Chapter 8
The Theory of Real Options in Real Estate

Abstract The limits of discounted cash flow analysis in its static version, even when using a probabilistic approach to risk analysis, are given by the impossibility to incorporate strategic assessments, that is, to take into account the interaction between current investment alternatives and future decisions. The theory of real options outlined in this chapter, is an attempt to overcome this limitation. It discards the static approach and allows to manage the uncertainty related to possible changes of scenario. It is used to determine a final investment value in which the values of future opportunities that may arise for the real estate investor are included, such as expansion, contraction, abandonment or postponement of the project.

In the previous paragraph it was pointed out that the main limitation of the NPV—as traditionally understood—consists in the inability to evaluate between current investment opportunities and future decisions. This is crucial for strategic investment where the benefits from future developments resulting from the assumption of current decisions, may represent most of the value of the investment.[1]

It then becomes necessary to adopt models and logics of evaluation to assign a specific value to the discretion of the investor to administer an investment, among threats and opportunities that characterize future scenarios. This discretion has been defined as adaptability/flexibility management.

Investment projects in real assets should be seen as the source for a number of opportunities that lead the management, in changing from a passive to an active setting, to recognize the necessity of certain conditions in certain scenarios.

The opportunities for expansion, contraction, abandonment or postponement of the launch of an investment project give the decision maker a managerial flexibility whose value should be taken into account in the overall evaluation of the project.

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[1] In practice it seems that many managers already realize that there is something wrong with the simple rule of the net present value (NPV) as it is commonly understood. That is, they become aware that the expectation of further information can create value and that this value is not reflected by the traditional calculation of the net present value…. It may happen that managers realize that the options of an enterprise are valuable and that it is appropriate to keep them open (Dixit and Pindyck 1994).

B. Manganelli, *Real Estate Investing*, DOI 10.1007/978-3-319-06397-3_8

The Real Options Theory is presented as an attempt to overcome the main limitations associated with the methods of risk treatment described so far. This approach describes a method which involves evaluating and managing strategic investment in an uncertain context and which extends the methods of option pricing from the financial contracts to the so-called real assets. The most interesting chance offered by this new theory is the ability to consider strategic evaluations, to quantify the risk related to consequences that result from future decisions based upon original design assumptions.

The real options analysis is a technique for evaluating investments that can be successfully used to deal with uncertainty related to possible changes of scenario.

Compared to the static approach—which considers the discounting of cash flows for investment in the most likely predictable future scenario—the real options approach, when it is possible to transform uncertainty into risk, allows for the risk analysis for different project solutions, to be carried out, namely, options.

In the real options theory, this is made with the concept of extended NPV (ENPV):

$$ENPV = NPV_b + OP$$

where

NPV_b net present value related to base investment, the evaluation is deterministic in this case, and the result does not consider future opportunities;

OP the value of future options.

The theory of real options, as mentioned above, means to evaluate an investment project with regards to opportunities offered to the investor and which may lead him to assume specific future conducts that are part of a broader concept of flexibility/adaptability. The cornerstones of the theory are:

1. Strategic investments are a source of real options;
2. The investor has the managerial flexibility/adaptability.

The real options approach, along with the evaluation methods regarding contexts of uncertainty, has the advantage of adapting to the characteristics of complexity that some investment projects have.

Through the theory of real options, the analysis of economic convenience of a specific initiative and the strategic considerations thus become a unique moment of reflection.

8.1 Investment as a Generator of Real Options

The evaluation of an investment project through the NPV is based on the formulation of assumptions related to their ability to be absorbed by the market, related to the cost of production, to credit policy and to the amount invested. This results in an expected cash flow that is discounted using an appropriate rate, expression of the riskiness of the project. When one decides on the basis of the NPV, the project cannot be subject to change in any of the dimensions that make up its original structure. Furthermore one assumes that the investor, upon leaving the strategic vision, confines himself to implement the planned project, even if unexpected events may cause appropriate amendments to the project itself.

The NPV therefore does not take into account, that the investor is a manager of real assets, and consequently he is able to respond to market changes, to make strategic choices that will enable him to benefit from the positive scenario evolution, and at the same time to intervene to contain the negative consequences of an unfavourable evolution of the variables that affect the value of the project.

Investment projects incorporate real options which are exercisable by the investor at the most appropriate time. These options gain value the moment certain conditions and potentialities have been pin-pointed and are considered as realistic. This value should be reflected in the overall evaluation of the project. The problem is to define the tools for the options assessment.

The procedural process, which is based on the theory of the options, is subject to the following considerations:

(a) There is a problem of definition and recognition of the opportunities offered to the investor once he embarks on a given investment project or at the very moment of the assessment;
(b) These opportunities must be adequately evaluated for each of them providing the quantitative information necessary to their evaluation.
(c) When the types and numbers of the different options are clear, one needs to question the interrelations between the different options that arise from the same project (the problem of joint options requires special precautions in the evaluation phase);
(d) Finally, it is necessary to quantify the total value of the identified options, so it is necessary to use mathematical-statistical models that interpret the structural complexity of the project.

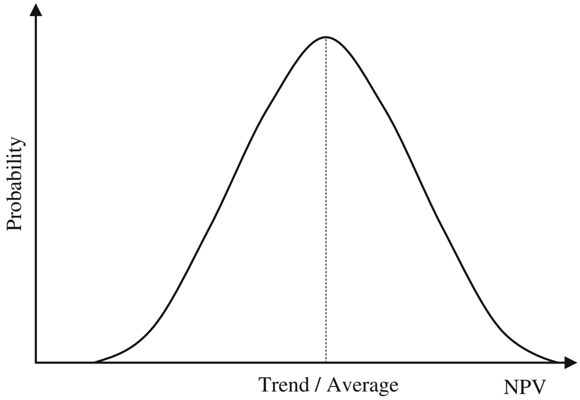
The fundamental elements that characterize an option are:

1. The deadline given by the validity timescale of the right to exercise an option;
2. The exercise price, which is the price to pay to apply the right;

As said, the evaluation of investment projects in an uncertain context involves the determination of many values of extended NPV as many as the number of scenarios resulting from the combination of the assumptions related to the development of the key variables of the project. The different values of NPV thus determined are represented through a frequency distribution, which can be associated with a specific probability function to be estimated, based on trust attributed to the different scenarios.

Figure 8.1 shows a possible probability distribution of the NPV of a project. In the absence of managerial flexibility i.e. real options, it is reasonable to assume that the distribution is normal (symmetric). This is actually an implicit assumption to the deterministic approach of the static NPV.

Fig. 8.1 Distribution of probability of the NPV of a risky project, in case of absence of managerial flexibility



The consideration of real options related to the investment project introduces asymmetries in the probability distribution of the NPV. This is because the opportunities to change the project are non-zero-sum games, which limit the distribution to the left (pessimistic scenario), but do not limit them on the right, since the same uncertainty related to the development of the project leaves the possibility that the project will also assume much larger proportions than those provided, open.

In other words, the presence of real options, given the management rights but not duties, extends the probability distribution to the right. It offers the opportunity to reap the benefits of the positive scenario evolution and at the same time to take action to contain the negative consequences of the unfavourable evolution of the variables that affect the value of the project.

The actual expected value of this asymmetric distribution exceeds the modal value of NPV in an amount equal to a premium that reflects the value of managerial flexibility, and Fig. 8.2 clearly proves it.

For the real options theory, the main elements that characterize the value of a project are the following:

- Initial investments related to the implementation of the project;
- Uncertain variables, of which the performance affects the decision related to the project;
- Gross value of the opportunity, characterized by the benefits resulting from the implementation of the option (it represents the final value of the project assuming the implementation of the option)
- Additional investment required to carry out the option;
- End of the option;
- Risk-free rate, corresponding to the yield of the bonds issued by the Treasury.

As a consequence of this, an investment opportunity must be carried out if the benefit arising from it is greater than the cost required. Since the value of the project is “optioned” i.e. subject to uncertainty, the evaluation of the cost effectiveness of the initiative is not amenable to the logic of the NPV.

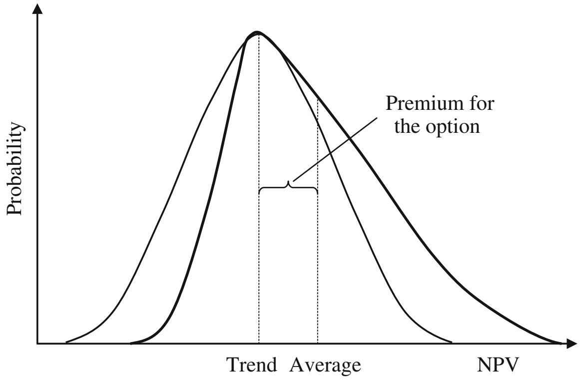


Fig. 8.2 The role of managerial flexibility in the probability distribution of the NPV

8.2 The Development of the Process

In the case of investment evaluation, a model represents all the logical and mathematical relationships among variables that characterize the project to be evaluated. The output of the model is in this case a number that summarizes the value of the project at the time of evaluation.

The process is instead the sequence of actions carried out by the investor from the moment he begins the analysis of an investment project to the obtainment of its final value. In this sense, the process also includes within itself the model that represents a crucial moment, but implies a dynamic and organic approach to the problem of resource allocation.

Therefore, adopting a perspective of the process, the evaluation of investments oriented to the enhancement of strategic opportunities could be structured along three main phases:

- Risk analysis;
- Strategic analysis;
- Quantitative analysis.

8.3 Risk Analysis

The main purpose of this phase is the identification and analysis of the uncertain variables, the evolution of which depends on the decisions related to the project. The moment of risk analysis aims to supply the quantitative information needed for the evaluation of the project.

The critical variables are generally represented by the evolution of the cash flows, the financial structure, the cost of construction, etc.

The determination of the variable, the evolution of which depends on the decision related to the implementation of the various opportunities, is a crucial moment for the application of the real options approach. It is necessary to determine one or more stochastic variables from whose value derives the decision of how to proceed with implementation of the project.

Another question, of course, is the definition of the optioned project. In this case, starting on the performance of the stochastic variable, one needs to build a relationship that allows to estimate the activity subject to option.

This confirms the basic difference between a deterministic approach (NPV) and a stochastic (theory of Options) approach. In the first case, it is assumed that the behaviour of the reference variables is determinable in advance, or it is describable with a probability distribution. In the second case, it is assumed that the uncertain reference variables follow a stochastic behaviour,[2] represented by a mathematical model, which determines the estimation of the trends and volatility of these variables even through the identification of the relevant parameters.

Regarding the applicability of deterministic and stochastic approaches, it is useful to reiterate that the first assumes that on the market there is a twin activity (compared to the size of the risk) whose yield is directly adopted to discount the cash flows of the investment to be evaluated (otherwise it would not respect the principle of the opportunity cost of the resources employed). In the theory of options, the value of the investment (in its deterministic component) is given by the value of the right to capture the twin activity, which corresponds to the size that the project would have in the case of exercise of the options.

8.4 The Process of Strategic Analysis

The second key part of the process of strategic allocation of resources through the real options approach regards the identification of the areas of managerial flexibility implicit in the project to evaluate and to define the relationships between the different options. The definition of algorithms that define the conditions for the exercise of real options and the analysis of the logical sequence for the analysis of the identified options constitute the two outputs of this phase.

8.4.1 Identification of Areas of Managerial Adaptability

The first step consists of a series of evaluations made by the investor from the structure of the project and from the characteristics of the decision-making process. At this first stage of analysis it is important to involve all those who in different ways participate, even indirectly, to the process of taking strategic

[2] The movement of a variable whose value evolves in time in a condition of uncertainty is defined as a stochastic process.

decisions. Typically, these can be analysts, designers, the construction company and other professionals who have expertise and discretionary power in some of the areas of interest of the project.

8.4.2 Identification of Options and Their Parameters

The purpose of this phase is the identification real options and the formalization of the same through characteristic parameters, such as the value of the optioned project, the exercise price, expiration, and volatility.[3]

The recognition of a real option depends on the existence of two basic conditions:

- It must be a discretionary lever—available to the investor—that causes variations in the results of the project;
- It should lead to the identification of the parameters of the option.

On a conceptual level, the different types of real options—arising from any investment project—can be classified within the following taxonomy:

- Deferment options;
- Expansion options;
- Contraction options;
- Conversion options;
- Abandonment options;
- Temporary suspension options.

Specifically, looking in particular at real estate investments, the various options can be explained as follows[4]:

8.4.2.1 Deferment Options

The deferment option relates to the decision about the moment in which to start the project. This opportunity can arise, in general, from any situation in which it is

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[3] "The investment projects incorporate real options, which are exercisable by the management at the most appropriate moment, these options have a value at the same time in which it recognizes a potential exercisability upon the occurrence of certain conditions, this value should be reflected in the overall evaluation of the project" (Micalizzi 1997).

[4] The study real options has been one of the most fruitful areas of research on the topic of real estate market. The approach to real options assumes that when an economic individual engages in an irreversible investment projects, he gives up the opportunity to postpone the start of the project at the time in which he has acquired new information that could affect the attractiveness of the investment. Not recognizing the value of leads to sub-optimal investment decisions (Yavas and Sirmans 2005).

believed that the implementation of a project can be delayed over time without undermining its technical feasibility. This opportunity has value if it is expected that something may happen within the period of deferment that increases the present net value of the project.

Typically there are therefore issues regarding the timing of the operation of an intervention of buildings, where the ownership of the building lot preserves intact the right to develop the transformation. The underlying asset (the optioned project) captured from the exercise of the option is given by the best expected value of the project in the case of positive scenario, that is, with a different point of view, from the cost savings associated with investing in case one decides not to proceed with the operation after finding an unfavourable scenario evolution.

When there is a possibility to defer the start of the investment, the investor has two options: to implement the project, having found a positive evolution scenario, giving up the initiative, saving the costs involved. This is typical of the real estate sector, regarding the decision to develop a project on an owned land, or the possibility of deferring waiting to check the stability of the prices and to see the uncertainty of the market resolved. The algorithm that represents the value of the option is:

$$Max(PV - In;\ 0)$$

where:

PV value of the expected benefit from the implementation of the project;

In Initial investment capitalized at time *n*, where n is the number of periods of deferment.

8.4.2.2 Expansion Options

This option allows to increase the share of the scale of a project a (with $a > 1$) making further investments *Ie*.

In the case of real estate investment, one may think to purchase additional units, for the expansion of the project, and so on.

After some time, a product added in a particular market may prove to be suitable to meet the needs of different markets or in different niches of the market itself. The importance of this option in a strategic optics is related to those initiatives in which the investor makes a series of initial investments that enable him to benefit particularly from the positive evolution of the scenario. The estimation can be done by applying an expansion parameter to the value of the existing project. As far as the exercise price is concerned, it coincides with the investment required to perform the expansion. This could involve the allocation of new facilities, improve the level of quality of the building, and so on.

Keep in mind that in the real estate field, a change in the size of the project, such as the amount built, may allow the developer to benefit from some economies of scale. The choice to increase the scale of the project therefore facing additional production costs, in the construction field, generally produces an increase of costs that are less than proportional to the increase in cubage, at least up to a certain threshold of intervention.[5]

The algorithm that summarizes the condition of exercise typically takes the following form:

$$\text{Max}\ (PV;\ a \times PV\ -\ Ie)$$

where

- PV gross value of the project in the absence of strategic opportunities;
- a expansion factor (greater than 1) of the value of the project;
- Ie investment linked to the exercise of the option (exercise price).

It is hardly necessary to point out that instead of the gross value of the project, a function, that allows to place in the analysis the role of a particular uncertain variable, can be used (e.g., cash flow).

8.4.2.3 Contraction Options

This option detects, in this case, an opposite sign to the expansion option; it offers the possibility of reducing, by a c % percentage, the size of the initial project, saving an IC share of part of the initial investment. The decision on the sizing of a residential park can be taken considering the opportunity of reducing it to respond to unpredictable changes in the market. This of course, defining beforehand (a priori) the operating procedures of such an operation. The exercise price must be considered equal to the savings in operating costs or the possible achievable value of the contracted part of the program. The option of contraction has value in the event that the savings of further investments is higher than the worst expected value of the benefits resulting from the completion of the project.

It is assumed that the investor may divide the total investment in two parts:

$$I = Io + In$$

where *Io* is the immediately affordable investment and *Io* is the investment deferred to the future. This is the typical situation in which one has to choose the right mix between maintenance costs and construction costs. The formula that represents the value is:

[5] Please note that the morphological and volumetric aspects of a building are able to heavily influence the cost of construction. For example, if the height of the building increases, the ratio between useful floor area and gross floor area decreases because of the larger space required by technological systems, circulation spaces, stairwells and elevators, the presence of the latter, as well as foundations, is typically able to produce threshold effects (Bravi 2003).

$$(PV - In) + Max(Ic - (c \times PV); 0)$$

where:

In the part of the investment that can be postponed at time *n*;
Ic investment saved as a result of the decision of contraction;
C percentage of reduction of the original project.

8.4.2.4 Conversion Options

The conversion option regards the possibility of using the consequences of an investment project that has been prematurely interrupted (adaptability of the product) or rather, of changing, for example, the intended use of the property. Especially in the early stages of research and development, when the product is still fuzzy, it could highlight the possibility to redirect the efforts put on the development of a product which proves to be substantially different from the one initially aimed at achieving.

Alternatively, there may be the possibility to change the production process and the used factors (adaptable process), achieving the same output. This is to capture the best alternative use, choosing the maximum between the value of the project in its current use and the value in its best alternative use. The exercise price is the renunciation of the benefits arising from the project in its current use (or from the conversion costs).

The operating condition is thus formalized in the following equation:

$$Max\,(x \times PV - In;\, 0)$$

where

x Value increase purchasable due to the conversion;
$x \times PV$ Value of the "optioned" project following the decision to proceed with the change of use;
In The expected cost at time *n* for conversion.

8.4.2.5 Abandonment Option

Not different in terms of the logical aspect, is the case of the abandonment option, when one considers that in this case it also takes on an active attitude on behalf of the investor, who able to understand the elements of flexibility that the structure and characteristics of the project present. At the time of the preliminary evaluation of the operation, it is therefore necessary to recognize and possibly estimate the value represented by the opportunity that part of the efforts incurred up to the date of the potential exercise of the option, can be partially recovered, in the event of abandonment of the original project. This option has value to the extent that the realizable value of the existing is higher than the worst-case scenario in case of continuation of the activity. An example is the case where the investment required for the project can be divided into lots, and the abandonment would result in the saving of costs related to subsequent lots which have not yet been realized (e.g., development of real estate in most units). The cost of each lot can be considered as the exercise price for acquiring the subsequent parts of the project.

The algorithm of the operating condition takes the following form:

$$Max\,(x \times PV - In;\; 0)$$

where

x	portion purchasable from the project;
$x \times PV$	value of the "optioned" project following the decision to proceed with the development;
In	the estimated cost at time n to continue the development of the project.

8.4.2.6 Temporary Suspension Option

This option, instead, is placed halfway between the abandonment option and the deferment options. It exists when it is technically possible, as well as convenient from an economic point of view, to stop the development of a project for a certain period, pending changes in market conditions. This is an opportunity for companies engaged in the construction of several residential complexes, which may temporarily suspend the activities of construction, or the business of marketing of real estate. This situation may allow a saving of some categories of variable costs, and has a value from the point of view of managerial adaptability if it is expected that within a certain range the variable operating costs are higher than the worst expected value of monetary income. In these circumstances, it may be better to bear the fixed costs and stop activity until the change of scenario reinstates the minimum condition of operating convenience.

The operating condition is the following:

$$(PV - Cf) - Min\,(Cv; F)$$

where:

F	annual revenues;
Cf	fixed costs per year;
Cv	variable operating costs.

If the annual revenues are less than the variable operating costs $Min\,(Cv;F) = F$, the decision maker has the convenience to suspend their activities, obtaining the value of the project, net of fixed costs.

In case of *Min (Cv;F)* = *Cv*, the investor prefers to work, supporting variable costs for the achievement of its revenues.

8.4.3 Verification of the Operating Conditions

In this phase, if more than one option comes out from the project it is necessary to define the order in which the different algorithms, that have been identified, should be applied. As a matter of fact, real options are often presented as compound options. In other words, if from a single project, different real options arise simultaneously, these must be combined within a unitary analysis scheme.[6] The problem of compound options, (i.e. an option made up from a plurality of options), requires special precautions in the evaluation stage (Example 8.1).

The final value certainly cannot be the algebraic sum of the values of the single options. The greater the number of existing options, the more likely it is that the incremental value of an additional option appears insignificant.

The degree of interaction between the options is a function of the type of action (*call or put*[7]) and of the distance between the operating times.[8]

The bond of interaction between compound options can be interpreted at two levels:

- With reference to a single project, there may be a temporal interaction, due to the dependence between current decisions and future development of the project itself (intra-project real option);

[6] The rule is that the first algorithm to be applied is the one for the real option, the exercise of which is less affected from the exercise of other real options.

[7] Financially speaking, a call option confers the right, but not the obligation, to its owner to purchase a stock at a stated price at a future time, conversely, a put option gives its owner the right to sell a stock at a set price at a future time.

[8] The sign of the interaction depends on the sequence in which they occur. The exercise of a first option can alter the structure of the underlying asset and therefore the value of subsequent options. The conditional probability of an option after the exercise of a previous one may be greater or less than the marginal probability of an exercise as a single option. If there are two real options of opposite sign (i.e. put or call), they are necessarily exercised in opposite situations. If, therefore, the exercise of the put option responds to the case of abandonment (i.e., the unfavorable evolution of the scenario), the exercise of the call may coincide with the development of the project (i.e. positive evolution of the scenario). In such situations, the conditional probability of exercising the second option, it being understood as the exercise of the first, it would be low and much less than the marginal probability of exercising the option when considered alone. If the two options were of the same type (two call or two put), the conditional probability of exercise of the second would be high, as well as the degree of interaction between the same. The degree of interaction depends on the distance of the periods of exercise for the two options considered. For two puts or two calls with exercise dates coincident and that do not interfere with each other, the combined value is approximately equal to the sum of the values considered individually (Trigeorgis 1996).

- Decisions taken in relation to a project, on the other hand, can generate effects on other projects: in this case one is dealing with the interaction of design (inter-project real option).

Sometimes the investment required for the completion of the initiative is not immediately supported. Rather, it is divided into shares at different times in the life of the project. This is the typical case of the development of real estate. In such circumstances, each instalment of the total investment can be seen as the afforded exercise price needed to acquire the subsequent options for continuation of the project. This is the case of the intra-project compound options.

The benefits that the investor can derive from a situation of this kind are clear. Each investment instalment, in fact, identifies a specific, strategic, decisional node in which the decision maker, facing the opportunity of evaluating alternative developments of the investment (abandonment of the project, conversion, temporary suspension), finds himself.

Temporal interaction contrasts with the case of inter-project options. This condition occurs whenever the implementation of an investment project becomes the prerequisite for the start-up of a second project.

Example 8.1[9] Table 8.1 shows the case of an investment related to the acquisition of a number of residential units that generates a cash flow of €30,000 (for the sake of simplicity, it is assumed that the investment runs out in a single period). Assuming that the investor has two different managerial levers, the first is the expansion of the project through an additional investment (Ie) which amounted to €14,000 for the improvement of the quality of housing, and the resulting increase of rental in an amount equal to an expansive factor of 1.4. The second is related to an investment, of implantation-type (Ic) for €11,000, which causes a decrease in the incidence of monetary costs on the flow, from 70 to 60 %.

The traditional analysis based on NPV, shown in Table 8.2, would draw a separate evaluation of the two opportunities. The first leads to an operating cash flow (Fc) equal to 28, and the second to an operating cash flow (Fc) equal to 29.

In neither case, therefore, does it seem appropriate to carry out additional investment, given that the base project ensures a flow of 30.

The two levers are considered as real options, with exercise prices Ic and Ie and values of the optioned project amounting to $(e \times Fb)$ and $(70\,\% - c) \times R_b$, where, e, equal to 40 %, represents the percentage of increase in cash flow, while c, equal to 60 %, represents the lowest level of the operating cost. The expansion leads to the following flow:

[9] The example is taken from Quigg (1995).

Table 8.1 Data relating to a real estate investment

Revenues (R_b)	€100,000	–
Operating costs (C_b)	€−70,000	[70 % of R_b]
Cash flow (F_b)	€30,000	–

Table 8.2 Cash flows of the optioned project

Expansion Option		
Expansion factor	1.4	[70 % of R_e]
Revenues (*Re*)	€140,000	
Operating costs (*Ce*)	€−98,000	
Initial investment (*Ie*)	€−14,000	
Cash Flow (*Fe*)	€28,000	
Option of cost reduction		
Incidence costs	60 %	[60 % of R_c]
Revenues (*Rc*)	€100.000	
Operating costs (*Cc*)	€−60.000	
Initial investment (*Ic*)	€−11.000	
Cash flow (*Fc*)	€29.000	

$$dF_e = F_b + Max\,(e \times F_b - Ie; 0) = 30{,}000 + Max\,(0.4 \cdot 30{,}000 - 14{,}000; 0)$$
$$= 30{,}000 + Max\,(-2{,}000; 0) = €30{,}000$$

The technology investment instead, leads to a cash flow equal to:

$$dF_c = F_b + Max\,[(70\,\% - c) \times R_b - Ic; 0] = 30{,}000 + Max(0.1 \times 100{,}000 - 11{,}000; 0)$$
$$= 30{,}000 + Max(-1{,}000; 0) = €30{,}000$$

So, by considering the investments separately, even in the form of options the same result as the previous one will be reached. If, on the other hand, one considers the interaction between them, or the possibility of proceeding with the investment related to the reduction of costs within the expansion investment, the value of the compound project can be expressed in the following way:

$$dF'_b = F_b + Max\,[e \times F_b - Ie + (70\,\% - c) \times (1 + e) \times R_b - Ic; 0]$$
$$= 30{,}000 + Max(0.4 \times 30{,}000 - 14{,}000 + 0.1 \times 1.4 \times 100{,}000 - 11{,}000; 0)$$
$$= 30{,}000 + Max(1{,}000; 0) = €31{,}000$$

It follows that the value of €1,000, given by $dF_b{}' - F_b$, is the result of the synergy existing between the two options considered together.

8.4.4 The Process of Quantitative Analysis

The purpose of this phase of the analysis is to reach the quantification of the value of strategic opportunities.

The quantitative analysis in turn consists of three steps. The first, is related to the calculation of the extended value of the project. Considered individually, the NPV summarises all the above considerations. In the second phase, the value of the interaction between the options, or the weight of strategic synergies that link together different real options, are considered. Lastly, the sensitivity analysis is developed in order to understand the impact of a change in volatility or a change in the expiry of the option.

8.4.5 The Calculation Models for Extended NPV

The real options approach includes a series of financial models intended to estimate the value of flexibility embedded in investment projects.

The main models are based on the notion of equivalent portfolios, which combine investments and debts so that the payoff of the option is replicated. Thus a hedging strategy which triggers the risk of neutrality and the consequent possibility of making the estimate in terms of the definite equivalent.

Mason and Merton (1985) assert that with the NPV, it is implicitly assumed that it is possible to identify market activity equal to the project under evaluation, i.e. having the same risk profile. The expected return on the twin activity is taken as the discount rate of the cash flows expected from the project. The evaluation of an option is developed on the same assumptions. Given the price of a twin activity to the project to be evaluated, the investor could build a properly balanced portfolio consisting of a number of units of the twin activity and an indebtedness that provides the same return on the project.

For the determination of extended NPV, it is possible to adopt evaluation models which directly approximate the stochastic process of the optioned project, in a continuous (Black and Scholes 1993)[10] or discreet manner.

In fact, non-financial investment and real estate in particular, normally assume a trend more similar to a discrete process than to a continuous one (Micalizzi 1995).

Among the discrete models for calculating the value of one of the most common options is the binomial (Cox et al. 1979; Cox and Rubinstein 1985). It is based on the assumption that the value of the optioned project evolves according to a stationary multiplicative binomial process. In a similar process, the S value of the project optioned at the beginning of a certain period can grow with probability q of

[10] The Black and Scholes formula can be used only in the case where the underlying asset includes only one option and the uncertainty is due to a single variable (Micalizzi and Renzetti 2000).

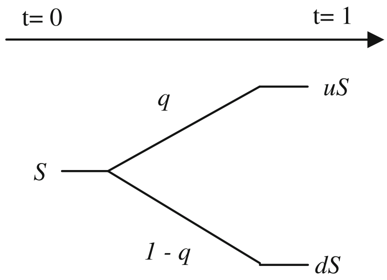


Fig. 8.3 Diagram of the binomial process

a multiplicative factor *u* (up), and may decrease with probability $1 - q$ of a reducing factor *d* (down) at the end of the period (Fig. 8.3).

The binomial method comes from the formulation of the value *V* of the extended project, assuming that the investor will replicate this value with the purchase of an asset at current value S_0 (or equivalently of an asset or portfolio of stocks perfectly correlated with *S*), borrowing an amount equal to *D* (debt) at the risk-free rate *i*.

The value of the replicating portfolio constructed in this way must equal the same revenues of the project *V* for every possible scenario. After a certain period the investor must repay the borrowed amount *D* and the related interest. Figure 8.4 shows the value of the portfolio.

By imposing the condition of equality between the flows of the two investment projects:

$$V^+ = nS_0^+ - (1+i)DV^- = nS_0^- - (1+i)D$$

It is possible to obtain the two parameters *n* and *D*:

$$n = \frac{(V^+ + V^-)}{\left(S_0^+ - S_0^-\right)}D = \frac{V^+S_0^- + V^-S_0^+}{\left(S_0^+ - S_0^-\right)(1+i)}$$

From which recalling that $V = nS_0 - D$:

$$V = \frac{[pV^+ + (1+i)V^-]}{(1+i)}p = \frac{(1+i)S_0 - S_0^-}{S_0^+ - S_0^-}$$

where *p* represents the equivalent probability (or pseudo probability) that arises from the favourable scenario at zero risk conditions. This can be interpreted as the certainty equivalent to the expected cash flow that can be discounted at the risk-free rate *(risk-neutralized probability)*.

The today value of the option to invest tomorrow in the asset *S* is equal to the net expected value (risk-neutral) of the asset in the favourable situation discounted at the risk-free rate.

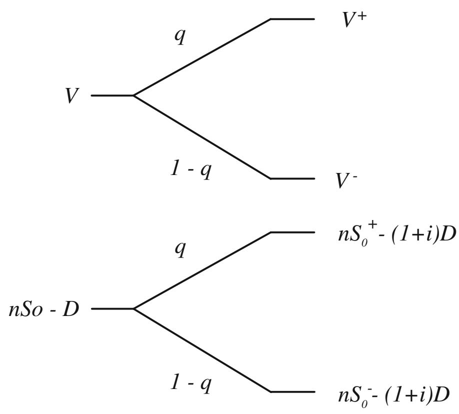


Fig. 8.4 Value of the optioned project in a binomial process

Example 8.2 An investor has the problem of evaluating the purchase of a land that is located in an area not yet explored from a real estate perspective. The decision must be taken quickly because the owner is negotiating with other counter-parties. There is a possibility that within a year the City Hall approves the urbanization plan that will lead to an overall development of the area.

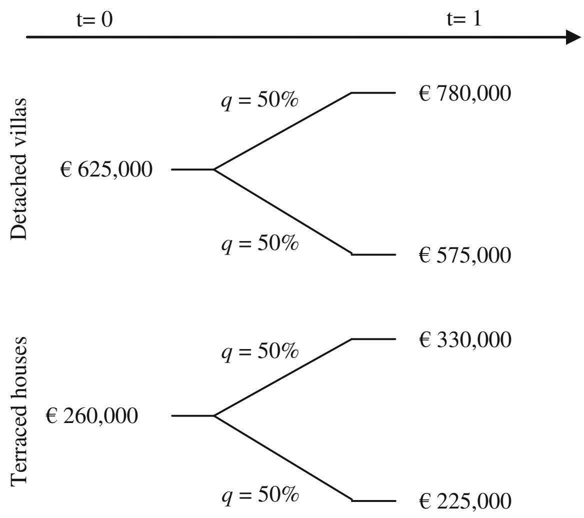
The investor must assess, in a short time, the purchase of a property whose value might change shortly. There is the possibility to develop two different projects: the first involves the construction of 8 detached villas of 250 m^2, while the second involves the construction of 11 terraced houses of 120 m^2. At present, the reference price per square meter of a villa is estimated at €2,500 while the terraced house can be sold at €2,100. The sale price of a villa according to the first project would therefore amount to €625,000, while the price of a terraced house would amount to €260,000. The total costs of production are known and equal to €500,000 per villa and €160,000 per terraced house. The amount is estimated as constant for the next 2 years.

However, if the company defers the final decision to a year later, the approval of the plan of urbanization of the area could lead to changes in the project, in light of a change in the selling prices of properties. In particular, given the uncertainty that weighs on sales prices, it is believed that one year after the decision the prices of the two building types might evolve in the manner described in Fig. 8.5 with a probability of 50 % respectively.

At time 0, the project has the following cash flows:

$$CF = nP - C$$

Fig. 8.5 Probable evolution of sales prices



where

n number of built real estate units;

P price of the units;

C total cost of production.

The flows corresponding to the two assumptions are:

- $CF_{villas} = 8 \times (625{,}000) - 8 \times (500{,}000) = 1{,}000{,}000$
- $CF_{houses} = 11 \times (260{,}000) - 11 \times (160{,}000) = 1{,}100{,}000$

Under present conditions, the most profitable project is that related to the construction of terraced houses. What would happen if the investor decides to defer the transformation for one year? (Table 8.3)

After a year he would opt for the project of detached villas in the event of a favourable evolution. Instead, he would choose to realize terraced houses if the scenario evolves unfavourably.

The problem is solved taking into account the value of the deferment. It is necessary to calculate the value at time 0 of a project that ensures at time 1 a flow of €2,280,000 in the case of favourable scenario and €660,000 in the unfavourable case scenario.

Considering a risk-free rate of 8 %, the risk-neutral probability of obtaining a flow of €2,280,000 is obtained as described in Table 8.4.

The extended value of the project is obtained by discounting at a risk-free rate values corresponding to the best use of the land in the two scenarios, one favourable and one unfavourable, weighing them with respect to risk neutral probabilities:

Table 8.3 Expected cash flows in the event of deferment

		Cash Flows
Detached villas		
Favourable scenario	8 × (780,000 – 500,000)	€2,280,000
Unfavourable scenario	8 × (575,000 – 500,000)	€600,000
Terraced houses		
Favourable scenario	11 × (330,000 – 160,000)	€1,870,000
Unfavourable scenario	11 × (220,000 – 160,000)	€660,000

$$V = \frac{[2{,}280{,}000 \times 0.48 + 660{,}000 \times (1 - 0.48)]}{1.08} = €1,331,111$$

The extended value of the land, in light of the possibility of deferring the choice between the two alternative uses, has a higher value than the best value of its current use. (€1.1 million).

It is hardly necessary to point out that if DCF had been used, the conclusions would have been distorted. If current market prices reflect expectations and are therefore equal to the present value of the expected cash flows, the rates implied in the prices of the market would be the following (k) in Table 8.5.

By applying the two rates obtained for the values corresponding to the best use of the land, it would lead to different current values (Tables 8.6, 8.7).

In order to be able to set up the analysis of the multi-period cash flows, taking into account the real options, a slightly more complex approach than the one described above should be taken. According to the model of Cox, Ross and Rubinstein (CRR), the prices follow a time pattern that simulates a binomial multiplicative process of the type in Fig. 8.6.

The binomial paradigm develops the modifications of the initial value of the investment through probabilistic multiplicative states, as defined by the coefficients $u > 1$ and $d < 1$ which represent, respectively, the evolution of the initial state to a favourable scenario or to an unfavourable scenario.

The measurement of the coefficients u and d is the result of the risk analysis of the investment, statistically calculated through a dispersion index that, in the case of normal distribution of the variable object of the analysis, coincides with its standard deviation (σ).

Once the risk has been estimated, the definition of the possible states of evolution occurs with mathematical expressions:

Table 8.4 Calculation of risk-neutral probability

Detached villas	$p = \frac{(1.08 \times 625{,}000 - 575{,}000)}{(780{,}000 - 575{,}000)} = 0.48$
Terraced houses	$p = \frac{(1.08 \times 260{,}000 - 220{,}000)}{(330{,}000 - 220{,}000)} = 0.55$

Table 8.5 Calculation of the rates implied in market prices

Detached villas	$\frac{[780{,}000 \times 0.50 + 575{,}000 \times 0.5]}{1+k} = 625{,}000$	k = 8.40 %
Terraced houses	$\frac{[330{,}000 \times 0.50 + 220{,}000 \times 0.50]}{1+k} = 260{,}000$	k = 5.77 %

Table 8.6 Calculating the value corresponding to the best use (k = 8.40 %)

Detached villas	$V = \frac{[2{,}280{,}000 \times 0.50 + 600{,}000 \times 0.50]}{1.084} = 1{,}328{,}413$
Terraced houses	$V = \frac{[1{,}870{,}000 \times 0.50 + 660{,}000 \times 0.50]}{1.0577} = 1{,}195{,}991$

Table 8.7 Calculating the value corresponding to the best use (k = 5.77 %)

Detached villas	$V = \frac{[2{,}280{,}000 \times 0.48 + 600{,}000 \times (1 - 0.48)]}{1.08} = 1{,}302{,}222$
Terraced houses	$V = \frac{[1{,}870{,}000 \times 0.55 + 660{,}000 \times (1 - 0.55)]}{1.08} = 1{,}227{,}315$

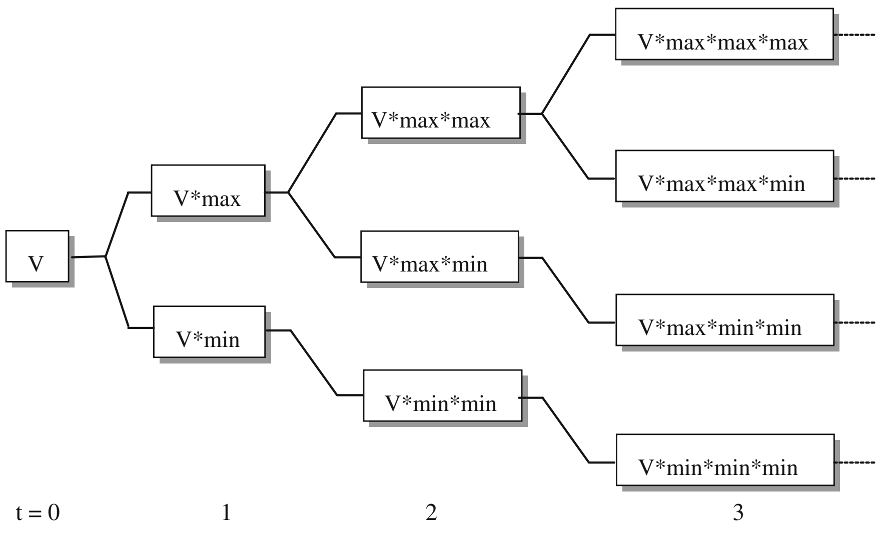


Fig. 8.6 Diagram of the multiplicative binomial process

$$\begin{cases} u = e^{\sigma\sqrt{dt}} \\ d = e^{-\sigma\sqrt{dt}} \end{cases}$$

where:

e Napier's number;

σ standard deviation, also called volatility;

dt time interval between successive evolutions of scenario.

With the use of the coefficients u and d the initial value of the investment is a typical tree structure (tree of scenarios), derived from the decision tree analysis. Following the risk analysis, there is the construction of the decision tree. Feasible options are identified and the corresponding values are quantified at this stage. The decision tree is used to measure the impact produced, based upon on financial analysis, by the possible options for the development of the investment. The measurement of the impact requires the preliminary definition of a maximizing function, conformed to the particular type of the identified option. With this function, the year in which the option is exercised and for every possible scenario that year, the comparison of the present value of the investment "with" and "without" the option[11] is carried out.

The operation described above, performed for all the n scenarios of the year in which the option is feasible, allows to determine a vector of the majors, of size $n \times 1$, which is then discounted following the tree of scenarios from right to left. The calculation is done by weighting the elements of the vector with the neutral risk probability coefficients (p and $1 - p$) and by bringing current events back to the results of the weights through a discount rate (r_i). The p and $1 - p$ probabilities, called risk neutral probabilities are calculated through the following expression:

$$p = \frac{(1 + r_i) - d}{(u - d)}$$

$$1 - p = 1 - \frac{(1 + r_i) - d}{(u - d)}$$

The two coefficients to be used for the weighting of the values of the various scenarios are such that, if it were not convenient to exercise any option in the scenarios of the analysis period, the initial value of the investment would be returned to year zero.

The result represents the present value of the investment, a value that includes the effect of the option or of the options that may be exercised.

[11] The model is used by Manganelli et al. (2014b).

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