Chapter 7
The Risk Analysis

Abstract The topic of risk management is one of the main problems that investors have to deal with. There are many variables that identify the risk of an investment, many are also subjective in nature. This makes the task of the analyst who wants to incorporate the risk factor in the assessment of convenience, a difficult one. The analyst will have to study and investigate the correlations between the specific objectives of the investment and the probability estimates of returns. In the analyst's help section, the traditional methods for the treatment of risk are explained, based on the subjective perception of the risk itself, as well as the more complex approaches that make use of the probabilistic analysis, instead.

7.1 The Variables and Risk Factors

The analysis of the allocation of resources, discussed above, has as its object the definition of the logical and fundamental tools for investment evaluation in contexts of certainty.

It is necessary at this point to introduce the possibility of uncertainty and imagine, in line with what happens in reality, that most of the investments generate uncertain cash flows both in terms of the entity and that of the time limits.

The risk theme in the investment evaluation is one of the main problems that investors have to face. The volatility of the financial markets and the increasing competitiveness, which occur both in mature sectors and in those characterized by strong technological innovation, are in fact continuously paying the problem of controlling and managing the uncertainty that usually characterizes the process of analysis that leads to the assumption of strategic decisions.

From a merely formal point of view, the concept of uncertainty is distinct from that of risk. An investment is considered risky even when the results produced by it cannot be determined with certainty, however, it can make various assumptions about future results, each of which is associated with a given probability value, so as to build the entire probability distribution of event.

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The uncertainty involves an unknown number of possible outcomes, with insignificant information about its chance of occurrence. It takes over when it is not possible to establish a priori a probability distribution to be associated with different outcomes of the event. By definition, the uncertainty is not measurable, so there is no way of communicating a "degree of uncertainty". From the point of view of the decision maker, this is the worst condition, the one in which anything can happen.

Surely, it is preferable to operate under conditions of risk, since, in this case, the possible outcomes are known and the analyst is able to estimate the relative probability.

If a situation is uncertain, but the possible consequences and their probability of occurrence are known objectively, the situation involves risk or objective uncertainty. When the possible outcomes are known, but their probabilities of occurrence are not objectively known, the situation involves uncertainty or subjective uncertainty, and finally, when the list of possible outcomes is not clearly defined, the situation involves ambiguities or unforeseen contingencies.

A good investor will always tend, as far as possible, to convert the elements of uncertainty in risk factors, forcing the analyst to a continuous process of research and arrangement of data in order to produce and incorporate in the analysis, new "objective" deployments of probability.

In this context, the term uncertainty will be used as a synonym of riskiness, regardless of the formal specification just made, thereby opposing the concept of certainty. Therefore, under conditions of uncertainty, an investment cannot be described by a single cash flow, but by a series of flows in each of which a given probability distribution can be associated.

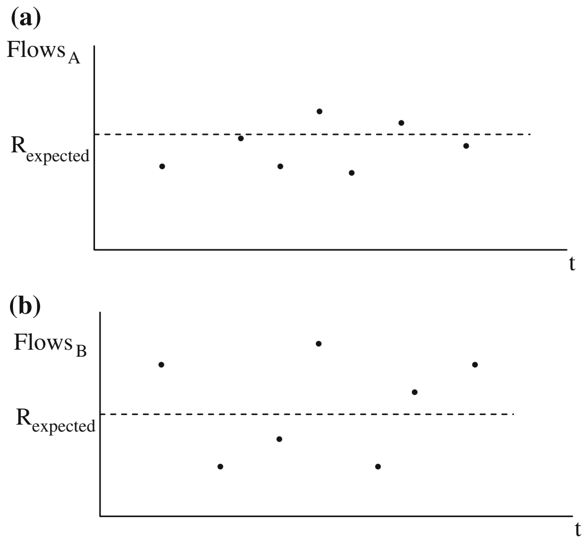
The measure of risk attached to the investment is given by the dispersion of the different expected results. On a general level, it can be said that an investment is even more risky as the possible outcomes resulting from it are scattered around the expected value. The dispersion of the expected values of a project is also identified with the term volatility. In order to provide the first concrete perception of the concept of dispersion or volatility, Fig. 7.1 compares the expected results (or yield) of two investment projects, A and B, which have the same level of average expected return.

As one can see from the comparison between the two graphs, the individual values of project A have a lower dispersion than the expected average, which on average are less distant from the line compared to the same points of project B. There is no doubt that a rational investor, faced with the choice between A and B, would prefer A, which has the same return at a lower risk.

The risk element is a feature of all kinds of investment. The major uncertainty factors of real estate market have already been illustrated and it has also been explained that the investor, facing the danger of failure of an operation, may have a different behaviour, even in terms of specific strategies and perception of risk.

The many variables that identify the investment risk, many of which are subjective, realize the difficulty of incorporating the risk factor in the analysis, through formalized procedures and techniques. The forecast of cash flows of a real estate

Fig. 7.1 Dispersion of the expected returns of the two projects



investment is a complex task. This does not exempt the analyst from the task of streamlining an effectively complex problem, through the study of the correlations between the specific objectives of the investment and the probability estimates of the possible returns.

While acknowledging the entrepreneur's innate capacity to deal with the risk factor in an intuitive way, it is useful to try to make the choices easier, and at the same time highlight the possible consequences (see footnote 1 in Chap. 1). The need to support the final decision therefore involves the effort to provide investors with all the necessary elements to frame the sources of risk and in assessing the probability of ticking off gains equal or bigger to the minimum acceptable.

The discount rate used to calculate the NPV, which is the minimum acceptable IRR, has been interpreted as the sum of several components, one of which is identified as a premium for the risk of investment. The problem is therefore to correlate the risk to profitability.

First, one needs to identify the risk factors that characterize investments in the real estate field. A very general classification of risk factors in business risks, financial risks and external risks was previously laid out. In relation to this preliminary distinction it is then possible to identify the risk factors with reference to other possible real estate investment classifications (even overlapping each other). If one looks at the object of the investment it may, for example, be useful to divide the risk factors in exogenous, which are common to any investment, or in specific risk factors, related to the specific asset. Exogenous factors are the general economic conditions, interest rates, employment and inflation, taxation and government policy. Specific risk factors are the location, financial-income characteristics of the lessees, the quality and the fungibility of the asset.

One can also distinguish economic risk from technical risk. The economic risk depends on factors that are extrinsic to the project to be evaluated. In the economic risk the tenant's risk[1] is contemplated. It concerns the possible difficulties related to the actual collection of the rents.

The characteristics of the conductors in terms of reliability and contractual power may affect the variability of returns. The losses on rentals or delays in the collection can result from non-payment, tenant's insolvency or contentiousness, conditions that often produce the need to renegotiate the fee or a temporary vacancy of the property.

One must also keep in mind that every time it is necessary to recover unpaid rent, it involves very expensive legal actions.

Another factor of economic risk, which could affect the cash flows expected from the investment, and somehow related to the tenants' risk, depends on the location of the building within the urban context. The importance of the city and/or of the area in the city in which the property is located, can tell us a lot about the income, cultural characteristics and mobility of the population, and therefore the reliability of the lessees and the resulting stability of returns. The concept of zone of prestige is obviously in function of the type of target property. The importance of a particular location in relation to the other depends on the presence, in terms of quantity and quality, of services, facilities and infrastructures, therefore on the ability to attract that particular category, be it residential, commercial or productive.

The last economic risk factor is related to the volatility of the market in general, and that of the local market. As far as the real estate market in general is concerned, the main determinants of supply and demand have already been illustrated, the factors that can determine a variation in prices, use or sale and therefore the expected cash flows from the investment.

The technical risk depends on factors that are intrinsic to the project, including the specific destination, the connotations of quality and appearance of the building, the characteristics of fungibility of the asset. This uncertainty often leads investors to anticipate the start of the project in order to collect information on the potential transaction. Events within the investment process are also part of this specific risk, namely those stages of development in which the risk is related to the possibility of inappropriate choices. For example, an inefficient construction management may cause an increase in construction costs, inadequate administration may result in a vacancy rate higher than expected, and finally an inadequate sales plan can seriously affect the longed-for gain.

Another conceptual category concerns the distinction between operational and financial risk. Operational risk refers to the variability of the results that derive from the organizational structure of the investor and in particular from his activities. The concept of financial risk includes within itself a whole series of elements

[1] In this regard Fiedler and Janda (1993) point out that "the extent of the risk could be not a science but an art".

that are relevant to the management of the financial and monetary aggregates. From a purely conceptual point of view it is possible to distinguish the financial risk into two basic components:

- Risk of leverage;
- Interest rate risk.

The risk of leverage is directly associated with the financial structure. The financial leverage ratio is the relationship between interest-bearing debt and equity, and indicates the extent to which the management company is financed by recourse to debt capital.

In dealing with the issue of funding it has been highlighted how a positive financial leverage expands the return of personal capital investment, if the cost of the loan is less than the income obtained from the building. However, the use of an ever growing loan amplifies the variability of the expected results and produces an increase in the financial risk. Figure 7.2 shows this situation qualitatively.

Another type of financial risk is one that materializes when excessive debt increases the probability of not achieving a net operating income sufficient for the payment of the amortization (risk of default).

Figure 7.3 illustrates the risk of insolvency that may occur in the case of a complex of apartments for rent. In the absence of funding, the balance between costs and revenues is reached at point A, to a relatively low level of employment. If one resorts to the loan, will need to maintain a very limited vacancy rate (point B) to ensure the solvency of the revenues.

Interest rate risk arises in the possibility that, given the volatility of the markets, a change of market interest rates leads to an unexpected cost for the investor because of the discrepancy between lending rates and deposit rates that characterize the assets and liabilities.

The analysis of operational and financial risk helps to understand the concept of irreversibility of decisions.

Ceteris paribus, if the weight of fixed costs gets larger, the costs of abandonment of the project increases, and it becomes less and less reversible. In such circumstances, a positive NPV may not be enough to convince the investor to take the risks that the structure of the project entails. The irreversibility of certain decisions is therefore a deterring factor for the investment.

Another distinction relates to the insurability of the risk. Those risks related to natural disasters and not (fires, floods, storms, etc.) and which may be transferred, by payment, to the insurance companies are defined as insurable.

The final level of analysis is related to the distinction between systematic risk and sectorial or non-systematic risk. The first relates to those events of general scope that cause an impact on the economy as a whole. An example of systematic risk factors is the sharp increase in the price of oil, an extended armed conflict, a sharp rise in the discount rate, the declaration of bankruptcy of a State.

Against this, there are also peculiar risks for a particular field. Within this risk category, so-called sector risk, are all the factors that come from competitors, industry and production factors.

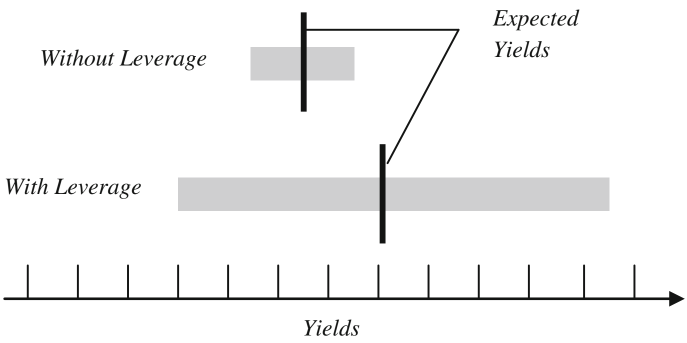
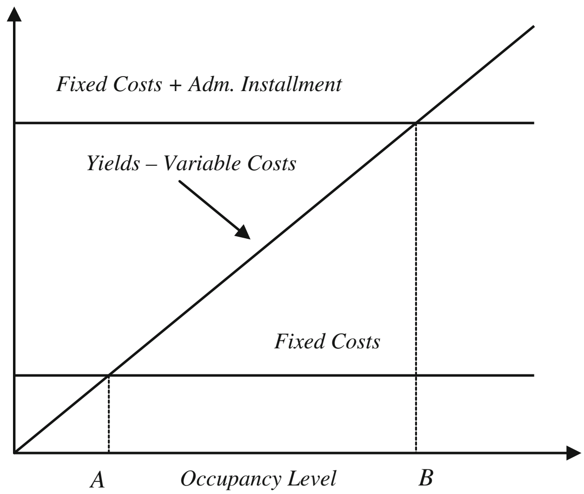


Fig. 7.2 Leverage and variability in yield

Fig. 7.3 Financial Leverage and insolvency risk



In particular, the sector risk can be thought of as the total of factors that cause a loss of competitiveness of the investment, especially in those fields where the technological component and the level of innovation of the production process are crucial. The specific source of risk for a Real Estate investment can be recognized in the well-known phenomenon of technological obsolescence.

7.2 The Analyst's Task

Despite the low incidence of the cost of the design on the total cost, the role of the Architect/Engineer is essential to the economic performance of building initiatives, more than each of the other operators. It is therefore necessary that the

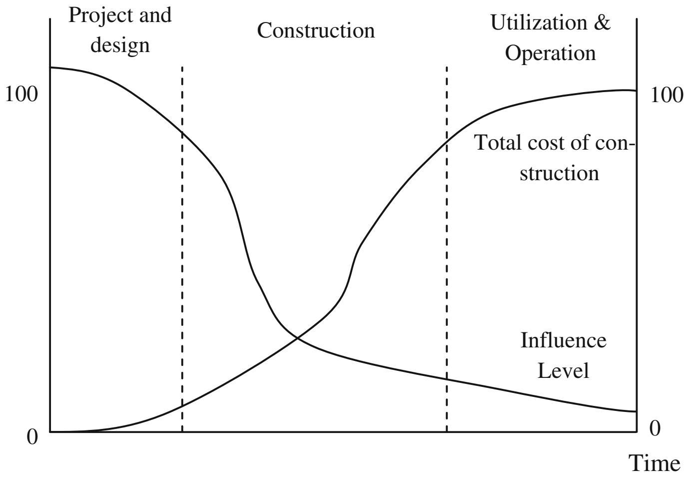


Fig. 7.4 Influence of the phases of the construction process on construction costs

designer coincides with the figure of the analyst, or at least has the knowledge required to work with the latter.

In fact, several authors state that during the completion of 20 % of the preliminary draft, 80 % of the cost has already been decided (Kelly 1984). Figure 7.4 illustrates this situation.

This shows how it should be, on the one hand, to spend a greater attention to the preliminary stages of the project and on the other, under the predominant influence of the project moment on the costs for construction and use of the buildings, that the Architect/Engineer is fully aware of the economic consequences of his choices.

The analyst, in estimating the costs, especially when it is necessary to refer to the concept of rapid estimation, must be firmly anchored to the principles of the estimation discipline and avoid that the investment is underestimated or overestimated. It is expressed in its normal consistency, in reference to all the factors that may intervene later. Any subsequent adjustment should be aimed at not saving the project, but at its completion.

The lack of knowledge of the detail of the character of the project, cannot lead to justifying, beyond a certain threshold, the deviation between the calculated costs in decision-making and those defined later.

Another fundamental aspect of the analyst's work is that of communication with the investor client. The problem becomes complicated especially when one has the need to provide a comprehensive overview about the risk of the intervention. As shown in the next chapter, there are several techniques that allow to incorporate the risk into the analysis, it is not easy to understand how they all handle the risk. Traditional approaches suffer the effects of the subjectivity of the analyst, and even the most accurate estimates may be unclear to the decision maker. It so happens that the analyst, in adjusting the estimates based on his sensitivity, does nothing more than make decisions for the client, barring any possibility of choice. A similar approach may be acceptable only when the analyst and the client share the same kind of perception of risk, but this is a utopian condition.

7.3 Risk Control

The increased attention to risk analysis in real estate is a relatively recent fact. It has become of significant interest to the business world in the last few decades and at unfavourable cyclical phases during which not all projects have automatically reached success.

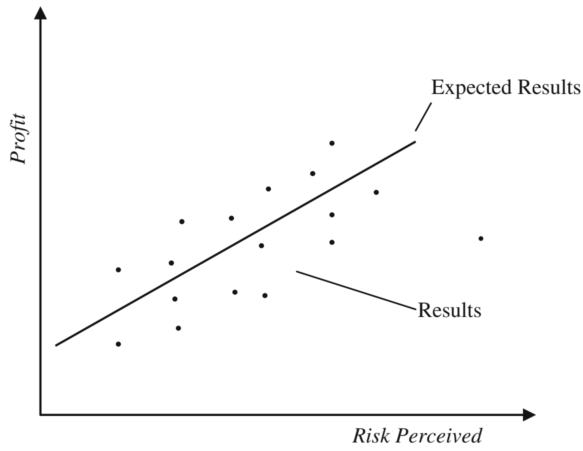
When it was understood that real estate investing is not immune to the dangers of failure, literature has been enriched by works on sophisticated techniques of risk control, traditionally adopted in other areas of the economy. These approaches, however, are often insufficient for real estate reality: the greatest benefits conferred by such analyses do not justify the cost and time needed to obtain them. For moderate-sized projects, and in cases where the results are predictable, the use of a less complex analysis is more appropriate. Many of these simple procedures for risk reduction are applied by the investor who is not always aware of the situation. It is about attitudes, rather than rules. The easiest way, but certainly not the most obvious, is to control the risk by investing in less risky projects, and by accepting only those opportunities that guarantee, with a very small margin of uncertainty, the achievement of certain results. Unfortunately, even in an imperfect market such as real estate, such behaviour means to preclude other possible extraordinary gains, given that there is a proportionality—as already mentioned—between perceived risk and expected returns.

In an efficient market, the observed results (profits made) are distributed randomly around the line of the expected results (Fig. 7.5), so that the upper and lower returns, compared with those expected, tend to compensate each other. On the other hand, if there were investment opportunities with high returns that do not correspond to equally high risks, investors would quickly enter this market, lowering the profits to levels comparable with investments of equal risk that could be implemented in other markets. This means that the only feasible way to minimize the risk in an efficient market is to choose the opportunity with the lowest expected return.

It is known, however, that the real estate market is not efficient at all. As a result, most capable real estate investors have the opportunity to take advantage of these inefficiencies and achieve extraordinary profits without being burdened with an equivalent dose of additional risks. The secret lies in the ability to identify those investment opportunities, the results of which are above the line in Fig. 7.5.

Another way to control the risk exposure is to exploit the relationship between the owned properties and those that could potentially be acquired. Because the factors that affect profits and market values do not uniformly affect all properties, the owners of more diversified portfolios have more stable gain patterns than those

Fig. 7.5 Risk perceived and real returns in an inefficient market



obtained by concentrating all the wealth in a single project. Diversification ensures risk reduction only if the investments are chosen so as to avoid correlations between the patterns of the profitability of the various assets.

Unfortunately, diversification presupposes the availability of a large amount of capital. It is important, having the possibility, to diversify even geographically, in order to minimize the impact of economic crises that affect specific market areas. Another possibility of diversification, but equally expensive, is to invest in different real estate projects, different in type and destination.

Diversification represents an impracticable road for all those investors, most of whom do not have a high enough budget. In this case, the solution is to combine the individual availabilities in common investment funds. Real estate funds are born with the spirit of allowing a remarkable variety to those who do not have the opportunity to make big capital investments.

The investor has other options to limit the risk associated with a given project. He could try, for example, to improve recruitment that makes about the future profitability of a property, reducing the gap between expected and observed outcomes of the operation. A result like this is obtained by increasing the activity of analysis, by using more accurate predictive techniques, and above all by doing meticulous market research.

The risk control action does not end with the final decision; the investor must act in such a way as to conduct the project, during the phases of development and management, towards the results of the predictions. In this sense, control over the supervision of the works and an effective administration, are useful actions.

In addition, the investor can use a number of tools that allow him to partially transfer the risk to other subjects. For example, he could make use of the particular contractual clauses that allow him to shift some risk factors to customers (this occurs with the risk of inflation, when engaging the rent index of consumer prices). Other possibilities of transfer of risk relate to the activation of special forms of pre-financing. Among these, for example, is the exchange in the purchase of land to be transformed, or, the start-up of options for the purchase of buildings or units being built (sales on paper) which is also very common, so as to have the time to carry out the analysis of convenience, obtain zoning approvals and negotiate a loan.

7.4 Traditional Methods for the Treatment of Risk

Instead of objectively quantifying the perception of risk, analysts and investors are traditionally concerned, according to subjective impressions, to add a premium to the return expected under conditions of normal risk. This additional premium results in shortened recovery times, in demands for higher returns and downsizing of expected cash flows. The effect of this strategy is the reduction in investment values and, often, the consequential waiver of the investment. The inability to quantify the risk element also makes inter-project comparisons complicated.

7.4.1 Risk-Adjusted Discount Rate

One of the traditional ways of response to a condition of uncertainty is to require a shorter payback period. The technique does not measure the risk, but consists of an "adjustment" in the discount rate in order to bring the payback period within the acceptable maximum limit, chosen on a subjective basis. The analysts simply set this parameter, depending on their personal impression.

This is obviously a rather rough method of evaluation, which, on the one hand, assumes that the investor gives up the opportunity to know the true riskiness of the project and to thus implement possible actions resulting from it, and on the other hand the prevalence of the analyst rather than the decision maker.

The problem remains also in the approach that incorporates the risk factor directly in the calculation of NPV (i.e. without reference to the payback period), when one establishes a modified discount rate.

This approach determines the rate as the sum of a risk free rate and a premium that is proportional to the expected perceived risk. The selection of the "risk free" rate is a theoretical problem, rather than a practical one. In general, the choice falls on the rate charged for short-term government bonds. The determination of the risk premium, however, should be based on the function of the investor's risk-return indifference, but, in practice, it so happens that analysts carry out this operation based on their personal perception.

The adjusted discount rate approach is probably the most commonly used, although it does not address the problem of the analyst's subjectivity, and despite the fact that it presents several other theoretical and practical difficulties.

An important issue concerns the calculation of the monetary value of time. This yield is incorporated in the risk-free rate, but inevitably, even the risk premium includes the element related to time equity. As a result, future risks are discounted more heavily compared to those in the short-term, although often the risks that are closest in time prove to be relatively more important. In a new construction project, for example, the probability of a growth spurt in construction costs or a slower market uptake than expected, is definitely higher than the probability of making significant errors in the estimation of future operating results (revenues and operating costs). Despite operating results that become progressively more predictable because of the development of the neighbourhood, the risk premium with which such operative results are discounted, is equal to the initial one and the corresponding discount factor increases proportionally.

Example 7.1 Consider a five-year investment where the risk premium is estimated at 5 %, while the risk-free rate is fixed at 4 %. Thus, the adjusted discount rate for the calculations is 9 %.

Table 7.1 shows the discount factors for the flow of each year.

The part of the discount factor set aside for the bearing of risk is given by the difference between the adjusted factor $(1.09)^n$ and the risk-free discount factor $(1.04)^n$. The growth of factor in the third column shows that also the risk premium suffers the incidence of time. The monetary value of time, already considered in the risk-free rate, is then set aside twice.

The method of the adjusted discount rate can theoretically be considered correct for those investments whose risks grow over time. For projects where the risk does not depend on time, this technique does not seem appropriate.

7.4.2 Certainty Equivalents

The problem of the double allocation in the approach of the adjusted risk discount rate is bypassed by the approach of the adjusted cash flows, or the *certainty equivalents*. The certainty equivalent is that amount of money that someone would accept, at a given date *t*, rather than taking a chance on a higher, but uncertain, return; in other words the amount that makes the choice between the same certain amount and the expected result from a risky investment, indifferent. Compared to the previous model in which the project is estimated by taking its expected cash flows and discounting them to a weighted average cost of capital, and adjusted according to the risk, with this approach adjustment for risk is made on cash flow projections rather than on the discount rate. Modified cash flows are discounted at the risk-free rate. This technique also avoids to quantify the perception of risk, but introduces a number of practical problems. First of all, is the problem related to how to "worsen" cash flows to such an extent as to leave the investor indifferent to

Table 7.1 Discount factors for the flow of each year

Years	Adjusted discount factor	Risk free discount factor	Discount factor for risk
1	1.0900	1.0400	0.0500
2	1.1881	1.0816	0.1065
3	1.2950	1.1249	0.1702
4	1.4116	1.1699	0.2417
5	1.5386	1.2167	0.3220

the superior gains relative to the most probable scenario (portrayed with a certain degree of risk), and the absolute certainty of obtaining lower earning of the adjusted estimate.

The method of the certainty equivalents is based on the calculation of the certain equivalent of an uncertain outcome. In situations of adversity to risk, two incomes, one of which is sure Q and one is unsure F, will be equivalent in terms of utility only if the second is greater than the first:

$$Q = \alpha \times F$$

with $0 \leq \alpha \leq 1$.

The coefficient α allows the investor to express the required premium in accordance with their degree of aversion to risk.

Substituting the respective certainty equivalents to Ft random flows, the formulation of NPV will became:

$$NPV = \sum_{i=0}^{n} \frac{Q_t}{(1+r)^t}$$

where the discount rate r expresses, as is well known, the expected return for risk free investments, in line with the nature of the flows to be discounted.

The value of the certainty equivalent Q to be included in the numerator of NPV formula can be derived in different ways.

One of the possible ways is based on the explanation of the utility function of the decision maker. First, one needs to make the client aware of the expectations about the economic environment in which one plans to invest. Once informed on the major elements of risk, the investor must express a preference between the expected cash flows for the analysed project and a large number of "low risk" alternatives. The series of comparisons ends when the investor identifies opportunities to limited gain, but certainly capable of satisfying him the same way as the risky investment.

Example 7.2 Table 7.2 shows the expected cash flow from a project to be managed for 5 years. The income of €120,000, which is expected to be gained from the sale of the estate, will materialize only if the market

Table 7.2 Cash flow and transformation factors related to the certainty equivalent

Years	Expected	Risk free equivalent alternative	Transformation factor
1	€15,000	€14,400	0.960
2	€15,000	€14,400	0.960
3	€15,000	€14,400	0.960
4	€15,000	€14,400	0.960
5	€15,000	€14,400	0.960
6	€120,000	€103,200	0.860

conditions evolve according to analyst expectations. The risk perceived by the investor, as well as his attitude to risk are reflected by the transformations shown in Table 7.2.

The transformation factor is obtained by dividing the risk free cash flows by those expected. For the investor, the expected cash flow of €15,000 with a certain risk, is equivalent to the sure flow of €14,400. The lower reliance in the entry of €120,000 due to the sale, results in a lower transformation factor.

It is understood that the present value calculation takes the following form:

$$PV = \sum_{t=1}^{n} \frac{CF_t}{(1+k)^t} = \sum_{t=1}^{n} \frac{\alpha \times CF_t}{(1+i)^t}$$

where k is the adjusted discount rate, i is the risk free rate and α represents the factor of transformation.

If one is discounting the risk free cash flows equivalent to a risk-free rate of 5 %, one gets the current value of €143,204 (Table 7.3).

To tick off the same current value even from the expected cash flows, one has to apply a rate of 7.7 %. This means that the risk premium can be quantified by subtracting the risk-free rate to 7.7 %:

$$Risk\ Premium = 7.7\,\% - 5\,\% = 2.7\,\%$$

This method allows to approximate the indifference curve of risk-return profile of the investor through the determination of its transformation factors. This operation can be carried out by requiring the investor to express a range of preferences of an appropriate number of combinations of risky and non-risky alternatives. Proceeding in this way it is possible to construct a map of preferences

Table 7.3 Calculation of the PV of the certainty equivalents

Years	Expected cash flows	Present value at 7.7 %	Risk free equivalent alternative	Present value at 5 %
1	€15,000	€13,928	€14,400	€13,714
2	€15,000	€12,932	€14,400	€13,061
3	€15,000	€12,007	€14,400	€12,439
4	€15,000	€11,149	€14,400	€11,847
5	€15,000	€10,352	€14,400	€11,283
6	€120,000	€82,814	€103,200	€80,860
		€143,181		€143,204

that express the individual's attitude to risk. From the map one can extract the transformation factors.

7.5 The Probabilistic Analysis

The NPV in its static formulation assumes that the cash flow and the parameters that define the structure are determined, that is, it is not possible for them to build a probability distribution in relation to the expected values. However, when the cash flows relating to a particular investment are not known with certainty, and one wants to quantify its riskiness, thus overcoming the limitations of traditional methods, it is necessary to study the factors that significantly influence the results. This study should provide a prediction on the measurement of these factors together with the associated probability of occurrence.

The exact estimates of cash flows are developed by considering a specific condition. One can construct a spectrum of forecasts, each of which reflects a set of assumptions about the social and economic conditions that may arise during the investment period. The uncertainty about the course of future events, results therefore, in the development of estimates that identify with a certain degree of "security", the possible or probable results. In practice it is necessary to associate a probability to each possible outcome. The set of possible results with the relative probability of occurrence is the so-called probability distribution.

If the estimate of this probability is performed with statistical techniques, then the risk is measured on objective probabilistic distributions.

For the variables to which one cannot apply any statistical measurement technique, it is necessary to quantify at least the impressions of the analyst, building a risk estimation expressed in terms of subjective probability distribution.[2] The effectiveness of these estimates is influenced by the expertise and experience of those who make them.

[2] The word "subjective" is intended to indicate that the probability estimate is not based on historical data, but on the individual opinion of an expert analyst.

7.5.1 Partitioning and Sensitivity Analysis

Risk control through a probabilistic approach goes, first of all, through the identification of the most significant sources of danger. The technique of partitioning the present value allows the analyst to determine the elements of risk that mostly influence the outcome of the investment.

The method consists in dividing the cash flow, after deduction of tax, in its fundamental components (cash flow before tax, amortization of debt, income taxes, and changes in the value of the property during the holding period, etc.). Expressing the components as a percentage of the total present value, one has the chance to understand the relative importance of each of them. The components with the highest impact on the present value are the ones that deserve further analysis.

The partitioning method emphasizes the relative importance of various types of cash flow and allows to identify the parts of the prediction that deserve further analysis.

The sensitivity analysis extends this process so as to highlight the consequences that, any probable error in the forecasts, they could have on the actual outcome of the investment. The technique consists in altering, one at a time, the components of the expected cash flows and study the impact on the index of return (IRR, NPV, etc.). The purpose of the analysis is to examine the variation of the result at the change of one of the assumptions underlying the project, assuming that the others are unchanged.

Typically, for each variable, one identifies new values (pessimistic, optimistic and intermediate scenarios) and then recalculate the parameters of convenience.

Example 7.3 The property, of which the estimated cash flows are listed in Table 7.4, can be purchased for €1,350,000. A loan of €1 million (repayable over 30 years with monthly payments at 12 %) is available. The investment therefore requires an immediate commitment of €350,000. It is assumed that the property is sold for €1.7 million, at the end of the sixth year.

From the revenue of the sale of the property, extinguishing the remaining debt of €966,373 and paying taxes (€178,000), one will get an additional cash flow at the end of the sixth year which amounts to €555,627:

€1,700,000 − €966,373 − €178,000 = €555,627

This amount is also divisible in:

- Extinguished Debt (€33,627)
- Value increase (€1,700,000 − €1,350,000)
- Taxes (€178,000)
- Recovered Capital (€350,000)

In Table 7.5 the present value is calculated by separately discounting (at 10 %) the components of cash flow. The present value of the components of

Table 7.4 Cash flow expected the next 6 years

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Actual gross revenue (€)	233,000	241,000	249,000	255,000	261,000	265,000
Operating expenses (€)	−18,000	−18,500	−19,200	−20,300	−21,100	−22,000
Actual net revenue (€)	215,000	222,500	229,800	234,700	239,900	243,000
Borrowing costs (€)	−123,482	−123,482	−123,482	−123,482	−123,482	−123,482
Financial flow (€)	91,518	99,018	106,318	111,218	116,418	119,518
Taxes (€)	−30,000	−30,000	−30,000	−30,000	−30,000	−30,000
Net flow (€)	61,518	69,018	76,318	81,218	86,418	89,518

Table 7.5 Present value of partitioned cash flows

Year	Gross effective income (€) $1/(1+i)^t$	Operating expensed (€) $1/(1+i)^t$	Borrowing costs (€) $1/(1+i)^t$	Taxes (€) $1/(1+i)^t$	Net flow (€) $1/(1+i)^t$
1	211,818	16,363	112,256	27,273	55,926
2	199,173	15,289	102,051	24,793	57,040
3	187,077	14,425	92,773	22,539	57,340
4	174,168	13,865	84,340	20,490	55,473
5	162,060	13,101	76,672	18,627	53,660
6	149,585	12,418	69,702	16,934	50,531
Total	1,083,881	−85,461	−537,794	−130,656	329,970

Present value of sale flows

Extinguished Debt × $1/(1+i)^6$ = €33,627 × 0.5645 = €18,982

Appreciation × $1/(1+i)^6$ = €350,000 × 0.5645 = €197,575

Taxes × $1/(1+i)^6$ = €178,000 × 0.5645 = €−100,481

Recovered capital × $1/(1+i)^6$ = €350,000 × 0.5645 = €197,575

Total flows present value = €643,621

the expected cash flow is presented as a percentage value Table 7.6. The last column provides a measure of the impact of individual components on the total present value. Note, for example, that the expected results depend very much on the management of the property (71.7 %) and on the increase of its market value during the holding period (30.7 %). It is therefore necessary to give more attention to the estimation of these items.

Table 7.7 shows the influence of an error of ±10 % on the estimated selling price.

Table 7.6 Components of present value

Components of flow present value	Percentage		
Actual gross income	€1,083,881	168.4 %	71.7 %
Operating expenses	€−85,461	−13.2 %	
Borrowing costs	€−537,794	−83.5 %	
Taxes	€−130,656		−20.3 %
Extinguished debt	€18,982		2.9 %
Appreciation	€197,575		30.7 %
Taxes on sale	€−100,481		−15.6 %
Recovered capital	€197,575		30.7 %
Total	€643,621		100 %

Table 7.7 Impact of a variation of ±10 % of the sale price

	Cash flow after taxes		
	Variation −10 %	Expected	Variation +10 %
Selling price	€1,530,000	€1,700,000	€1,870,000
Remaining debt	€966,373	€966,373	€966,373
Sales tax	€160,200	€178,000	€195,800
Sale flow	€403,427	€555,627	€707,827

The present value of the equity position and the value of investment respond significantly to changes in the selling price (Table 7.8).

In Table 7.9, the same procedure is developed assuming an error of ±10 % in the prediction of gross revenue.

Figure 7.6 shows the relationships between the percentage changes in selling prices and gross revenues and the corresponding changes in the value of investment.

By observing the graph of Fig. 7.6, it is obvious that the results of the investment are relatively more sensitive to possible errors made in the estimation of revenues.

In general, besides the example proposed, even small variations of some parameters can significantly change the results. The decision maker, of course, must take into great consideration those key elements of the prediction that, by varying even slightly, may worsen the outcome of the investment by moving them below the minimum threshold of profitability.

Table 7.8 Response to changes in the selling price

Selling price	Flows present value	NPV	Investment value	IRR
Expected	€643,621	€293,621	€1,643,621	26.0 %
−10 %	€612,239	€262,239	€1,612,239	24.9 %
	(−4.8 %)	(−10.7 %)	(−1.91 %)	(−4.2 %)
+10 %	€674,967	€324,967	€1,674,967	27.1 %
	(+4.8 %)	(+10.7 %)	(+1.91 %)	(+4.2 %)

Table 7.9 Responding to changes in gross revenue

Gross revenue	Flows present value	NPV	Investment value	IRR
Expected	€643,621	€293,621	€1,643,621	26.0 %
−10 %	€535,215	€185,215	€1,535,215	20.1 %
	(−16.8 %)	(−36.9 %)	(−6.6 %)	(−22.8 %)
+10 %	€751,992	€401,992	€1,751,992	32.2 %
	(+16.8 %)	(+36.9 %)	(+6.6 %)	(+23.8 %)

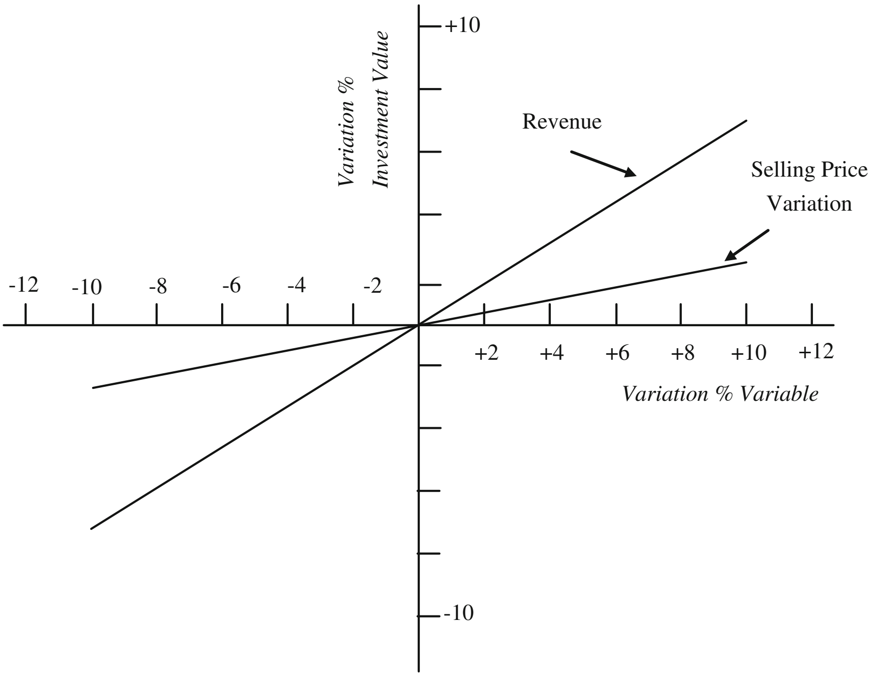


Fig. 7.6 Sensitivity analysis

Figure 7.6 could be completed by adding the sensitivity curves of all the entries in the cash flow. The variables that most affect the results will show a graph with a steeper trend.

Of course, the analyst will have to pay more attention to critical variables. So, if the expected cash flows prove sensitive to changes in the vacancy rate, for example, they will need to develop a more refined analysis of marketability, if the outcome of construction works was sensitive to the time required for the alienation of single real estate units, then it is necessary to work carefully on the sales. In the case of real estate investments, the variables of greatest influence are the cost of the property, the interest rate, the cost of construction, the time for obtaining planning permission, the vacancy rate, the sales plan. The operating expenses, urban development costs, the intermediation and expenses techniques, taxation, etc. instead have a smaller impact on the results.

The sensitivity analysis, however, has some limitations:

- The need of a framework in which risk factors and the reachable level by the fundamental values, are clearly defined;
- Separate analysis of each variable does not take into account the fact that uncertainty acts simultaneously on several factors;
- The theoretical results of the analysis does not consider the probability of occurrence of events;
- An indication of the variability of the investment does not provide elements of choice.

Techniques such as sensitivity analysis and partitioning, identify only the "key" variables that mostly influence the results of the DCF, but say nothing about the reliability of the data used in the estimate, or rather, on the probability of occurrence of certain variations compared to the expected values.

7.5.2 Decision Tree Analysis

When the decision maker evaluates complex projects, the sequentiality and interdependence of the choices pose problems not related to the linearity inherent to the static NPV.

A project can in fact be described as complex or multi-step when its implementation is:

- Fragmented over time at different stages;
- Uncertain in terms of in and out cash flows;
- Conditioned by exogenous variables that have a temporary nature.

The Real Estate investing has these features and requires a structured decision-making process, based, on one hand, on the correct definition of the critical steps that significantly affect the performance of the project, and on the other, the understanding of the interrelationships that exist in decision-making moments in temporal sequence.

The approach drawn up by the Decision Tree Analysis is particularly suited to guiding the decision-making process of complex projects.[3]

The logic underlying the model implies that the structure of the project to be evaluated is translated into a flowchart called "decision tree".

Each branch of the tree is associated to the possible values that the project can take on as a result of certain hypothetical scenarios, to which specific probabilistic attributes are given. This analysis allows us to capture the interrelationships between decisions taken at different times and mutually dependent on each other. In fact, in the presence of a chain of mutually conditioned choices, where the proposed development at a given stage depends on the actions taken in the earlier stage, to identify the optimal sequence of decisions, one needs to work backwards, starting from the last branches of the tree.

In this way, it is possible to reflect on the "nodes" from which the different decision alternatives ramify. This approach is also known as "rollback method". The decision tree analysis defines an NPV that takes into account the possibility of managerial adaptability inherent to the project. This consideration will be dealt with later to introduce the topic of real options and the role of managerial adaptability/governance in contexts of uncertainty.

It is perhaps useful to premise that, although the decision tree analysis allows to reflect on the interrelationships between the various moments in the life of a project, this is done according to a deterministic approach. This approach requires that, at time 0, it must be possible to explain and calculate the consequences of such interrelationships.

In practice, if a project has two critical areas in terms of decision-making, since the latter depends on the outcome of the first product, the decision tree analysis requires that the result of the interaction is made explicit at the time of evaluation.

Structuring the project in the manner described implies that the cash flows, income and expenses are the only relevant parameters for decision-making. In other words, it is assumed that the decision maker is motivated by considerations of pure maximization of the monetary value of the project, which implies two often decisive considerations in reality:

- The tactical and strategic profile of the project is not directly included in the scheme of analysis;
- It is assumed that the decision maker is risk-neutral, which allows to exclude the consideration of the utility function compared to the investor's cash flows, from the analysis.

Example 7.4 shows that an application of decision tree analysis in the decisions on a project that has the characteristics of complexity mentioned above, helps to understand the logic.

[3] About Decision Tree Analysis see: Magee (1964), De Ambrogio (1977) and Zaderenko (1970).

Example 7.4 Consider the case of a company that is planning the construction of a series of buildings with increased use of innovative plants.

One performed a preliminary market research to investigate the size and the evolution of the demand. The company proceeds with the design and promotion of the intervention. Uncertainty about the behaviour of potential users means that the construction activity is initially limited to a small number of buildings. The eventual success of the pilot project then pushes the company to expand production by investing in the completion of the project. The project is complex (as defined), and consists of the following phases:

Phase 1—At time T_0 the company commissioned a market research the cost of which is equal to €20,000.

Phase 2—Given the results of the research, the company decides whether to proceed with the project. The decision implies the beginning of construction of the first building. The total investment cost amounts to €1,000,000. The alternative is the abandonment of the project.

Phase 3—After one year from the start of pilot construction, having analysed the first sales figures, the company evaluates the potential response of the target.

If the sales figures are considered satisfactory, the construction of other buildings begins. This involves an investment estimated at €10 million. Alternatively, the company gives up the completion of the project.

Phases 4, 5, and 6—In the event that one decides to proceed, one has to formulate the assumptions regarding cash flows obtainable in the next 3 years, according to three different scenarios (optimistic 4/realistic 5/pessimistic 6). The project structure and its decision-making process are shown in Fig. 7.7.

The decision nodes are shown with a circle and are located at time 0, at time 1 and at time 2. Each node is associated to consequences in terms of cash flows, which take the form of outputs related to the first two nodes and revenue from time $t = 3$.

Two or more branches of the tree derive from each node, indicating the different possibilities that open up on the occurrence of certain hypothetical scenarios. This, as mentioned above, allows us to capture the sequence and interdependence of the decisions reached at different times.

Further task of the decision maker is to associate to each branch and then to each scenario the relative probability of the event. The sum of the probabilities associated with the branches that arise from the same decision node is of course equal to 1.

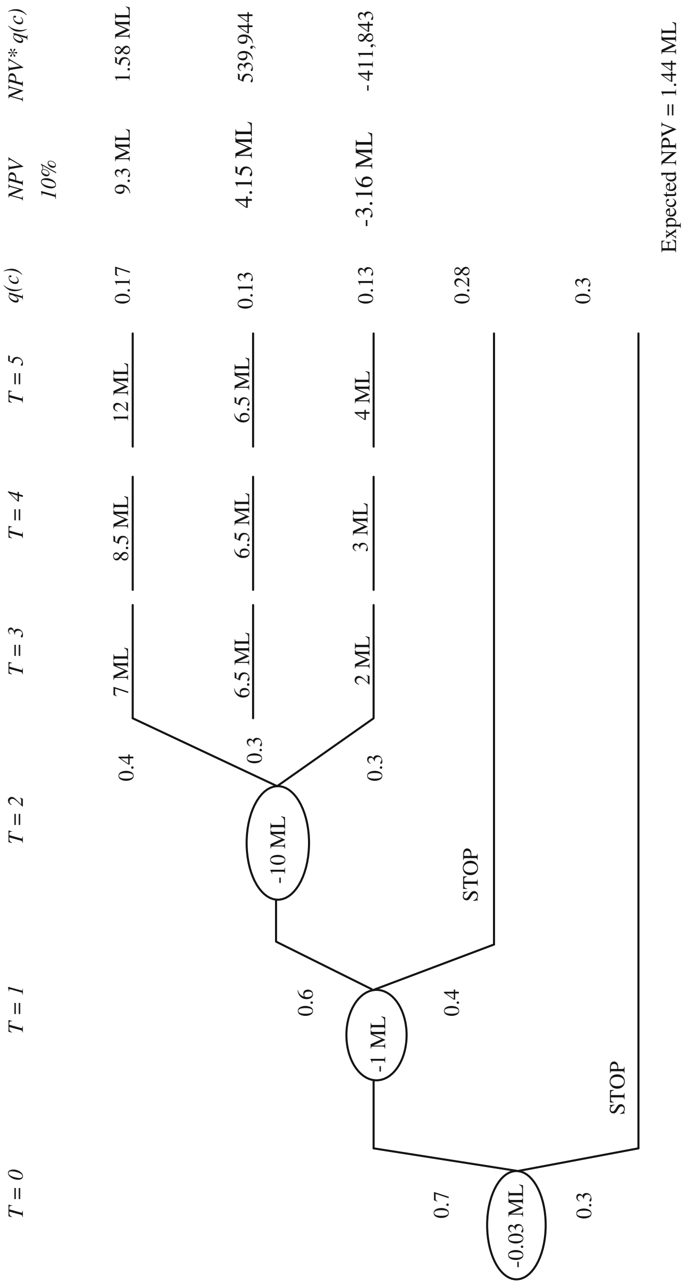


Fig. 7.7 Decision tree and calculation of PV

Entering into the calculation of the present value of the project, the first step is to determine the joint probability associated with each event (third last column of Fig. 7.7).

The sequence of events most favourable to the company is assumed: the market research provides a positive response for the project (q = 70 %); the experimentation on a limited number of buildings shows a high number of units sold compared to expectations (q = 60 %); expanding production, the company makes a more optimistic sequence of cash flows (q = 40 %).

The probability of occurrence of the whole sequence is equal to the joint probability of the different events that compose it, i.e. to the product of the simple probabilities:

$$q(c) = q(1) \times q(2) \times q(3) \times \ldots. \times q(n)$$

where:

$q(c)$ joint probability, or the probability of occurrence of the entire described sequence of events;

$q(t)$ simple probability related to the event tth, as t varies from 1 to n;

n number of events.

In this case it will be:

$$q(c) = 0.7 \times 0.6 \times 0.4 = 0.17$$

Therefore, the probability that from the beginning of market research the most favourable sequence of events for the company will take place, is 17 %.

The penultimate column of Fig. 7.7 shows the calculation of the NPV for each of the 5 sequences of events that may occur. With reference to the most favourable sequence, regardless of the role of chance, its NPV is calculated by discounting the cash flows related to single events (−30,000; −1 ML; −10 ML, 7 ML, 8.5 ML, 12 ML) that make up the sequence by an appropriate discount rate of 10 %. The NPV is equal to 9.3 ML. The same procedure is repeated for all other scenarios.

The last column of Fig. 7.7 is dedicated to the calculation of the expected overall NPV, of the project. The values of NPV, calculated for each sequence of events, are weighted for the joint probability relating to the same sequence.

The algebraic sum of the weighted NPV, thus provides the expected value of the entire project, amounting to 1.44 ML.

This result suggests to proceed with the implementation of the project. However, some observations are necessary. The range of variation of possible values of NPV appears to be very wide, which indicates a high variability of the results. It should also be emphasized that there is an overall joint probability of $0.13 + 0.27 + 0.30 = 0.7 = 70\ \%$ that the project could lead to a negative result. In other words, the apparent opportunity to proceed with the investment could be contradicted by the consideration of a low risk appetite of the decision maker.

The principal advantage attributable to the decision tree analysis is the opportunity to express the structure of the project through a chart that highlights the fundamental key steps (nodes). The methodology in question should be seen as a first attempt to introduce, the concept of strategic analysis into the traditional methods of capital budgeting.

The attempt remains unfinished. In fact, the limit that this approach does not exceed consists in the necessity to explain the probability distribution of the scenarios that characterize the different expected results, which leads the latter within the methods of evaluation of the deterministic type.

7.5.3 The Monte Carlo Simulation

In the previous paragraph, it was shown the usefulness of the decision tree analysis in the case of valuation of investments with a strong interrelation between the decisions taken in different moments of the life of the project. In such circumstances, the application of the traditional formula of the static NPV, could require to make some simplification to the structure of the decision-making process of a size that radically changes the meaning, and therefore the informative extent of the result.

Another type of investments, difficult to fit into the logic of the static NPV, consists of projects characterized by strong uncertainty about the values of a number of parameters that are considered relevant.

Although it is possible to associate a given probability distribution to each of these parameters, it can be difficult to estimate directly all possible combinations of values that the various parameters may assume as a consequence of their respective probability distributions. This will prevent us from associating a probability distribution to the different results generated by the project as a whole.

Consider now the case in which the decision maker is faced with the problem of estimating the expected cash flows from an investment, and the uncertainty concerns both the determination of cash flows and of the capital necessary for beginning the project.

Although the decision maker is in a position to assign probabilistic weights to the values of each of the three parameters considered individually, this is not sufficient to determine the expected NPV of the project. The three parameters to be estimated can vary simultaneously within the predetermined range of values. In such circumstances, the problem is therefore to make a probability distribution of possible values of NPV explicit, in light of the combinations that can be generated between the probability distributions associated to the key parameters.

The approach of Monte Carlo simulation allows to afford the problem, simulating a statistically high number of possible combinations of the values that key parameters may take as a result of the attribution of certain probability distributions. Each of the combinations is randomly generated, but in respect of the probability distribution assigned to each variable, it gives rise to a particular NPV. After associating the relative frequency to each NPV, it is possible to construct a given probability distribution of the NPV, and to determine the net present value expected for the project.

The steps for the application of Monte Carlo simulation in the economic evaluation of real estate investment are summarized as follows:

1. *Definition of the relevant parameters*

The first step is to identify the critical variables that have an impact on the overall result of the project. The selection of variables may be carried out by the sensitivity analysis, which aims to highlight the sensitivity of the result to the total variation of input parameters of the analysis, including the level of revenue, monetary operating costs, weight of taxation, the initial outlay required to implement the project and future cash receipts (or payments) related to the sale of the assets.

2. *Definition of the result to be reached*

It is about to mathematically explicit the formula or the model that allows to determine the outcome of the project to change the input deemed relevant.

For example, assuming that the cash flow of the operating management is the only parameter considered to be significant (or deemed by the sensitivity analysis) for the purposes of the possible impact generated on the configuration of result, and with reference to the application of the approach of the NPV, one can use a configuration of this type of parameter:

$$FC_t = FC + / - \Delta_{FC}$$

where:

CF net cash flow tax consequences;
Δ_{CF} change in the level of cash flow.

In this formula, for the year 0 it will also be necessary to subtract the initial required investment.

3. *Attribution of probability distributions*

For each of the selected parameters it is necessary to determine a range of values which are thought to occur, and these values must therefore be accompanied by probabilistic attributions, so as to define the complete probability distribution for each variable in the model.

4. *Launch of the simulation*

By using a processor, one generates a series of random numbers, each of which is associated with a particular value of the relevant parameters. If the relevant parameters are three, the first random number calls into question a particular value of the first parameter, the second random number calls a value of the second parameter, and so on. The first three random numbers generate, therefore, one of the possible combinations of values of the three parameters. Through this combination, it is possible to determine the first value of NPV generated by the simulation. Repeating the generation of the series of random numbers it follows the determination of a high number of NPV, each of which is characterized by a determined frequency, both absolute and relative.

Clearly, once the probability distribution of the NPV is constructed, it is possible to determine the expected value using the formula:

$$NPV_{Expected} = \sum_{t=1}^{N} (NPV_t \times RF_t)$$

where:

NPV_t tth value of NPV generated by the simulation;
RF_t relative frequency associated;
N number of simulations.

The Monte Carlo simulation solves all those evaluation problems where the large number of variables that characterizes the value of the project prevents an organic consideration of the possible combinations of value that the variables can take. Therefore, the simulation develops the probability distribution of a given function or objective outcome one wants to achieve, based on the consideration of a large number of possible combinations between the different values that the individual variables take.

The limit of this method is the need to explain the probability distributions of the individual variables included in the analysis, and thus to introduce into the evaluation, as is the case in the Decision Tree Analysis, a high dose of subjectivity.

7.6 Mathematical: Statistical Criteria

The cornerstone of traditional methods of risk control lies in the ability of the analyst to draw a "subjective" judgment about the riskiness of investment opportunities. In fact, even the probabilistic approach remains subjective when the probability distribution of the scenarios that characterize the different expected results is not based on historical data, but on the individual opinion of the analyst. Although in real estate one normally operates on the basis of subjective probability distributions—not having time series data able to build, through statistical techniques, objective distributions—the analytical formalization of such distributions still allows to manage the risk with mathematical and statistical methods.

The risk associated with the forecast of cash flow is defined as the probability of having discrepancies between actual and expected outcomes, or in other words, the measurable possibility of an error with respect to the most likely outcome. This definition is the starting point for the modern risk analysis. It allows us to express the perception of risk in numerical form, i.e. through summary measures that respond to two fundamental questions:

1. What is the value of central tendency valid for the entire project?
2. What is the "quality" of performance determined in terms of risk, or offset and dispersion of the values around the value of central tendency?

The first question requires the determination of the average expected return; the answer to the second question is given by the introduction of statistical measures of dispersion, formed by the variance and standard deviation.

The average expected rate of return may be calculated using the following formula:

$$E(R_x) = \sum_{s=1}^{n} P(s) \times R_{xs}$$

where the symbols have the following meanings:

$E(R_x)$	average expected return of the project X;
s	scenario;
n	total number of forecast scenarios;
$P(s)$	probability associated with each scenario;
R_{xs}	rate of return of the project x at the occurrence of the scenario s.

The expected average rate of return is a measure of central tendency of the different values that one expects the project to take on, it represents a summary measure that takes into account the different scenarios, in an appropriate way, and is therefore of utmost importance to assess the overall performance to be associated with the initiative.

The formula calculates the weighted average of the various returns that one thinks the project will generate; the weighting factor is constituted by the probability of the occurrence of the different scenarios.

Example 7.5 The probability distribution of the annual cash flows resulting from the development of a real estate project is presented in Table 7.10.

In the third column the most likely cash flow expected from the project ($CF_{average}$) is calculated.

The distribution presented in Table 7.10 is called discrete because the possible outcomes are limited only to six discrete values of the cash flows.

Table 7.10 Cash flows and associated probability

Expected cash flows (€)	Associated probability	Weighted average(€) (flows · probabilities)
125,000	0.04	5,000
150,000	0.16	24,000
175,000	0.30	52,500
200,000	0.30	60,000
225,000	0.16	36,000
250,000	0.04	10,000
	Total = 1.00	$CF_{average}$ = 187,500

Each of the values in the first column of the example identifies a particular expected scenario. The probability (second column) represents the measure of the verisimilitude that a given expected scenario can turn into reality. However, the scenarios identified in this way define the whole range of possible values that the project can take, since the sum of the probabilities is equal to 1. In other words, it is assumed that these six possibilities exhaust all possible outcomes associated with the project. This clearly cannot correspond to reality: a more truthful distribution should include all the intermediate values that Table 7.10 does not consider. That example is therefore a discretization of a continuous distribution of probability and represented by the curve in Fig. 7.8.

A scenario summarizes the adoption of certain assumptions concerning variables considered relevant to the project, and these variables results from the occurrence or not of certain, often external, events, with respect to the scope of the intervention. In the attribution of probabilities to different scenarios, the degree of discretionary decision-maker varies depending on two factors:

1. The availability of objective data on the occurrence of events that affect the appearance of the forecast scenarios;
2. The ability of the decision maker to use the data, linked in turn to the use of statistical tools able to synthesize the impact on the overall value of the project.

In order to reduce the uncertainty of the estimates, an analysis of historic performance with respect to projects that have a level of risk similar to that of the project to be evaluated, is often carried out. The objective is to arrive at a more objective estimate of the expected return associated with different scenarios that characterize the evolution of the relevant variables of an investment project.

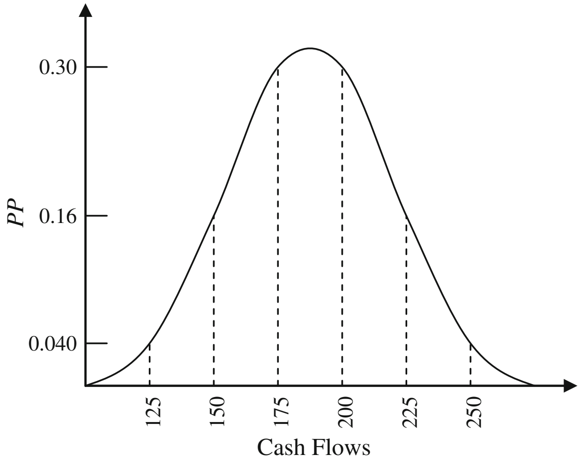


Fig. 7.8 Probability distribution of cash flows

The assumption of historical data for forecasting purposes, however, suffers a fundamental limitation on the hypothesis of constancy, even for the future, of the conditions of variability that have affected the returns of the project within the historical period of observation. If one assumed the uncertainty about the future is determined by new events and is different from those that characterized the historical performance of the project, it is certainly preferable to express the expected return of the project through a probability distribution that is subjectively determined.

If, on the other hand, there is trust in a certain constancy of the factors that have influenced the past performances, the historical data can provide useful information for the economic evaluation of the initiative, in which case the historical performance of project Y can be determined in the following way:

$$R'y = \sum_{t=1}^{N} Ryt/N$$

where the symbols have the following meanings:

$R'y$ average historical performance of project Y;
N number of observations;
Ryt rate of return of project Y in year t.

Example 7.6 Table 7.11 shows the values and the historical returns associated with project Y. The data was provided at the start of 2012 assuming that the value of the project is equal to 100.

In the example, the future value of the project is the result of a probabilistic estimate. The data for the project represents the performance

Table 7.11 Historical data relating to the project Y

Observations	Years	Values	Rates of return (%)
1	2012	100	2.04
2	2011	98	−2.97
3	2010	101	6.32
4	2009	95	1.06
5	2008	94	3.30
6	2007	91	7.06
7	2006	85	−3.41
8	2005	88	7.32
9	2004	82	10.81
10	2003	74	4.32
11	2002	71	1.43
12	2001	70	−2.78
13	2000	72	16.13
14	1999	62	6.90
15	1998	58	18.37
16	1997	49	4.26
17	1996	47	6.82
18	1995	44	4.76
19	1994	42	20.00
20	1993	35	9.38
	1992	32	

historically observed within a period of 20 years. For each observation the values that project Y took on at the end of the year, are associated. The rate of return for each year is calculated by dividing the annual change of the value of the project (final value − initial value) by the value taken on at the end of the previous year.

Referring to the data of Table 7.11 the historical average return is calculated:

$$R'y = 1.2103/20 = 0.060515$$

Performance history is therefore the average value of a time series. However, investment decisions cannot be separated from the joint consideration of the risk-return profile, representing the risk as a parameter of "qualitative" discrimination of the performance associated with a given project.

The analysis of the returns must therefore be completed with the analysis of risk. Since the variability of returns is a synonym of risk, one must calculate the dispersion of estimated values around the value that expresses the central tendency.

The most commonly used statistical measures in this regard are the variance and the standard deviation. The variance measures the extent of variability or dispersion from the average of the measured/estimated values. It is given by the square of the standard deviation, which is the arithmetic average of the squares of the distances from their average values.

The higher the variance associated with the returns of an investment project, the higher the risk it is associated with. Variability is therefore a synonym of uncertainty.

The variance can be calculated both on the expected returns from a given project, and on historically observed values. With regards to the first case, the variance formula is the following:

$$E\left(\sigma_x^2\right) = \sum_{S=1}^{n} P(s) \times [Rxs - E(Rx)]^2$$

where the symbols have the following meanings:

$E(\sigma_x^2)$ expected variance of investment X returns;
s scenario;
n total number of scenarios;
$P(s)$ probability associated with the occurrence of scenario s;
Rxs rate of return of project x at the occurrence of scenario s;
$E(Rx)$ average expected return of the project X.

That is:

$$\sigma_y^2 = \sum_{t=1}^{n} P(t) \times [Ryt - R'y]^2$$

with the following meanings:

σ_y^2 variance of returns of the project Y;
n total number of observations;
Ryt rate of return of project Y observed in period t;
$R'y$ average return of project Y.

The standard deviation (σ) is given by the square root of the variance, and it provides a measure of dispersion that is very useful when it comes to comparing investment alternatives whose cash flows are very different. Given that the variance and standard deviation provide coinciding information on the dispersion of

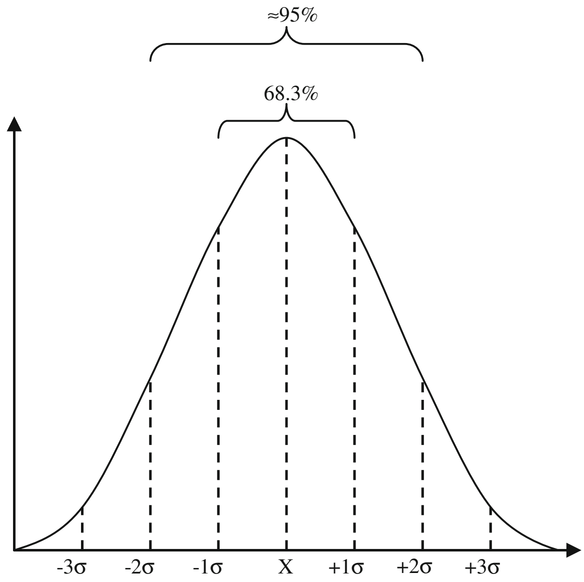


Fig. 7.9 Probability curve distributed symmetrically around the average (Gaussian)

returns of an investment around the average, the use of the standard deviation has the advantage of expressing the riskiness of the project in the same unit of measurement used to express the expected or observed values.

The standard deviation has mathematical properties that make it particularly useful. If the probability is distributed symmetrically with respect to the average (normal distribution or Gaussian), then 68.3 % of all possible values is within one standard deviation from the expected value, while two standard deviations comprise 95 % of the possible results (Fig. 7.9).

Once the average and the standard deviation are determined, the probability of occurrence of values included in a certain range can be calculated by referring to the table of standardized values in Table 7.12.

Table 7.12 shows the distance from the average (expressed in terms of standard deviation) of each expected value. The relationship is algebraically expressed as follows:

$$Z = \frac{X - \bar{X}}{\sigma_X}$$

where X is the generic value of a normal distribution, it is the central point of distribution (expected value) and σ_X is the standard deviation.

The expected value and standard deviation define the entire frequency curve of a normally distributed variable. If the distribution is not symmetrical, then it would be necessary to introduce other parameters that greatly complicate the problem.

The two parameters, expected return and expected variance, taken jointly, represent the measures used to properly express the risk-return profile. Then, by

Table 7.12 Normal standardized distribution

Number of standard deviations (Z)	Left or right area	Number of standard deviations (Z)	Left or right area
0.00	0.5000	1.55	0.0606
0.05	0.4801	1.60	0.0548
0.10	0.4602	1.65	0.0495
0.15	0.4404	1.70	0.0446
0.20	0.4207	1.75	0.0401
0.25	0.4013	1.80	0.0359
0.30	0.3821	1.85	0.0322
0.35	0.3632	1.90	0.0287
0.40	0.3446	1.95	0.0256
0.45	0.3264	2.00	0.0228
0.50	0.3085	2.05	0.0202
0.55	0.2912	2.10	0.0179
0.60	0.2743	2.15	0.0158
0.65	0.2578	2.20	0.0139
0.70	0.2420	2.25	0.0122
0.75	0.2264	2.30	0.0107
0.80	0.2119	2.35	0.0094
0.85	0.1977	2.40	0.0082
0.90	0.1841	2.45	0.0071
0.95	0.1711	2.50	0.0062
1.00	0.1587	2.55	0.0054
1.05	0.1469	2.60	0.0047
1.10	0.1357	2.65	0.0040
1.15	0.1251	2.70	0.0035
1.20	0.1151	2.75	0.0030
1.25	0.1056	2.80	0.0026
1.30	0.0968	2.85	0.0022
1.35	0.0885	2.90	0.0019
1.40	0.0808	2.95	0.0016
1.45	0.0735	3.00	0.0013
1.50	0.0668		

Table 7.13 Measures of dispersion

Expected cash flows (€)	Expected cash flow (€)	$CF_s - CF$ (€)	P(s)	$(CF_s - CF)^2 \times P(s)$ (€)
125,000	187,500	−62,500	0.04	156,250,000
150,000	187,500	−37,500	0.16	225,000,000
175,000	187,500	−12,500	0.30	46,875,000
200,000	187,500	12,500	0.30	46,875,000
225,000	187,500	37,500	0.16	225,000,000
250,000	187,500	62,500	0.04	156,250,000
			$\sum = 1.00$	V = 856,250,000
				σ = 29,262

measuring the ratio between the standard deviation and expected return, one gets a third key risk indicator: the coefficient of variation (CV). In symbols:

$$CV_X = \frac{\sigma_X}{R_X}$$

This coefficient expresses the amount of risk per unit of performance. The usefulness of this indicator, in particular, is due to the case where the decision maker must choose between alternative investment projects that have measures of expected return and standard deviation which are very different amongst each other. The use of CV makes it possible to classify the different initiatives according to a criterion of minimizing the risk considering an equal state of returns.

Example (continued) 7.6 In Table 7.13 measures of dispersion for the cash flows are calculated.

Figure 7.10 illustrates the results. The expected value of €187,500 is the midpoint of the distribution, and the 68.3 % of all possible values is included in the standard deviation (€29,262). Therefore, the real cash flow will be between €158,238 (187,500 − 29,262) and €216,762 (187,500 + 29,262), with a probability of 0.683.

$$CV = \frac{€29{,}262}{€187{,}500} = 0.156$$

Example 7.7 shows how one can incorporate the risk in the investment decision through the exploitation of probability parameters introduced.

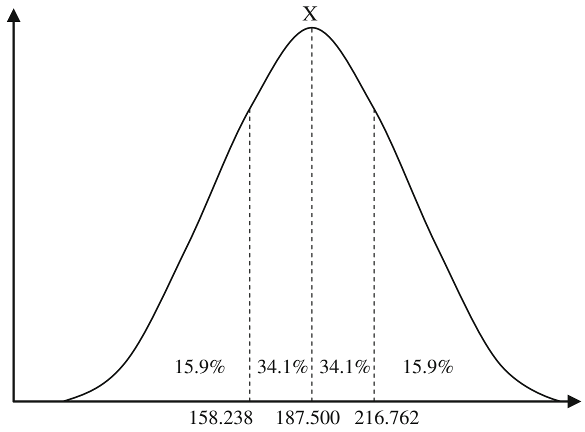


Fig. 7.10 The probability that the cash flow is included within ±1σ from the average

Example 7.7 An investment project requires an initial outlay of €25,000. After 3 years, the investor expects to collect a net cash flow of €50,000. The standard deviation of the flow in the third year is estimated at €7,000, and it is assumed that the possible cash flows are symmetrically distributed around the expected flow. The income for the first 2 years of the holding period is void. The opportunity cost of capital for the investor is set at 10 %. Since it is assumed that the distribution of possible cash flows is symmetrical, the distribution of present values will be distributed symmetrically. The central point of the distribution of present values corresponds to the discounted value of expected cash flow.

$$\overline{PV} = \frac{\overline{CF}}{(1+i)^3} = \frac{€50{,}000}{1.10^3} = €37{,}566$$

It is also true for the determination of the standard deviation of the current value:

$$\sigma_{PV} = \frac{\sigma_{CF}}{(1+i)^3} = \frac{€7{,}000}{1.10^3} = €5{,}259$$

A present higher value than the current cost of initial investment (€25,000) means that the performance of the project exceeds the discount rate adopted (10 %). The probability of this happening is corresponds to the percentage of the area that is to the right of the point where the present value is €25,000 (Fig. 7.11).

The calculation can be done by entering the table of the standardized values with parameter Z, determined as follows:

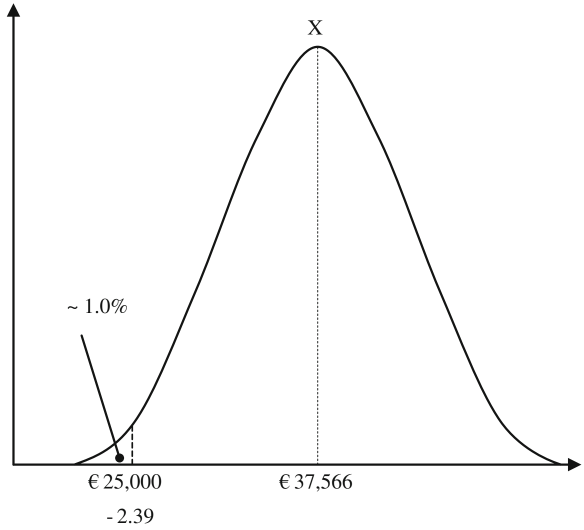


Fig. 7.11 The probability that the real present value of the initial investment proves smaller

$$Z = \frac{PV_x - PV}{\sigma_{PV}} = \frac{€25{,}000 - €37{,}566}{€5{,}259} = -2.39$$

where:
PV_x present value of the minimum acceptable;
PV Expected present value (arithmetic mean);
σ_{PV} standard deviation of the present value.

Table 7.10 indicates that the area to the left of PV = €25,000 corresponds to approximately 0.8 % of the total area underlying the distribution. The probability that this project generates at least a 10 % return is therefore greater than 99 %.

The expected value of the distribution of the indexes of profitability is calculated:

$$IP = \frac{€37{,}566}{€25{,}000} = 1.50$$

By specifying the maximum levels of acceptable risk for different values of the index of profitability, a risk profile that allows the evaluation of the project regardless of the initial financial commitment, is built. If the dispersion of possible outcomes indicates that the project is too risky with respect to the performance per

Table 7.14 Risk profile of two investors

	Acceptable probability that the index profitability is lower than 1	
Expected profitability index	Investor A	Investor B
0.0	0.0000	0.0000
1.2	0.0150	0.0150
1.4	0.0230	0.0260
1.6	0.0290	0.0345
1.8	0.0330	0.0430
2.0	0.0350	0.0460
2.2	0.0350	0.0490
2.4	0.0350	0.0500

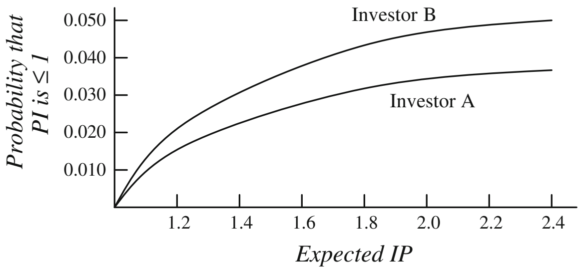


Fig. 7.12 Risk-return profiles

euro invested, then the opportunity is automatically discarded. Otherwise, the investor will take the final decision based on his strategies and on considerations regarding the management of the portfolio, the availability of capital, the legislative constraints, etc.

The construction of the risk profile (combinations of acceptable risk and expected returns) allows the investor to interface his personal attitude with the riskiness of the projects with the analyst's work.

Example 7.8 Table 7.14 shows the risk profiles of the two investors expressed in terms of probability of earning less than the minimum acceptable performance.

It is assumed that the cash flows have been discounted at the rate representing the minimum acceptable return for the individual investor. Thus, for a profitability index equal to 1.2, investor A will accept, at the utmost, a probability of 1.5 % that the real IP is lower than 1.

The higher the expected IP (i.e., the expected return), the higher the level of acceptable risk (Fig. 7.12).

Example 7.8 (not very close to a real condition especially in real estate) refers to an investment characterized by a single flow of incoming cash, placed at the end of the holding period.

In reality, the cash flows (positive and negative) will mature periodically for a certain number of years. It is therefore necessary to estimate the average and the standard deviation of cash flows for each of the years of the investment period. It is very likely that the revenues relating to the various annuities depend, at least in part, on what has happened in previous periods. This bond, defined as serial correlation, is measured through the coefficient of variation, a parameter between +1 and −1. When the coefficient of variation is zero, the cash flows are completely independent from each other, and if the coefficient is 1 or −1, then there is perfect correlation between the flows. In this second case, the deviation from the outcome is equal to the sum (financial) of the deviations related to all the years to come. The serial correlation does not affect the expected present value, but drastically alters the standard deviation of the related probability distribution. In general, the higher the degree of correlation between the annual flows, the greater the dispersion of possible outcomes around the average.

The standard deviation of the present value of a series of perfectly correlated cash flows (coefficient of variation = 1) is equal to:

$$\sigma_{PV} = \sum_{t=1}^{n} \sigma_{CF_t} / (1+i)^t$$

where

σ_{PV} is the standard deviation of the distribution of the present value;
σ_{CF} is the standard deviation of the distribution of the cash flows for the period t;
i is the discount rate used to obtain the expected present value.

With regards to the serial independence of cash flows, the present value does not change. Instead, its standard deviation is different. If the flows are completely independent, the deviation of the present value can be calculated using the following:

$$\sigma_{PV} = \sqrt{\sum_{t=1}^{n} \frac{\sigma_{CF_t}^2}{(1+i)^{2t}}}$$

where the symbols have the known meanings.

Table 7.15 Average and standard deviation of the expected cash flows

Year	Cash flows average	Standard deviation
1	€150,000	€54,000
2	€150,000	€45,000
3	€400,000	€90,000

Table 7.16 Average and standard deviation of the CF assuming perfect correlation

Years	Expected flows	×	Present value (i = 12 %)	=	Average present value
1	€150,000		0.8929		€133,940
2	€150,000		0.7972		€119,580
3	€400,000		0.7118		€284,720
					€538,240
Years	Flows standard deviations	×	Present value (i = 12 %)	=	Present value standard deviation
1	€54,000		0.8929		€48,220
2	€45,000		0.7972		€35,870
3	€90,000		0.7118		€64,060
					€148,150

The dependence between serial correlation and standard deviation is shown in Example 7.9.

Example 7.9 To launch a real estate development intervention, it is necessary to anticipate a capital of €500,000. The analyst develops the average estimation and standard deviation for the cash flows expected during the investment period (Table 7.15).

The distribution of annual cash flows is considered to be symmetrical and the correlation coefficient is assumed equal to +1. The minimum acceptable return for the investor is set at 12 %.

Table 7.16 calculates the average and the standard deviation of the present value using the formulas that are valid in the case of perfect correlation.

By performing the calculation under the assumption of fully independent flows, the standard deviation of the present value is:

$$\sigma_{PV} = \sqrt{\frac{(54{,}000)^2}{(1.12)^2} + \frac{(45{,}000)^2}{(1.12)^4} + \frac{(90{,}000)^2}{(1.12)^6} = €87{,}846}$$

Comparing this value with the standard deviation of the present value calculated in Table 7.16, it is obvious that, moving from a perfect correlation serial to a zero correlation, there was a reduction of €60,313 (−41 %) of the standard deviation of the present value.

When the cash flows are neither independent, nor perfectly correlated, then it is said that there is a partial correlation. In this case (certainly more in line with realistic conditions) calculations become complicated.

Help, in this sense, comes from a model developed by Hillier, which is particularly useful in the field of real estate investment (Hillier 1963). The approach is to group the annual cash flows according to their degree of correlation. Basically, a first group of flows is characterized by a very strong correlation, and a second group of flows is characterized by a near zero correlation. The two groups are then treated as if the corresponding flows were completely independent and perfectly correlated.

The present value of the segmented income flows is not affected by the changes introduced by the Hillier model, while the standard deviation formula takes the following form:

$$\sigma_{PV} = \sqrt{\left[\sum_{t=1}^{n} \frac{\sigma_{CF_t}}{(1+t)^t}\right]^2 + \left[\sum_{t=1}^{n} \frac{\sigma^2_{CF_t}}{(1+i)^{2t}}\right]}$$

where the first calculation is applied to the cash flows considered as perfectly correlated, and the second calculation is applied to the flows considered to be independent.

Example 7.10 To improve risk assessment, the analyst decides to separate the projections of the cash flows of Example 7.9, dividing them into inputs and outputs. Because rents are heavily dependent on residential factors, it is assumed that these revenues are highly correlated over time. The same also applies to the selling price of the property at the end of the holding period.

The operating expenses, however, are not normally influenced by location factors: it is expected that they vary with respect to the projections, only based on random events. If it is assumed that costs are serially independent and revenues perfectly correlated, this condition can be summarized in Table 7.17.

The expected present value of investment remains unchanged from Example 7.9:

Table 7.17 Cash flows and standard deviations

	Planned revenues (€)		Planned expenses (€)	
Year	Revenues	Standard deviation	Expenses	Standard deviation
1	270,000	37,800	120,000	16,200
2	270,000	35,000	120,000	10,000
3	520,000	60,000	120,000	27,000

$$PV = \frac{150{,}000}{1.12} + \frac{150{,}000}{1.12^2} + \frac{150{,}000}{1.12^3} = €538{,}220$$

The standard deviation of the expected present value, however, takes a very different value. Applying the formula of Hillier the standard deviation is:

$$\sigma_{PV} = \sqrt{\left[\frac{37{,}000}{1.12} + \frac{35{,}000}{1.12^2} + \frac{60{,}000}{1.12^3}\right]^2 + \left[\frac{(16{,}200)^2}{1.12^2} + \frac{(35{,}000)^2}{1.12^4} + \frac{(27{,}000)^2}{1.13^6}\right]}$$

7.6.1 The Average-Variance Approach

Average and variance are two important parameters with which one can proceed for the financial evaluation of investments when the uncertainty of the expected cash flows can be quantified by defining their probability distribution. Example 22 shows an application of the criteria for the measurement of risk previously described in the traditional formulation of NPV.

The approach of the average-variance theorizes that between two investment strategies, the one that has a higher expected return and a minor standard deviation should be preferred.

Example 7.11 Suppose one has two alternative investment projects, A and B, the expected cash flows of which represent variables in three alternative scenarios (Table 7.18). Assuming a weighted average capital cost of 11 %, one has to proceed to the determination of the different values of NPV in order to achieve a forecast of the different scenarios. Based on these, it appears that each of the two projects is associated with three possible NPV values, provided with relative probability distribution. Each project can therefore be dealt with, by using tools of average and variance.

Table 7.18 Expected values for projects A and B based on the hypothesis of three different scenarios

Projects	Scenarios	Probability	F_0	F_1	F_2	NPV
A	Pessimistic	0.2	−100	30	40	−40
	Normal	0.5	−100	50	70	2
	Optimistic	0.3	−100	80	100	53
B	Pessimistic	0.3	−100	30	50	−32
	Normal	0.4	−100	60	60	3
	Optimistic	0.3	−100	80	80	37

Based on the previous formula, replacing *Rxs* with the *NPV* associated with the different scenarios for project A, the expected *NPV* is equal to:

$$NPV(A) = -40 \times 0.2 + 2 \times 0.5 + 53 \times 0.3 = -8 + 1 + 15.9 = 8.9$$

and the standard deviation is equal to:

$$\sigma_{PV}(A) = \sqrt{\left[(-40 - 8.9)^2 \times 0.2 + (2 - 8.9)^2 \times 0.5 + (53 - 8.9)^2 \times 0.3\right]}$$
$$= 32.9$$

With regard to project B instead:

$$NPV(B) = -32 \times 0.3 + 3 \times 0.4 + 37 \times 0.3 = -9.6 + 1.2 + 11.1 = 2.7$$

$$\sigma_{PV}(B) = \sqrt{\left[(-32 - 2.7)^2 \times 0.3 + (3 - 2.7)^2 \times 0.4 + (37 - 2.7)^2 \times 0.3\right]}$$
$$= 26.7$$

Summarizing, therefore:

Project	Expected NPV	σ_{PV}
A	8.9	32.9
B	2.7	26.7

As one can see from the table, investment project A has an expected NPV higher than that of B. The data relating to investment risk, however, is in favour of project B, since the standard deviation of the expected NPV is equal to 26.7, lower than project A.

The average-variance criterion asserts that individual preferences are characterized only by these two parameters. Individuals prefer the alternative which has a higher expected return at a constant risk or, alternatively, a minor risk equal to the expected return.[4] The problem of Example 7.11, with reference to the illustrated approach, does not, therefore, identify, a preferable solution.

The choice between investment A and B requires the consideration of the degree of aversion to risk of the investor. This is possible through the construction of a utility function.

7.6.2 The Expected Utility Theory

Among the psychological models of human behaviour, the theory of Expected Utility represents a fundamental model at least within the framework of the classical decisional theory.[5] This model, based on the theory that the operator is rational and predictable in his actions, was born at the end of the '40s (Von Neumann and Morgenstein 1974). It has been widely applied as a model of economic behaviour, at least until new theories overcame certain limits and properly integrated psychological aspects of individual assessment within the analysis as a whole [e.g., the theory of the Prospectus (Kahneman and Tversky 1979)].

Fundamental to this model is the assumption of rationality underlying the behaviour of individuals. This theory, in conditions of uncertainty of the result to achieve, i.e. risk,[6] defines' "utility" as the cardinal measure of consumer preference. The assumption of rational behaviour leads to the hypothesis that individuals in reality, move according to predetermined patterns based on the fact that they always prefer to have a greater wealth than a minor and therefore the marginal utility of wealth (in this case the NPV) is always positive.

The model postulates that the decisions of economic operators conform to a function of expected utility that is able to bind a corresponding measure of utility to each choice. The model can be used for classifying risky alternatives: the

[4] It should be noted that an investor could be defined as "rational" when its process of resource allocation is based on the principle of maximizing the return on an equal risk, or risk minimization, given an equal level of expected return.

[5] Usually, the models of decision theory, which reflect the main points of the economic outlook, are identified in this context.

[6] The individual is called upon to make a decision without knowing with certainty, ex ante, what may happen in the world, but he knows the list of possible events, to which he associates a probability of occurrence.

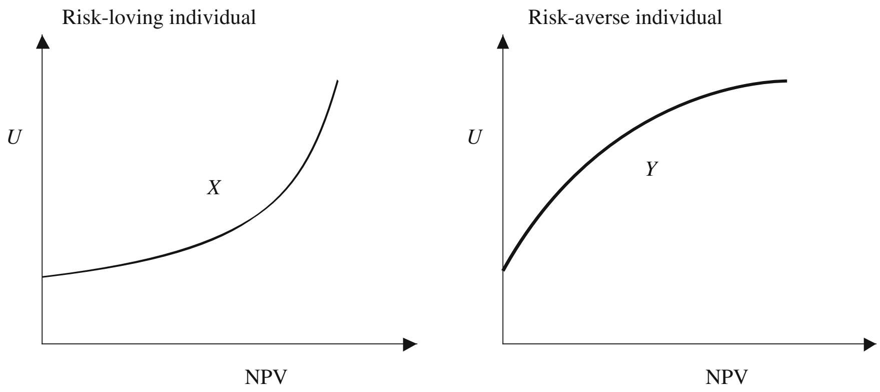


Fig. 7.13 Utility functions—NPV

measure of utility assigned to each alternative is expressed as a function of the possible results and the probability that these results occur. The individual will choose the alternative associated with the highest expected utility.

The utility function can have different forms (Fig. 7.13) according to the perceived risk by the investor:

- It is concave when describing the preferences of a risk-averse individual (Y);
- It is convex when describing the preferences of a risk-loving individual incline (X);
- It is linear when describing the preferences of a risk neutral individual.

The utility function can be considered as an immediate derivation (or vice versa) of the indifference curve, already defined as the set of points that identify combinations of risks and returns that are indifferent for the investor.

The inclination of an indifference curve expresses the degree of aversion to risk that the investor has. The higher the inclination, the higher the required risk-return substitution rate. In other words, at an increase in the inclination of the curve, the investor will be willing to take on increasing amounts of risk on condition that the returns achieved are proportionately higher.

The concept of "risk premium" is also attributable to the utility theory, where "risk premium" is defined as the maximum payment that the individual is willing to pay to eliminate risk and obtain the expected gain from the risky condition[7] with certainty. In other words, the risk premium measures how much the individual is willing to pay to eliminate the risk of choice. This concept is reminiscent of the certainty equivalent, which instead represents the willingness to accept a sure amount in lieu of a higher uncertain gain resulting from a risky investment. The

[7] Wikipedia defines it as the minimum amount of money by which the expected return on a risky asset must exceed the known return on a risk-free asset, or the expected return on a less risky asset, in order to induce an individual to hold the risky asset rather than the risk-free asset.

risk premium (RP) is in fact measurable as the difference between the expected value (EV) and the certainty equivalent (CE):

$$PR = EV - CE.$$

A risk-averse investor will exhibit a positive risk premium, then the higher the premium, the higher its level of risk aversion.

According to this approach for each possible outcome (*NPV*) of a project, a value based on the investor's utility function *U*(*NPV*) is assigned. The expected utility is obtained as the weighted average of the utility associated with each possible outcome, with the weight determined by the respective probabilities.

The construction of the investor's utility function allows one to measure the certainty equivalent, but above all, it allows for an intra-project comparison. In practice, according to this theory, if called upon to choose between several alternative projects, an individual will compare the levels of expected utility *EU* associated with the various alternatives and will choose the one with the highest expected utility.

The expected utility theory is based on certain axioms thanks to which the underlying logic of decision-making behaviour is greatly simplified. It assumes that individuals are perfectly rational and act using complete and homogeneous information sets. These assumptions allow a simple mathematical modelling of the decision-making process based on some external constraints. On the other hand, as already mentioned, the empirical research and the study of the psychological processes of judgment and decision-making have shown that investors systematically make errors, of reasoning and preferences, which are difficult to reconcile with the assumption of rationality behind the choices made.

Furthermore, there is the operational limit related to the need to clarify the utility functions of the investor.

Example 7.12 For investment Z two possible scenarios are expected: one with NPV = 20 (pessimistic) and the other with NPV = 80 (optimistic) to which exactly the same probability is associated (0.50).

The expected value of the NPV is = 20 × 0.50 + 80 × 0.50 = 50.

If the investor's utility function is like that of Fig. 7.14 which highlights an aversion to risk, the expected utility corresponding to the pessimistic scenario is 30, while it is equal to 55 in the event of an optimistic scenario. The expected utility is then equal to

$$EU = 30 \times 0.50 + 55 \times 0.50 = 42.5$$

Instead, the utility associated with the expected value is equal to 47, i.e. greater than *EU*.

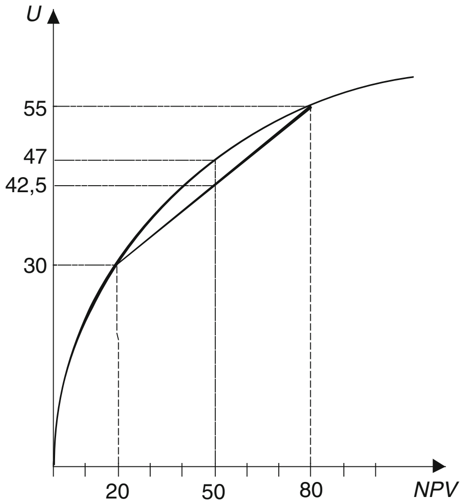


Fig. 7.14 Examples of utility functions—NPV

Table 7.19 Cumulative function of the probability of (S)

NPV	Probability	Cumulative function
10	0.2	0.2
12	0.25	0.45
15	0.3	0.75
18	0.15	0.90
20	0.1	1

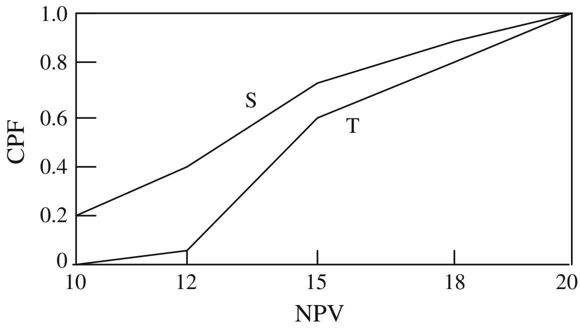
Table 7.20 Cumulative function of the probability of (T)

NPV	Probability	Cumulative function
10	0	0
12	0.1	0.1
15	0.5	0.6
18	0.2	0.8
20	0.2	1

The certainty equivalent is the *NPV* that is achievable without any risk. It gives the investor a utility equal to that expected from the risky project under consideration.

In the example the certainty equivalent is measured from the vertical that intersects the utility curve with the value 42.5: $EC = 41$. The risk premium is equal to $\text{RP} = 50 - 41 = 9$. Since, as in the example, a positive risk premium characterizes the risk aversion of the investor, it is clear that this kind of attitude is represented by a concave utility function.

Fig. 7.15 Cumulative functions of probability of the projects (*S*) and (*T*)



7.6.3 The Approach of Stochastic Dominance

The mathematical derivation of utility functions can be overcome by using the concept of stochastic dominance.

There exists a stochastic dominance in a project (A) with to a project (B) when the level of utility derived from A is greater than the level of B for any amount of expected result of the project. Instead of deriving the mathematical structure of the utility function, it is therefore necessary to verify some basic characteristics. Firstly, it is necessary to represent the values of NPV using the Cumulative Probability Function (CPF). This function indicates the probability that the NPV is equal to or lower than a certain value. Analytically, the CPF is derivable by summing the probabilities expressed as relative frequencies of the values of NPV.

Example 7.13 Tables 7.19 and 7.20 show the probability distribution of NPV relating to projects S and T. The respective cumulative probability functions are shown in Fig. 7.15. As is clearly seen in this figure, the cumulative function of the probability of T is always to the right of that of S. This means that for each level of NPV, the project (T) with respect to (S) has a lower probability and that its value is less than the identified NPV.

For example, if one considers the NPV threshold of 15, project (T) has a 60 % chance of a total value lower than this level, compared to 75 % of S. In such circumstances, regardless of the derivation of the utility function of the decision maker, it can be said that project (T) stochastically dominates project (S), and so (T) is the preferred project. This case is known as the so-called first-degree stochastic dominance (Copeland and Weston 1992; Goodwin and Wright 1994).

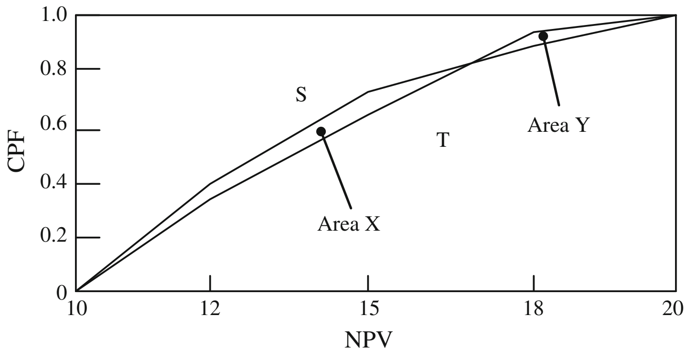


Fig. 7.16 Examination of a case of second-degree stochastic dominance

Consider now the case of two projects with cumulative functions of probability that intersect, for which there is no condition for stochastic dominance of the first degree. The situation is described in Fig. 7.16.

In this case, to verify the hypothesis of stochastic dominance it is necessary to consider both the extent of the NPV intervals in which a project partially dominates the other and the size of that domain.

A similar verification can be done by comparing the extent of the areas included between the two, respectively area X with area Y. The first area, in fact, shows the extent to which project (B) stochastically dominates project (A); area Y, on the other hand, indicates the dominance of (A) to (B). The examination of the graph clearly shows that area X has a greater extension than that of Y; this configures a hypothesis of stochastic dominance of second degree of project (B) on project (A).

Whereas the graphical analysis does not allow us to determine which of the areas is wider than the other, it is necessary to use the calculation of the integral, analytically determining the amplitude of the two areas. In the formula, for a risk-averse investor, project (A) stochastically dominates project (B) when the following condition occurs:

$$\int_0^{NPV_i} U_B(NPV)d(NPV) \geq \int_0^{NPV_i} U_A(NPV)d(NPV)$$

with the inequality settled in a strict sense at least for a NPV.

The latter case, which also regards stochastic dominance, has the same limitations of average variance. Only when one is in a clear condition of stochastic dominance is it not necessary to explain the utility function of the decision maker.

7.6.4 Evaluation Based on Differentiated Assumptions

It is an evaluation technique implicitly used in the illustrated examples, based on which one determines different values of NPV for each project to be evaluated, by following three basic hypotheses: pessimistic, optimistic, and maximum likelihood. The aim is to define the width of the spectrum of different possibilities that one imagines might occur. The evaluation based on differentiated assumptions differs from the sensitivity analysis because the latter acts in single mode on the different variables that determine the value of the project, while the former takes into account multiple scenarios within which a specific interaction between the different variables is assumed.

The fundamental limits of this approach are twofold. First, the spectrum that goes from the pessimistic assumption to the optimistic one is often so wide as to result ineffective for the development of guidelines for decision-making. Secondly, this method, in any case, leaves space to the subjective component of the decision maker, this limitation, however, is quite common amongst the evaluation methods discussed in this chapter.

7.6.5 Mathematical Programming

Mathematical programming is based on the expression of the results of the project to be evaluated through a mathematical function, called "objective", which relates to the main variables.

The function is associated with a set of constraints, represented by disequations that define the "area of eligibility" of the values associated to single variables or to groups of variables included in the "objective" function.

A problem of optimization of multiple function variables that are subject to constraints of non-linear disequations or equations, arises.[8]

This methodology is certainly rigid when considering the interrelationships between the single variables, but has the background disadvantage linked to the need to analytically express the relationships between the variables that explain the value of an investment project.

The formal rigidity is followed by a difficulty of application, as well as a lack of responsiveness to the typical structure of a decision-making process, which is similar to that which has been presented above.

[8] On this topic, refer to Comincioli (1990).

7.7 Limits and Perspectives of Evaluation Techniques

In view of the traditional capital budgeting an investor faced with options/alternative projects, manages the decision-making process on the basis of an eminently economic nature, not being able to evaluate his choices in relation to the consistency of them with the business strategy.

The NPV for example, (a crucial tool of capital budgeting) responds well to the question of whether it is appropriate to spend today, or invest savings in order to spend more in the future. Its genesis is an instrument for the evaluation of investments in the context of relative certainty (in the risk free version) and its fundamental insight is that of the financial value of time. To this the advantage of simplicity that made this tool easily applicable to many different contexts, must be added. The limitations on the use of the NPV are related to the lack of consideration of the risk factor. The management of the risk factor is essential and unavoidable in real estate, unless it has to deal with, as often happens, a committed project (or build-to-suit project) implemented in a context of relative certainty. In all other cases it is necessary to introduce the assumption of uncertainty, or to imagine that, in line with what happens in reality, most real estate investments generate cash flows that are uncertain both in terms of the entity and in terms of temporal manifestation.

Several techniques based on the discounted cash flows that can influence the risk in the evaluation process, have been illustrated.

The DCF analysis, in its static version, calculates an NPV by modifying the denominator of the formula, the discount rate k (which reflects the concept of opportunity cost of capital) given by the sum of a risk-free rate (r) and a premium for the risk (p).

Rate k therefore expresses a subjective risk factor. This step is one of the main reasons why this approach is criticized. Another critical issue is that relating to the adoption of a constant discount rate in projects whose risk profile appears variable during the implementation phases. In the case of construction of a building, for example, the first steps involve a risk that is definitely higher than in the latest phases. These problems affect the sustainability of the results and therefore the efficacy of using this methodology. As a result, the investors will turn to the application of discount rates, constant but very high, based on subjective estimates and not always justified.

Exceeding this limit occurs with the use of a probabilistic approach to risk analysis. It is true, however, that the fundamental prerequisite for the beginning of a probability analysis is the availability of a sufficient number of cases to allow the construction of a frequency distribution for the single key variables. However, the difficulty of collecting data and information in an inefficient and non-transparent market such as real estate is known, especially in Italy. This is the main reason for which the probabilistic analysis, when it is carried out, is often developed from subjective probability distributions; in other cases, the use of traditional techniques is preferred.

The limit of NPV that even a probabilistic approach to risk analysis is able to overcome, is given by the impossibility to incorporate feedback of a strategic nature in contexts of uncertainty, that take into account the interaction between current investment alternatives and future decisions.

A decision is defined as strategic when it has a long-term horizon, it is irreversible, and it can deeply modify the outcome of the investment. An investment that provides, in its development, choices of this kind is often the bearer of a wide area of value in itself, which is difficult to quantify using traditional approaches of capital budgeting. It can therefore be said that the strategic analysis takes care of all those decisions that ensure the achievement of long-term objectives, while the capital budgeting techniques provide the tools needed to measure the impact that decisions can cause.

Traditional DCF approaches allow to choose between investment alternatives that are homogeneous in terms of strategy, and those that differ in risk, in the amount and in the temporal distribution cash flows. Very often, however, developments related to an investment decision are such that they can only be evaluated at a later stage. These are investments that are broken down into several, temporally consecutive segments, whose value depends on the outcome observed in the previous segment.

There are two cases of dependence that can derive from a project:

- Temporal interaction: it refers to the consequences that a project generates in the future on itself, i.e. the interaction between current decisions and future opportunities, in a context of environmental uncertainty;
- Design interaction: concerns the relations generated between the actions and decisions taken in relation to a specific project and the consequences related to the implementation or the evolution of other projects.

Both the temporal interaction and design interaction have a common root in the concept of adaptability/managerial governance of the project.

Adaptability is intended for all changes which may affect the project, depending on the future market or industry, and that may be the subject of a decision by the management. The adaptability seems to well epitomize the levers available to the investor, and ultimately the strategic dimension of a project.

Consider, for example, the possibility of delaying the construction of a building in relation to the possible intervention of adverse conditions. In this regard, it is necessary, firstly, to reflect on the fact that if time has a financial value, rightly considered by the NPV, it also has an informative value. The informative value of time is a resource of great importance in all those areas where rapid technological and competitive evolution requires a dynamic management.

This leads to assign a particular value to those projects whose implementation may be delayed, without compromising their feasibility. In such circumstances, in fact, the investor can alternatively avoid taking a wrong decision or optimize the implementation of the project in light of changed strategic-competitive scenarios.

The reasons for dissatisfaction with the traditional tools of capital budgeting are rooted in their inadequacy for evaluating investment projects characterized by decisions dependent on the outcome of uncertain variables and the consequent possibility/necessity of an intervention by the investor, after the implementation of the project.

Ultimately, it is possible to conclude that the NPV does not allow to weigh the strategic importance of investment decisions, and this mainly because it is not able to take into account the interaction between current investment alternatives and future decisions.

When an investor decides to produce a very innovative building in an area that has not yet been exploited, the result of the investment cannot be searched in the cash flows that are directly generated by the initiative itself. In such circumstances, in fact, the value of the investment lies mainly in the future opportunities it may open.

As a rule, it is possible to say that the purpose of many strategic investments is to collect information about a particular product or market. Just as a construction company finances market research aimed at estimating the absorption capacity of a given segment of potential users, they can also develop a pilot project, the costs of which are known but the expected benefits are uncertain.

Even in the case where the NPV of the basic investment is negative, the investor can still proceed with the operation if estimates that the value of future opportunities justify the initial sacrifice incurred in order to acquire them.

In many cases, the logic of the NPV must therefore be corrected so as to take into account the creation value represented by the managerial flexibility implicit in the initiative itself.

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