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Berezin Transform and a Deformation Decomposition

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Abstract. We present a new form for a deformation decomposition of the Berezin transform in polynomial quantization on para-Hermitian symmetric spaces. For rank one spaces, we write a full deformation decomposition explicitly

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In construction of quantizations in the spirit of Berezin on symplectic spaces G/H the main role belongs to the Berezin transform. Program for such a construction is the following: (a) to express it in terms of Laplace operators on G/H, in fact it is the same as to determine the Plancherel formula for a canonical representation on G/H; (b) to write its asymptotic decomposition when $h \to 0$ (h being the Planck constant). Two first terms of the decomposition give the corresponding principle. Berezin carried out it for Hermitian symmetric spaces G/K, see [1, 2]. We succeeded in solving these problems for para-Hermitian symmetric spaces of rank one. Moreover, for *polynomial quantization* we can write a *full* asymptotic decomposition explicitly.

1. Para-Hermitian symmetric spaces

Let G/H be a *semisimple symmetric space*. Here G is a connected semisimple Lie group with an involutive automorphism $\sigma \neq 1$, and H is an open subgroup of G^σ, the subgroup of fixed points of σ.

We consider that groups act on their homogeneous spaces *from the right*, so that G/H consists of right cosets Hg.

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Let $\mathfrak{g}$ and $\mathfrak{h}$ be the Lie algebras of G and of H respectively. Let $B_\mathfrak{g}$ be the Killing form of G. There is a decomposition of $\mathfrak{g}$ into direct sum of $+1$, -1-eigenspaces of σ:

$$\mathfrak{g} = \mathfrak{h} + \mathfrak{q}.$$

The subspace $\mathfrak{q}$ is invariant with respect to H in the adjoint representation Ad. It can be identified with the tangent space to G/H at the point $x^0 = He$.

The dimension of Cartan subspaces of $\mathfrak{q}$ (maximal Abelian subalgebras in $\mathfrak{q}$ consisting of semisimple elements) is called the rank of G/H.

Now let G/H be a *symplectic* manifold. Then $\mathfrak{h}$ has a non-trivial center $Z(\mathfrak{h})$. For simplicity we assume that G/H is an orbit $\mathrm{Ad}G \cdot Z_0$ of an element $Z_0 \in \mathfrak{g}$. In particular, then $Z_0 \in Z(\mathfrak{h})$.

Further, we can also assume that G is *simple*. Such spaces G/H are divided into 4 classes (see [3, 4]):

(a) Hermitian symmetric spaces;
(b) semi-Kählerian symmetric spaces;
(c) para-Hermitian symmetric spaces;
(d) complexifications of spaces of class (a).

Dimensions of $Z(\mathfrak{h})$ are 1,1,1,2, respectively. Spaces of class (a) are Riemannian, of other three classes are pseudo-Riemannian (not Riemannian).

We focus on spaces of class (c). Here the center $Z(\mathfrak{h})$ is one-dimensional, so that $Z(\mathfrak{h}) = \mathbb{R}Z_0$, and Z_0 can be normalized so that the operator $I = (\mathrm{adZ_0})_\mathfrak{q}$ on $\mathfrak{q}$ has eigenvalues ± 1. A symplectic structure on G/H is defined by the bilinear form $\omega(X,Y) = B_\mathfrak{g}(X, IY)$ on $\mathfrak{q}$.

The ± 1-eigenspaces $\mathfrak{q}^\pm \subset \mathfrak{q}$ of I are Lagrangian, H-invariant, and irreducible. They are Abelian subalgebras of $\mathfrak{g}$. So $\mathfrak{g}$ becomes a graded Lie algebra:

$$\mathfrak{g} = \mathfrak{q}^- + \mathfrak{h} + \mathfrak{q}^+,$$

with commutation relations $[\mathfrak{h},\mathfrak{h}] \subset \mathfrak{h}$, $[\mathfrak{h},\mathfrak{q}^-] \subset \mathfrak{q}^-$, $[\mathfrak{h},\mathfrak{q}^+] \subset \mathfrak{q}^+$.

The pair $(\mathfrak{q}^+,\mathfrak{q}^-)$ is a Jordan pair [5] with multiplication

$$\{XYZ\} = (1/2)\,[[X,Y],\,Z].$$

Let r and $\varkappa$ be the rank and the genus of this Jordan pair. This rank r coincides with the rank of G/H.

Set $Q^\pm = \exp \mathfrak{q}^\pm$. The subgroups $P^\pm = HQ^\pm = Q^\pm H$ are maximal parabolic subgroups of G. One has the following decompositions:

$$G = \overline{Q^+HQ^-} \tag{1}$$

$$= \overline{Q^-HQ^+}, \tag{2}$$

where bar means closure and the sets under the bar are open and dense in G. Let us call (1) and (2) the *Gauss decomposition* and (allowing some slang) the *anti-Gauss decomposition* respectively. For an element in G all three factors in (1) and (2) are defined uniquely.

These decompositions generate actions of G on $\mathfrak{q}^\pm$, namely, $\xi \mapsto \widetilde{\xi} = \xi \bullet g$ on $\mathfrak{q}^-$ and $\eta \mapsto \widehat{\eta} = \eta \circ g$ on $\mathfrak{q}^+$ by (1) and (2) respectively:

$$\exp\xi \cdot g = \exp Y \cdot \widetilde{h} \cdot \exp\widetilde{\xi}, \tag{3}$$

$$\exp\eta \cdot g = \exp X \cdot \widehat{h} \cdot \exp\widehat{\eta}, \tag{4}$$

where $X \in \mathfrak{q}^-$, $Y \in \mathfrak{q}^+$. These actions are defined on open and dense sets depending on g. Therefore, G acts on $\mathfrak{q}^- \times \mathfrak{q}^+ : (\xi, \eta) \mapsto (\widetilde{\xi}, \widehat{\eta})$. The stabilizer of the point $(0,0) \in \mathfrak{q}^- \times \mathfrak{q}^+$ is $P^+ \cap P^- = H$, so that we get an embedding

$$\mathfrak{q}^- \times \mathfrak{q}^+ \hookrightarrow G/H. \tag{5}$$

It is defined on an open and dense set, its image is also an open and dense set. Therefore, we can consider ξ, η as coordinates on G/H, let us call them *horospherical coordinates*.

Take $\xi \in \mathfrak{q}^-$, $\eta \in \mathfrak{q}^+$ and decompose $\exp\xi \cdot \exp(-\eta)$ (the "anti-Gauss") according to the "Gauss":

$$\exp\xi \cdot \exp(-\eta) = \exp Y \cdot h \cdot \exp X,$$

where $X \in \mathfrak{q}^-$, $Y \in \mathfrak{q}^+$. The obtained $h \in H$ depends on ξ and η only, denote it by $h(\xi, \eta)$.

The determinant of $\mathrm{Ad}h(\xi, \eta)$ to power -1 is a polynomial in ξ, η. Moreover, it is the $\varkappa$th power of an irreducible polynomial $N(\xi, \eta)$ of degree r in ξ and in η separately:

$$\left\{ \det \left(\mathrm{Ad}h(\xi, \eta) \right) |_{\mathfrak{q}^+} \right\}^{-1} = N(\xi, \eta)^{\varkappa}.$$

The G-invariant measure on G/H is:

$$dx = dx(\xi, \eta) = |N(\xi, \eta)|^{-\varkappa} \, d\xi \, d\eta$$

where $d\xi$, $d\eta$ are Euclidean measures on $\mathfrak{q}^-$, $\mathfrak{q}^+$, respectively.

2. Maximal degenerate series representations

For $\lambda \in \mathbb{C}$, we take the character of H:

$$\omega_\lambda(h) = \left| \det(Ad\, h)|_{\mathfrak{q}^+} \right|^{-\lambda/\varkappa}$$

and extend this character to the subgroups $P^\pm$, setting it equal to 1 on $Q^\pm$. We consider induced representations of G:

$$\pi_\lambda^\pm = \mathrm{Ind}_{P^\mp}^G \, \omega_{\mp\lambda}.$$

Let $\mathcal{D}_\lambda^\pm(G)$ be the space of functions $f \in C^\infty(G)$ satisfying the uniformity property

$$f(pg) = \omega_{\mp\lambda}(p) f(g), \quad p \in P^\mp.$$

The representation $\pi_\lambda^\pm$ acts on it by translations from the right:

$$\left(\pi_\lambda^\pm(g) f \right)(g_1) = f(g_1 g).$$

Realize them in the *noncompact picture*: we restrict functions from $\mathcal{D}_\lambda^\pm(G)$ to the subgroups $Q^\pm$ and identify them (as manifolds) with $\mathfrak{q}^\pm$, we obtain

$$\left(\pi_\lambda^-(g)f\right)(\xi) = \omega_\lambda(\widetilde{h})f(\widetilde{\xi}), \quad \left(\pi_\lambda^+(g)f\right)(\eta) = \omega_\lambda(\widehat{h}^{-1})f(\widehat{\eta}),$$

where $\widetilde{\xi}$, $\widetilde{h}$, $\widehat{\eta}$, $\widehat{h}$ are taken from decompositions (3), (4).

Let us write intertwining operators. Introduce operators A_λ and B_λ by:

$$(A_\lambda\varphi)(\eta) = \int_{\mathfrak{q}^-} |N(\xi,\eta)|^{-\lambda-\varkappa} \varphi(\xi)\,d\xi,$$

$$(B_\lambda\varphi)(\xi) = \int_{\mathfrak{q}^+} |N(\xi,\eta)|^{-\lambda-\varkappa} \varphi(\eta)\,d\eta.$$

The operator A_λ intertwines π_λ^- with $\pi_{-\lambda-\varkappa}^+$ and the operator B_λ intertwines π_λ^+ with $\pi_{-\lambda-\varkappa}^-$.

Their composition is a scalar operator:

$$B_\lambda A_{-\lambda-\varkappa} = c(\lambda)^{-1}\cdot \mathrm{id}, \tag{6}$$

where $c(\lambda)$ is a meromorphic function.

We can extend $\pi_\lambda^\pm$, A_λ and B_λ to distributions on $\mathfrak{q}^-$ and $\mathfrak{q}^-$.

3. Symbols and transforms

In this section we give main constructions of a quantization in the spirit of Berezin on *para-Hermitian* symmetric spaces G/H, see [6]. We consider a variant of the quantization, which we call the *polynomial quantization*. We introduce two types of symbols of operators: covariant and contravariant ones, the Berezin transform etc.

As a (an analog of) supercomplete system we take the kernel of the intertwining operators from Section 2, i.e., the function

$$\Phi(\xi,\eta) = \Phi(\xi,\eta)_\lambda = |N(\xi,\eta)|^\lambda.$$

It has a reproducing property, which is formula (6) written in another form:

$$\varphi(s) = c(\lambda)\int_{G/H} \frac{\Phi(\xi,\eta)}{\Phi(u,v)}\,\varphi(u)\,dx(u,v).$$

The role of the Fock space is played by a space of functions $\varphi(\xi)$ depending on one of horospherical coordinates ξ, η.

For an initial algebra of operators we take the algebra $\pi_\lambda^-(\mathrm{Env}(\mathfrak{g}))$, where $\mathrm{Env}(\mathfrak{g})$ is the universal enveloping algebra of $\mathfrak{g}$. For an operator $D = \pi_\lambda^-(X)$, $X \in \mathrm{Env}(\mathfrak{g})$), the function

$$F(\xi,\eta) = \frac{1}{\Phi(\xi,\eta)}\,(\pi_\lambda^-(X)\otimes 1)\Phi(\xi,\eta)$$

is called the *covariant symbol* of D. Since ξ, η are horospherical coordinates on G/H, covariant symbols becomes functions on G/H and, moreover, *polynomials* on

$G/H \subset \mathfrak{g}$. It is why we call this variant of quantization by *polynomial* quantization. For generic λ, the space of covariant symbols is the space of all polynomials on G/H.

In particular, the covariant symbol of the identity operator is the function on G/H equal to 1 identically. For the operator $\pi_\lambda^-(X)$, corresponding to an element X of the Lie algebra $\mathfrak{g}$, its covariant symbol is a linear function $B_\mathfrak{g}(X, x)$ of $x \in G/H \subset \mathfrak{g}$ with coordinates ξ, η, up to a factor depending on λ.

The operator D is recovered by its covariant symbol F:

$$(D\varphi)(\xi) = c \int_{G/H} F(\xi, v)\, \frac{\Phi(\xi, v)}{\Phi(u, v)}\, \varphi(u)\, dx(u, v), \tag{7}$$

where $c = c(\lambda)$ is taken from (6).

The multiplication of operators gives rise to the multiplication of covariant symbols. Namely, let F_1, F_2 be covariant symbols of operators D_1, D_2, respectively. Then the covariant symbol $F_1 * F_2$ of the product $D_1 D_2$ is

$$F(\xi, \eta) = \frac{1}{\Phi(\xi, \eta)}\, (D_1 \otimes 1)(\Phi(\xi, \eta) F_2(\xi, \eta)),$$

or

$$(F_1 * F_2)(\xi, \eta) = \int_{G/H} F_1(\xi, v) F_2(u, \eta)\, \mathcal{B}(\xi, \eta; u, v)\, dx(u, v),$$

where

$$\mathcal{B}(\xi, \eta; u, v) = c\, \frac{\Phi(\xi, v)\Phi(u, \eta)}{\Phi(\xi, \eta)\Phi(u, v)}\, .$$

Let us call this function $\mathcal{B}$ the *Berezin kernel*. It can be regarded as a function $\mathcal{B}(x, y)$ on $G/H \times G/H$. It is invariant with respect to G:

$$\mathcal{B}(\mathrm{Ad}\, g \cdot x,\ \mathrm{Ad} g \cdot y) = \mathcal{B}(x, y).$$

Now we define *contravariant symbols*. A function (a polynomial) $F(\xi, \eta)$ is the contravariant symbols for the following operator A (acting on functions $\varphi(\xi)$):

$$(A\varphi)(\xi) = c \int_{G/H} F(u, v)\, \frac{\Phi(\xi, v)}{\Phi(u, v)}\, \varphi(u)\, dx(u, v) \tag{8}$$

(notice that (8) differs from (7) by the first argument of F only). This operator is a Toeplitz type operator.

Thus, we have two maps: $D \mapsto F$("co") and $F \mapsto A$("contra"), connecting polynomials on G/H and operators acting on functions $\varphi(\xi)$.

If a polynomial F on G/H is the covariant symbol of an operator $D = \pi_\lambda^-(X)$, $X \in \mathrm{Env}(\mathfrak{g})$, and the contravariant symbol of an operator A simultaneously, then $A = \pi^-_{-\lambda-\varkappa}(X^\vee)$, where $X \mapsto X^\vee$ is the transform of $\mathrm{Env}(\mathfrak{g})$, generated by $g \mapsto g^{-1}$ in the group G. Therefore, A is obtained from D by the conjugation with respect to the bilinear form

$$(F, f) = \int_{\mathfrak{q}^-} F(\xi)\, f(\xi)\, d\xi.$$

In terms of kernels, it means that the kernel $L(\xi,u)$ of the operator A is obtained from the kernel $K(\xi,u)$ of the operator D by the transposition of arguments and the change of λ by $-\lambda-\varkappa$. So, the composition $\mathcal{O}: D \mapsto A$(“contra” $\circ$ “co”) is

$$\mathcal{O}:\ \pi_\lambda^-(X) \longmapsto \pi_{-\lambda-\varkappa}^-(X^\vee).$$

This map commutes with the adjoint representation. Such a map was absent in Berezin's theory for Hermitian symmetric spaces.

The composition $\mathcal{B}$(“co” $\circ$ “contra”) maps the contravariant symbol of an operator D to its covariant symbol. Let us call $\mathcal{B}$ the *Berezin transform.* The kernel of this transform is just the Berezin kernel.

Let us formulate unsolved problems for spaces of arbitrary rank (for $r>1$):

1) to express the Berezin transform $\mathcal{B}$ in terms of Laplacians $\Delta_1,\dots,\Delta_r$ (in fact, it is the same that to decompose a canonical representation into irreducible constituents);
2) to compute eigenvalues of $\mathcal{B}$ on irreducible constituents;
3) to find a full asymptotics of $\mathcal{B}$ when $\lambda\to-\infty$ (an analog of the Planck constant is $h=-1/\lambda$).

These problems are solved for spaces of rank one, see Section 4.

4. Polynomial quantization on rank one spaces

In this section we lean on [7]. We consider here the spaces G/H, where $G=\mathrm{SL}\,(n,\mathbb{R})$, $H=\mathrm{GL}\,(n-1,\mathbb{R})$. They have dimension $2n-2$, rank $r=1$ and genus $\varkappa=n$. These spaces G/H exhaust all para-Hermitian symmetric spaces of rank one up to the covering. Further we assume $n\geqslant 3$.

Let $\mathrm{Mat}(n,\mathbb{R})$ denote the space of real $n\times n$ matrices x. The Lie algebra $\mathfrak{g}$ of G consists of x with $\mathrm{tr}\,x=0$. By Section 1, the space G/H is a G-orbit in $\mathfrak{g}$.

But now it is more convenient for us to change a little the realization of G/H.

The group G acts on $\mathrm{Mat}\,(n,\mathbb{R})$ by $x\mapsto g^{-1}xg$. Let us write matrices x in the block form according to the partition $n=(n-1)+1$:

$$x=\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}$$

where $\alpha\in\mathrm{Mat}\,(n-1,\mathbb{R})$, β is a vector-column in $\mathbb{R}^{n-1}$, γ is a vector-row in $\mathbb{R}^{n-1}$ and δ is a number.

Let x^0 be the following matrix:

$$x^0=\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$$

The G-orbit of x^0 is just G/H. This manifold is the set of matrices x whose trace and rank are equal to 1. The stabilizer H of x^0 consists of matrices $\mathrm{diag}\{a,b\}$, where $a\in\mathrm{GL}\,(n-1,\mathbb{R})$, $b=(\det\,a)^{-1}$, so that $H=\mathrm{GL}\,(n-1,\mathbb{R})$.

Subalgebras $\mathfrak{q}^-$ and $\mathfrak{q}^+$ consist respectively of matrices

$$X = \begin{pmatrix} 0 & 0 \\ \xi & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & \eta \\ 0 & 0 \end{pmatrix},$$

where ξ is a row $(\xi_1, \ldots, \xi_{n-1})$, and η is a column $(\eta_1, \ldots, \eta_{n-1})$ in $\mathbb{R}^{n-1}$. Embedding (5) is

$$x = \frac{1}{N(\xi,\eta)} \begin{pmatrix} -\eta\xi & -\eta \\ \xi & 1 \end{pmatrix},$$

where $N(\xi,\eta) = 1 - \xi\eta = 1 - (\xi_1\eta_1 + \cdots + \xi_{n-1}\eta_{n-1})$.

A G-invariant metric ds^2 on G/H up to a factor is $\mathrm{tr}\,(dx^2)$. It generates the measure dx, the Laplace–Beltrami operator Δ, the symplectic form ω and the Poisson bracket $\{f,h\}$. In coordinates ξ, η we have:

$$\begin{aligned}
ds^2 &= -2N(\xi,\eta)^{-2}\Big\{\sum \xi_i\, d\eta_i \sum \eta_i d\xi_i + N(\xi,\eta) \sum d\xi_i\, d\eta_i\Big\}. \\
dx &= |N(\xi,\eta)|^{-n}\ d\xi\, d\eta \quad (d\xi = d\xi_1 \ldots d\xi_{n-1}), \\
\Delta &= N(\xi,\eta) \sum (\delta_{ij} - \xi_i\eta_j)\, \frac{\partial^2}{\partial\xi_i\, \partial\eta_j}, \\
\omega &= \frac{1}{N(\xi,\eta)} \sum \left(\delta_{ij} + \frac{1}{N(\xi,\eta)}\, \eta_i\, \xi_j\right) d\xi_i \wedge d\eta_j, \\
\{f,h\} &= N(\xi,\eta) \sum (\delta_{ij} - \xi_i\eta_j) \left(\frac{\partial f}{\partial\eta_i}\, \frac{\partial h}{\partial\xi_j} - \frac{\partial f}{\partial\xi_i}\, \frac{\partial h}{\partial\eta_j}\right).
\end{aligned}$$

The Berezin kernel is

$$\mathcal{B}(x,y) = c(\lambda)\, \frac{\Phi(\xi,v)\Phi(u,\eta)}{\Phi(\xi,\eta)\Phi(u,v)} \ = c(\lambda)\, |\mathrm{tr}(xy)|^\lambda,$$

where

$$c(\lambda) = \left\{2^{n+1}\pi^{n-2}\Gamma(-\lambda-n+1)\Gamma(\lambda+1) \left[\cos\left(\lambda + \frac{n}{2}\right)\pi - \cos\frac{n\pi}{2}\right]\right\}^{-1}.$$

The Berezin transform is written in terms of the Laplace–Beltrami operator Δ as follows

$$\mathcal{B} = \frac{\Gamma(-\lambda+\sigma)\,\Gamma(-\lambda-\sigma-n+1)}{\Gamma(-\lambda)\,\Gamma(-\lambda-n+1)}, \tag{9}$$

the right-hand side should be regarded as a function of $\Delta = \sigma(\sigma+n-1)$.

Now let $\lambda \to -\infty$. Then (9) gives

$$\mathcal{B} \sim 1 - \frac{1}{\lambda}\, \Delta.$$

Hence we have

$$F_1 * F_2 \sim F_1F_2 - \frac{1}{\lambda}\, N^2\, \frac{\partial F_1}{\partial\xi}\, \frac{\partial F_2}{\partial\eta},$$

so that for $\lambda \to -\infty$ we have

$$F_1 * F_2 \longrightarrow F_1 F_2, \tag{10}$$

$$-\lambda\,(F_1 * F_2 - F_2 * F_1) \longrightarrow \{F_1,\, F_2\}, \tag{11}$$

in the right-hand sides of (10) and (11) the pointwise multiplication and the Poisson bracket stand, respectively. Relations (10) and (11) show that for the family of algebras of covariant symbols the *correspondence principle* is true. As the Planck constant, one has to take $h = -1/\lambda$.

Moreover, we can write not only two terms of the asymptotics but also a full asymptotic decomposition (a deformation decomposition) of $\mathcal{B}$ explicitly. In order to have a transparent formula, one has to expand not in powers of $h = -1/\lambda$ but use "generalized powers" of $-\lambda - n$. Then decomposition turns out to be a series terminating on any irreducible subspace of polynomials on G/H.

Namely, we have the following decomposition of the Berezin transform:

$$\mathcal{B} = \sum_{k=0}^{\infty} \frac{1}{k!} \cdot \frac{\Delta\,[\Delta - 1\cdot n]\,[\Delta - 2\cdot(n+1)]\,\ldots\,[\Delta - (k-1)(k-2+n)]}{(-\lambda - n)^{(k)}}\,,$$

where

$$a^{(m)} = a(a-1)\ldots(a-m+1).$$

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