

Nonlinear Dynamic Analysis of Ball Bearings Due to Varying Number of Balls and Centrifugal Force

V. Vakharia, V.K. Gupta and P.K. Kankar

Abstract This paper aims to study the influence of varying number of balls on nonlinear dynamic behavior of an unbalanced rotor supported by ball bearings. Centrifugal force due to rotation of balls is included in the model. An oscillating spring-mass-damper is considered to formulate the contact between races and rolling elements. Stiffness for the spring-mass-damper system is calculated using Hertzian elastic contact deformation theory. The results show appearance of regions of periodic, sub-harmonic and chaotic behavior on response of the system. Invariably, the route to chaos is seen to be intermittency mechanism by period doubling behavior. Poincare maps and frequency responses are used to explain and to study the system behavior.

Keywords Nonlinear dynamics · Ball bearing · Unbalanced rotor · Centrifugal force · Poincare map

1 Introduction

Rolling element bearing is one of the essential components in rotating equipments. In present scenario, bearing applications require understanding of nonlinear dynamic effects. Nonlinearity often arises due to Hertzian force deformation relationship, the varying compliance nature, the internal clearance, varying number of balls etc. The behavior of nonlinear system often demonstrates unexpected behavior

V. Vakharia · V.K. Gupta · P.K. Kankar (✉)
Mechanical Engineering Discipline, PDPM Indian Institute of Information Technology,
Design and Manufacturing Jabalpur, Jabalpur 482005, India
e-mail: pavankankar@gmail.com

V. Vakharia
e-mail: vinayvakharia4343@gmail.com

V.K. Gupta
e-mail: vkgupta@iiitdmj.ac.in

patterns that are extremely sensitive to initial conditions. The excitation due to unbalance force in rotor bearing system cannot be eliminated completely [1, 2] even with good balancing of rotor and therefore it is the major parameter to consider for study. Due to consideration of unbalance force the system is bi-periodically excited. The amount of unbalanced force changes continuously with operating parameters and thus changing the dynamic behavior of the whole system. Moreover, when ball bearing operates at high speed the effect of centrifugal force cannot be neglected. Jang and Jeong [3] considered the effect of centrifugal force of ball with waviness on races for balanced rotor. However effect of centrifugal force of ball along with unbalanced rotor and nonlinear spring-damping model has not been considered so far.

The aim of present study is to analyze the nonlinear dynamic response of unbalanced rotor supported on ball bearings considering centrifugal force. Sources of nonlinearity such as Hertzian contact force, varying compliance frequency are considered and formulate in Mathematical model. Effect of number of balls is considered at constant rotational speed.

2 Mathematical Modeling

A rolling element bearing supporting horizontal rotor considering unbalance force is shown in Fig. 1. To analyze nonlinear dynamic response of ball bearing the bearing components are modeled as non-linear spring-mass-damper system, in which the outer race of the bearing is fixed rigidly and inner race, is fixed rigidly with shaft. A constant radial force is assumed to be acting on the system.

It is assumed that the spring damper model considered here has equally spaced balls rotating on the surfaces of the inner and outer races. When the rolling element

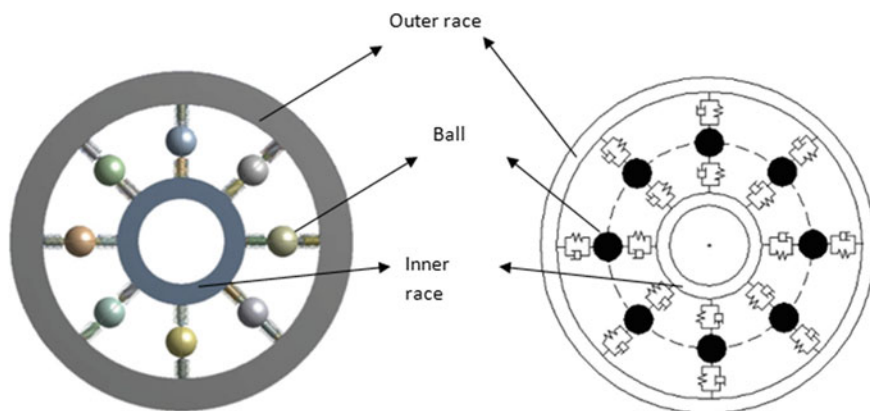


Fig. 1 Rolling element bearing model considering Mass-spring-damper

set and the cage rotates with a constant angular velocity (ω_{cage}), a parametrically excited vibration is generated and transmitted through the outer race.

$$\omega_{cage} = \omega_{rotor} \frac{r}{(R+r)} \tag{1}$$

The characteristic frequency of this vibration is called the varying compliance frequency (VC) and is given by

$$\omega_{VC} = \omega_{cage} \times N_b \tag{2}$$

where N_b is number of balls.

For the i th ball, local Hertzian contact force and deflection are related as [4]

$$P_{\theta_i} = k(r_{\theta_i})^{3/2} \tag{3}$$

where, k is the nonlinear stiffness due to point contact, θ_i is the angular position i th ball and r_{θ_i} is radial displacement of the i th ball which is given by [5]

$$r_{\theta_i} = x \cos \theta_i + y \sin \theta_i \tag{4}$$

Considering the internal radial clearance, γ_0 , the contact deformation becomes,

$$r_{\theta_i} = x \cos \theta_i + y \sin \theta_i - (\gamma_0) \tag{5}$$

Substituting r_{θ_i} in the Eq. (3), we get

$$P_{\theta_i} = k.H\{(x \cos \theta_i + y \sin \theta_i) - (\gamma_0)\}^{3/2} \tag{6}$$

where, $H\{(x \cos \theta_i + y \sin \theta_i) - (\gamma_0)\}^{3/2}$ is a Heaviside step function.

Total restoring force in x and y direction considering Hertzian contact force (P_{θ_i}) and centrifugal force (P_c) is written as

$$P_x = \sum_{i=1}^{N_b} \left[k.H\{(x \cos \theta_i + y \sin \theta_i) - (\gamma_0)\}^{3/2} - P_c \right] \cos \theta_i \tag{7}$$

$$P_y = \sum_{i=1}^{N_b} k \left[k.H\{(x \cos \theta_i + y \sin \theta_i) - (\gamma_0)\}^{3/2} - P_c \right] \sin \theta_i \tag{8}$$

The centrifugal force is given by [6]

$$P_c = 2.26 \times 10^{-11} \times D_b^2 \times \omega_{cage} \times D_m \tag{9}$$

where, D_b is ball diameter, ω_{cage} is angular velocity of the cage and D_m is pitch circle diameter of the bearing.

2.1 Nonlinear Damping Force

A viscous damping model is considered in present formulation in which the dissipative forces are proportional to the time derivate. The resulting equation [7] is given by

$$P_{d_i} = c(r_{\theta_i})\dot{r}_{\theta_i}^q \quad (10)$$

where, $c(r_{\theta_i})$ depends on contact geometry, material properties of elastic bodies, lubricant property and contact surface velocities.

2.2 Equation of Motion

The equation considering effect of inertia, restoring and damping force and constant vertical force acting on the inner race are represented as [5]

$$M_R \ddot{x} + \sum_{i=1}^{N_b} P_{d_i} \cos \theta_i + \sum_{i=1}^{N_b} \left[k.H\{(x \cos \theta_i + y \sin \theta_i) - (\gamma_0)\}^{3/2} - P_c \right] \cos \theta_i = P_r + P_u \cos(\omega t) \quad (11)$$

$$M_R \ddot{y} + \sum_{i=1}^{N_b} P_{d_i} \sin \theta_i + \sum_{i=1}^{N_b} k \left[k.H\{(x \cos \theta_i + y \sin \theta_i) - (\gamma_0)\}^{3/2} - P_c \right] \sin \theta_i = P_u \sin(\omega t) \quad (12)$$

where, M_R is the rotor mass supported on bearings, P_r is the radial load on bearing and P_u is force due to unbalance.

The system is defined by Eqs. (11) and (12) which are two coupled non-linear second order differential equations.

3 Results and Discussion

In present study, ball bearing 6205 is considered whose parameters are shown in Table 1. Shaft is assumed to be running at 5000 rpm and an unbalance force of 10 % of radial load is considered in the current study. The Runge-Kutta methods of fourth order are used to solve coupled equations. The time step for the investigation

Table 1 Parameters of bearing 6205 (SKF)

S. no.	Parameters	Value
1	Outer race diameter (D_o)	28.262 mm
2	Inner race diameter (D_i)	18.738 mm
3	Pitch circle diameter (D_m)	26.725 mm
4	Ball diameter (D_b)	4.762 mm
5	Radial load	6 N
6	Radial internal clearance	10 μm

is taken as $\Delta t = 10^{-5}$ s with the initial displacements as $x_0 = 10^{-6}$ m, $y_0 = 10^{-6}$ m and initial velocities are assumed to be zero i.e. $\dot{x}_0 = 0$ m/s, $\dot{y}_0 = 0$ m/s. Poincaré maps and frequency responses are used to illustrate and study the effect of system diversity.

For number of balls $N_b = 6$, peak amplitude of vibration for X direction response appears at rotational speed ($X = 83$ Hz) as shown in Fig. 2a. Other major peaks appears at the interaction of varying compliance frequency and rotational speed

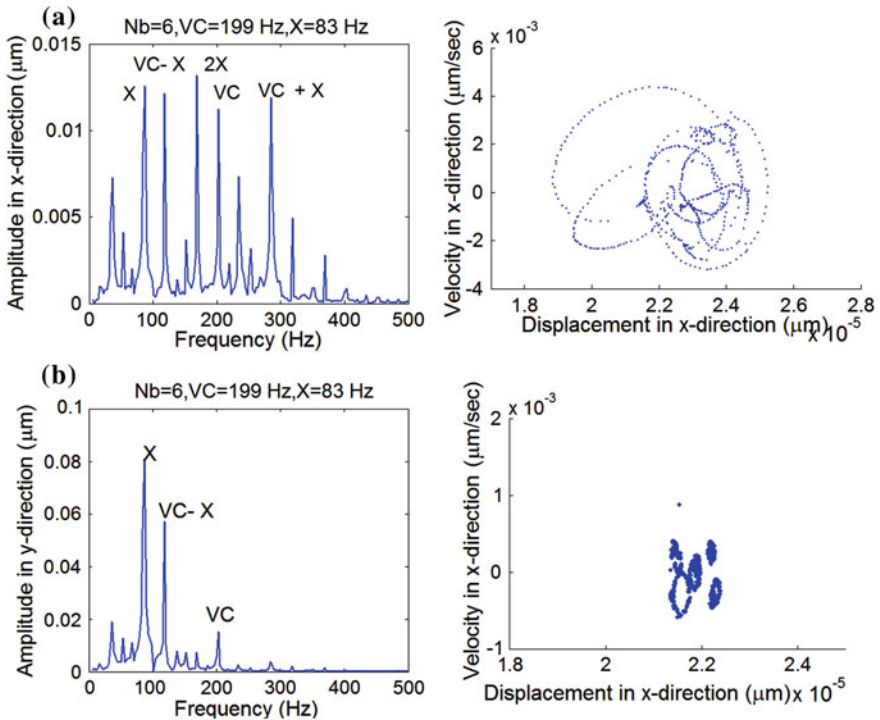


Fig. 2 FFT and poincare map at $N_b = 6$ **a** X direction **b** Y direction

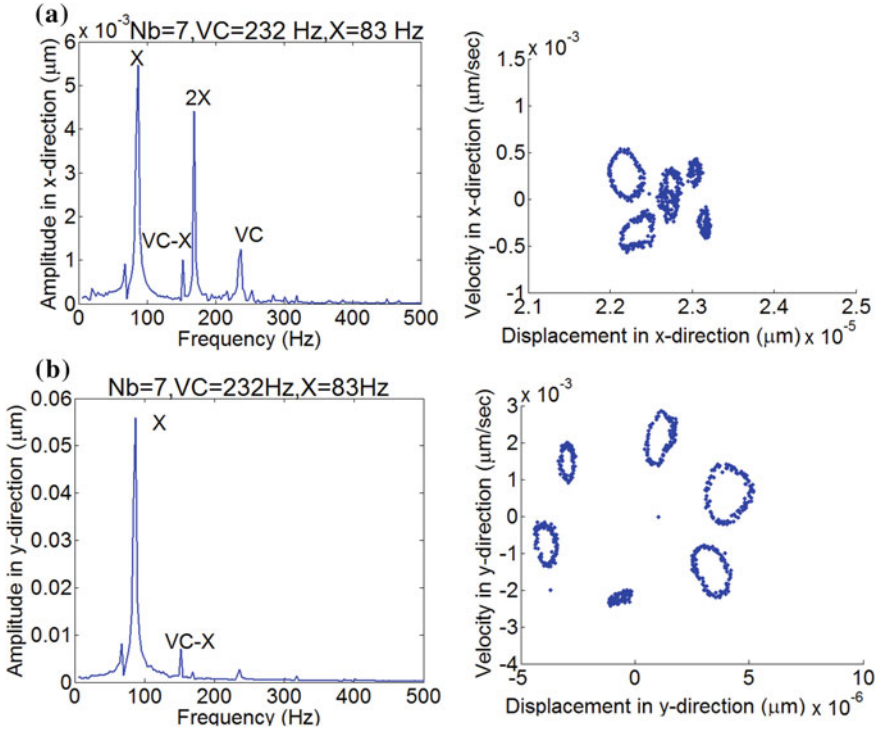


Fig. 3 FFT and poincare map at $N_b = 7$ **a** X direction **b** Y direction

($X = 83.33$ Hz) and their harmonics at $VC - X(=116$ Hz), $2X$, $VC + X(= 282$ Hz). Fractal structure is observed through Poincaré map which reveals chaotic nature. For Y Direction response, peak amplitude appears at rotational speed ($X = 83.33$ Hz) and other peak appears at $VC - X(=116$ Hz), $VC(= 199$ Hz) shown in Fig. 2b. The Poincaré map shows fractal structure for $N_b = 6$ as shown Fig. 2b.

Figures 3, 4 and 5 shows quasi periodic response from Poincaré map for $N_b = 7$, 8 and 9 respectively. Major peak is obtained corresponding to shaft rotational frequency ($X = 83$ Hz) and amplitude decreases as number of balls increases. This is due to increase in stiffness of the system since number of balls coming in load zone increases. System further stabilizes when number of balls is changed to 10, which is shown in Fig. 6. At this moment, systems exhibits multi orbit periodic response which is clearly indicated in Poincaré map. The results are summarized in Table 2.

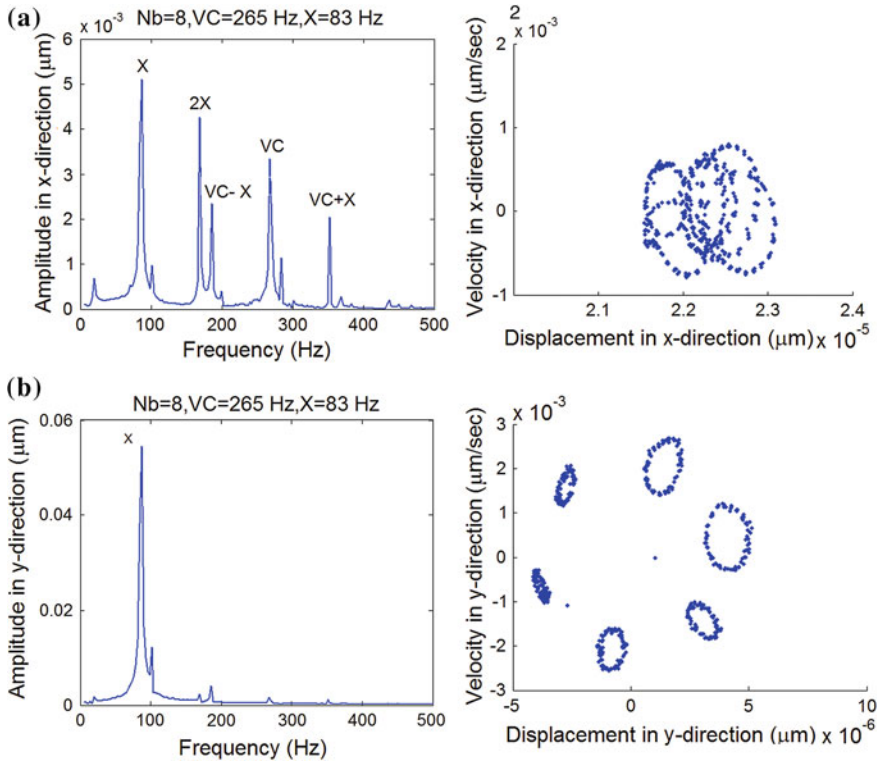


Fig. 4 FFT and poincare map at $N_b = 8$ **a** X direction **b** Y direction

4 Concluding Remarks

This study presents an analytical model to investigate the effects of unbalance and centrifugal force acting on ball bearing model. Unbalance force corresponding to 10 % magnitude of radial load (P_r) with internal clearance of 10 μm are considered in the present study. Number of balls are varied and effect of variation has been analyzed at constant rotational speed of shaft. Following conclusions can be drawn from present study.

- Nonlinear dynamic responses are found to be associated with rotational frequency (X) and its super harmonics due to unbalance force.
- Results show interaction of rotational frequency (X) due to unbalance force and varying compliance frequency (VC) due to parametric excitation.

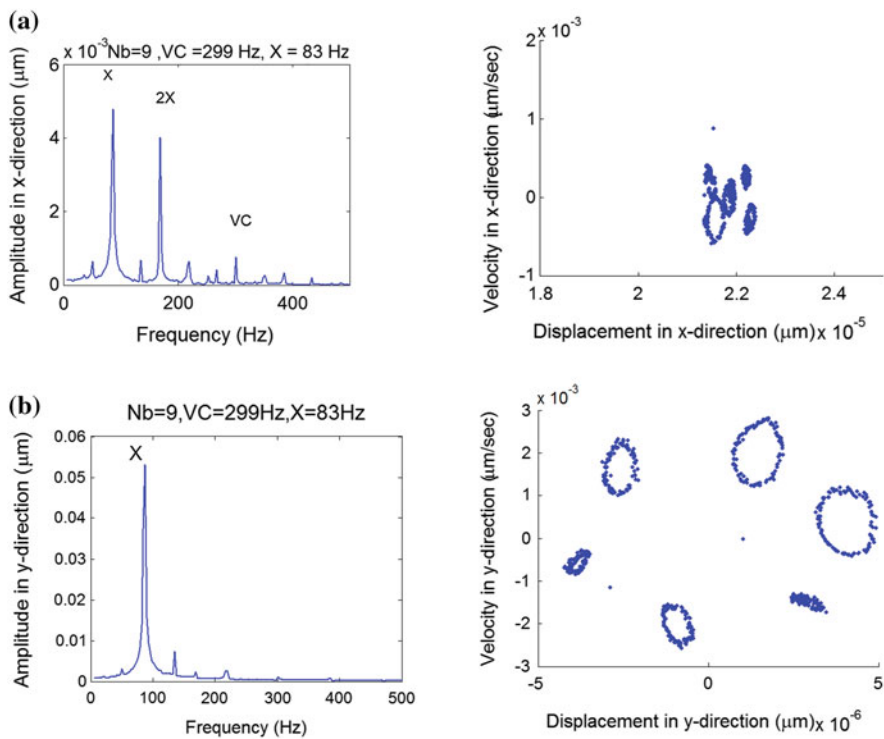


Fig. 5 FFT and poincare map at $N_b = 9$ **a** X direction **b** Y direction

- When number of balls are increased peak amplitude decreases implying a stiffer system and same was reported by Harsha et al. [4] and Kankar et al. [8]. It can be predicted from the results that increasing number of balls reduces the effect of frequency modulation.
- For $N_b = 5-6$, chaotic nature is observed from poincare map. When number of balls increases to $N_b = 7-9$, quasi-periodic behavior is observed and further increase in the number of balls results in entering of system in stable zone. This can be attributed to increase in stiffness due to increase in number of balls.
- From this analysis an insight is obtained to choose appropriate number of balls so that system can be operated in stable zone and severe amplitude of vibration (chaotic) can be avoided.

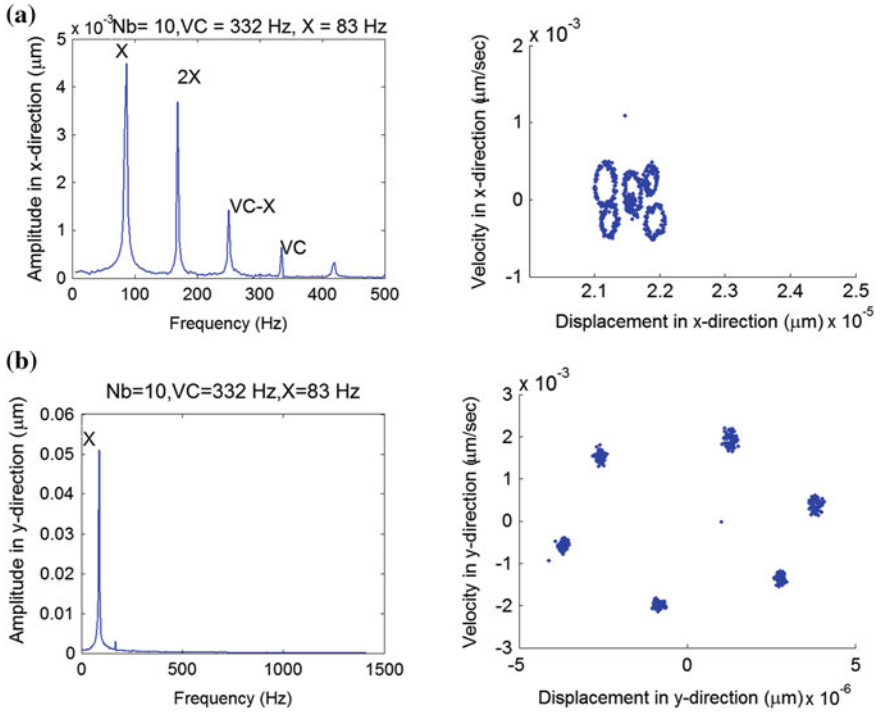


Fig. 6 FFT and poincare map at $N_b = 10$ **a** X direction **b** Y direction

Table 2 Influence of number of balls on stability

No. of balls	Major peak appears at (X direction response)	Major peak appears at (Y direction response)	Other frequency peaks in Spectrum appears (X direction response)	Other frequency peaks in spectrum appears at (Y direction response)	Nature of system
6	2X	X	X, VC - X, 2X, VC, VC + X	VC - X, VC	Chaotic
7	X	X	2X, VC	VC - X	Quasi-periodic
8	X	X	2X, VC - X, VC, VC + X	-	Quasi-periodic
9	X	X	2X, VC	-	Quasi-periodic
10	X	X	2X, VC - X, VC	-	Multi-orbit periodic

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