

# Noise Technologies for Operating the System for Monitoring of the Beginning of Violation of Seismic Stability of Construction Objects

T. A. Aliev, N. F. Musayeva and U. E. Sattarova

**Abstract** We analyzed the difficulties of operating the system for monitoring of seismic stability and technical condition of building structures. Technologies are proposed for solving the problem of monitoring at early stages of violation of seismic stability with application of sets of informative attributes consisting of variance, correlation, static and dynamic noise characteristics. The results of computational experiments are given.

## 1 Introduction

It is known that a system of monitoring of seismic stability and prediction of changes in the technical condition of high-rise buildings and building structures generally has to contain the following subsystems: primary sensors subsystem; primary data transmission and collection subsystem; subsystem for processing of obtained data and submitting the processing results to the specialized service [1].

Primary data transmission and collection subsystem is to provide data digitalization and transmission from all sensors to the local server, where all data is stored. Data digitalization provides acceptable resolution and frequency for each type of sensors. The server receives requests, processes the received requests and provides only authorized external access. The subsystem for data processing and

---

T. A. Aliev  
Institute of Cybernetics, Baku, Azerbaijan  
e-mail: telmancyber@rambler.ru

N. F. Musayeva (✉) · U. E. Sattarova  
Azerbaijan University of Architecture and Construction, Baku, Azerbaijan  
e-mail: musanaila@gmail.com

U. E. Sattarova  
e-mail: ulker.rzaeva@gmail.com

submittal contains the software providing visualized representation of processed primary data for the operator. Operator position subsystem provides information on the state of the control object by operator's request.

Only correct and adequate processing of signals that come from sensors measuring vibration, oscillation, tilts, deflation, rolls, strain in building structures, etc. and their timely submittal to the operator will ensure carrying out of operational complex of measures on preventing early wear, damage, and defects, such as cracks, bends, tilts, deformation, deflation, etc., and allow controlling physical ageing and obsolescence. On the other hand, in most cases for many high-rise buildings, there are certain troubles in prediction of changes in the object condition at early stages of fault origin with application of known methods of calculation of dispersion, correlation, spectral, static and dynamic characteristics. They allow one to detect only explicit faults at best. Analysis of emergency origin demonstrates that emergency situations are always preceded by hidden microfaults that emerge as microwears, microsagging, microvibration, microcracks, etc. in some units of the investigated building structure under. Their timely detection makes it possible to predict possible changes in the condition of a building structure, which can be used for warning and prevention of grave failures. The paper thereby considers one of alternative solutions to the problem of monitoring of seismic stability and the technical condition of high-rise buildings, building structures and strategic objects in seismically active regions with application of the technology and methods for development of robust sets of informative attributes.

## 2 Problem Statement

As is known, the prediction process can be generally represented as a combination of three elements: (1) pattern set  $Z$  formed from estimates of statistical characteristics, auto- and cross-correlation functions, spectral characteristics, static and dynamic characteristic corresponding to each  $i$ -state of all  $k$  of possible states of the building structure; (2) set  $V$  formed from current similar informative attributes that carry information on the current state; (3) rules of identification of  $F$  that compares each element of set  $Z$  to an element of set  $V$ , and vice versa, compares each element of set  $V$  to an element of set  $Z$ . The totality of elements of  $Z$  and  $V$  forms data support of prediction subsystem.

In this case, set  $Z_X$  corresponding to the normal state of a building structure, when signals  $X_i(t)$  are received from sensors, can be represented as follows:

$$Z_X = \begin{bmatrix} D(\varepsilon_1) = 0 & D(\varepsilon_2) = 0 & \dots & D(\varepsilon_n) = 0 \\ R_{X_1 X_1}^\circ(\mu) & R_{X_1 X_2}^\circ(\mu) & \dots & R_{X_1 X_n}^\circ(\mu) \\ R_{X_2 X_1}^\circ(\mu) & R_{X_2 X_2}^\circ(\mu) & \dots & R_{X_2 X_n}^\circ(\mu) \\ \dots & \dots & \dots & \dots \\ R_{X_n X_1}^\circ(\mu) & R_{X_n X_2}^\circ(\mu) & \dots & R_{X_n X_n}^\circ(\mu) \\ a_{11} & a_{12} & \dots & a_{1n} \\ b_{11} & b_{12} & \dots & b_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ a_{k1} & a_{k2} & \dots & a_{kn} \\ b_{k1} & b_{k2} & \dots & b_{kn} \\ c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots \\ c_{m1} & c_{m2} & \dots & c_{mn} \\ W_{11} & W_{12} & \dots & W_{1n} \\ W_{21} & W_{22} & \dots & W_{2n} \\ \dots & \dots & \dots & \dots \\ W_{m1} & W_{m2} & \dots & W_{mn} \\ \dots & \dots & \dots & \dots \end{bmatrix}, \quad (1)$$

where  $D(\varepsilon_i) = 0$  is the value of variance of noise  $\varepsilon_i(t)$ , which in the normal operation of the building structure equals zero;  $R_{X_i X_j}^\circ(\mu)$ ,  $i, j = \overline{1, n}$  are estimates of auto- and cross-correlation functions;  $a_{ij}$ ,  $b_{ij}$ ,  $i = \overline{1, k}$ ,  $j = \overline{1, n}$  are Fourier spectral expansion coefficients;  $c_{ij}$ ,  $i = \overline{1, m}$ ,  $j = \overline{1, n}$  are coefficients of results of solving static identification problem;  $W_{ij}$ ,  $i = \overline{1, m}$ ,  $j = \overline{1, n}$  are transfer functions;  $n$  is the quantity of input parameters,  $m$  is the quantity of output parameters,  $k$  is the quantity of coefficient in Fourier series.

Sets, which are similar to set  $Z_X$ , are composed for each deviation from the normal state of the building structure and allow predicting a failure state as a result of identification.

However, signals  $g_i(t) = X_i(t) + \varepsilon_i(t)$  received from sensors measuring vibration, oscillation, tilts, deflation, rolls, etc. are in most cases distorted with noise, i.e. stationary state, normalcy of distribution law are violated in those signals, correlation between the useful signal and the noise is absent, etc. Then, instead of set  $Z_X$ , set  $Z_g$  is obtained, elements of which contain noise errors:

$$Z_g = \begin{bmatrix} D(\varepsilon_1) & D(\varepsilon_2) & \dots & D(\varepsilon_n) \\ R_{g_1 g_1}^{\circ \circ}(\mu) & R_{g_1 g_2}^{\circ \circ}(\mu) & \dots & R_{g_1 g_n}^{\circ \circ}(\mu) \\ R_{g_2 g_1}^{\circ \circ}(\mu) & R_{g_2 g_2}^{\circ \circ}(\mu) & \dots & R_{g_2 g_n}^{\circ \circ}(\mu) \\ \dots & \dots & \dots & \dots \\ R_{g_n g_1}^{\circ \circ}(\mu) & R_{g_n g_2}^{\circ \circ}(\mu) & \dots & R_{g_n g_n}^{\circ \circ}(\mu) \\ a_{11}^* & a_{12}^* & \dots & a_{1n}^* \\ b_{11}^* & b_{12}^* & \dots & b_{1n}^* \\ a_{21}^* & a_{22}^* & \dots & a_{2n}^* \\ b_{21}^* & b_{22}^* & \dots & b_{2n}^* \\ \dots & \dots & \dots & \dots \\ a_{k1}^* & a_{k2}^* & \dots & a_{kn}^* \\ b_{k1}^* & b_{k2}^* & \dots & b_{kn}^* \\ c_{11}^* & c_{12}^* & \dots & c_{1n}^* \\ c_{12}^* & c_{22}^* & \dots & c_{2n}^* \\ \dots & \dots & \dots & \dots \\ c_{m1}^* & c_{m2}^* & \dots & c_{mn}^* \\ W_{11}^* & W_{12}^* & \dots & W_{1n}^* \\ W_{21}^* & W_{22}^* & \dots & W_{2n}^* \\ \dots & \dots & \dots & \dots \\ W_{m1}^* & W_{m2}^* & \dots & W_{mn}^* \\ \dots & \dots & \dots & \dots \end{bmatrix}, \tag{2}$$

where  $D(\varepsilon_i) = 0$  is the value of variance of noise  $\varepsilon_i(t)$ , which is different from zero in the presence of noise;

$R_{g_i g_j}^{\circ \circ}(\mu) = R_{X_i X_j}^{\circ \circ}(\mu) + A_{X_i X_j}^{\circ \circ}(\mu)$ ,  $i, j = \overline{1, n}$  are estimates of auto- and cross-correlation functions of noisy signals,  $A_{X_i X_j}^{\circ \circ}(\mu)$  is the value of errors of estimates  $R_{g_i g_j}^{\circ \circ}(\mu)$ ;  $a_{ij}^* = a_{ij} + \lambda_{a_{ij}}$ ,  $b_{ij}^* = b_{ij} + \lambda_{b_{ij}}$ ,  $i = \overline{1, k}$ ,  $j = \overline{1, n}$  are Fourier spectral expansion coefficients for noisy signals,  $\lambda_{a_{ij}}$ ,  $\lambda_{b_{ij}}$  is the value of errors of coefficients  $a_{ij}$ ,  $b_{ij}$ ;  $c_{ij}^* = c_{ij} + \lambda_{c_{ij}}$ ,  $i = \overline{1, m}$ ,  $j = \overline{1, n}$  are coefficients of results of solving identification problem of noisy signals static,  $\lambda_{c_{ij}}$  is the value of error  $c_{ij}$ ;  $W_{ij}^* = W_{ij} + \lambda_{W_{ij}}$ ,  $i = \overline{1, m}$ ,  $j = \overline{1, n}$  are transfer functions of noisy signals,  $\lambda_{W_{ij}}$  is the value of error of estimates of are transfer functions  $W_{ij}$ ;  $n$  is the quantity of input parameters,  $m$  is the quantity of output parameters,  $k$  is the quantity of coefficient in Fourier series.

Building the sets  $V_{g_i}$ , which characterize the current state of the construction object, presents similar difficulties.

Works [1–8] therefore offer algorithms for robust correlation and spectral monitoring, monitoring with application of noise characteristics, as well as algorithms for improvement of conditioning of correlation matrices in solving of static identification problems and problems of dynamics of high-rise buildings, construction and strategic objects in seismically active regions. Using those algorithms, one can build robust matrices  $Z_X^R$ ,  $V_{X_i}^R$ , elements of which are close to

elements of matrices  $Z_X, V_{X_i}$ . Considered below are algorithms for building robust matrices  $Z_X^R, V_{X_i}^R$ .

### 3 Technology and Algorithms for Monitoring of High-Rise Buildings, Construction and Strategic Objects in Seismically Active Regions by Means of Sets of Informative Attributes

Robust matrices  $Z_X^R, V_{X_i}^R$  should be built in four stages. At the first stage, estimates of values of noise variances  $D^*(\varepsilon_i)$  and robust estimates of auto- and cross-correlation functions  $R_{X_i X_i}^R(\mu)$ . Robust coefficients  $a_n^R, b_n^R$  of Fourier spectral expansion are calculated at the second stage. At the third stage, with application of robust correlation matrices  $\vec{R}_{XX}^R(0), \vec{R}_{XY}^R(0)$  formed from robust estimates of auto- and cross-correlation functions  $R_{X_i X_i}^R(0), R_{X_i Y}^R(0)$ , static identification problem is solved and robust coefficients  $c_{ij}^R$  are calculated. At the fourth stage, with application of robust correlation matrices  $\vec{R}_{XX}^R(\mu), \vec{R}_{XY}^R(\mu)$  formed from robust estimates of auto- and cross-correlation functions  $R_{X_i X_i}^R(\mu), R_{X_i Y}^R(\mu)$ , dynamic identification problem is solved and robust  $W_{ij}^R$  transfer functions are calculated.

*First stage.* Calculation of values of noise variances  $D^*(\varepsilon_i)$  and robust estimates of auto- and cross-correlation functions  $R_{X_i X_i}^R(\mu)$ .

1. Values of autocorrelation functions  $R_{g_i g_i}^{\circ \circ}(\mu), i = \overline{1, n}$  at time shift  $\mu = 0$  are calculated from the following formula:

$$R_{g_i g_i}^{\circ \circ}(\mu) = \frac{1}{N} \sum_{k=1}^N g_i^{\circ}(k\Delta t) g_i^{\circ}(k\Delta t). \tag{3}$$

2. Values of autocorrelation functions  $R_{g_i g_i}^{\circ \circ}(\mu), i = \overline{1, n}$  at time shift  $\mu = 1 \cdot i\Delta t$  are calculated from the following formula:

$$R_{g_i g_i}^{\circ \circ}(\mu) = \frac{1}{N} \sum_{k=1}^N g_i^{\circ}(k\Delta t) g_i^{\circ}((k+1)\Delta t). \tag{4}$$

3. Values of autocorrelation functions  $R_{g_i g_i}^{\circ \circ}(\mu)$ ,  $i = \overline{1, n}$  at time shift  $\mu = 2 \cdot i \Delta t$  are calculated from the following formula:

$$R_{g_i g_i}^{\circ \circ}(\mu) = \frac{1}{N} \sum_{k=1}^N \overset{\circ}{g}_i(k \Delta t) \overset{\circ}{g}_i((k + 2) \Delta t). \tag{5}$$

4. Values of variances  $D^*(\varepsilon_i)$  of noises  $\varepsilon_i(t)$  are calculated from the following formula:

$$D^*(\varepsilon_i) = R_{g_i g_i}^{\circ \circ}(\mu = 0 \cdot \Delta t) - 2R_{g_i g_i}^{\circ \circ}(\mu = 1 \cdot \Delta t)R_{g_i g_i}^{\circ \circ}(\mu = 2 \cdot \Delta t). \tag{6}$$

5. Robust estimates of autocorrelation function  $R_{g_i g_i}^R(\mu = 0)$  are calculated by means of the following formula:

$$R_{X_i X_i}^R(\mu = 0) = R_{g_i g_i}^{\circ \circ}(\mu = 0) - D^*(\varepsilon_i). \tag{7}$$

*Second stage.* Calculation of robust coefficients  $a_n^R$ ,  $b_n^R$  of Fourier spectral expansion.

Robust estimates  $a_n^R$  are determined in the following way.

1. Noise variance  $D^*(\varepsilon)$  and the mean value of relative error of readings  $\bar{\lambda}_{rel}$  are determined

$$\begin{aligned} \bar{\lambda}_{rel} &= \frac{\sqrt{\frac{1}{N} \sum_{i=1}^N \left[ \overset{\circ}{g}(i \Delta t) \overset{\circ}{g}(i \Delta t) + \overset{\circ}{g}(i \Delta t) \overset{\circ}{g}((i + 2) \Delta t) - 2 \overset{\circ}{g}(i \Delta t) \overset{\circ}{g}((i + 1) \Delta t) \right]}}{\sqrt{\frac{1}{N} \sum_{i=1}^N \overset{\circ}{g}^2(i \Delta t)}} \\ &= \frac{D^*(\varepsilon)}{D(g)}; \end{aligned} \tag{8}$$

2. Values  $\Pi^+$ ,  $\Pi^-$ ,  $N^+$  and  $N^-$  are determined

$$\Pi^+ = \overline{\overset{\circ}{g}(i \Delta t) \cos n\omega(i \Delta t)}^+, \tag{9}$$

$$\Pi^- = \overline{\overset{\circ}{g}(i \Delta t) \cos n\omega(i \Delta t)}^- \tag{10}$$

as well as the number of positive  $N^+$  and negative  $N^-$  sums of  $\overset{\circ}{g}(i \Delta t) \cos n\omega(i \Delta t)$ .

3. Conditions  $N^+ = N^-$  and  $\Pi^+ = \Pi^-$  are checked, during fulfilment of which application of conventional algorithms is recommended.

- 3.1. When conditions  $N^+ \neq N^-$  and  $\Pi^+ = \Pi^-$  hold true, formula for determination of robust estimates  $a_n^R$  is represented as follows

$$a_n^R = \frac{2}{N} \left\{ \sum_{i=1}^N \overset{\circ}{g}(i\Delta t) \cos n\omega(i\Delta t) - \frac{1}{4} |N_{a_n^+} - N_{a_n^-}| A(i\Delta t) \right\}, \quad (11)$$

where  $A(i\Delta t) = \overline{\lambda_{rel} \overset{\circ}{g}(i\Delta t) \cos n\omega(i\Delta t)}$

3.2. When conditions  $N^+ > N^-$  and  $\Pi^+ \neq \Pi^-$  hold true, estimates  $a_n^R$  are determined from the expression

$$a_n^R = \frac{2}{N} \left\{ \sum_{i=1}^N \overset{\circ}{g}(i\Delta t) \cos n\omega(i\Delta t) - \frac{1}{4} (N_{a_n^+} - N_{a_n^-}) A^+(i\Delta t) - \frac{1}{2} [N - (N_{a_n^+} - N_{a_n^-})] [A^+(i\Delta t) - A^-(i\Delta t)] \right\}, \quad (12)$$

where  $A(i\Delta t) = \overline{\lambda_{rel} \overset{\circ}{g}(i\Delta t) \cos n\omega(i\Delta t)}$ ,  $A^+(i\Delta t) = \overline{\lambda_{rel} \overset{\circ}{g}(i\Delta t) \cos n\omega(i\Delta t)}^+$ ,  $A^-(i\Delta t) = \overline{\lambda_{rel} \overset{\circ}{g}(i\Delta t) \cos n\omega(i\Delta t)}^-$ .

3.3. If  $N^+ < N^-$  and  $\Pi^+ \neq \Pi^-$  take place, estimates  $a_n^R$  are determined from the expressions:

$$a_n^R = \frac{2}{N} \left\{ \sum_{i=1}^N \overset{\circ}{g}(i\Delta t) \cos n\omega(i\Delta t) - \frac{1}{4} (N_{a_n^-} - N_{a_n^+}) A(i\Delta t) - \frac{1}{2} [N - (N_{a_n^-} - N_{a_n^+})] [A^+(i\Delta t) - A^-(i\Delta t)] \right\}. \quad (13)$$

3.4. If conditions  $N^+ = N^-$  and  $\Pi^+ \neq \Pi^-$  hold true, estimates  $a_n^R$  are determined from the formula

$$a_n^R = \frac{2}{N} \left\{ \sum_{i=1}^N \overset{\circ}{g}(i\Delta t) \cos n\omega(i\Delta t) - \frac{1}{2} N [A^+(i\Delta t) - A^-(i\Delta t)] \right\}. \quad (14)$$

Robust estimates  $b_n^R$  are determined in the following way.

1. Noise variance  $D^*(\varepsilon)$  and the mean value of relative error of readings  $\overline{\lambda_{rel}}$  are determined from the expression (8).
2. Values  $\Pi^+$ ,  $\Pi^-$  are determined

$$\Pi^+ = \overline{\overset{\circ}{g}(i\Delta t) \sin n\omega(i\Delta t)}^+, \quad (15)$$

$$\Pi^- = \overline{\overset{\circ}{g}(i\Delta t) \sin n\omega(i\Delta t)}^- \quad (16)$$

as well as the number of positive  $N^+$  and negative  $N^-$  sums of  $\overset{\circ}{g}(i\Delta t) \sin n\omega(i\Delta t)$ .

3. Conditions  $N^+ = N^-$  and  $\Pi^+ = \Pi^-$ , are checked, during fulfilment of which application of conventional algorithms is recommended.

3.1 When conditions  $N^+ = N^-$  and  $\Pi^+ = \Pi^-$  hold true, formula for determination of robust estimates  $b_n^R$  is represented as follows

$$b_n^R = \frac{2}{N} \left\{ \sum_{i=1}^N \overset{\circ}{g}(i\Delta t) \sin n\omega(i\Delta t) - \frac{1}{4} |N_{b_n^+} - N_{b_n^-}| B(i\Delta t) \right\}, \quad (17)$$

where  $B(i\Delta t) = \overline{\lambda_{rel} \overset{\circ}{g}(i\Delta t) \sin n\omega(i\Delta t)}$ .

3.2 When conditions  $N^+ > N^-$  and  $\Pi^+ \neq \Pi^-$  hold true, estimates  $b_n^R$  are determined from the expression

$$b_n^R = \frac{2}{N} \left\{ \sum_{i=1}^N \overset{\circ}{g}(i\Delta t) \sin n\omega(i\Delta t) - \frac{1}{4} (N_{b_n^+} - N_{b_n^-}) B^+(i\Delta t) - \frac{1}{2} [N - (N_{b_n^+} - N_{b_n^-})] [B^+(i\Delta t) - B^-(i\Delta t)] \right\}, \quad (18)$$

where  $B(i\Delta t) = \overline{\lambda_{rel} \overset{\circ}{g}(i\Delta t) \sin n\omega(i\Delta t)}$ ,  $B^+(i\Delta t) = \overline{\lambda_{rel} \overset{\circ}{g}(i\Delta t) \sin n\omega(i\Delta t)^+}$ ,  $B^-(i\Delta t) = \overline{\lambda_{rel} \overset{\circ}{g}(i\Delta t) \sin n\omega(i\Delta t)^-}$ .

3.3 If  $N^+ < N^-$  and  $\Pi^+ \neq \Pi^-$  take place, estimates  $b_n^R$  are determined from the expressions:

$$b_n^R = \frac{2}{N} \left\{ \sum_{i=1}^N \overset{\circ}{g}(i\Delta t) \sin n\omega(i\Delta t) - \frac{1}{4} (N_{b_n^-} - N_{b_n^+}) B(i\Delta t) - \frac{1}{2} [N - (N_{b_n^-} - N_{b_n^+})] [B^+(i\Delta t) - B^-(i\Delta t)] \right\}. \quad (19)$$

3.4 If conditions  $N^+ = N^-$  and  $\Pi^+ \neq \Pi^-$  hold true, estimates  $b_n^R$  are determined from the formula

$$b_n^R = \frac{2}{N} \left\{ \sum_{i=1}^N \overset{\circ}{g}(i\Delta t) \sin n\omega(i\Delta t) - \frac{1}{2} N [B^+(i\Delta t) - B^-(i\Delta t)] \right\}. \quad (20)$$

*Third stage.* Building robust correlation matrices  $\vec{R}_{\overset{\circ}{X}\overset{\circ}{X}}^R(0)$ ,  $\vec{R}_{\overset{\circ}{X}\overset{\circ}{Y}}^R(0)$ , solving static identification problem and calculation of robust coefficients  $c_{ij}^R$ .

1. Values of autocorrelation functions  $R_{\overset{\circ}{g}_i \overset{\circ}{g}_i}(\mu)$ ,  $i = \overline{1, n}$  at time shift  $\mu = 0$  are calculated from the formula (3).
2. Values of autocorrelation functions  $R_{\overset{\circ}{g}_i \overset{\circ}{g}_i}(\mu)$ ,  $i = \overline{1, n}$  at time shift  $\mu = 1 \cdot i\Delta t$  are calculated from the formula (4).
3. Values of autocorrelation functions  $R_{\overset{\circ}{g}_i \overset{\circ}{g}_i}(\mu)$ ,  $i = \overline{1, n}$  at time shift  $\mu = 2 \cdot i\Delta t$  are calculated from the formula (5).
4. Values of variances  $D^*(\varepsilon_i)$  of noises  $\varepsilon_i(t)$  are calculated from the formula (6).



5. Robust estimates of autocorrelation function  $R_{g_i g_i}^R(\mu = 0)$  are calculated by means of the formula (7).
6. Robust correlation matrix  $\vec{R}_{XX}^R(0)$  is formed by means of the following expression:

$$\vec{R}_{X_o}^R(0) = \begin{bmatrix} R_{g_1 g_1}^{\circ \circ}(0) - D^*(\varepsilon_1) & R_{g_1 g_2}^{\circ \circ}(0) & \dots & R_{g_1 g_n}^{\circ \circ}(0) \\ R_{g_2 g_1}^{\circ \circ}(0) & R_{g_2 g_2}^{\circ \circ}(0) - D^*(\varepsilon_2) & \dots & R_{g_2 g_n}^{\circ \circ}(0) \\ \dots & \dots & \dots & \dots \\ R_{g_n g_1}^{\circ \circ}(0) & R_{g_n g_2}^{\circ \circ}(0) & \dots & R_{g_n g_n}^{\circ \circ}(0) - D^*(\varepsilon_i) \end{bmatrix}$$

$$\approx \begin{bmatrix} R_{X_1 X_1}^{\circ \circ}(0) & R_{X_1 X_2}^{\circ \circ}(0) & \dots & R_{X_1 X_n}^{\circ \circ}(0) \\ R_{X_2 X_1}^{\circ \circ}(0) & R_{X_2 X_2}^{\circ \circ}(0) & \dots & R_{X_2 X_n}^{\circ \circ}(0) \\ \dots & \dots & \dots & \dots \\ R_{X_n X_1}^{\circ \circ}(0) & R_{X_n X_2}^{\circ \circ}(0) & \dots & R_{X_n X_n}^{\circ \circ}(0) \end{bmatrix} \approx \vec{R}_{XX}^R(0).$$

(21)

7. Similarly, correlation matrix  $\vec{R}_{XY}^R(0)$  is formed.
8. The following matrix equation is solved

$$\vec{R}_{XX}^R(0) \cdot \vec{C}^R = \vec{R}_{XY}^R(0) \tag{22}$$

and robust coefficients  $c_{ij}^R$  of static identification equation are calculated from the expression

$$\vec{C}^R = \left[ \vec{R}_{XX}^R(0) \right]^{-1} \vec{R}_{XY}^R(0). \tag{23}$$

*Fourth stage.* Building robust correlation matrices  $\vec{R}_{XX}^R(\mu)$ ,  $\vec{R}_{XY}^R(\mu)$ , solving dynamic identification problem and calculation of robust transfer functions  $W_{ij}^R$ .

1. Value of autocorrelation functions  $R_{g g}^{\circ \circ}(\mu)$  at time shift  $\mu = 0$  is calculated from the following formula:

$$R_{g g}^{\circ \circ}(\mu) = \frac{1}{N} \sum_{i=1}^N \overset{\circ}{g}(i\Delta t) \overset{\circ}{g}(i\Delta t). \tag{24}$$

2. Value of autocorrelation functions  $R_{g g}^{\circ \circ}(\mu)$  at time shift  $\mu = 1 \cdot i\Delta t$  is calculated from the following formula:

$$R_{gg}^{\circ}(\mu) = \frac{1}{N} \sum_{i=1}^N g^{\circ}(i\Delta t) g^{\circ}((i+1)\Delta t). \tag{25}$$

3. Value of autocorrelation functions  $R_{gg}^{\circ}(\mu)$  at time shift  $\mu = 2 \cdot i\Delta t$  is calculated from the following formula:

$$R_{gg}^{\circ}(\mu) = \frac{1}{N} \sum_{i=1}^N g^{\circ}(i\Delta t) g^{\circ}((i+2)\Delta t). \tag{26}$$

4. Value of variance  $D^*(\varepsilon)$  of noise is calculated from the following formula

$$D^*(\varepsilon) = R_{gg}^{\circ}(\mu = 0 \cdot \Delta t) - 2R_{gg}^{\circ}(\mu = 1 \cdot \Delta t) + R_{gg}^{\circ}(\mu = 2 \cdot \Delta t). \tag{27}$$

5. Robust estimates of autocorrelation function  $R_{gg}^R(\mu)$  are calculated by means of the following formula:

$$R_{gg}^R(\mu = 0) = R_{gg}^{\circ}(\mu = 0) - D^*(\varepsilon). \tag{28}$$

6. Robust correlation matrix  $\vec{R}_{XX}^R(0)$  is formed by means of the following expression:

$$\begin{aligned} \vec{R}_{gg}^R(\mu) &= \begin{vmatrix} R_{gg}^{\circ}(0) - D^*(\varepsilon) & R_{gg}^{\circ}(\Delta t) & \dots & R_{gg}^{\circ}[(N-1)\Delta t] \\ R_{gg}^{\circ}(\Delta t) & R_{gg}^{\circ}(0) - D^*(\varepsilon) & \dots & R_{gg}^{\circ}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ R_{gg}^{\circ}[(N-1)\Delta t] & R_{gg}^{\circ}[(N-2)\Delta t] & \dots & R_{gg}^{\circ}(0) - D^*(\varepsilon) \end{vmatrix} \\ &\approx \begin{vmatrix} R_{XX}^{\circ}(0) & R_{XX}^{\circ}(\Delta t) & \dots & R_{XX}^{\circ}[(N-1)\Delta t] \\ R_{XX}^{\circ}(\Delta t) & R_{XX}^{\circ}(0) & \dots & R_{XX}^{\circ}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ R_{XX}^{\circ}[(N-1)\Delta t] & R_{XX}^{\circ}[(N-2)\Delta t] & \dots & R_{XX}^{\circ}(0) \end{vmatrix} \approx \vec{R}_{XX}^{\circ}(\mu). \end{aligned} \tag{29}$$

7. Similarly, correlation matrix  $\vec{R}_{XY}^R(\mu)$  is formed.

8. The following matrix equation is solved

$$\vec{R}_{XX}^R(\mu) \vec{W}(\mu) = \vec{R}_{XY}^R(\mu) \tag{30}$$

and robust transfer functions  $W_{ij}^R$  of matrix equation of dynamic identification are calculated from the expression

$$\vec{W}_{ij}^R = \left[ \vec{R}_{XX}^R(\mu) \right]^{-1} \vec{R}_{XY}^R(\mu). \quad (31)$$

After all four stages are complete, robustness conditions are provided, i.e. the following equalities hold true:

$$D^*(\varepsilon_i) \approx D(\varepsilon_i), \quad (32)$$

$$R_{g_i g_j}^R(\mu) \approx R_{X_i X_j}^R(\mu), \quad (33)$$

$$a_{ij}^R \approx a_{ij}, \quad b_{ij}^R \approx b_{ij}, \quad (34)$$

$$c_{ij}^R \approx c_{ij}, \quad W_{ij}^R \approx W_{ij}. \quad (35)$$

It allows forming robust correlation matrices  $Z_X^R, V_{X_i}^R$

$$Z_X^R = \begin{bmatrix} D^*(\varepsilon_1) & D^*(\varepsilon_2) & \dots & D^*(\varepsilon_n) \\ R_{X_1 X_1}^R(\mu) & R_{X_1 X_2}^R(\mu) & \dots & R_{X_1 X_n}^R(\mu) \\ R_{X_2 X_1}^R(\mu) & R_{X_2 X_2}^R(\mu) & \dots & R_{X_2 X_n}^R(\mu) \\ \dots & \dots & \dots & \dots \\ R_{X_n X_1}^R(\mu) & R_{X_n X_2}^R(\mu) & \dots & R_{X_n X_n}^R(\mu) \\ a_{11}^R & a_{12}^R & \dots & a_{1n}^R \\ b_{11}^R & b_{12}^R & \dots & b_{1n}^R \\ a_{21}^R & a_{22}^R & \dots & a_{2n}^R \\ b_{21}^R & b_{22}^R & \dots & b_{2n}^R \\ \dots & \dots & \dots & \dots \\ a_{k1}^R & a_{k2}^R & \dots & a_{kn}^R \\ b_{k1}^R & b_{k2}^R & \dots & b_{kn}^R \\ c_{11}^R & c_{12}^R & \dots & c_{1n}^R \\ c_{12}^R & c_{22}^R & \dots & c_{2n}^R \\ \dots & \dots & \dots & \dots \\ c_{m1}^R & c_{m2}^R & \dots & c_{mn}^R \\ W_{11}^R & W_{12}^R & \dots & W_{1n}^R \\ W_{21}^R & W_{22}^R & \dots & W_{2n}^R \\ \dots & \dots & \dots & \dots \\ W_{m1}^R & W_{m2}^R & \dots & W_{mn}^R \\ \dots & \dots & \dots & \dots \end{bmatrix}, \quad (36)$$

for which the following equalities hold true:

$$Z_X^R \approx Z_X, \quad (37)$$

$$V_{X_i}^R \approx V_{X_i}. \quad (38)$$

Thus, as a result of application of robust technology for formation of state matrices  $Z_X^R$ ,  $V_{X_i}^R$ , a possibility arises to detect faults at early stages for reliable prediction of the technical condition of high-rise buildings or building structure.

## 4 The Results of Computational Experiments

Computational experiments have been carried out to prove the validity of the developed algorithms. The experiments were carried out in the following way.

First, a technology was developed for modeling the signals coming from the sensors installed in the most informative spots of high-rise buildings and building structures; the results of their processing are used to estimate the seismic stability and technical condition of high-rise buildings, construction objects and strategic objects. It is demonstrated that according to their nature, the signals received from the sensors can be divided into four groups: (1) deterministic, when the signal is non-random, undistorted and is some function of time; time relationship of the signal can be defined with any mathematical expression; (2) deterministic (non-random) signal distorted by a noise, which is a random function; (3) random signal not containing distortions; (4) random signal distorted by a noise, which is a random function.

Therefore, we model a bank of useful signals  $X(t)$  with different sampling intervals  $\Delta t$  in the form of sums of sine functions and cosine functions with specified amplitudes  $a_k$ ,  $b_k$  and frequencies  $k\omega_0$ :

$$X(i\Delta t) = \sum_{k=1}^{k_{end}} [a_k \cos(k\omega_0(i\Delta t)) + b_k \sin(k\omega_0(i\Delta t))]. \quad (39)$$

The bank of useful signals  $X(t)$  and noises  $\varepsilon(t)$  is also modeled by means of the random number generator in the form of random sequences with beta-, exponential-, gamma-, normal-, rayleigh- and weibull distribution laws at different values of parameters of those distributions. Noisy signals  $g(i\Delta t) = X(i\Delta t) + \varepsilon(i\Delta t)$  are formed with different distribution laws and “useful signal-noise”. Thus, the bank contains both the signals, for which the classical conditions hold true, and the signals, for which the classical conditions are violated. Charts and histograms have been built for all of those signals. MATLAB computing environment was used in the modeling of the signals of all four groups and in the computational experiments.

We developed a technology for experimental research of random signals by means of noise characteristics. In each computational experiment, variances were calculated for random noises, using the traditional algorithm, i.e. the formula

$$D(\varepsilon) = \frac{1}{N} \sum_{i=1}^N \overset{\circ}{\varepsilon}(i\Delta t) \overset{\circ}{\varepsilon}(i\Delta t). \tag{40}$$

Then, the variance  $D^*(\varepsilon)$  of the noise  $\varepsilon(v\Delta t)$  was calculated for the noisy signal  $g(v\Delta t)$  from the formula (6). The value of relative error was taken as the indicator of efficiency of the experiment:

$$proc = (D(\varepsilon) - D^*(\varepsilon))/D(\varepsilon)^*100\%. \tag{41}$$

Given below are the results of one experiment with the useful signal  $x_0 = 40 \sin(t)$  and the random noise complying with exponential-distribution  $f(\varepsilon|\mu) = \frac{1}{\mu} e^{-\frac{\varepsilon}{\mu}}$  with parameter  $\mu = 5$

Our numerous experiments confirmed the efficiency of the developed algorithm.

A technology was developed and the results of computational experiments are given for correlation monitoring of seismic stability and technical condition of high-rise buildings, construction objects and strategic objects in seismically active regions.

To conduct the computational experiments, the following were calculated: (1) estimates of the correlation functions  $R_{XX}^{\circ \circ}(\mu)$ ,  $R_{gg}^{\circ \circ}(\mu)$  of the useful signal  $X(t)$  and the noisy signal  $g(t)$ ; (2) the value of variance  $D^*(\varepsilon)$  of the noise  $\varepsilon(i\Delta t)$  from the expression (27); (3) robust estimates of the normalized auto-correlation function from the expression (28). The comparative analysis followed, and the values of relative errors of autocorrelation functions  $R_{gg}^{\circ \circ}(\mu)$  of the noisy signals  $g(t)$  and robust estimates  $R_{gg}^R(\mu)$  were calculated from the following expressions:

$$\Delta R_{XX}^{\circ \circ}(\mu) = \left| \frac{R_{gg}^{\circ \circ}(\mu) - R_{XX}^{\circ \circ}(\mu)}{R_{XX}^{\circ \circ}(\mu)} \right| \cdot 100\%; \tag{42}$$

$$\Delta R_{XX}^R(\mu) = \left| \frac{R_{gg}^R(\mu) - R_{XX}^{\circ \circ}(\mu)}{R_{XX}^{\circ \circ}(\mu)} \right| \cdot 100\%. \tag{43}$$

Given below are the results of the computational experiment with the input useful signal  $X(i\Delta t) = 40 \sin(i\Delta t) + 100$  and the noise  $\varepsilon(t)$  complying with the normal distribution law with mathematical expectation  $m_\varepsilon \approx 0$  and variance  $D(\varepsilon) \approx 90$  (Table 2), i.e. the classical conditions are observed. In the computational experiment with the useful signal  $X(i\Delta t) = 50 \sin(i\Delta t) + 5 \cos(0.5 \cdot i\Delta t) + 2 \sin(i\Delta t) + 5 \cos(13 \cdot i\Delta t) + 15$ , the noise  $\varepsilon(t)$  complying with the lognormal distribution with mathematical expectation  $m_\varepsilon \approx 0$  and variance  $D(\varepsilon) \approx 665$  (Table 3), the classical conditions are violated.

**Table 1** Results of the calculation of the variance estimate for the noise of the noisy signal

N	Estimates	$\Delta t = \pi/$	$\Delta t = \pi/$	$\Delta t = \pi/$	$\Delta t = \pi/$
		50	100	200	400
1	$D(\varepsilon)$	27.7153	23.6353	28.2664	25.9530
2	$D^*(\varepsilon)$	23.8590	25.0362	26.8281	25.6202
3	<i>proc, %</i>	15.5922	5.9272	5.0884	1.2820

**Table 2** Results of the calculation of the robust autocorrelation function with the classical conditions for the useful signal and noise holding

	$R_{XX}^{\circ \circ}(\mu)$	$R_{gg}^{\circ \circ}(\mu)$	$R_{gg}^R(\mu)$	$\Delta R_{XX}^{\circ \circ}(\mu)$	$\Delta^R R_{XX}^{\circ \circ}(\mu)$
1	2	3	4	5	6
<i>Experiment N1</i>	800.000	888.7157	808.0540	11.0895	1.0067
$D(\varepsilon) = 89.7642$	799.6052	801.1015	801.1015	0.1871	0.1871
	798.4214	794.1490	794.1490	0.5351	0.5351
$D_\varepsilon = 80.6617$	796.4496	798.2796	798.2796	0.2298	0.2298
	793.6918	790.7896	790.7896	0.3657	0.3657

**Table 3** Results of the calculation of the robust autocorrelation function with the classical conditions for the useful signal and noise violated

	$R_{XX}^{\circ \circ}(\mu)$	$R_{gg}^{\circ \circ}(\mu)$	$R_{gg}^R(\mu)$	$\Delta R_{XX}^{\circ \circ}(\mu)$	$\Delta^R R_{XX}^{\circ \circ}(\mu)$
1	2	3	4	5	6
<i>Experiment N6</i>	1377.0	1963.9	1366.1	42.6213	0.7927
$D(\varepsilon) = 665.69$	1376.6	1320.6	1320.6	4.0687	4.0687
	1375.3	1275.0	1275.0	7.2900	7.2900
$D^*(\varepsilon) = 597.8$	1373.2	1317.9	1317.9	4.0313	4.0313
	1370.4	1279.5	1279.5	6.6343	6.6343

The following conclusions were made based on the analysis of the obtained results.

- (1) The conditions (Tables 1 and 2, Column 1)

$$D^*(\varepsilon) \approx D(\varepsilon) \tag{44}$$

hold true.

- (2) For the estimates of the correlation functions  $R_{gg}^{\circ \circ}(\mu)$  of the noisy signals, the following inequality holds true (Tables 1 and 2, Columns 2, 3)

$$R_{gg}^{\circ \circ}(\mu) \neq R_{XX}^{\circ \circ}(\mu). \tag{45}$$

- (3) For the robust estimates of the correlation functions  $R_{gg}^R(\mu)$ , the equality (11) holds true (Tables 2 and 3, Columns 3, 4).

A technology was developed and the results of computational experiments are given for spectral monitoring of seismic stability and technical condition of high-rise buildings, construction objects and strategic objects in seismically active regions. For this purpose, the following were calculated:

- (1) estimates of the coefficients of Fourier series of the useful signal, using the conventional algorithms:

$$a_0 = \frac{2}{N} \sum_{i=1}^N \overset{\circ}{X}(i\Delta t), \quad (46)$$

$$a_n = \frac{2}{N} \sum_{i=1}^N \overset{\circ}{X}(i\Delta t) \cos n\omega(i\Delta t), \quad (47)$$

$$b_n = \frac{2}{N} \sum_{i=1}^N \overset{\circ}{X}(i\Delta t) \sin n\omega(i\Delta t); \quad (48)$$

- (2) estimates of the coefficients of Fourier series of the noisy signal, using the conventional algorithms:

$$a_n^* = \frac{2}{N} \sum_{i=1}^N \overset{\circ}{g}(i\Delta t) \cos n\omega(i\Delta t), \quad (49)$$

$$b_n^* = \frac{2}{N} \sum_{i=1}^N \overset{\circ}{g}(i\Delta t) \sin n\omega(i\Delta t), \quad (50)$$

- (3) actual errors of estimates of the coefficients of Fourier series  $a_n^*$ ,  $b_n^*$  from the following expression:

$$\lambda_{a_n} = \frac{2}{N} \sum_{i=1}^N \overset{\circ}{\varepsilon}(i\Delta t) \cos n\omega(i\Delta t), \quad (51)$$

$$\lambda_{b_n} = \frac{2}{N} \sum_{i=1}^N \overset{\circ}{\varepsilon}(i\Delta t) \sin n\omega(i\Delta t); \quad (52)$$

- (4) robust estimates of the coefficients of Fourier series of the noisy signals, using the unconventional algorithms (8)–(20);
- (5) relative errors of estimates of the coefficients of Fourier series of the noisy signals and robust estimates of the coefficients of Fourier series:

**Table 4** Results of the calculation of the robust estimate of the coefficient  $\mathbf{a}_n$  of spectral decomposition of the noisy signal

$n$	$\varpi$	$a_n$	$a_n^*$	$a_n^R$	$N_{b_n}^+ - N_{b_n}^-$	$N P_{a_n}$	$P_{a_n}^+$	$P_{a_n}^-$
0	0	0.000	-0.000	0.000	0	0.00	0.00	0.00
25	$\pi/100$	10.010	11.176	9.837	450	27940.402	28.926	22.367
50	$2\pi/100$	0.020	0.954	0.013	256	2383.933	24.692	26.352
75	$3\pi/100$	20.000	23.977	18.982	1622	59942.184	30.398	24.101
100	$4\pi/100$	-0.130	-1.549	-0.189	-550	-3873.065	28.699	24.406
125	$5\pi/100$	30.000	35.628	28.853	2174	89070.086	30.764	15.059

**Table 5** Results of the calculation of the robust estimate of the coefficient  $\mathbf{b}_n$  of spectral decomposition of the noisy signal

$n$	$\varpi$	$b_n$	$b_n^*$	$b_n^R$	$N_{b_n}^+ - N_{b_n}^-$	$N P_{b_n}$	$P_{b_n}^+$	$P_{b_n}^-$
0	0	0.000	-0.000	0.000	0	0.00	0.00	0.00
25	$\pi/100$	15.000	17.75	14.913	1024	44367.03	27.31	19.03
50	$2\pi/100$	25.000	29.46	24.093	1847	73,641.44	28.67	15.37
75	$3\pi/100$	-0.130	-0.41	-0.174	-108	-1028.90	22.29	21.75
100	$4\pi/100$	-20.00	-24.30	-19.66	-1665	-60740.66	18.25	27.47
125	$5\pi/100$	-0.050	1.48	0.040	513	3694.92	22.60	26.19

$$\Delta a = |a_n - a_n^*|/a_n * 100, \Delta b = |b_n - b_n^*|/b_n * 100\%; \tag{53}$$

$$\Delta a^R = |a_n - a_n^R|/a_n * 100, \Delta b^R = |b_n - b_n^R|/b_n * 100\%. \tag{54}$$

Given below are the results of the computational experiment with the input useful signal in the form of the sum of harmonic oscillations

$X(i\Delta t) = 10 \cos(i\Delta t) + 20 \cos(3 \cdot i\Delta t) + 30 \cos(5 \cdot i\Delta t) + 15 \sin(i\Delta t) + 25 \sin(2 \cdot i\Delta t) - 20 \sin(4 \cdot i\Delta t) + 35$ , with the noise  $\varepsilon(t)$  complying with the normal distribution law with mathematical expectation  $m_\varepsilon \approx 0$  and variance  $D(\varepsilon) \approx 130$ , the value of the relative error of the readings  $g(i\Delta t)$  is  $\lambda_{rel} = D^*(\varepsilon)/D(g) = 0.253$  (Tables 4 and 5).

Thus, application of the robust technology of spectral analysis allows one to ensure that the equality (34) holds true.

A technology was developed and the results of computational experiments are given for improving the conditionality of correlation matrices in solving the static identification problems of high-rise buildings, construction objects and strategic objects in seismically active regions. For this purpose, the following were calculated:

- (1) estimates of the correlation functions of the useful and noisy input and output signals  $R_{X_l X_l}^\circ(\mu), R_{X_l Y}^\circ(\mu), R_{Y Y}^\circ(\mu), R_{g_k g_l}^\circ(\mu), R_{g_k \eta}^\circ(\mu), R_{\eta \eta}^\circ(\mu) k, l = \overline{1, n}$ ;
- (2) robust estimates of auto-correlation functions of noisy input and output signals  $R_{X_k X_k}^R(\mu), R_{X_k Y}^R(\mu)$  from the expression (7);



- (3) matrices  $\vec{R}_{\overset{\circ}{X}\overset{\circ}{X}}(0)$ ,  $\vec{R}_{\overset{\circ}{X}\overset{\circ}{Y}}(0)$ ,  $\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0)$ ,  $\vec{R}_{\overset{\circ}{g}\overset{\circ}{\eta}}(0)$  of the useful and noisy signals, the robust matrix  $\vec{R}_{\overset{\circ}{X}\overset{\circ}{X}}^R(0)$  were formed from the expression (21), their determinants  $\Delta_{\overset{\circ}{X}\overset{\circ}{X}}(0)$ ,  $\Delta_{\overset{\circ}{g}\overset{\circ}{g}}(0)$ ,  $\Delta_{\overset{\circ}{X}\overset{\circ}{X}}^R(0)$  and the condition numbers  $H(\vec{R}_{\overset{\circ}{X}\overset{\circ}{X}}(0))$ ,  $H(\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0))$ ,  $H(\vec{R}_{\overset{\circ}{X}\overset{\circ}{X}}^R(0))$  were calculated;
- (5) coefficients of the mathematical model of the useful and noisy signals, robust estimates were calculated from the expressions (22)–(23).

A comparative analysis was carried out. For that purpose, the following were calculated:

- (1) the values of relative errors for the estimates of the elements of correlation matrices of noisy signals and robust estimates  $\Delta R_{\overset{\circ}{g}\overset{\circ}{g}}(0)$ ,  $\Delta R_{\overset{\circ}{g}\overset{\circ}{\eta}}(0)$ ,  $\Delta R_{\overset{\circ}{X}\overset{\circ}{X}}^R(0)$ ,  $\Delta R_{\overset{\circ}{X}\overset{\circ}{Y}}^R(0)$  from the expressions (42), (43);
- (2) matrices of relative errors for the elements of correlation matrices of noisy signals and robust correlation matrices  $\Delta \vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0)$ ,  $\Delta \vec{R}_{\overset{\circ}{g}\overset{\circ}{\eta}}(0)$ ,  $\Delta \vec{R}_{\overset{\circ}{X}\overset{\circ}{X}}^R(0)$ ,  $\Delta \vec{R}_{\overset{\circ}{X}\overset{\circ}{Y}}^R(0)$ ;
- (3) relative errors of the determinants  $p\Delta_{\overset{\circ}{g}\overset{\circ}{g}}(0)$  and  $p\Delta_{\overset{\circ}{X}\overset{\circ}{X}}^R(0)$  of the correlation matrices of noisy signals and robust correlation matrices;
- (4) relative errors of the coefficients of the mathematical model of noisy signals and robust coefficients of the model  $pc_{\overset{\circ}{g}\overset{\circ}{g}}, pc_{\overset{\circ}{X}\overset{\circ}{X}}^R$ .

The results of the computational experiment with violated classical conditions are given. Three useful input signals were formed  $x_1(i\Delta t) = 30 \sin(i\Delta t) + 13 \cos(1.2 \cdot i\Delta t - 3) - 35 \sin(1.1 \cdot i\Delta t) + 100$ ,  $x_2(i\Delta t) = 50 \sin(i\Delta t - 0.5) + 120$ ,  $x_3(i\Delta t) = 70 \sin(i\Delta t + 0.3) - 45 \cos(0.4 \cdot i\Delta t) + 150$  and the output signal  $y(i\Delta t) = 120 + 10x_1(i\Delta t) + 15x_2(i\Delta t) - 10x_3(i\Delta t)$ . The noise  $\varepsilon_1(t)$  complies with the exponential distribution with mathematical expectation  $m_{\varepsilon_1} \approx 7$  and mean square deviation  $\sigma_{\varepsilon_1} \approx 7$ . The noise  $\varepsilon_2(t)$  complies with the gamma-distribution with mathematical expectation  $m_{\varepsilon_2} \approx 9$  and mean square deviation  $\sigma_{\varepsilon_2} \approx 5$ . The noise  $\varepsilon_3(t)$  complies with the normal distribution with mathematical expectation  $m_{\varepsilon_3} \approx 0$  and mean square deviation  $\sigma_{\varepsilon_3} \approx 25$ . The noise  $\phi(t)$  complies with the normal distribution with mathematical expectation  $m_{\phi} \approx 0$  and mean square deviation  $\sigma_{\phi} \approx 100$ .

Based on the analysis of the obtained results, we conclude that the values of coefficients of the mathematical model of noisy signals differ substantially from those of coefficients of useful signals, and the values of robust coefficients of the mathematical model practically match those of coefficients of useful signals:

$$c_{\overset{\circ}{g}\overset{\circ}{g}} \neq c_{\overset{\circ}{X}\overset{\circ}{X}}, \quad (55)$$

**Table 6** Results of the calculation of the robust correlation matrix in solving of the statics identification problems

$\vec{R}_{XX}^{\circ}(0) =$	$\vec{R}_{gg}^{\circ}(0) =$	$\vec{R}_{XX}^R(0) =$
$\begin{bmatrix} 465.9 & 674 & 383.8 \\ 674 & 1250 & 1107.5 \\ 383.8 & 1107.5 & 3251.9 \end{bmatrix}$	$\begin{bmatrix} 539.2 & 692 & 375.9 \\ 692 & 1288 & 1082.3 \\ 375.9 & 1082.3 & 3806 \end{bmatrix}$	$\begin{bmatrix} 486 & 692 & 375.9 \\ 692 & 1262.7 & 1082.3 \\ 375.9 & 1082.3 & 3257.5 \end{bmatrix}$
	$\Delta\vec{R}_{gg}^{\circ}(0) =$	$\Delta\vec{R}_{XX}^R(0) =$
	$\begin{bmatrix} 15.7437 & 2.6673 & 2.0647 \\ 2.6673 & 3.0434 & 2.2712 \\ 2.0647 & 2.2712 & 17.0386 \end{bmatrix}$	$\begin{bmatrix} 4.3188 & 2.6673 & 2.0647 \\ 2.6673 & 1.0150 & 2.2712 \\ 2.0647 & 2.2712 & 0.1720 \end{bmatrix}$
$\vec{R}_{XY}^{\circ}(0) =$	$\vec{R}_{g\eta}^{\circ}(0) =$	$\vec{R}_{XY}^R(0) =$
$[10930 \quad 14420 \quad -12070]^T$	$[11350 \quad 14650 \quad -12400]^T$	$[11350 \quad 14650 \quad -12400]^T$
	$\Delta\vec{R}_{gg}^{\circ}(0) =$	$\Delta\vec{R}_{XY}^R(0) =$
	$[3.8577 \quad 1.6525 \quad 2.7756]$	$[3.8577 \quad 1.6525 \quad 2.7756]$
$H(\vec{R}_{XX}^{\circ}(0)) = 66.8700$	$H(\vec{R}_{gg}^{\circ}(0)) = 39.6712$	$H(\vec{R}_{XX}^R(0)) = 63.8403$
$\Delta_{XX}^{\circ}(0) = 2.3387e + 008$	$\Delta_{gg}^{\circ}(0) = 5.7031e + 008$	$\Delta_{XX}^R(0) = 2.5448e + 008$
$\Delta 1_{XX}^{\circ}(0) = 2.3387e + 009$	$\Delta 1_{gg}^{\circ}(0) = 6.4381e + 009$	$\Delta 1_{XX}^R(0) = 2.9239e + 009$
$\Delta 2_{XX}^{\circ}(0) = 3.5081e + 009$	$\Delta 2_{gg}^{\circ}(0) = 6.7348e + 009$	$\Delta 2_{XX}^R(0) = 3.4545e + 009$
$\Delta 3_{XX}^{\circ}(0) = -2.3387e + 09$	$\Delta 3_{gg}^{\circ}(0) = -4.4097e + 009$	$\Delta 3_{XX}^R(0) = -2.4542e + 09$
	$p\Delta_{gg}^{\circ}(0) = 143.8550$	$p\Delta_{XX}^R(0) = 8.8119$
	$p\Delta 1_{gg}^{\circ}(0) = 175.2828$	$p\Delta 1_{XX}^R(0) = 25.0222$
	$p\Delta 2_{gg}^{\circ}(0) = 91.9798$	$p\Delta 2_{XX}^R(0) = 1.5276$
	$p\Delta 3_{gg}^{\circ}(0) = 88.5518$	$p\Delta 3_{XX}^R(0) = 4.9366$
$c0_{XX}^{\circ} = 120; c1_{XX}^{\circ} = 10;$	$c0_{gg}^{\circ} = -143.32; c1_{gg}^{\circ} = 11.2888$	$c0_{XX}^R = -108.2; c1_{XX}^R = 11.5$
$c2_{XX}^{\circ} = 15; c3_{XX}^{\circ} = -10$	$c2_{gg}^{\circ} = 11.81; c3_{gg}^{\circ} = -7.7321$	$c2_{XX}^R = 13.6; c3_{XX}^R = -9.64$
	$pc0_{gg}^{\circ} = 219.4335; pc1_{gg}^{\circ} = 12.8879$	$pc0_{XX}^R = 190.18$
	$pc2_{gg}^{\circ} = 21.273; pc3_{gg}^{\circ} = 22.6787$	$pc1_{XX}^R = 14.8976$
		$pc2_{XX}^R = 9.5021$
		$pc3_{XX}^R = 3.5614$

$$c_{k_{XX}}^R \approx c_{k_{XX}}^{\circ}. \tag{56}$$

It means that the adequacy of the mathematical model is ensured (Table 6).

A technology was developed and the results of computational experiments are given for improving the conditionality of correlation matrices in solving the dynamic identification problems of high-rise buildings, construction objects and

strategic objects in seismically active regions. For this purpose, the following were calculated:

- (1) estimates of the correlation functions  $R_{XX}^{\circ\circ}(\mu)$ ,  $R_{gg}^{\circ\circ}(\mu)$ ,  $\vec{R}_{XX}^{\circ\circ}(\mu)$  from the expressions (24)–(28);
- (2) matrices  $\vec{R}_{XX}^{\circ\circ}(\mu)$ ,  $\vec{R}_{gg}^{\circ\circ}(\mu)$  of the useful and noisy signals and the robust matrix  $\vec{R}_{XX}^R(\mu)$  were formed from the expression (29);
- (3) determinants  $\Delta_{XX}^{\circ\circ}(\mu)$ ,  $\Delta_{gg}^{\circ\circ}(\mu)$ ,  $\Delta_{XX}^R(\mu)$  and the condition numbers  $H(\vec{R}_{XX}^{\circ\circ}(\mu))$ ,  $H(\vec{R}_{gg}^{\circ\circ}(\mu))$ ,  $H(\vec{R}_{XX}^R(\mu))$  of the matrices  $\vec{R}_{XX}^{\circ\circ}(\mu)$ ,  $\vec{R}_{gg}^{\circ\circ}(\mu)$ ,  $\vec{R}_{XX}^R(\mu)$ .

The following were calculated for the purpose of comparative analysis:

- (1) the values of relative errors for the estimates of the elements  $\Delta R_{gg}^{\circ\circ}(\mu)$ ,  $\Delta R_{XX}^R(\mu)$  of the correlation matrix of noisy signals and robust correlation matrix;
- (2) matrices of relative errors for the elements of the correlation matrix of noisy signal  $\Delta \vec{R}_{gg}^{\circ\circ}(\mu)$  and robust correlation matrix  $\Delta \vec{R}_{XX}^R(\mu)$ .

Given below are the results of the computational experiments for solving the dynamic identification problem when the classical conditions do not hold true for the useful signal and the noise. The input useful signal  $X(t)$  is obtained through interpolation by Fourier series downsampled with the parameter  $r = 100$  of the normally distributed random sequence. The noise  $\varepsilon(t)$  complies with the exponential distribution with mathematical expectation  $m_\varepsilon \approx 10$  and mean square deviations  $\sigma_\varepsilon \approx 10$ . The correlation matrix was built for the values  $\mu = 0, 1, \dots, 9$ . The results of the calculations are given in Table 7.

The analysis of the obtained results allows us to conclude that the correlation matrix of the noisy signal differs from the correlation matrix  $\vec{R}_{XX}^{\circ\circ}(\mu)$  of the useful signal (Table 7, Rows 1, 2). The robust correlation matrix  $\vec{R}_{XX}^R(\mu)$  practically matches the correlation matrix  $\vec{R}_{XX}^{\circ\circ}(\mu)$  of the useful signal (Table 7, Rows 1, 3). The value of the determinant  $\Delta_{XX}^R(\mu)$  and the condition number  $H(\vec{R}_{XX}^R(\mu))$  of the robust correlation matrix  $\vec{R}_{XX}^R(\mu)$  is closer to the value of the determinant  $\Delta_{XX}^{\circ\circ}(\mu)$  and the condition number  $H(\vec{R}_{XX}^{\circ\circ}(\mu))$  of correlation matrix  $\vec{R}_{XX}^{\circ\circ}(\mu)$  of the useful signal (Table 7, Row 6).

**Table 7** Results of the calculation of the robust correlation matrix in solving of the dynamics identification problems

$\bar{R}_{xx}^s(\mu)$	773.6555	773.4497	772.9718	772.2215	771.1990	769.9044	768.3380	766.5004	764.3921	762.0139
	773.4497	773.6555	773.4497	772.9718	772.2215	771.1990	769.9044	768.3380	766.5004	764.3921
	772.9718	773.4497	773.6555	773.4497	772.9718	772.2215	771.1990	769.9044	768.3380	766.5004
	772.2215	772.9718	773.4497	773.6555	773.4497	772.9718	772.2215	771.1990	769.9044	768.3380
	771.1990	772.2215	772.9718	773.4497	773.6555	773.4497	772.9718	772.2215	771.1990	769.9044
	769.9044	771.1990	772.2215	772.9718	773.4497	773.6555	773.4497	772.9718	772.2215	771.1990
	768.3380	769.9044	771.1990	772.2215	772.9718	773.4497	773.6555	773.4497	772.9718	772.2215
	766.5004	768.3380	769.9044	771.1990	772.2215	772.9718	773.4497	773.6555	773.4497	772.9718
	764.3921	766.5004	768.3380	769.9044	771.1990	772.2215	772.9718	773.4497	773.6555	773.4497
	762.0139	764.3921	766.5004	768.3380	769.9044	771.1990	772.2215	772.9718	773.4497	773.6555
$\bar{R}_{gg}^s(\mu)$	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662
	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711
	784.1673	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662
	786.6325	784.1673	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711
	784.1673	786.6325	784.1673	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662
	786.6325	784.1673	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711
	784.1673	786.6325	784.1673	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662
	786.6325	784.1673	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711
	784.1673	786.6325	784.1673	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662
	786.6325	784.1673	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711	786.5662	898.1711
$\bar{R}_{xx}^R(\mu)$	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652
	786.5662	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652
	784.1673	786.5662	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652
	786.6325	784.1673	786.5662	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652
	784.1673	786.6325	784.1673	786.5662	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652
	786.6325	784.1673	786.5662	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652
	784.1673	786.6325	784.1673	786.5662	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652
	786.6325	784.1673	786.5662	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652
	784.1673	786.6325	784.1673	786.5662	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652
	786.6325	784.1673	786.5662	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652	788.9652

(continued)

**Table 7** (continued)

$\Delta \bar{R}_{gg}^e(\mu)$	1.6958	1.4484	1.8662	1.6782	1.8849	1.2514	1.9473	2.1641	1.1676
	16.0945	1.6958	1.4484	1.8662	1.6782	1.8849	1.2514	1.9473	2.1641
	1.6958	1.6958	1.4484	1.8662	1.6782	1.8849	1.2514	1.9473	2.1641
	1.4484	16.0945	1.6958	1.4484	1.8662	1.6782	1.8849	1.2514	1.9473
	1.8662	1.4484	16.0945	1.6958	1.4484	1.8662	1.6782	1.8849	1.2514
	1.6782	1.4484	1.6958	16.0945	1.6958	1.4484	1.8662	1.6782	1.8849
	1.8849	1.8662	1.4484	1.6958	16.0945	1.6958	1.4484	1.8662	1.6782
	1.2514	1.8849	1.8662	1.4484	1.6958	16.0945	1.6958	1.4484	1.8662
	1.9473	1.8849	1.6782	1.4484	1.6958	16.0945	1.6958	1.4484	1.8662
	2.1641	1.2514	1.8849	1.6782	1.8662	1.4484	1.6958	16.0945	1.6958
	1.1676	2.1641	1.2514	1.8849	1.8662	1.4484	1.6958	16.0945	1.6958
$\Delta \bar{R}_{XX}^e(\mu)$	1.9789	1.6958	1.4484	1.6782	1.8849	1.2514	1.9473	2.1641	1.1676
	1.6958	1.9789	1.4484	1.8662	1.6782	1.8849	1.2514	1.9473	2.1641
	1.4484	1.6958	1.9789	1.6782	1.8662	1.6782	1.8849	1.2514	1.9473
	1.8662	1.4484	1.6958	1.4484	1.8662	1.6782	1.8849	1.2514	1.9473
	1.6782	1.8662	1.4484	1.9789	1.6958	1.4484	1.8662	1.6782	1.8849
	1.8849	1.6782	1.4484	1.9789	1.6958	1.4484	1.8662	1.6782	1.8849
	1.2514	1.8849	1.8662	1.4484	1.9789	1.6958	1.4484	1.8662	1.6782
	1.9473	1.2514	1.6782	1.8662	1.4484	1.9789	1.6958	1.4484	1.8662
	2.1641	1.9473	1.8849	1.6782	1.8662	1.4484	1.9789	1.6958	1.4484
	1.1676	2.1641	1.2514	1.8849	1.6782	1.8662	1.4484	1.9789	1.6958
$H(\bar{R}_{XX}^e(\mu))$			$H(\bar{R}_{XX}^e(\mu))$		$\Delta_{XX}^e(\mu)$		$\Delta_{gg}^e(\mu)$		$\Delta_{XX}^e(\mu)$
	2.158e + 05	77.5210	2.322e + 04	2.741e-04	2.577e + 22	4.759e + 09			

## 5 Conclusion

The developed noise technologies allow one to identify the early period of changes in the technical condition and seismic stability of high-rise buildings, building structures and strategic objects. The use of those technologies in monitoring systems makes it possible to warn the relevant services to take the necessary preventive measures at the initial stage of failure origin. This will lead to considerable reduction of the number of disastrous failures caused by delayed diagnostics in the modern monitoring systems.

## References

1. Sushev, S.P.: Monitoring of stability and residual life of high-rise buildings and structures with application of “Strela” mobile diagnostics complex. Unique and special technologies in construction (UST-Build 2005). In: M.: CNTCMO, pp. 68–71 (2005)
2. Aliev, T.: Digital Noise Monitoring of Defect Origin, 235 p. Springer, London (2007)
3. Aliev, T.: Robust Technology with Analysis of Interference in Signal Processing, 199 p. Kluwer Academic/Plenum Publishers, New York (2003)
4. Aliev, T.A., Musaeva, N.F., Guluyev, G.A., Sattarova, U.E.: Noise indication of change of dynamical condition of production objects. *Mechatronics, automation, control*, No. 8, pp. 2–5 (2011)
5. Aliev T.A., Musaeva N.F., Guluyev G.A., Sattarova U.E. Noise Technology of Indication and Identification of the Latent Period of Transition of an Object from a Normal Condition to an Emergency One. *Mechatronics, automation, control*, No. 9, pp. 13–18 (2010)
6. Aliev, T.A., Guluyev, G.A., Rzayev, A.H., Pashayev, F.H.: Correlated indicators of microchanges in technical state of control objects. *Cybernetics and systems analysis*, No. 4. Springer, New York (2009)
7. Aliev, T.A., Abbasov, A.M., Guluyev G.A., Rzayev, A.H., Pashayev, F.H.: Positionally-binary and spectral indicators of microchanges in technical conditions of objects of the control. *Automatic Control and Computer Sciences*, No. 3. Allerton Press, Inc., New York (2009)
8. Musaeva, N.F.: Methodology of Calculating Robustness as an Estimator of the Statistical Characteristics of a Noisy Signal, *Automatic Control and Computer Sciences*, vol. 39, no. 5, pp. 53–62. Allerton Press Inc., New York (2005)