with the proposed WACFMGC method. Both methods WACFMGC and WA reach lower values of MAX in comparison to reference methods. However, the WACFMGC method achieves the smallest value of MAX for $\alpha \geq 1.9$.

The P2 experiment is more difficult to the weighted averaging methods for two reasons. The first is the extremely low GSNR level and the second reason is the nearly the same level of GSNR. In this case, the weighted averaging may have a problem estimating the proper weight values. However, the results are similar to those of the P1 experiment to the benefit of the WACFMGC method. Only for $\alpha = 1.8$ the WA and WACFMGC methods have the same RMSE but the WACFMGC has the smallest MAX error. If $\alpha \geq 1.85$ the performance of WACFMGC is better than the WA method. The advantage from the use of this method is the ability to work with other types of noise than the Gaussian but it requires extra computational burden (at given ϵ, 3-4 iterations are performed).

Table 2. RMSE for averaged signals with artificial noise (P2 experiment)

α	AM	TM 25%	WA	WACFMGC $\sigma = 1$, $p = 0.01$
		RMSE [nV]		
1.80	160.03	48.75	**28.50**	**28.50**
1.85	77.11	48.92	26.24	**26.16**
1.90	63.82	51.01	25.08	**24.70**
1.95	59.71	49.49	23.44	**23.15**
2.00	49.64	48.26	21.12	**21.00**
		MAX [nV]		
1.80	3501.36	171.18	115.52	**111.38**
1.85	466.88	139.75	85.37	**82.76**
1.90	217.85	151.73	**91.22**	91.75
1.95	463.62	130.81	83.11	**76.75**
2.00	149.89	132.65	76.94	**74.17**

The results of ABR averaging of real signals are shown in Figure 5. Signal (a) was obtained using the arithmetic mean (equally weights), (b) using the weighted averaging method, (c) using the weighted averaging with criterion function minimization with the dissimilarity function based on the General Cauchy distribution with $p = 0.01$ and $\sigma = 1$, and (d) using the trimmed mean method.

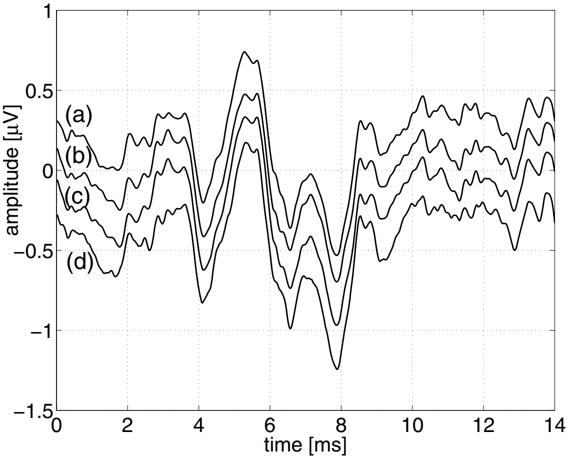


Fig. 5. Results of ABRs averaging for 1800 real sweeps: (a) AM method, (b) WA method, (c) WACFMGC method, (d) TM method. For better presentation signals (b), (c) and (d) are shifted by -0.2, -0.4 and -0.6 μV respectively

5 Conclusion

A new method of robust weighted averaging of periodic signals is presented in this work. It is applied to auditory brainstem responses averaging. The proposed method operation is based on minimization of scalar criterion function. The robust dissimilarity functions derived from the generalized Cauchy are applied. The special cases of the proposed method are obtained with use of the myriad and the meridian cost functions, respectively. The robustness of the proposed methods can be controlled either with the two parameters. Two patterns of simulation experiments were proposed to evaluate the respective methods. The methods were tested with the artificial signals and the real ABRs. The best capability of the noise suppression (the smallest value of RMSE which means the highest signal-to-noise improvement) in majority of cases had the method based on the generalized Cauchy distribution cost function. The obtained results show the usefulness of the presented WACFMGC weighted averaging method for ABRs processing. The presented methods can help to improve averaging of evoked potentials like ABRs or other biomedical signal cycles when the data are highly non-stationary and the signals are disturbed by the impulsive noise.

Acknowledgement. This work is supported by SUT grant BK-216/RAu3/2013 The authors would like to thank J.Zając and W.Kowalski (from EMAG, Katowice) for providing them with their ABRs data and the anonymous reviewers for their valuable suggestions.

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