Vibration Response Analysis of Mistuned Bladed Disk Consisting of Directionally Solidified Blade

Yasutomo Kaneko, Kazushi Mori and Hiroharu Ooyama

Abstract Recently, DS (Directionally Solidified) and SC (Single Crystal) alloys have been widely applied for gas turbine blades instead of CC (Conventionally Casting) alloys. In this study, the mistuning analysis of the bladed disk consisting of DS blades is carried out, considering the deviations of the elastic constant and the crystal angle of the DS blade. The FMM is used to analyze the mistuned bladed disk. The maximum amplitude of the mistuned bladed disk of the DS blade is estimated by the Monte Carlo simulation combining with the response surface method, and the calculated results are compared with those of the CC blade.

Keywords Mistuning · Bladed disk · Resonance · Directionally solidified blade · Response surface method

1 Introduction

Recently, DS (Directionally Solidified) and SC (Single Crystal) alloys have been widely applied for gas turbine blades instead of CC (Conventionally Casting) alloys to meet the requirement of the high temperature operation and to improve the thermal efficiency [1, 2]. The SC blade consists of one columnar grain, and therefore creep rupture and thermal fatigue resistance can be improved by eliminating the grain boundary. Since the elastic constants of the SC blades are anisotropic, the vibration

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analysis of the SC blade has to be carried out, taking account of the anisotropy. On the other hand, the DS blade consists of several columnar grains where the growing direction of the columnar crystal is set to the direction of the centrifugal force to yield a large creep resistance by eliminating a grain boundary perpendicular to the loading direction. The vibration analysis of the DS blade also has to be carried out, taking account of the anisotropy because the direction of the columnar crystal growth is parallel to the loading direction.

In the vibration analysis of the DS blade by FEA (Finite Element Analysis), it seems that all of the columnar grains included in the blade have to be modeled exactly [3, 4]. In practice, however, it is difficult to know the exact growing direction and the geometry of the columnar grain. It is reported that if the volume and the rotation angle in the transverse plane of each columnar crystal are known, the vibration analysis of the DS blade can be carried out with the practical accuracy by use of the elastic constants averaged by the Reuss and Voigt method [5]. However, it is also difficult to get such data of the actual DS blade in advance. Therefore, in the actual mechanical design of the DS blade, the vibration analysis is carried out, assuming that the number of the columnar grains included in the blade is large, and the DS blade can be considered transverse isotropic [1]. From the background above-mentioned, one of authors investigated the effect of the anisotropy of elastic constants on the vibration characteristics of the SC and DS blade systematically [6, 7], and it was shown that even if the number of the columnar grains included in the DS blade is small, the DS blade can be treated as a transversely isotropic material, and the vibration analysis can be carried out using the elastic constants averaged by the Wells' method [8]. On the other hand, as far as authors know, research on the vibration response of mistuned bladed disks consisting of DS blades is limited [9], and the mistuning characteristics of a bladed disk with DS blades have not been clarified.

In this study, taking account of the above-mentioned situation, the vibration response characteristics of the mistuned bladed disk consisting of DS blades are investigated, carrying out the parametric study on the material constants and the crystal direction of the DS blade. First, using the sensitivity of the blade frequency against the deviation of material constants and crystal angles, a response surface of the blade frequency is calculated. Second, combining the Monte Carlo simulation with the response surface, the vibration response analysis of the mistuned bladed disk is carried out. In the analysis, the large-scaled FE model used in the actual blade design is utilized. However, because it is difficult to carry out the vibration response analysis using the large-scaled FE model directly due to the enormous computational time, the reduced order model FMM is applied [10–13]. The vibration response analysis of the mistuned bladed disk consisting of CC blades is also carried out in the same procedure. Comparing both results, the vibration response characteristics of the mistuned bladed disk consisting of DS blades are evaluated.

2 Analysis Method

This paper treats a DS blade as a transverse isotropic material [6, 7], and employs a coordinate system in Fig. 1 to define the blade geometry and the material property. The coordinate system expressed by the capital letter X_1, X_2, and X_3 is fixed to each columnar grain, and the axes are directed to the crystal axes of [100], [010], and [001]. In Fig. 1a, the elastic constants of E_0 and E_{90} denote the Young's modulus in the direction of the X_3 and X_1 axis, respectively, while G_0 corresponds to the shear rigidity around the X_3 axis. In designing the SC or DS blade, although it is assumed that the X_3 axis is coincident with the direction of the blade height, the crystal growing direction deviates slightly from the X_3 axis, due to the error in casting. This deviation of the crystal growing direction is defined by α and β as shown in Fig. 1b.

The mistuning analysis of the DS blade is carried out as follows.

(1) The sensitivity analysis of the natural frequency of the freestanding blade is, first, carried out by FEA, changing the elastic constants (E_0, E_{90}, G_0) and the crystal angles (α, β) from the nominal value. The response surface of the natural frequency of the freestanding blade against the deviation of the elastic constants and the crystal angles are created, using the results of the sensitivity analysis.

(2) The elastic constants and the crystal angles of blades on a disk are selected at random from the normal distribution given, and a mistuned bladed disk is assembled, utilizing the response surface.

(3) The eigenvalue analysis of a mistuned bladed disk is carried out, using the sophisticated reduced model FMM, because it is not practical to apply the FEA directly to the eigenvalue analysis of the mistuned bladed disk. The frequency response analysis of the mistuned bladed disk is carried out by the conventional modal analysis method, using the modal parameter obtained by FMM [10–13].
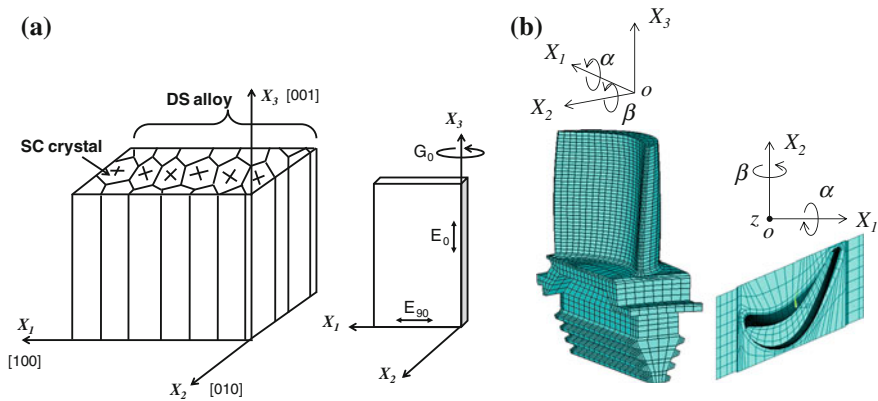
Fig. 1 Coordinate and material property of SC and DS alloy. **a** Coordinate and material property. **b** Deviation angle of crystal direction α and β

(a)   **(b)**
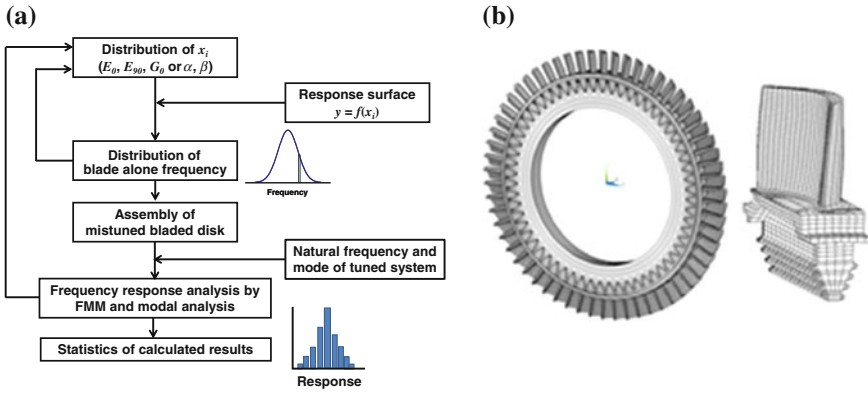
Fig. 2 Procedure of frequency response analysis of mistuned bladed disk with DS blade.
a Procedure of mistuning analysis. **b** FEA model used in analysis

(4) The frequency response analysis is carried out many times for mistuned bladed
disks assembled at random, and the calculated results are processed statisti-
cally (Monte Carlo simulation).

In the mistuning analysis of a bladed disk consisting of DS blades by use of the
response surface, the deviation of the blade alone frequency (the frequency dif-
ference caused by the deviation of the material constants and crystal angles) is
defined as a response, while deviations of 3 independent material constants (E_0, E_{90},
G_0) and 2 crystal angles (α, β) are explanatory variables for the response surface.
However, if 5 deviations are adopted as explanatory variables to the response
surface at the same time, the number of the explanatory variables becomes too large
to understand the mistuning phenomena. Therefore, in this study, the effect of the
deviation of the material constant and the crystal angle is analyzed separately.

Figure 2 shows the procedure of the frequency response analysis of a mistuned
bladed disk, and the FEA model. The FEA model of a freestanding blade is used in the
sensitivity analysis against the deviation of the elastic constants and the crystal angles,
while the FEA model of a tuned bladed disk is used to prepare the input data of FMM.

3 Analysis Results

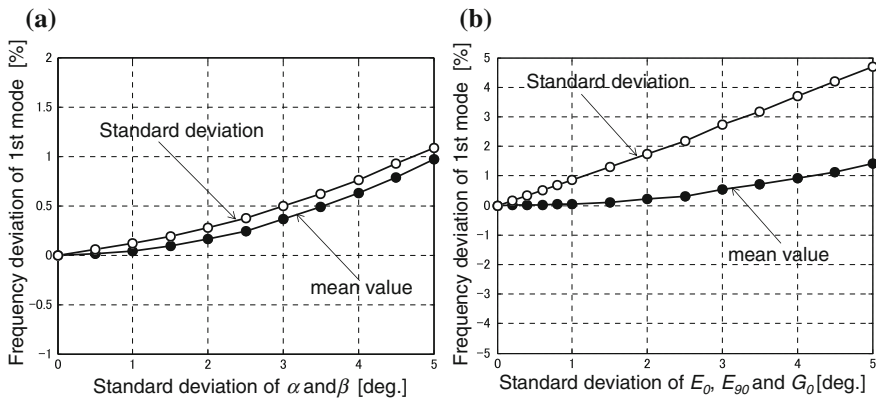### 3.1 Effect of Deviation of Crystal Angle and Material Constant on Vibration Characteristics of DS Blade

Figure 3a denotes a response surface of the blade alone frequency to the deviation
of the crystal angles (α, β), while Fig. 3b to the deviation of the material constants
(E_0, E_{90}, G_0). Figure 4 shows a mean value and a standard deviation of the blade
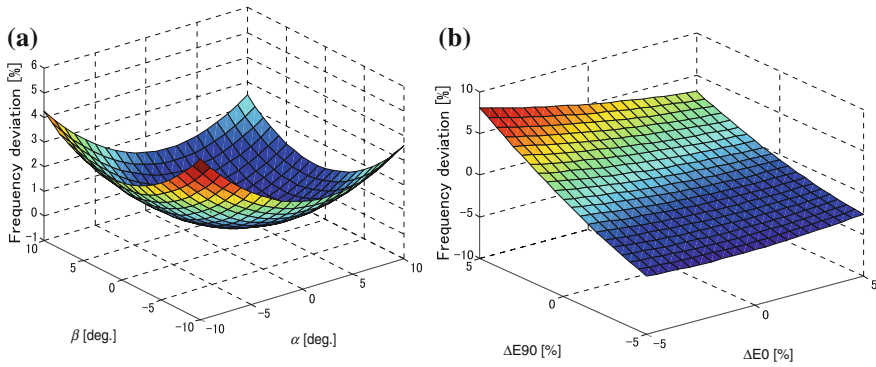
Fig. 3 Response surface of natural frequency of the 1st mode. **a** To crystal angles (α, β). **b** To material constants (E_0, E_{90}, G_0)

alone frequency against a standard deviation of the crystal angles and the material constants. The frequency distribution in Fig. 4 is calculated by the response surface of the blade alone frequency and the Monte Carlo simulation, assuming that the deviation of the crystal angles and the material constants follows a normal distribution, and generating 10,000 random variables (explanatory variables) for the normal distribution with a standard deviation given. The standard deviations of crystal angles (α, β) are assumed to be identical for the analysis of the blade alone frequency deviation against the deviation of the crystal angles in Fig. 4a, while the standard deviations of material constants (E_0, E_{90}, G_0) are assumed to be identical for the analysis against the deviation of the material constants in Fig. 4b.

As shown in Fig. 3a, the response surface of the blade alone frequency to the crystal angles becomes a convex curved surface for the 1st mode frequency. Therefore, whether the direction of the deviation of the crystal angles is positive or

Fig. 4 Frequency deviation of the 1st mode due to deviation of crystal angles and material constants. **a** Frequency deviation due to α and β. **b** Frequency deviation due to ΔE_0, ΔE_{90} and ΔG_0

negative, the blade alone frequency becomes higher. As a result, the distribution of the blade alone frequency caused by the deviation of the crystal angles is distorted toward the positive side, and not only a standard deviation but also a mean value of the blade alone frequency increases when the deviations of crystal angles increase, as shown in Fig. 4a. It is also said that because the blade alone frequency of the 1st mode increases more than 4 % when crystal angles change up to around ±10° as shown in Fig. 3a, the effect of the deviation of the crystal angles on the blade alone frequency is not negligible.

On the other hand, the response surface of the blade alone frequency to the material constants becomes an almost flat surface as shown in Fig. 3b. Therefore, when the deviations of the material constants are small, the response surface can be approximated by an inclined plane, and the distribution of the blade alone frequency becomes symmetric with respect to a mean value. However, as the deviations of the material constants increase, the response surface is distorted from the plane. As a result, not only a standard deviation but also a mean value of the blade frequency increases with increase of the standard deviation of the material constants as shown in Fig. 4b.

3.2 Mistuning Analysis of Bladed Disk with DS Blade

Figure 5a shows the natural frequencies of the tuned bladed disk consisting of DS blades, where the abscissa denotes the number of the nodal diameters of the vibration modes, and the ordinate denotes the natural frequency of the bladed disk. In the vibration response analysis of the mistuned bladed disk, using the natural frequencies and vibration modes of the tuned bladed disk as input data, frequency response analyses are carried out many times by FMM and the Monte Carlo method as shown in Fig. 2. From these results, the statistical characteristics of the vibration response against the deviation of the crystal angles and the material constants are evaluated. Standard deviations of crystal angles (α, β) are assumed to be identical for the mistuning analysis against the deviation of the crystal angles, while standard deviations of material constants (E_0, E_{90}, G_0) are assumed to be identical for the mistuning analysis against the deviation of the material constants. In the Monte Carlo simulation, the frequency response analysis is repeated 1,000 times. The logarithmic decrement of the bladed disk is assumed to be 0.03 for all vibration modes, and the unit harmonic excitation force is exerted on the tip sections of all blades in the tangential direction. The number of the harmonics of the excitation force (the engine order of the excitation force) H is 10. Figure 5b shows an example of the calculated frequency response of the mistuned bladed disk.

Figure 6a shows the relationship between the standard deviations of crystal angles and the statistic of the maximum response of the blades, while Fig. 6b shows the relationship between the standard deviations of material constants and the statistic of the maximum response of the blades. The ordinate of Fig. 6 is normalized by the resonant amplitude of the tuned bladed disk.
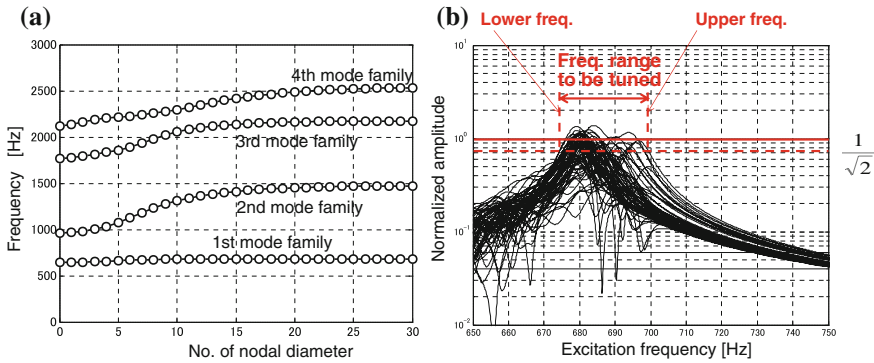
Fig. 5 Natural frequency and frequency response of bladed disk with DS blade. **a** Natural frequency of tuned bladed disk. **b** Frequency response of mistuned bladed disk (Standard deviation of α, $\beta = 4°$)
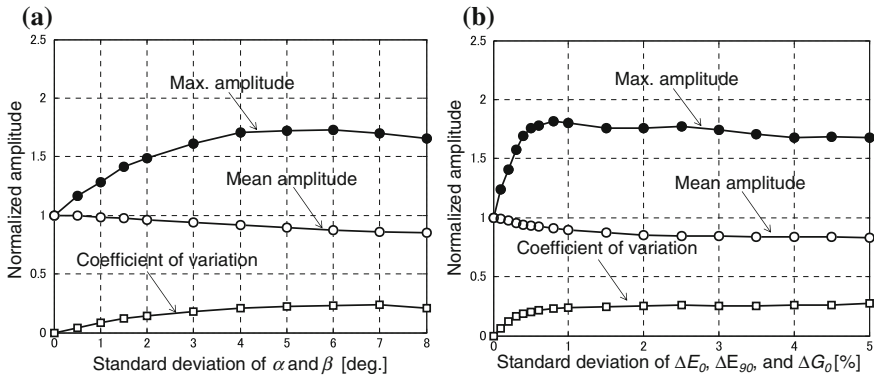
Fig. 6 Statistics of maximum amplitude of mistuned bladed disks. **a** Max. amplitude and deviation of crystal angle. **b** Max. amplitude and deviation elastic constants

As shown in Fig. 6a, the maximum amplitude caused by deviations of the crystal angles reaches its maximum (around 1.7 times as large as the tuned system) in the vicinity of standard deviation of 5°, and if the standard deviations of the crystal angles become greater than this, the maximum amplitude begins decreasing gradually. Similarly, it is shown in Fig. 6b that the maximum amplitude caused by deviations of the material constants reaches its maximum (around 1.8 times as large as the tuned system) in the vicinity of standard deviation of 1 %, and if the standard deviations of the material constants become greater than this, the maximum amplitude begins decreasing gradually. Comparing Fig. 6 with Fig. 4, it is said that the standard deviations of the crystal angles and material constants causing the maximum amplitude of the blade are corresponding to 1 % standard deviation of the blade alone frequency. In other words, it is construed that the maximum amplitude
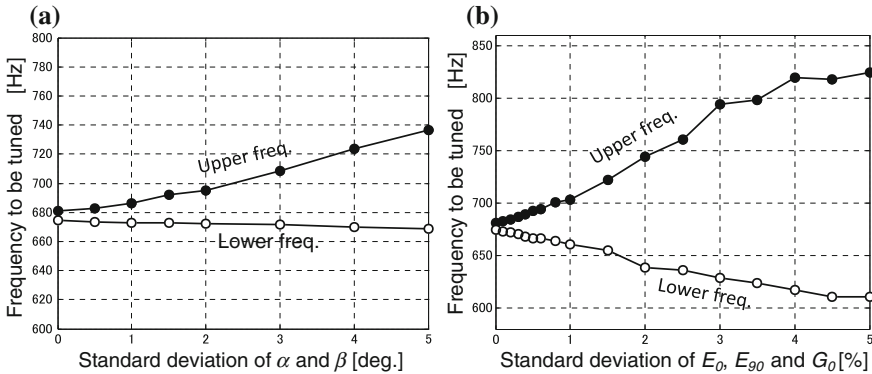
Fig. 7 Range of resonant frequency to be tuned for DS blade. **a** Due to deviation of crystal angle. **b** Due to deviation of material constant

of the blade reaches its maximum (around 1.7–1.8 times as large as the tuned system) when the standard deviation of the crystal angles and the material constants increases up to that equivalent to 1 % standard deviation of the blade alone frequency. If the standard deviations of the crystal angles and the material constants become greater than this, the maximum amplitude of the blade begins decreasing.

Figure 7 shows the relationship between the resonant frequency range and the standard deviations of the crystal angles and the material constants. The resonant frequency range means a frequency band to be tuned from the excitation force in designing blade, and is defined provisionally as a range, in which the vibration response becomes greater than the resonant amplitude of the tuned system divided by $\sqrt{2}$, as shown in Fig. 5b. From Fig. 7, it is shown that the resonant frequency range increases monotonically as the deviations of the crystal angles and the material constants increase.

3.3 Mistuning Analysis of Bladed Disk with CC Blade

The mistuning analysis of bladed disks consisting of the CC blades is also carried out according to the procedure in Fig. 2 as well as that of the DS blades.

Figure 8a shows the relationship between the standard deviations of material constants and the statistic of the maximum response of the blades, and the results for the DS blade is also plotted for comparison in the figure. The ordinate of Fig. 8a is normalized by the resonant amplitude of the tuned bladed disk. As shown in Fig. 8a, for the bladed disk with CC blades, it is said that the maximum amplitude caused by deviations of the material constants reaches its maximum (around 1.8 times as large as the tuned system) in the vicinity of standard deviation of 1.5 %, and if the standard deviations of the material constants become greater than this, the
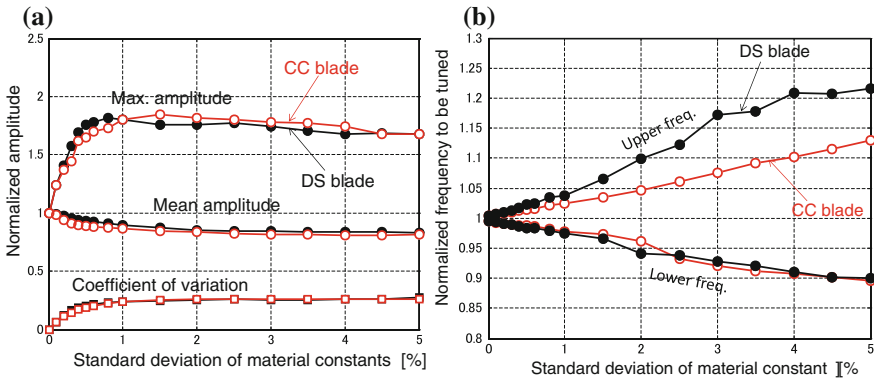
Fig. 8 Maximum Amplitude and resonant frequency range of mistuned bladed disk. **a** Maximum amplitude. **b** Normalized resonant frequency range

maximum amplitude begins decreasing gradually. From the calculated results of the CC blade corresponding to Fig. 4b, it is said that the standard deviations of the material constants causing the maximum amplitude of the blade is corresponding to 1 % standard deviation of the blade alone frequency. In other words, it is interpreted like the bladed disk with DS blades that the maximum amplitude of the bladed disk with CC blades reaches its maximum (around 1.8 times as large as the tuned system) when the standard deviation of the material constants increases up to that equivalent to 1 % standard deviation of the blade alone frequency. If the standard deviations of the material constants become greater than this, the maximum amplitude of the blade begins decreasing. In short, for both of bladed disks, the maximum amplitude of the blade reaches its maximum when the standard deviation of the blade alone frequency is around 1 %.

For the mistuned bladed disk with DS blades, although the deviations of the crystal angles should be considered as well as the deviations of the material constants, the maximum amplitude of the blade reaches its maximum when the standard deviation of the blade alone frequency is around 1 %, and the maximum amplitude does not increase beyond this even if the deviations of the blade alone frequency increases. Therefore, in the design of an un-tuned blade, where the resonance is acceptable, it seems that the same safety factor can be applied to the mistuning effect for both of the DS blade and the CC blade.

On the other hand, Fig. 8b shows the relationship between the normalized resonant frequency range and the standard deviations of the material constants. The resonant frequency range is defined as a range, in which the vibration response becomes greater than the resonant amplitude of the tuned system divided by $\sqrt{2}$ as shown in Fig. 5b. The result of the DS blade is also plotted in Fig. 8b for comparison. As shown in Fig. 8b, the resonant frequency range of the DS blade caused by the deviation of material constants becomes wider than that of the CC blade. In addition, the resonant frequency range of the DS blade is widened by the deviation

of crystal angles. Therefore, in the design of a tuned blade, where the resonant frequency should be tuned to avoid the excitation frequency, a blade designer should consider that the range of the resonant frequency of the DS blade caused by the deviation of the material constants and the crystal angles becomes wider than that of the CC blade.

4 Conclusions

In this study, the vibration response characteristics of the mistuned bladed disk consisting of DS blades were investigated, carrying out the parametric study on the material constants and the crystal direction of the DS blade. From the calculated results, following conclusions are derived.

(1) For the DS blade, the response surface of the blade alone frequency to the crystal angles becomes a convex curved surface for the 1st mode frequency. Therefore, whether the direction of the deviation of the crystal angles is positive or negative, the blade alone frequency becomes higher. On the other hand, the response surface of the blade alone frequency to the material constants becomes an almost flat surface for both of the DS blade and the CC blade.

(2) The maximum amplitude of the bladed disk reaches its maximum when the standard deviation of the material constants increases up to that equivalent to 1 % standard deviation of the blade alone frequency for both of the DS blade and the CC blade. The maximum amplitude of the blade decreases if the standard deviations of the blade alone frequency become greater than this. Therefore, in the design of an un-tuned blade, the same safety factor can be applied to the mistuning effect for both of the DS blade and the CC blade.

(3) The resonant frequency range of the DS blade becomes wider than that of the CC blade. Therefore, in the design of a tuned blade, a blade designer should consider that the range of the resonant frequency of the DS blade caused by the deviation of the material constants and the crystal angles becomes wider than that of the CC blade.

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