

Planet Eccentricity Error on a Planetary Gear Transmission: Influence on Load Sharing

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Abstract The efficacy of the planetary gear transmissions is strongly conditioned by the leveling of the load sharing achieved among the different planet paths. There are two main causes accepted as the most important sources of uneven load sharing, which are the errors in the positioning of the planets and the eccentricity of the gears. Several solutions have been implemented in these mechanical systems in order to improve the load sharing among planets, such as configurations with a floating member or the use of a flexible ring. In this work a dynamic planetary transmission model is presented, which has been developed by the authors and successfully used to simulate the effect of cracking and pitting on various variables in ordinary gear transmissions. This model has been extended in order to allow internal gearing, extending the simulating platform to include the static modeling capability of the planetary transmission behavior, including the presence of defects in gear positioning. The planetary transmission model presented in this work is an evolution of the previous one, and it allows now for the study of the planetary transmission behavior in dynamic regime. This model has been applied to the study of the load sharing in the presence of eccentricity errors in the planets. An assessment of the results is performed, and a comparison between the positioning and eccentricity errors in terms of their effects is also presented.

Keywords Gear · Planetary · Error · Eccentricity

1 Introduction

Compactness is one of the most important advantages of planetary gear transmissions with respect to the alternatives. For high torque levels, instead of increasing the size and width of gears, a better solution consists on dividing the load among

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several paths. Thus, the load per tooth width remains constant, but the total transmitted torque can be multiplied.

Moreover, planetary gear transmissions present axial symmetry, and planets are usually equally angular-spaced. These two factors lead to a very low value (almost zero) of the radial forces in the supporting elements, or in any case considerably lower than in ordinary gear transmissions. This is because the meshing forces in the central elements (sun, planets and ring) sum zero due to the system symmetry. Thus, radial support is no longer needed for the sun, which can be floating around its nominal position, fact which helps to compensate manufacturing errors and defects on the transmission.

Planetary gears failure is commonly associated with fatigue (due to bending) at the teeth, but also in the ring, especially when the latter is designed with low stiffness. Failure can also occur due to pitting and bearings defects [1]. These problems can be aggravated because usually the design of planetary systems is made following ordinary gears criteria. This neglects the special dynamic conditions that planetary gears must face with. This is the reason why a deeper research must be done in this particular transmission type, following two main lines of action: experimental vibration measurements [2] and developing of computer models to foresee the behavior of the system [3], in absence of experimental data. The last one enables to increase the reliability and decrease the time and cost of the design phase.

Ideally, each path supports the same load. However, real planetary gear transmissions present some deviations, due to manufacture errors and tolerances, which lead to a different load sharing ratio (LSR) among each path. This causes dynamic malfunction and reliability problems because the nominal load per teeth width may be surpassed. The LSR has been studied previously from both experimental [4, 5] and computational modeling approaches. This last approach includes models that go from simple analytical [6] to complex hybrid models that combine analytical and finite element methods [7].

Defects in planetary gear transmissions are also a recurrent topic that can be found in the literature. Causes and defects that produce an uneven LSR have been recently studied, and solutions to avoid it have also been proposed [6]. The two main reasons that produce an uneven LSR are: (i) errors in planets position and (ii) gears eccentricity. It is generally accepted that a three-planet configuration is the best option to deal with deviation, manufacturing and assembly errors, decreasing the divergence in the LSR. The authors developed a planetary model for the quasi-static study of the LSR on a transmission affected by planet positioning errors [8]. In this paper the planetary transmission model is extended to allow for dynamic simulation, and is applied to the study of the planet eccentricity error effect on the LSR.

2 Planetary Gear Transmission Model

The detailed model of the planetary gear transmission is thoroughly described in [9], where also a validation study in terms of the meshing stiffness is presented. This is the reason why only a brief description is presented next, mainly the key

issues on gear modeling, namely, profile definition, potential contact points' localization and computation of contact forces.

The profile definition of the external gears is made by cutting tools, following the vectorial approach described by Litvin and Fuentes [10]. This is a realistic procedure of defining the tooth profile, because is the one used during its manufacture. Besides, the analytical definition implies a great adaptability of the model, allowing shifted gears, but also undercutting conditions. A tip rounding arc is also added to the top of the teeth, following Vedmar procedure [11], in order to avoid singularities in the contact at that points.

As the profile definition has been described in an analytical way, it is also possible to compute potential contact points and distances analytically as well. To this end, contact over the line of action (LOA) is defined and the geometrical overlapping between profiles is computed for each potential contact point. Thus, numerical procedures which may incur in a rise of the computational effort are avoided and the accuracy is increased.

Finally, contact forces are computed using its relation with deformations. By solving a non-linear constrained system of equations, contact forces are computed for a specific position. The relation between contact forces and deformations is obtained using a modification of Andersson procedure [12]. This method computes contact efforts by following a similar approach to the one applied by Vijayakar [13]. The deformations are obtained as a combination of two terms: global (also called structural) and local. The structural term is due to linear deformation in the region far from the contact point, representing the deformation (shearing and bending) of the tooth and the whole gear body. On the other hand, the local term is used to describe the non-linear deformation in the region where the contact is taking place. Thus, the three problems described in Fig. 1 are combined, taking into account Saint-Venant theory for statically-equivalent loads: far away from the application point, differences among their effects can be neglected.

Therefore, a boundary is established at a distance h in order to consider both structural and local deformations. Structural deformation is then computed using Finite Element Method (FEM), loading a single-point load on the potential contact point (Fig. 1a). These deformations are valid only for regions far away from the contact point. Consequently, a correction is needed for regions closed to the contact point, where the deformation is going to be computed via Weber-Banashek formulation. Thus, a subtraction is applied near the contact point (depth $< h$), as it can be seen in Fig. 1b. Total structural deformation is then obtained as a combination of a and b in Fig. 1. Then, local deformation is applied to the region close to the contact point (Fig. 1c), computed by means of a non-linear analytical, which is dependent on depth h and length of contact zone L , functions shown next. Total deformation can be described as

$$\mathbf{u}_{Tj} = \mathbf{u}_{Local}^{G1}(f_j) + \mathbf{u}_{Local}^{G2}(f_j) + \mathbf{u}_{Struc,j}^{G1}(f_{1:N}) + \mathbf{u}_{Struc,j}^{G2}(f_{1:N}) \quad (1)$$

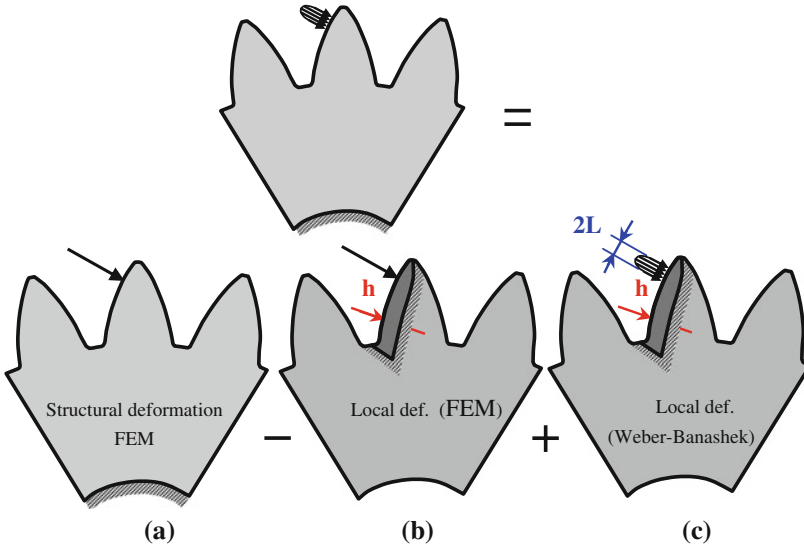


Fig. 1 Combination of structural and local deformations

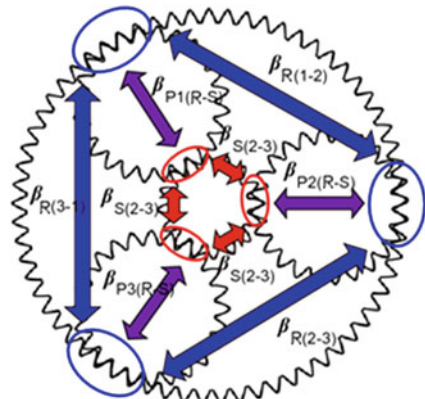
With the local non-linear analytical formulation defined as:

$$u_{local}(q) = \frac{2(1 - \nu^2)f}{\pi E} \frac{f}{b} \left[\ln \left(\frac{h}{L} + \sqrt{1 + \left(\frac{h}{L} \right)^2} \right) - \frac{\nu}{1 - \nu} \left(\frac{h}{L} \right)^2 \left(\sqrt{1 + \left(\frac{h}{L} \right)^2} - 1 \right) \right] \tag{2}$$

Computation of contact forces using this approach presents some advantages. Usually, contact zone is two orders of magnitude smaller than teeth size. This means that using FE, a very fine mesh would be needed in the contact region. Also, as the contact zone changes during motion, a remesh is needed for each position. With the hybrid approach used in this work, a much less fine mesh is accepted because is needed only for computing structural deformations and no remesh is required. This reduces considerably the computational effort.

As it has been stated previously, total deformation depends on both deflections of teeth and the whole gear. This means also that deformation coupling between adjacent teeth is taken into account. However, in planetary gear transmissions this deformation coupling implies an extra difficulty. This is because one single gear meshes with more than one wheel: each planet meshes with both sun and ring; and sun and ring mesh with all the planets at once. Consequently, the deformation of the gear bodies will result not only in coupling among the N potential contact points in one mesh, but also between meshes. Figure 2 shows the flexibility coupling between each mesh pair $\beta_{Pi(R-S)}$ by the planet deformation itself as well as $\beta_{R(i-i+1)}$ y $\beta_{S(i-i+1)}$ for coupling due to deformations of ring and sun.

Fig. 2 Deformation coupling among multiple gears



3 Dynamic Model

A planar dynamic model of lumped masses is presented in this section. Taking as a starting point the quasi-static model, the dynamic model must also include the dissipative phenomena, such as friction and damping. It is difficult to find in the literature a systematic modeling of these kind of efforts for gear transmissions, specially for its dynamic implementation. They can be classified by means of their dependency on losses by friction, rolling and deformation (dependent) and losses due to movement of fluid mass (independent). According to Höhn [14], friction is the most important loss component if we compare it with rolling. Moreover, from the point of view of the system excitation, the importance of the friction force is even greater, not only because it acts out of the line of action, but due to its direction change at the pitch point.

In this work, friction forces and damping forces due to solid deformation and lubricant effect have been taken into account. Friction forces are implemented using a Coulomb model, where the contact forces are multiplied by the friction coefficient. The definition of the friction coefficient takes into account its zero value at the pitch point, but also lubricant effects, such as sliding velocity, viscosity, load amplitude and rugosity.

Contact forces in bearings are described in detail in [10]. These, together with meshing forces (including contact, damping due to lubricant and friction) have been implemented in MATLAB® and integrated in SIMULINK®. Equations of motion are shown next

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{f}_{ext}(t) - \mathbf{C}\dot{\mathbf{q}} - \mathbf{f}_{bearings}(\mathbf{q}) - \mathbf{f}_{mesh}(\mathbf{q}, \dot{\mathbf{q}})) \tag{3}$$

A block diagram which represents Eq. (3) is shown in Fig. 3.

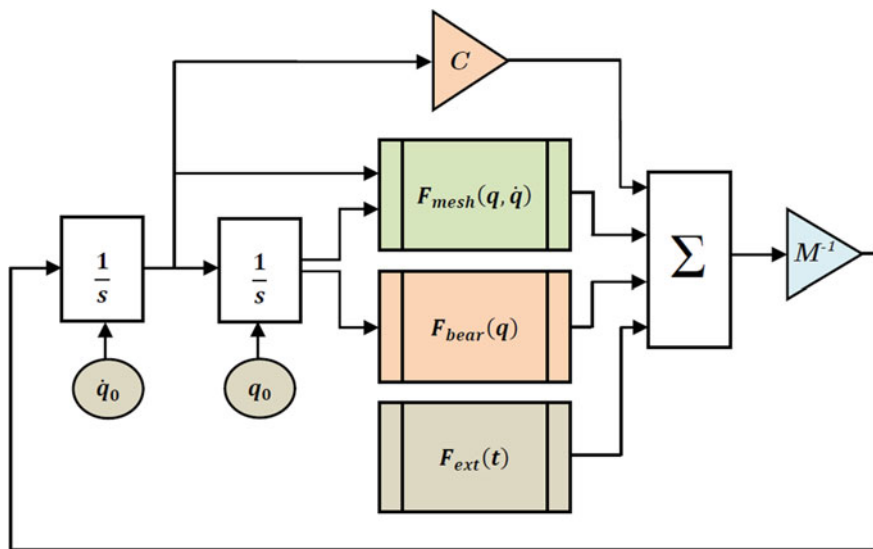


Fig. 3 Block diagram of the model for SIMULINK® implementation

4 Dynamic Behavior: LSR

LSR has been studied paying particular attention to the influence of position errors and run-out of the planets. Run-out is produced when there are a displacement between the geometrical center of the wheel and the rotation center. This eccentricity (run-out) error, from a kinematic point of view, is a position error which varies harmonically with respect to the angular position of the gear. Thus, the harmonic variation affects to both radial and tangential components of the gear position.

The planetary transmission modeled in this work has one stage, and it is based on an real planetary gear transmission for agricultural machines. In Table 1 some of the most significant parameters are presented. The three planets are equally spaced, and the meshing phase difference between planets is $2\pi/3$ radians.

The LSR has been defined as the ratio between the sun torque due to its meshing with each planet and the total input torque through the sun. Hence, three LSR are defined, one for each planet, as

$$LSR_i = \frac{T_{mesh(P_i-S)}}{T_{input}} \tag{4}$$

Two different cases have been studied, namely, constrained system and floating system. The first one does not allow translational displacements of the central elements. The second one allows translational displacements in the sun.

Table 1 Planetary gear transmission parameters

| | Sun | Planet | Ring | |
|------------------------------|-----------------------|---------------------|-----------------------|-----------------------|
| Teeth | 16 | 24 | 65 | |
| Modulus m (mm) | 4.23 | 4.23 | 4.23 | |
| Width (mm) | 25 | 25 | 25 | |
| Pressure angle (cut tool) | 25° | 25° | 25° | |
| Addendum (cut tool) | 1.35 m | 1.35 m | – | |
| Deddendum (cut tool) | 1.15 m | 1.25 m | – | |
| Tip round (cut tool) | 0.05 m | 0.05 m | – | |
| Base rounding | – | – | 0.05 m | |
| Teeth tip round | 0.05 m | 0.05 m | 0.05 m | |
| Axis radius | 20 mm | 20 mm | 156.4 mm | |
| Elastic modulus | 207 Gpa | | | |
| Poisson coefficient | 0.3 | | | |
| | Sun | Planet | Ring | Carrier |
| Mass (kg) | 1.19 | 1.23 | 28.10 | 3.64 |
| Inertia (kg m ²) | 9.92×10^{-4} | 24×10^{-4} | 68.2×10^{-2} | 20.4×10^{-3} |
| Rugosity | 0.8 μ m | | | |
| Viscosity lub. | 50 mPas | | | |

4.1 Constrained System

Figure 4a displays the LSR for the three planets with a run-out error of 20 μ m for planet 1 in a constrained system. It can be noted the harmonic variation of the LSR, with a maximum amplitude deviation due to the tangential component of the position error. This corresponds well with the findings explained in [8], where the great influence of the tangential positioning error in the LSR is shown. The minimum amplitude deviation of the LSR, where the three planets carry similar values

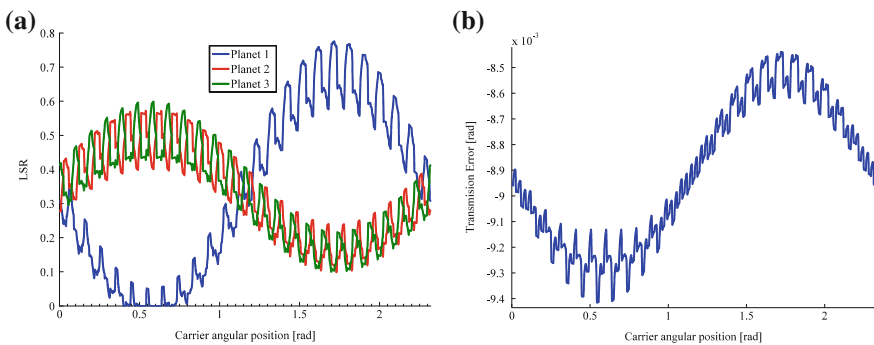


Fig. 4 Run-out = 20 μ m, fixed sun, torque = -600 Nm; **a** LSR; **b** Transmission error

of load can be found where the run-out only presents a radial component of positioning error. The radial effect over the LSR has been found 40 times smaller than the one caused by the tangential component.

Figure 4b shows the transmission error measured at the sun. Different areas can be distinguished and identified over the cycle: central part is due to radial component of the run-out and lower and upper deviations are due to negative and positive tangential error, respectively. Again, the harmonic variation of the eccentricity through its components can be appreciated.

4.2 Floating System

LSR in a floating planetary gear transmission with run-out errors stay very close to the ideal situation (without errors), as it can be seen in Fig. 5, with the exception of the contacts that take place out of the line of action. These contacts correspond to *involute-tip rounding arc* contacts, where the pressure angle does not remain constant. This means that there is no symmetry in the forces balance, and as a consequence a change in the LSR is produced. The sun describes an orbit around its nominal position, which allows to compensate the position deviations of the planet, but cannot absorb the out of line of contact effect.

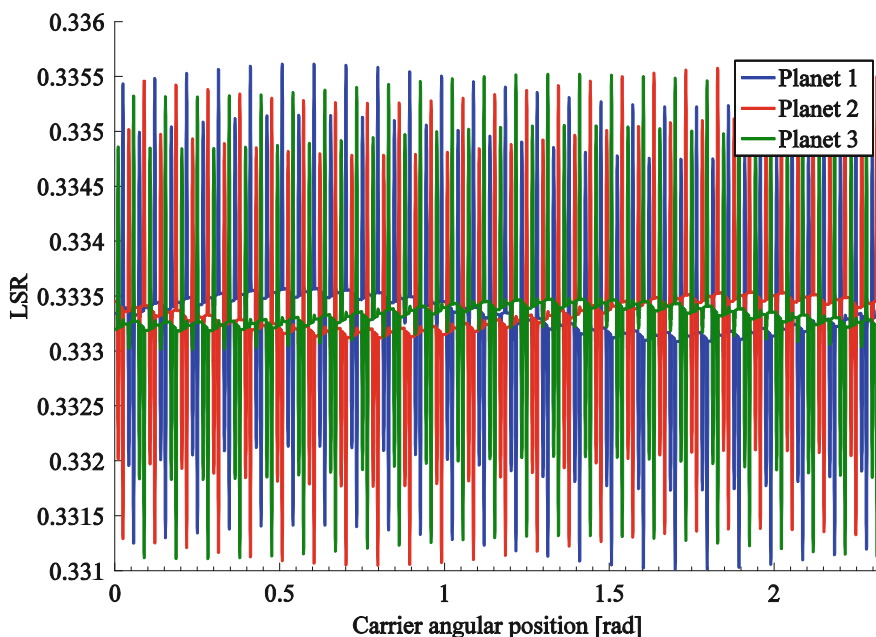


Fig. 5 LSR with floating sun. Run-out = 20 μm , torque = -600 Nm

Fig. 6 Dynamic orbit of the sun with run-out = 20 μm

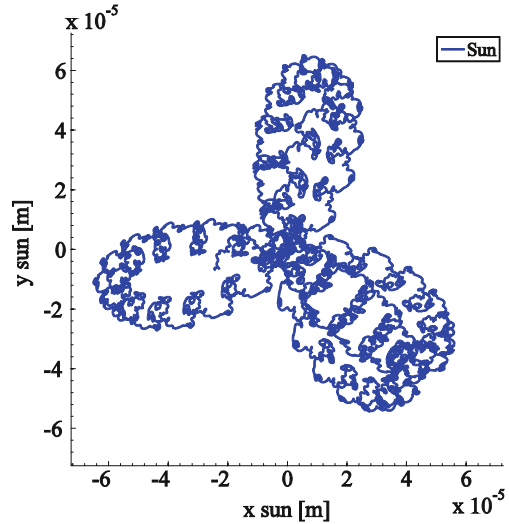


Figure 6 shows the dynamic orbit described by the sun in presence of a defected planet with a run out error. The main difference in terms of sun orbit between a system with a planet incorrectly positioned (by means of a tangential positioning error) and a system with an eccentric planet is the shape of this orbit. In the first case the orbit is circular, whereas in the second one it presents a lobular shape, due to the negligible contribution of the radial component.

5 Conclusions

This work presents an upgraded planar dynamic model for planetary spur gear transmissions. This model is especially designed for the detailed study of the LSR in different situations, including position errors and run-out. Load forces are computed following a hybrid procedure which includes FEM and analytical formulation, taking into account also the coupling between different gear meshes.

In the constrained configuration, the LSR shows a strong harmonic variation caused by the different component (radial or tangential) affected at each angular position of the gears. This effect can also be appreciated in the transmission error measured in the sun gear. Tangential component of run-out has a great influence on the LSR in constrained systems. This might even cause the complete loss of contact of one or several loading paths, depending on the external torque and on amplitude of run-out. Run-out also affects the transmission error of the whole system, where the specific effects of both components can also be seen.

In the floating configuration, the LSR remains almost constant and close to the ideal ratio, with the exception of the contacts out of the line of action, where a maximum variation of 0.1 % with respect to the ideal ratio is reached. In this

configuration, the sun describes an orbit, absorbing the run out error. Thus, the order of magnitude of the orbit radius is similar to the run-out in the planet. This orbit presents a lobular shape, again due to the different components and its effect over the LSR.

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