Chapter 2
A Random Journey from Monism to the (Dream of) Unity of Science

If the world were clear, art would not exist.
Albert Camus

2.1 History

Among the founders of Western culture, the Presocratic thinkers sought explanations of the natural world that required only one or very few fundamental principles or substances. Hence, Presocratic philosophers were the first (in Western culture, at least) to search for one substance capable either of explaining every natural phenomena or of unifying all phenomena within a single framework. In this endeavour, they put forward the first reductionist vision of the world, which we may interpret as a search for some kind of harmony in the world and, therefore, for a unifying picture. This idea has largely dominated scientific progress and development ever since and proven undeniably fruitful. Their motivation was that the marvelous and amazing variety of nature, experienced by the human senses, needs some kind of order to be appreciated and analysed. In particular, the Presocratics were attracted by a form of monism which may be considered the philosophical precursor to the most extreme version of reductionism. This search for first-principles explanations of "everything" has greatly influenced the subsequent developments of science and knowledge and, in particular, has become a cornerstone of physics, until the present day (Klein and Lachièze-Rey 1996).

Thales proposed water as the primary element: he thought of water as the material origin of everything. For the first time, an explanation of all phenomena was proposed in terms of a unique element, creating a departure from the mythic tradition, which had multiplied causes and ad-hoc explanations. The tragic expedient of the *Deus ex machina* is a splendid illustration of the former approach. Thus, for the first time, Thales expressed three ideas which remained thereafter at the core of scientific endeavour:

S. Chibbaro et al., *Reductionism, Emergence and Levels of Reality*,
DOI: 10.1007/978-3-319-06361-4_2,

- the search for the material causes, or mechanisms, underlying natural phenomena;
- the use of rational arguments to understand the world; and
- the reduction of answers to few or just to a sole fundamental substance.

Following this path, Anaximenes proposed a different cosmogony, in which air was regarded as the fundamental substance. Interestingly, from a methodological point of view, he put forward a logical framework in which every known natural phenomenon (earth, wind, clouds, etc.) could in principle be deduced from his primary substance.

A while later, Heraclitus chose fire as the fundamental element. He preferred an intrinsically more unstable element, in order to underline the dynamical features of the universe: evolution and becoming. Heraclitus clearly sided with "*becoming*" rather than with "*being*". To complete his cosmogony and explain the huge variety of natural phenomena, he complemented his fundamental substance with a fundamental law: a dialectic law asserting that a hidden conflict between opposites pervades nature. This fight was supposed to explain the observed harmony and stability of the world, despite the unstable nature of the fundamental element. As pointed out by Heisenberg (1958), Heraclitus' view appeals to contemporary physics: *if we replace the word "fire" by the word "energy" we can almost repeat his statements word for word from our modern point of view.*[1]

Some years later, Empedocles closed out the pioneering era of the monistic approach, proposing a multiplicity principle, which adds earth, as a fundamental element, to the elements considered by his predecessors. Water, air, fire and earth thus gave birth to the four-element cosmogony, which prevailed in the christian view of the whole Middle Ages, until the Renaissance. Interestingly enough, the idea of a small family of elements responsible for all phenomena constitutes the standard approach of theoretical particle physics, for which the whole world is made up of three families of elementary particles (6 quarks + 6 leptons).

Still in the Classical period, philosophers envisioned another concept which complemented the primordial form of monism described above and which remains at the heart of modern science: considering an abstract principle as a fundamental substance. This conceptual progression was achieved in the sixth century B.C. by Anaxagoras, who advanced intelligence as the first principle and organising force of the universe. At the same time, the Ionic philosopher Anaximander abandoned the idea of fundamental natural elements and made another leap forward, concluding that the fundamental substance should be a complete abstraction. To this purpose, he proposed the $\alpha\pi\epsilon\iota\rho o\nu$, the not-finished or the infinite. This was the first attempt to define an ultimate *One*, as the fundamental and primordial reason of everything. New insights into the relation between the One and the Multiple were given by Parmenides who declared: "*the being is, the not-being is not*" (Diels and Kranz 1951–1952). In this way, the metaphysical framework was introduced, for the first time, and lasted almost intact until modern times. For instance, the laws of physics are thought to be

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[1] With an even more audacious analogy, we can see the conflict between opposites essentially as the *in fieri* quantum wave/particle duality.

eternal[2]: their constancy permits a unitary vision of nature. Unstable and fluctuating truths do not allow a rational prediction of phenomena. This vision was shared by Poincaré (1913), who thought it necessary that the laws of nature not be subject to modification. He proposed this view under a methodological assumption, but he admitted that these laws ought to be generalised or transformed in case they were falsified.

Another approach and *Weltanshauung* was developed in parallel to the monistic approach: atomism. Leucippus and Democritus were the foremost proponents of this concept. While atomism avoided the definition of a unique fundamental substance, it established for the first time a reductionist approach to nature, in which every natural phenomenon is explained through the features of a small set of elementary objects.[3]

On the other hand, other great philosophers showed an anti-reductionist attitude, even mocking the reductionist attempts to describe the world. Among these, Socrates was probably the most influential and the most scathing: *It is strange that men wanted to understand the first principles of things, and from that understand everything, with conceit as infinite as their object of study* (Klein and Lachièze-Rey 1996).

The fundamental role of numbers is another important idea that has been most influential from ancient times through the Middle Ages. First underlined by Pythagoreans, the essential relevance of numbers as creative principles has been long recognised from a Mystic point of view. While the Mystic approach has not given important scientific results, it has been significant in the methods of modern scientists like Einstein and Dirac. They admitted the creative and scientific role of mathematical aesthetics and formal consistency, together with the empirical verification. It is plain that the search for a unified and harmonious account of the world has always animated and motivated scientists' and philosophers' works, even if in most diverse fashions. Furthermore, this metaphysical spirit transformed into a search for order and eventually for unity in nature, hence can be seen as one of the sources of the reductionist approach. A striking example (as well as one of the most important) of the step from the Mystic to the scientific point of view is afforded by Kepler, who significantly entitled one of his main works "*harmonia mundi*". Kepler desired primarily to point out the harmonious side of nature, by discovering the laws of the motion of planets. In order to accomplish this aim, he mingles mathematics, which continues to be the first principle underlying nature, with geometry, music, astrology and astronomy. In this way, he was able to reduce to a small set of statements (in this case three mathematical laws) a complex and in some sense universal natural behaviour (planetary orbits). This work prepared the formalisation established by Galileo, Descartes and Newton who, disposing of the spiritualistic and metaphysical elements, moved toward an actual reductionist vision of science. Ultimately, Kepler's work definitively sanctioned the link between mathematics and the harmony investigated by science, while dispensing with Mystical visions.

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[2] This vision is not naive and is compatible with possible major changes occurring in nature on the scale of the age of the universe.

[3] Of course, it is meaningly to try to establish whether Leucippus or Democritus were "reductionists" in modern terms. Our historical references should be considered *cum grano salis*.

Another important view emerges along with these developments of modern science. The synthesis motivated by the search for harmony in the world accompanies the attempt to unify our knowledge of all phenomena, as disparate as they may be. Thus, it is possible to appreciate how reductionism is intimately related to unification. An example of the unifying qualities of the quest for harmony is given by the Balmer series (Cartier 1995),[4] proposed to give order to a series of experimental results, reducing them to a rigorous and elegant mathematical formula. Yet, the harmony underlying this formula was recognised by Bohr in the quantum atom, and the Balmer series has been shown to be a part of the more general (and thus unifying) theory of quantum mechanics. It is interesting to note that even the quantum theory of fields, in particle physics, appears in some sense to remain in the wake of this harmony representation of the world, although it is an atomistic theory by definition. Indeed, symmetries are used as principles of mathematical harmony, related to different kinds of particles.

Loosely speaking, two main streams have animated the scientific and metaphysical developments since the ancient Greek philosophers, in very different forms: on the one hand, the demand for unity and order (in some sense the necessity of a reduction of phenomena to basic simple levels); on the other hand, the desire and search for harmony in the world, also with regard to the existence of many levels of complexity. Several further important elements have been put forward since then: the importance of intuition and creativity in the process of scientific and philosophical thinking, as well as the formidable instrument of unification that is mathematical language (Jona-Lasinio 2005; Dorato 2010).

2.2 Reductionism: The Philosophical Point of View

It is impossible to discuss, even only superficially, the vast literature concerning the subject of reductionism and liminal domains. In order to complete the present brief overview, we direct the reader to the following important references (Humphreys and Bedau 2006; Boyd and Gasper 1991; Adler et al. 2002).

2.2.1 General Introduction

As suggested by the historical sketch above, reductionism's roots can be traced back to the desire to unify different parts of the same science or even different sciences, within a more general scheme encompassing them. Thus, reductionism and the unity of science have commonly been associated with each other. In particular, the idea and strategy was usually to reduce some higher-level science (such as biology) to

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[4] It relates the classification of spectral rays of hydrogen following the frequency, afterwards generalised by Rydberg. It remains true in a given approximation.

a lower-level science such as chemistry or physics; or, more basically, to reduce a theory to some other theory, considered more fundamental. Paradigmatic examples in physics include the relations between classical thermodynamics and statistical mechanics (Nagel 1979), between classical Newtonian physics and the theory of relativity, or between classical mechanics and quantum mechanics (Dirac 1929). Thus, philosophers of science and scientists (who have often been the same persons until as recently as the first half of the nineteenth century) have usually been concerned with inter-theoretic relations (i.e. with relating theories of one domain with those of some other domain). This form of reductionism can be therefore called "epistemic reductionism". This clarifies an important point, related to the epistemic reductionistic approach but, at the same time, distinct from it: since the birth of modern thought, which for philosophical and scientific purposes can be traced back to the rationalist work of Galileo and Descartes, it has seemed natural and obvious to regard the world as being hierarchically structured. That means that we implicitly consider nature to be structured in several more or less fundamental levels, which are related to each other. It is worthwhile reflecting on the fact that every modern attempt to understand natural and human facts or evidence is associated with the effort to classify them, in order to make some sense out them.[5] Indeed, it seems that taxonomy is necessary to understand a given phenomenon, at least in western culture. This metaphysical disposition was further formalised by Descartes (1987), who proposed a rigorous method to face reality and its problems. He put forward a recipe based on the following metaphysical analytical approach: every problem or phenomenon should be decomposed into its smallest parts that preserve its properties. Here the idea is that it is always possible to dissect a phenomenon into smaller pieces, keeping the sum of the parts equal to the whole. The method consists in subdividing the problem into parts which are individually tractable, in order to eventually solve the original problem. This strategy can be related in some sense to reductionism, yet it is simply a manifestation of the human failing of not being able to deal with too many different issues at the same time. This approach may be seen as the basis of so-called methodological reductionism, which states that the only (or best) way to generate scientific knowledge is to decompose complex problems into simple tractable ones. However, it does not claim to be a philosophical position concerning inter-theoretic relations. Besides, Descartes' method remains even nowadays an essential ingredient of every scientific or simply rigorous approach, given that it seems quite hard (perhaps meaningless) to analyse complex phenomena holistically. In Descartes there is an idea of unified science but not necessarily in a hierarchically sense, from higher (less fundamental) sciences to lower (more fundamental) ones. Descartes and Galileo pointed out a humanistic vision of the unity of science, in which the method is the same, but

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[5] If this is strikingly so in modernity and in sciences, it is also true in earlier literary and philosophical studies: the highly symbolic, complex but, at the same time, very organised hierarchy proposed by Dante for heaven and hell; Aristotle's metaphysical vision of sciences organised in three areas (theoretical, practical and productive) which were devoted to different purposes and formed part of a unified hierarchy with the theoretical at the top.

total freedom and independence is left to each discipline. For natural sciences they proposed a unified language as well: the language of mathematics. The precise role of mathematics in natural sciences is a very subtle and deep issue (Bouveresse 2011; Dorato 2010), which remains debatable. However, in a loose sense, this kind of unity of science seems difficult to question, and has proven to hold in general (Klein and Lachièze-Rey 1996).

Before discussing in some detail the contemporary formalisation of intertheoretic reductionism, it is worth discussing a philosophical current that has strongly influenced scientific and philosophical thought in the twentieth century: logical empiricism or positivism. In a very broad sense, this philosophical current aims to completely systematise and unify sciences, even humanistic sciences, appealing to logic as its main instrument. The birth of logical empiricism can be traced back to the 1920s and the vivacious atmosphere of Germanic culture; notably the Vienna Circle and the Berlin Circle (Bechtel and Hamilton 2007). The philosophy of the logical positivists is related to the positivism introduced by Comte, the early nineteenth-century French philosopher who was sceptical about philosophical systems and of metaphysics in general. For this reason, he emphasised the importance of "positive knowledge"—that is, knowledge grounded in observation and experimental verification—even for the disciplines that had remained mostly speculative. In this context he is considered the father of modern social sciences. Logical empiricism can be seen as a much more radical version of classical empiricism, first developed by Hume, which has been also influenced by Mach's positivism, which implied a radical empiricism which considered sensorial experience as the only source of knowledge (Bechtel and Hamilton 2007). The adjective *logical* refers to the main tool considered by these empiricists, to proceed from individual observations to generalised scientific claims.[6] Some also argued that the different sciences could be unified through theory reduction. In that sense, logical empiricists appear to accord with Dirac's and Bohr's formal interpretation of the physical world (Bohr 2011; Dirac 1929) and tried to offer this physicists' approach to encompass all knowledge.[7] Their project culminated in the attempt to provide a common account of the methodology of all sciences and link them into a unique theoretical construct, which gave rise to the International Encyclopedia of Unified Science, edited jointly by Neurath, Carnap, and Morris (Bechtel and Hamilton 2007). The goal, according to Neurath (1938), was to dovetail the scientific disciplines, so that advances in one of them would bring about advances in the others as well. As previously explained, the main tool for such dovetailing of different sciences was logical analysis, necessary to formalise and systematise all the concepts of the different sciences and, eventually, the global theoretical claims of various sciences. The editors of the International Encyclopae-

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[6] In this sense, they were strongly influenced by the advances of mathematical logic of the late 19th and early 20th centuries due to Frege, Peano, Russell, Whitehead, and others.

[7] In Chap. 6, we will see how the philosophical ideas of the fathers of quantum mechanics are elaborate and not monolithic. On the other hand, a logical empiricist like Schlick developed a realistic approach.

dia envisioned an axiomatised integration of the whole body of knowledge provided by the various sciences and were convinced that this project would lead to a global improvement of each science as well as to prospects for their integration.

2.2.2 Philosophical Model of Theory-Reduction

2.2.2.1 Hempelian Explanation

Inter-theoretic reductionism entered naturally into the doctrine of logical empiricists, as one of the steps needed to set science free from metaphysical issues, and in relation to the issue of the unity of science (AA.VV 2011). In order to accomplish this difficult task, logical empiricists emphasised the role of logical analysis, trying to study by this means all claims made by the different sciences. The crucial idea was to represent scientific claims, including observations and theoretical statements, within a unique framework, whenever possible. In particular, Nagel identified "experimental laws" as a possible relevant candidate for this purpose, regarded as the middle way between theory and experiments and which were able to provide an empirical summary of the phenomena observed. Galileo's law of the quadratic dependence on time of the distance travelled by a falling object is considered an example of experimental law by Nagel. Moved by aversion to metaphysical and ontological issues, logical empiricists wanted to minimise the definition of theoretical objects needed to account for empirical evidence. In this framework, new predictions in the form of unknown observation statements are deduced directly from laws (with the addition of particular conditions), and thus their generality is measured by the number of new statements effectively included. This is the well-established deductive–nomological (D–N) or covering-law model of explanation (Hempel and Oppenheim 1948; Hempel 1965).

Logical empiricists proposed the generalisation of this approach toward the relation between different theories (Kemeny and Oppenheim 1956; Putnam and Oppenheim 1958), requiring that the same observable predictions be obtained within the less general theory (in their language, the reduced theory) as well as in the more general (the reducing) one. This approach was later formalised to show how to derive (hypothetically, at least) the laws of one discipline or science from those of another (Woodger 1952; Nagel 1979; Quine 1964; Kuipers 2001).

2.2.2.2 Nagelian Model of Reduction

The reductionist schema used in most philosophical models of reduction is based upon the seminal and most influential work of Nagel (1979), who stated that: *A reduction is effected when the experimental laws of the secondary science (and if it has an adequate theory, its theory as well) are shown to be logical consequences of the theoretical assumptions (inclusive of the coordinating definitions) of the primary science.*

It is worth recalling that, in all these philosophical discussions, reduction is concerned with a *secondary* science (less fundamental or higher-level) which can be reduced (in some sense) to a *primary* science, considered more fundamental or lower-level.

Thus, roughly, the Nagelian model of reduction can be summarised as follows: given two theories T and T1, T1 is reduced to T if the laws of T explain the laws of T1, where the explanation is interpreted in terms of Hempelian deductive-nomological model. In other words, T1 is reducible to T if the laws of T1 are derivable from the laws of T.

A first issue arose immediately from the obvious fact that laws in different sciences or even at different levels of the same science make use of different vocabularies. Therefore, one should ask whether the relation between semantically different domains is meaningful and to what extent. To answer this question Nagel first proposed that the homogeneous (same vocabulary) relations be distinguished from the heterogeneous ones. In addition, he advocated the use of some "*rules of correspondence*", now commonly called *bridge principles*, that equate the vocabulary. Let us follow Nagel's words: *If the laws of the secondary science contain terms that do not occur in the theoretical assumptions of the primary discipline . . ., the logical derivation of the former from the latter is prima facie impossible*. The two supplementary conditions to be met in order for the reduction to take place are:

1. connectability: assumptions of some kind must be introduced which postulate suitable relations between whatever is represented by "A" (the missing term in the reduced theory) and traits represented by theoretical terms already present in the primary science; and
2. derivability: with the help of these additional assumptions, all the laws of the secondary science, including those containing the term "A", must be logically derivable from the theoretical premises and their associated coordinating definitions in the primary discipline (Nagel 1979).

As an example of heterogeneous reduction, Nagel takes the reduction of thermodynamics to statistical mechanics. In this case, the concept of temperature is purely macroscopic and simply does not exist in the microscopic world.

The second issue concerning this theory reduction model is the fact that the regularities captured in higher-level laws exist only under certain conditios. To overcome this, he proposed that reduction also required statements of additional possible elements (in mathematical language they are boundary conditions). Therefore, in conclusion, a reduction scheme is then conceived to have the the following form:

- Lower-level laws (in the basic, reducing science)
- Bridge principles
- Boundary conditions
- Higher-level laws (in the secondary, reduced science).

A very standard example is the derivation of Gay-Lussac law from the kinetic theory of gases, as part of an overall reduction of classical thermodynamics to the newer and more basic science of statistical mechanics (Nagel 1979).

Feyerabend, however, argued that meaningful bridge principles cannot be established (Feyerabend 1962, 1985), since words in different theories have different meanings hence they remain *incommensurable* even when they have the same form. Kuhn too insisted on this difficulty, focussing on the so-called reduction of Newtonian to Einsteinian mechanics (Kuhn 1996). In order to address these criticisms, Schaffner (1967, 1969) revised the Nagelian model, describing "reduction functions" rather than bridge laws and proposing a revised Nagelian-type model, encompassing in a formal way the examples given by Feyeraband and Kuhn.[8]

Several comments are in order.

- As pointed out by Sklar (1967), the history of science strikingly demonstrates a point often overlooked by both scientists and philosophers—there are no actual successful homogeneous reductions of theories which do not concern old issues of scarce interest in present-day science. For instance, one might propose the inclusion of Galilean experimental results on falling objects within Newtonian mechanics. This obliges us to consider two other important questions.
- The project of the logical positivists does not seem to be grounded much in scientific facts.[9] Their ideal of knowledge is not necessarily related to reality but unified by formal logics and in practice reduced to physics. Let these enthusiastic reductionists talk: Reichenbach (1959) maintained that "*today it is possible to say that chemistry is a part of physics, just as much as thermodynamics or the theory of electricity*", and Putnam and Oppenheim (1958) argued for "*the possibility that science may one day be reduced to microphysics (in the sense in which chemistry seems today to be reduced to it …*"). Or consider the claim of Nagel (1979) that "*certain parts of 19th century chemistry (and perhaps the whole of this science) is reducible to post-1925 physics*". Such statements are not made by philosophers alone. Dirac (1929) wrote that "*… the underlying physical laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are thus completely known, and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble*". And Feynman et al. (1964) celebrated the Schrödinger equation as "*… one of the great triumphs of physics. By providing the key to the underlying machinery of atomic structure it has given an explanation for atomic spectra, for chemistry, and for the nature of matter*". Nonetheless, this strict hierarchy of levels (mind, psychology, biology, chemistry …) reducing eventually to physics leaves little room for the independence of higher levels, and the ontological significance of those levels' properties and elements[10] appear to be swept away. We will shortly return to this point.

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[8] Other Nagelian-type models have been suggested and discussed subsequently (Hull 1974; Ruse and Wilson 1986). However, it is worth emphasising that these models all differ from Nagel's original one, but not from a substantial and philosophical point of view. In particular, they all contain some kind of (not clearly specified) bridge principle.

[9] However, contrarily to a certain *vulgata*, logical positivists **were well aware** of these limits (Bouveresse 2011).

[10] In a different view, their causal powers.

- The reduction of thermodynamics to statistical mechanics is of a different nature and deserves much more attention: it will be discussed in detail later. For the time being, it suffices to note that this supposed success has unfortunately become paradigmatic and even today it is presented in many philosophical discussions and books almost like a dogma of successful reduction. However, this belief is completely misguided, as we shall show [see also (Sklar 1995)].
- Another issue concerning the philosophical position of logical empiricists deserves consideration. It is widely accepted that connecting theories are problematic and that bridge laws are questionable (Fodor 1974). Nevertheless, the prevalent idea in the philosophical literature is that one should rely on identity relations grounded in logical argument. A lucid presentation of this approach is given by Suppes, one of the first proponents of a theory-reduction model: "*To show in a sharp sense that thermodynamics may be reduced to statistical mechanics, we would need to axiomatise both disciplines by defining appropriate set-theoretical predicates, and then show that given any model T of thermodynamics we may find a model of statistical mechanics on the basis of which we may construct a model isomorphic to T*" (Suppes 1957). Consequently, Suppes required an isomorphism between each model of the reduced theory and a corresponding one of the more general one; therefore before proceeding to a reduction between two theories, one should logically formalise and axiomatise both of them, determine the suitable identity relations and eventually carry out a deductive-nomological reduction.

  This is important since the current approach of philosophers of science rests by and large on the logical analysis of theories (Humphreys and Bedau 2006). In this work, we mainly address this kind of reductionnism: deducibility in a broad logical sense. This can be said to be of Nagelian type, even considering its many recent improvements (Kim 1993, 2000; Butterfield 2011a,b; Bouveresse 2011). In the following, we will give explicit examples which show that reduction both in Suppes or Nagel terms is in fact impossible. However, even admitting that certain "technical" difficulties could be overcome sooner or later, the axiomatisation of theories remains necessary. Now, by definition, science is a work in progress, thus any attempt to axiomatise any of its branches, in order to effect a logical reduction of current theories, appears unrealistic. In many cases, dialectical or dynamical relations, rather than formal logical ones, appear more relevant (Sève 1998; Sève and Guespin-Michel 2005; Sanchez-Palencia 2013).

2.3 Reduction in Physics and Philosophy

Physicists look at inter-theoretic relations in the opposite manner. As recognised by Nickles (1973) in a philosophical paper, there are two notions of reduction:

1. Reduction$_1$, namely the reduction of philosophers discussed so far;
2. Reduction$_2$, namely physicists' reduction of theories (Batterman 2002).

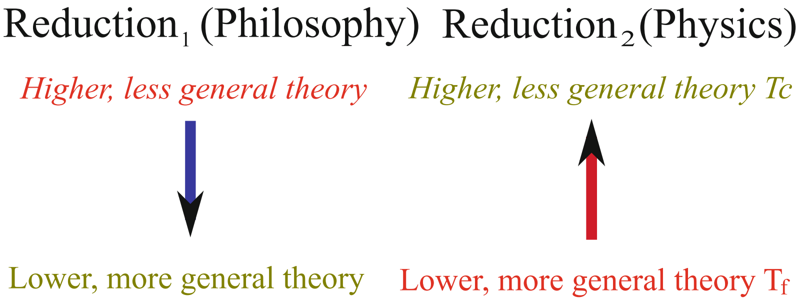

Fig. 2.1 Illustration of the two types of reduction: reduction$_1$, the “philosophical” notion; reduction$_2$, the “physical” notion. The first concerns the reduction of the more specific theory to the more general, in the attempt to unify and explain the phenomena of interest; the latter views the more general theory as encompassing the more specific theory, thus reducing to that in the overlapping areas. Philosophers have noticed that the physicists’ notion of reduction implicitly entails a horizontal time direction, meaning that the more general theory is often more recent than the more specific one, since it is developed in the light of new observations incompatible with the previous theory

In physics, the reduction of a theory (or law) T_f (f for finer) to another T_c (c for coarser) means that the theory T_f is more general and encompasses theory T_c and, in some particular case, T_f implies T_c. Furthermore, theory T_f is usually more recent than T_c, so that it can be viewed as a generalisation of the coarser theory T_c. Thus the meaning of reduction is similar to the deductive one (Nagel 1979) but the roles of reduced and reducing theories in physics are reversed with respect to the philosophical notion of reduction, Reduction$_1$. Despite possible semantic or ontological issues, it is always highly desirable for a new proposition to be consistent with the existing more specific theory, if the latter is still valid. Ineed, the reproduction of previously established results is the first, necessary step in the validation of a new theory or law. In Fig. 2.1, the two kinds of reductions are depicted. To give a simple, classical example that illustrates this point, consider the relation between Newtonian and relativistic mechanics. Newtonian mechanics was, and is, known to be in excellent agreement with macroscopic experiments, hence with basically all experiments carried out till the first half of the 19th century. The special theory of relativity, as well as the subsequent general theory of relativity, intended to address open issues in Newtonian mechanics. However, they clearly were generalisations of Newtonian mechanics, supposed to reduce to that in all cases in which Newtonian mechanics makes correct statements. In this sense, physicists refer to Newtonian mechanics as a limiting case of the theory of relativity: relativity (T_f in this case) reduces to Newtonanian mechanics (T_c) when the speed of light is considered infinite.

This shows that, while in Reduction$_1$ the more specific upper-level theory is reduced to the more general lower-level one (e.g. the reduction of gas laws to the more general theory of statistical mechanics), in Reduction$_2$ the more general theory is newer and reduces to the older theory, now recognised to be partially incorrect (e.g., the reduction of Einstein's formula for momentum to Newton's formula). In brief, Reduction_1 moves from special to general, whereas Reduction_2 moves from general to special (See Fig. 2.1).

Let us begin to formalise this relation of limits which is important indeed to understand inter-theoretic relations. Since natural sciences, particularly physics, are highly mathematical in nature, a characteristic parameter (typically dimensionless and called an order parameter) suggesting the correct limiting cases in which a theory is valid, possibly recovering the domain of validity of a coarser one, may usually be identified. The coarser theory can be seen as an asymptotic limit of the finer one, in a mathematical sense. Symbolically we can express this limiting operation as:

$$\lim_{\delta \to 0} T_f(\delta) = T_c \tag{2.1}$$

It is important to underline here that first, the limit operation applied to theories should only be taken in a formal symbolic sense, not in a rigorous one; and second, we are leaving aside for the moment all epistemological issues (such as "incommensurability") which may arise (and indeed usually do) in such a reducing limit. Let us reconsider the case of Newtonian mechanics and special relativity. As mentioned above, the relevant parameter is c, which must tend to infinity for special relativity to reduce to the Newtonian mechanics. Here, m_0 is the rest mass and the classical expression $p = mv$ is therefore recovered in the asymptotic case $c \to \infty$. The same limit procedure can be explicitly found in many other inter-theoretic relations. Following Berry (1994), we propose a list of asymptotic limits[11] which connects different theories:

- special relativity → Newtonian mechanics: $\delta = \frac{v}{c}$, where v is the velocity, c is the speed of light.
- general relativity → special relativity: $\delta = \frac{Gm}{ac^2}$, where G is the universal gravitational constant, m is the mass and a is the relevant length.
- statistical mechanics → thermodynamics $\delta = \frac{1}{N}$ and $\tilde{\delta} = \frac{1}{V}$ with $\delta/\tilde{\delta} = const.$ where N is the number of microscopic particles in the volume V.
- viscous fluids → non-viscous fluids: $\delta = \frac{1}{Re}$; where Re is the Reynolds number ($Re = \frac{UL}{\nu}$ is the relevant adimensional number, with ν the kinematic viscosity, U and L are the typical velocity and length of the flow respectively).
- wave optics → geometrical optics: $\delta = \frac{\lambda}{a}$; where λ is the wavelength and a the relevant macroscopic length.
- quantum mechanics → classical mechanics: $\delta = \frac{h}{S}$; where h is the Planck's constant and S is the relevant action of the system.

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[11] These limits are often called asymptotic since asymptotic analysis (Bender and Orszag 1978) is commonly used to describe the limit behaviour, which in our case is how a theory behaves when the appropriate order parameter approaches the limit value. The theory is trivial when the limit is regular but can be quite sophisticated and complex when it is singular (Primas 1981).

Some of these limits will be analysed in detail in the following chapters; the last two have also been discussed at length elsewhere (Berry 1981, 2001; Batterman 2002).

It is important, however, to note that the limit concerning relativistic mechanics is *regular*: there are no singularities, and therefore no infinities arise. In this case, it is appropriate to talk of inter-theoretic reduction, even though there is room for subtle discussions from the semantic and ontological points of view illustrated e.g. by Feyerabend and Kuhn. More precisely, let us consider one example given by Batterman (2002). In the symbolic Eq. 2.1, if formulas in T_f smoothly approach the corresponding formulas in T_c, the limit can be said to be regular. In these cases, the result obtained by taking $\delta = 0$ equals that given by the $\delta \to 0$ limit. The limit is said to be singular if the behaviour for $\delta = 0$ differs from the behaviour for $\delta \to 0$. In these cases the finer theory cannot very simply and directly reproduce the coarser one. A simple and clear illustration of these cases is afforded by the quadratic equation

$$x^2 - x + \delta 9 = 0 \tag{2.2}$$

which has two roots for any value of δ, and these two roots smoothly converge to the two roots of the case with $\delta = 0$, which are 0 and -1. Differently, the equation

$$\delta x^2 - x + 9 = 0 \tag{2.3}$$

has two roots only if $\delta > 0$, whereas at $\delta = 0$ the equation changes drastically its nature, becoming an equation of first degree with 9 as its unique root. This is an example of a singular limit, and even though not all singular limits are due to changes in the degree or order of the equations, this kind of singularity is paradigmatic. In practice, the relevant equations, which are usually partial differential equations, either change in order or become ill-posed in the asymptotic limit. Berry (2002) explains the essence of singular limits with an amusing example. Biting into an apple and finding half a maggot is unpleasant, but finding one-third of a maggot is worse. The less you find, the more you might have eaten. However a small maggot fraction $\delta \ll 1$ is qualitatatively different from no maggot ($\delta = 0$).

It is also important to stress that regular limits are almost absent in inter-theoretic relationships, the relativistic-Newtonian mechanics case being a fortunate exception. On the other hand, singular limits are much more interesting than regular limits from a scientific and philosophical point of view, because they are typically related to the discovery of new properties.

In the list given above, the inter-theoretic asymptotic limits are singular apart from general relativity, which approaches special relativity regularly, and special relativity, which in turn approaches Newtonian mechanics regularly. The corresponding emerging features, among others, include: (a) critical phenomena, when statistical mechanics approaches thermodynamics; (b) turbulence in fluids; (c) interference and caustics when wave theory approaches geometric optics and (d) fluctuations universality and chaos when quantum systems reduce to classical ones. In all of those cases in which a regular asymptotic limit cannot be taken, one cannot speak of reduction and new features appear. In those situations, it is possible to talk of *emergent* properties and hence of some kind of *emergence* (Batterman 2002).[12]

2.4 Emergence

2.4.1 Introduction

The issue of *emergence* is the appearance of novel phenomena, even though this definition is far from clear-cut. The word comes from latin and appeared in the french and english vocabularies in the fifteenth century (Adler et al. 2002). It indicates *appearance* (of novel properties). Since the eighteenth century the word has been used as a technical term in physics, geology and in evolutionary biology, where emergence indicates the appearance of a new and functional organ in a vegetal or animal line (Adler et al. 2002). The philosophical use is more recent and indicates effects which are not mechanically explained by their causes. The first to use the word in this sense is Lewes, who began the movement called "British emergentism" (Alexander 1920; Morgan 1923; Broad and Paul 1925). He distinguished emergent facts, which cannot be predicted on the grounds of past experience, from resultant facts, which can. In the twentieth century the word emergence has been widely (perhaps too widely) used for epistemological purposes and not only in inter-theoretic relations. From the very beginning, emergence opposed reductionism. It is possible to bring this opposition to life by emphasising two contrasting views of emergence and reduction given by two biologists: *nonoverlapping magisteria* (keeping different levels separate), by Gould (1997), and *consilience* (trying to connect all the levels), by Wilson (1998). It is also fair to say that a new furore about emergence, reductionism and supervenience has characterised the last decades, notably within the debates on the philosophy of the mind, in which the concept of supervenience has been differentiated from that of emergence (Humphreys 1997b; Humphreys and Bedau 2006).

Before entering into details about reduction and emergence, it is worth saying something about the relationship between fundamental scientific theories and contingent conditions or *contexts*, which are bound to play an important role in the emergence of new features within inter-theoretic relations.[13] A context fixes the

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[12] For readers more interested in philosophy, it is worth noting that emergence and Nagelian reduction are distinct notions but, in our opinion, they are related and it seems reasonable to argue that the presence of emergent properties make inter-theoretic reduction in the Nagelian sense impossible to carry out. Even though deeper analysis is certainly required, the arguments discussed in the next chapter appear in favour of this thesis.

[13] This fact, recognised by logical empiricists like Nagel, has been rigorously expressed by Primas (1981, 1998) and has been made more palatable to scientists in recent studies (Bishop and Atmanspacher 2006).

boundaries between what can be considered relevant or irrelevant in a given situation, notably in an experiment or observation. Therefore, the context allows us to fix the relevant level of description of a specified reality, hence the appropriate theory or model. Fundamental theories are generally formulated in terms of universal principles, assuming that these principles can apply to many different phenomena and possibly to everyone, as the idea of universality implies. It is plain that, given this broad scope, these theories are constructed in a context-independent way. In this sense, one may be led to accept the sceptical view, (Van Fraassen 1989; Cartwright 1983) according to which *the fundamental laws of physics do not describe true facts about nature*, and therefore the laws of physics do not state true facts, which is another way of saying that there are no laws of nature at all. However, this is due to the fact that fundamental laws refer to independent reality whereas phenomenological laws refer to empirical reality (Primas 1998; Bishop and Atmanspacher 2006). Starting from fundamental laws, one can build an operational theory which is a phenomenological model and can be used in the empirical context, disregarding all the details that are not relevant. The existence of a context which in turn identifies the relevant model is related to the emergence of new features. In practice, in inter-theoretic approaches, one starts from a given fundamental theory and restricts it, according to the theoretical context of interest, to obtain a coarser, less fundamental theory. From a formal point of view concerning the mathematical sciences, this operation amounts to an asymptotic expansion which is often singular, context-dependent and leads to the emergence of features qualitatively different from those characterising the fundamental theory. This point will be discussed further in the following chapters.

2.4.2 Reduction versus Emergence

A good collection of views on emergence, along with many influential and respected papers can be found in Humphreys and Bedau (2006). It is instructive to go deeper in the definition of emergence, following Kim (2000), who gives the following list of points, meant to illustrate the tenets of the "central doctrine of emergentism":

1. *Emergence of complex higher-level entities*: Systems with a higher level of complexity emerge from the coming together of lower-level entities in new structural configurations.
2. *Emergence of higher-level entities*: all properties of higher-level entities arise from the lower-level properties and relations that characterise their constituents. Some properties of these higher, complex systems are "emergent", and the rest merely "resultant".
3. *The unpredictability of emergent properties*: emergent properties are not predictable from exhaustive information concerning their "basic conditions". In contrast resultant properties are predictable from lower-level information.

4. *The inexplicable/irreducibility of emergent properties*: Emergent properties, unlike those that are merely resultant, are neither explicable nor reducible in terms of their basal conditions.
5. *The causal efficacy of the emergency*: Emergent properties have causal powers of their own; novel causal powers irreducible to the causal powers of their basal constituents.

Some comments are in order.

- Points 1–2 insist on the importance of the *whole*-to-*part* relationship, which is related to a hierarchical vision of the world subdivided into different levels. These points are hence related to the intuitive definition of reduction: the whole is nothing but the sum of the parts. While the hierarchical world is in general acceptable and difficult to question, it is not always relevant for inter-theoretic relations. It has been shown convincingly, in physics at least (Batterman 2002), that emergent properties without a whole-to-part relationship may be found when inter-theoretic relations can be formalised as asymptotic limits, and when asymptotic analysis holds. On the contrary, what seems really important for formalised sciences is the presence of a singular limit in the inter-theoretic relation underlying the emergence of new properties.
- Points 3–4 try to identify the characteristics of a genuinely emergent property. These two issues seem indeed to be relevant. It is difficult to conceive of emergent properties without thinking of something that cannot be predicted or explained in terms of the finer theory when approaching the coarser one. This point will be discussed at length in the following chapters in order to clarify the sense in which this explanation can be given, when it can be given.
- Point 5 is profoundly grounded in the debates on the philosophy of the mind. In this book, however, only inanimate matter is considered, which makes it hard to attribute causal power to finer or coarser levels or theory. In these cases it seems better (and also safer) to talk of explanation rather than of causal relations.

We can thus use here the scheme of classifications proposed by Bishop and Atmanspacher (2006), allowing different kinds of reduction and emergence to be discussed consistently:

1. At a certain level, the description of properties (including its laws) offers both necessary and sufficient conditions to rigourously derive the description of properties at a higher level. This is the strictest possible form of reduction.
2. At a certain level, the description of properties (including its laws) offers necessary but not sufficient conditions to derive the description of properties at a higher level. This version indicates that contingent contextual conditions are required in addition to the lower-level description for the rigourous derivation of higher-level properties. In this case, we speak of emergence.
3. At a certain level, the description of properties (including its laws) offers sufficient but not necessary conditions to derive the description of properties at a higher-level. This version includes the idea that a lower-level description offers multiple

realisations of a particular property at a higher level—a feature characteristic of supervenience.

4. At a certain level, the description of properties (including its laws) offers neither necessary nor sufficient conditions to derive the description of properties at a higher-level. This represents a form of radical emergence, insofar as there are no relevant conditions connecting the two levels whatsoever.

Note that class (2) complements class (3), since in many cases it turns out that higher-level features both supervene on and emerge from lower-level properties. Class (4) is not particularly attractive to those interested in explanatory relations between different levels of description, since it regards as loose the implications between those levels of descriptions. By contrast, class (1) represents the conventional wisdom of reduction: lower-level theories imply and completely set higher-level theories. Even though some examples in the literature were originally thought to exemplify class (1), in reality they do not bear closer scrutiny, and this class seems to be almost empty. Therefore, fewer and fewer philosophers admit class (1), a notable exception being Kim (1993). However, this vision remains probably the received view among some physicists.

2.4.3 Emergence and Reduction in Natural Sciences

This book concerns formal natural sciences, or exact natural sciences, namely physics and chemistry. In the following chapters, we shall discuss several examples of inter-theoretic relations which are non-trivial and for which the tenet (1) of strict reduction will be shown to be clearly inappropriate. Furthermore, it will be shown that some new features emerge *en route* from the basic theory to the higher-level theory.

Natural sciences, theoretical physics in particular, are highly mathematicised. Nowadays, it is frequent to see the cross-fertilisation from the frontiers of theoretical physics in some fields of mathematics. This allows a detailed analysis of inter-theoretic relations and thus may constitute an important testing ground for more general speculative arguments put forward by philosophers. Unfortunately, too often philosophers are anchored to a very simple idea of exact scientific theory and miss the mathematical and physical subtleties underlying them. This is particularly damaging, since these subtleties are nevertheless used to formulate examples for reduction and emergence from physics and chemistry (Nagel 1979; Kemeny and Oppenheim 1956; Feyerabend 1962; Bunge 1985; Humphreys 1997b; Batterman 2002) among the best known. However, the powerful method of investigation permitted by mathematics comes at a cost: **physics and chemistry** treat only the simplest natural object: inanimate matter. The more complex the phenomena considered, the less formalised description the that must be used. Nevertheless, we hope and believe that considerations arising from natural sciences are useful also in more complex domains of study and perhaps also for speculative analysis.

In inter-theoretic relations concerning phenomena at the boundary between two levels of representation of reality, the fundamental equations (the lower-level theory) do not suffice to represent the higher-level context-dependent empirical reality, and have to be complemented by suitable context or boundaries. This operation is generally accomplished via some multi-scale mathematical approach. Through an asymptotic expansion it is possible to produce the limit of the basic lower-level theory which should hold in the higher-level one. Nevertheless, this limit most often turns out to be singular, with the relevant terms diverging to infinity, which makes the desired asymptotic expansions impossible in the basic theory.

This singularity shows that the basic theory is not sufficient to cover higher-level phenomena. This means that there is a gap between the levels that cannot be bridged by the sole "language" taken from the fundamental theory. New semantics should be introduced, in order to obtain a closed and complete representation of the higher level. One then wonders whether natural science teaches anything about category (3), supervenience. It is fair to say that this category has been put forward in the framework of the philosophy of the mind, and is associated in particular with Kim (1984). We shall discuss this point in the context of "special sciences" in the next chapter.

2.4.4 Emergence and Reduction in Special Sciences

Let us now briefly review some arguments about reductionism in sciences other than physics, particularly in the philosophy of the mind, which has witnessed heated debates over the last decades. The implications of our study for these issues will be discussed in the conclusion.

As pointed out in the last section, the subjects of emergence and reduction are strongly related, and are also related to a very general issue: that concerning the unity or plurality of science. Indeed, specific scientific theories try to explain a vast variety of phenomena in terms of a very large number of different disciplines, sub-domains, etc. One may then ask whether this structural differentiation reflects a real and deeper differentiation at the level of objects and properties or whether, on the contrary, it is simply due to an effort to classify and organise the topic of interest, which would imply the existence of a fundamental unifying science, capable of encompassing all the sciences. It is clear that this view is related to some form of reductionism.

Let us state precisely a subtle but fundamental point, the problem of the status of a scientific theory. Science aims to describe the world and thus gives an interpretation that refers to our understanding of observable behaviours or patterns. This form of interpretation of the empirical reality is epistemic. On the other hand, a "realistic" interpretation deals with the nature of existence and refers to a theory about "real things", i.e. about those objects which exist independently of any observational context. A particular metaphysical doctrine which seems very reasonable and widely accepted (probably by all physicists) is physicalism. Physicalism states that all the entities in the world are physical and that all properties are either physical or related to physical properties. For instance, "*All individuals are constituted by, or identical to, microphysical individuals, and all properties are realised by, or identical to, microphysical properties*" (Gillett 2003).

Here microphysical means the lowest-level of the physics description, hence could be that of elementary particles, but this notion, commonly used in philosophical discussions, is vague and often higher-level properties like chemical ones are included in the microphysical ones. Nevertheless, the physicalist thesis seems to be confirmed in actual science and thus to refuse it appears smacks of sophistry. Indeed, in natural sciences at least, there is no example of a non-physical entity. All experimental work in the different areas of physics (from particle physics to astrophysics) and chemistry agree on these grounds.

Given the physicalist framework, it is opportune to give an account of direct reductionist views, which can be referred to as reductive physicalism or ontological reductionism, and which naturally lead to an ontological minimalism. Reductive physicalism means: (a) there are a small number of different fundamental constituents of the world; (b) every other object, state, process or property is composed of these fundamental entities, and are nothing but fundamental physical entities. It is fair enough to say that many physicists share this view still today [(Weinberg (1987) being one of its champions].

Although ontological minimalism has the merit of order and simplicity, it dissatisfies those who think that biological processes and particularly human actions are not so directly related to the properties of the fundamental building blocks of matter. It seems hopeless to counter such strongly rooted beliefs about the ontological status of the world. In some sense, ontological minimalism is a form of metaphysical position which substitutes some kind of god with elementary particle physics. At variance with this position, some authors like Davidson claim that there are no possible bridge laws between the level of mind and that of physics. He asserts that such bridge laws are not merely hard to discover (and hence have not yet been discovered): he states that they do not exist. Davidson indeed believes that mental states and process are "anomalous" in the sense that they cannot be expressed in terms of laws (*nomos* in greek), by their very nature (Davidson and Block 1980). He bases his views on two main facts which differentiate human regularities from natural ones: human regularities are normative and present numerous exceptions, while natural laws are factual and generally true.

This purely metaphysical position is hardly satisfactory and philosophers have pursued the non-reductionist thesis (Darden and Maull 1977; Machamer et al. 2000; Craver 2007; Bechtel 2008). Fodor, in an article of note (Fodor 1974), argue on this neat distinction between physics, which should be the fundamental science by definition, and other sciences named "special sciences".

In this framework, he also maintains that bridge laws between all special science regularities and physical laws are impossible to find. In particular, he claims that physics may describe some of the natural regularities, but not all of them and that, consequently, special sciences enjoy an autonomous status, independent of the laws of physics. It is worth noting that Fodor uses purely qualitative arguments. Complex phenomena, like the economy, include so many intricate physical actions that it becomes impossible to find any bridge laws. Actually, the main point of that paper is to introduce the idea of multi-realisability, which would later become very popular in the philosophy of the mind, stating that, given a property described by some special sciences, it can be realised by physical properties in several different manners. Assuming that bridge laws between physics and special sciences do not exist, Fodor postulates that the properties of special sciences should be multi-realisable. Fodor's main conclusion is that the world is compatible with a non-reductive physicalism, in contrast with the reductive physicalism or ontological minimalism: properties expressed in terms of special sciences are realised by a combination of physical properties, but they can be realised in various different ways.

Non-reductive physicalism is different from Descartes' dichotomy: on the one hand, it states that there is an ontological unity, since every property is ultimately physical but, on the other hand, it denies unity, because aggregation of physical properties described by special sciences escape a purely physical description. There is therefore a systematic dependence of special properties on physical ones, without that implying the identification of the relevant descriptions of the different phenomena. This kind of dependence has been specified as supervenience (Kim 1993, 2000). A plethora of definitions have been proposed to distinguish various kinds of supervenience: weak, strong, or global supervenience. From a logical point of view, accepting supervenience has direct consequences. Indeed, adopting the physicalist thesis "*all individuals are constituted by, or identical to, microphysical individuals, and all properties are realised by, or identical to, microphysical properties*", means, in terms of causal theory, adopting the causal inheritance principle (Kim 2000) or the "realisation thesis" (Gillett and Rives 2001):

> An instance of a property, or combination of properties, P realises an instance of a property M if and only if P plays the causal role of M by virtue of P having all the causal powers individuative of M, but not vice versa.

These arguments imply that higher-level properties do not contribute any causal powers to individuals. At variance with common sense, all causal powers[14] lie in the microphysical properties. In this sense supervenience leads naturally to a new form of reductionist physicalism. Higher properties may be multi-realised, but are not autonomous, which makes them mere epiphenomena. Some authors have thought of possible ways out, in the same framework. Notably, Gillett (2003) suggests that a possible way to circumvent the loss of causal power at higher level is to show that physics is not causally closed or "complete".[15] However, this would seem a challenging task, for all experiments in physics suggest the opposite.

In light of this argument, supervenience has lost most of its appeal to philosophers engaged in non-reductive approaches. Emergence, a category which became popular at the beginning of the nineteenth century during so-called British Emergentism, has

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[14] In the notion given by Kim (2000).

[15] Basically the claim that all microphysical events are determined, in so far as they are determined, by prior microphysical events and the laws of physics.

risen again in popularity (Humphreys 1997a,b; Gillett 2003; Humphreys and Bedau 2006).[16]

Bedau proposed a weak form of emergence based on two admittedly vague but nevertheless useful hall marks of emergent phenomena, defined as follows:

1. Emergent phenomena are somehow comprised of, and generated from, underlying processes.
2. Emergent phenomena are somehow autonomous from underlying processes.

We therefore have a weak form of epistemic emergence. Higher-level properties are difficult or impossible to explain in terms of the theories describing the lower level, and they are completely determined by physical mechanisms. This keeps the ordered and discrete hierarchical structure implicit in reductive-physicalism, and led Gillet to convincingly demonstrate that weak emergence cannot confer causal efficacy on higher-level properties like those concerning the mind.

Humphreys (1997a) has proposed a strong form of emergence in order to reach a true non-reductive physicalism (allowed neither by weak emergence nor by supervenience). Quite uncommonly for a philosopher, he looked for a genuine empirical fact supporting his views. In his vision, higher-level properties are given by a fusion of lower-level properties, which thus realise higher-level properties without being involved in the causal chain. In this sense, when lower-level properties blend to permit a higher-level property to emerge, they no longer exist and, therefore, do not play any causal role.

Concluding remarks are in order. In the following, we shall analyse some examples of foundational nature, concerning theory reduction in physics. In our opinion, these examples also reveal the weakness of epistemic reductionism, intended in a nomological-deductive sense, although attempts to conciliate reductionism with some forms of emergence have been recently made (Butterfield 2011a,b). Furthermore, we indicate as "naive reductionism" a position similar to homogeneous or simple heterogeneous Nagelian reductionism, in which a given level of description can be deduced simply by the underlying lower level. The "extreme reductionist" pushes to the limits this position, claiming that everything can be deduced from the most fundamental level of reality, that pertaining to sub-nuclear particles.

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[16] Earlier we discussed emergence in the framework of natural sciences. Philosophers have discussed it much more in terms of logical relations and in the framework of causal efficacy. They differ on views and definitions, but most agree on the point that emergence has to be related to downward causation (that is to say that physics is not causally complete). In order for mental properties to be causally efficient, they have to cause changes in the physical world.

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