Proportion and Continuous Variation in Vitruvius's *De Architectura*

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It is important to balance Vitruvius's discussion of the architectural orders, centered on temples, with his sections on civil and, in particular, domestic architecture. It is in this domain, the subject of Book 6 (Chapters 3 and 4) of the *De Architectura*, that the relationships implied by the term *symmetria* appear explicitly, in both functional and aesthetic terms and without interference from the question of whether the recommended ratios are affected by the transformation of wooden temples to stone ones. Based on a review of his rules for designing *atria*, the Vitruvian conception of order as *genus* appears not as a fixed set of ideal relationships laid down once and for all, but as a series of variations in proportion. While certainly not obeying the concept of "function" as developed in the seventeenth century, these variations can nevertheless be shown to follow continuous curves interpolated from sets of derived values. In this respect, the Vitruvian project finds contemporary expression in today's CAD/CAM software.

The Atrium of the Country House

The instructions that Vitruvius gives for the plan of country houses begin with the atrium, the large central court around which the parts of the *domus* are distributed (Fig. 1). The *compluvium*, the unroofed space in the center of the atrium, owes its name to the fact that it allows rainwater to collect in the *impluvium*, or cistern,

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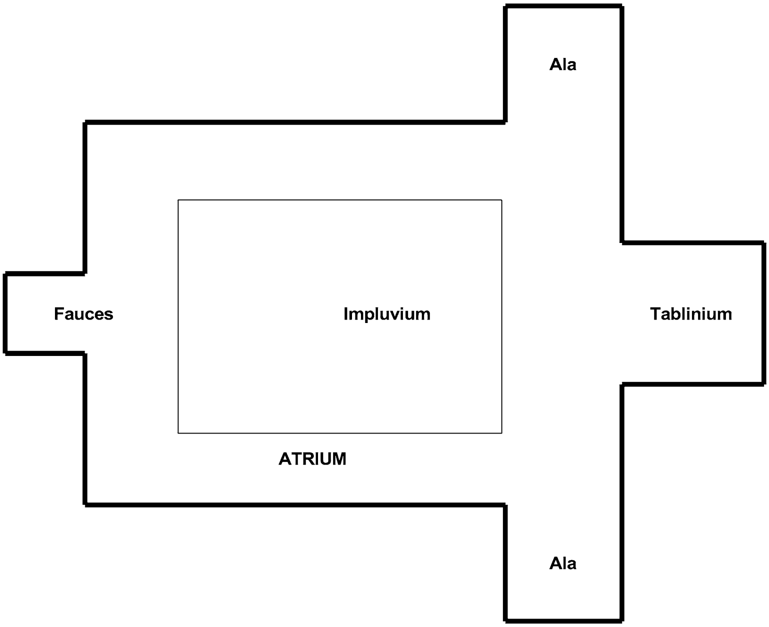

Fig. 1 Parts of the Roman Atrium, from *De Architectura*, Book 6, Chapter 3.4

below. In addition to the lateral wings, the *alae*, the other principal elements of Vitruvius's account are the *fauces*, a passage leading from the vestibule of the house, and the *tablinium*, at the other end of the atrium. As its name suggests, this room may have served to house the *tabulae*, or wax covered tablets inscribed with the accounts of the house, but it may also have served other purposes. Varro, for example, relates that it was used to host meals in summer.[1]

Vitruvius's rules for designing *atria* consist essentially of a series of instructions, in which the principal dimensions of the component spaces depend on each other according to the following sequence, with the preceding value determining the subsequent:

- length of the atrium
- width of the atrium
- width of the wings (*alae*)
- width of the *tablinium*
- width of the *fauces*

However, instead of the preceding value being linked to the next by a fixed proportion, Vitruvius subjects the four relationships between these five elements to what we would call dependant variables. We will look at these case by case.

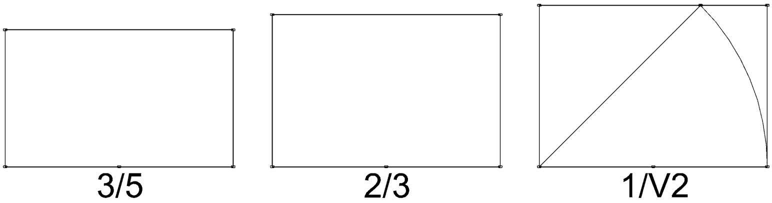

Fig. 2 Variations of the Roman Atrium (ratio *A*/*a*), from *De Architectura*, Book 6, Chapter 3

The Length-to-Width Ratio of the Atrium, *A:a*

The first relation is described as depending on a typological choice between three genres of atrium of increasing width. Given a length *A*, the width *a* of the atrium is calculated by choosing one of three proportions, formulated in the following manner:

- dividing the length in 5 parts, 3 will be given to the width
- dividing the length in 3 parts, 2 will be given to the width
- a square being constructed on the width, the length will be equivalent to the diagonal.

The series poses a problem of consistency: in the first two formulations, it is the length *A* that determines the width *a*, while in the third it is the width that determines the length. The third formulation, moreover is different in kind, as it is geometrical, while the first two are numerical. In any event, the instructions seem to correspond to a choice between one of three types of rectangle (Fig. 2).

Aisle Width to Atrium Length, *L:A*

Next, Vitruvius moves on to the rules for calculating the width of the aisles *L*. These are determined by the length of the atrium *A*. More precisely, the aisle widths are given in terms of a proportion, aisle width to atrium length (*L*:*A*), which itself varies as a function of the actual length of the atrium. Vitruvius's instructions are given in Table 1. We might call this a "second order" variation, $L=f(L:A)$, where the ratio *L*:*A* itself depends on *A*. Another way of expressing this relationship is by the formula $L=f(g(A))$.

Auguste Choisy was the first to note, in his 1909 translation of Vitruvius, that the proportions of the *alae*, as they diminish with respect to the length of the atrium, seem to imply a continuous variation. If the mean points of the five atrium lengths *L* (35, 45, 55, 70, and 90 ft) are plotted on a graph against the corresponding ratios of atrium length to aisle width *L:A*, the resulting points very closely approximate a curve, which Choisy identified as a hyperbola (Fig. 3).[2] He also found evidence of

Table 1 Vitruvius's instructions for determining the width of the aisle *L* in relation to the length of the atrium *A*

Atrium length	Ratio *L:A as recommended by Vitruvius*	Equivalent fractions
From 30 to 40 ft	1:3	2/6 (0.333)
From 40 to 50 ft	1:3.5	2/7 (0.285)
From 50 to 60 ft	1:4	2/8 (0.250)
From 60 to 80 ft	1:4.5	2/9 (0.222)
From 80 to 100 ft	1:5	2/10 (0.200)

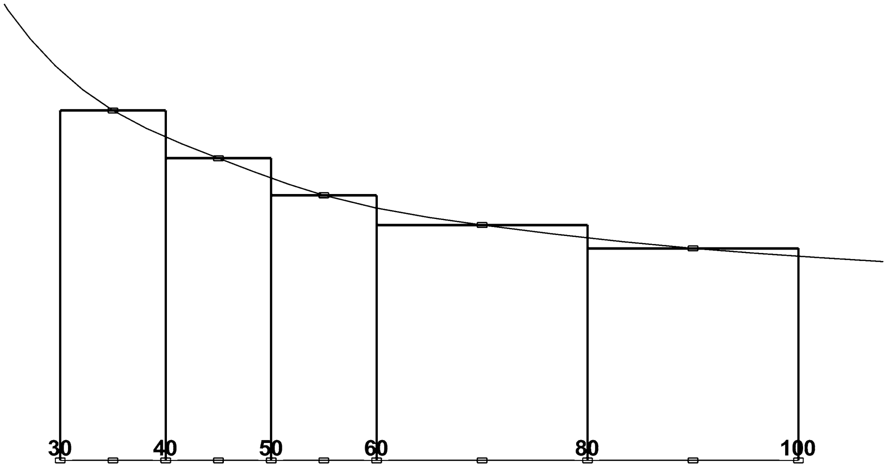

Fig. 3 Variations of the ratio *L*/*A*, from *De Architectura*, Book 6, Chapter 3.4

an attempt to approximate continuous variation in two other sequences that Vitruvius had recommended: the optical corrections for the width of columns (Book 3, Chapter 3) and for the height of architraves (Book 3, Chapter 6), implying a parabola and hyperbola respectively.[3]

That Vitruvius's instructions-in-series translate into a continuous variation is likely. As Choisy had suggested, the recommendations in these cases may be simplified rules-of-thumb derived from a learned mathematical tradition. But which tradition? Where did these recommendations originate? In one interesting analogy, Choisy related the curve implied by the rules for *atria* to the *scamilli impares*, the "unequal benches" mentioned by Vitruvius in Book 3 (Chapter 4). The *scamilli*, understood today either as small leveling blocks or as the ordinates of a full-scale construction drawing, are described in the text as the means of producing the subtly rising curve of the stylobate, or temple platform. It is by reference to the process of interpolating a curve, in this case that of a light chain hanging from the edges of the stylobate (inverted to produce a shallow mound), that the technique can be linked to the description of the atrium.[4]

In a very stimulating article, Gilbert Hallier has picked up this question, adducing other examples of this phenomenon.[5] Hallier also refers to the design of sundials, such as the monumental one drawn on the pavement of the Campus Martius, near the Ara Pacis. The curves of the dial—some 150 m long—would have been traced by the tip of the shadow of the obelisk through the day at different times of the year. Here we are indeed dealing with curves plotted point by point. Moreover, those curves are hyperbolas, formed by the intersection of the horizontal dial plane with the cone of solar rays passing by the tip of the gnomon.

This suggestion, however, probably reaches too far. Although the properties of hyperbolas were known at least since the time of Menaechmus in the fourth century BCE, we have no evidence that ancient astronomers had conceived the lines of sundials in this way. Book 9 of *De Architectura*, the sole surviving ancient treatise on sundials, makes no mention of the kinds of curves produced by the moving shadow of the gnomon. Nor does the word *hyperbola* appear in the second-hand references that we have of the treatise by the astronomer Diodorus of Alexandria concerning a method for drawing meridian lines.[6]

Hallier probably also goes too far in the other direction, attributing the curve of variation implied in the ratio $(L{:}A){:}A$ to the geometrical tradition stemming from Apollonius of Perga and Pappus of Alexandria. Apollonius had no doubt gathered most of the elements for solving the problem of constructing a conic through five points, but, as Heath explains, such constructions are not found in his *Treatise on Conic Sections*.[7] Much later, some three centuries after Vitruvius, Pappus would produce a method for constructing an ellipse from five given points, working from a problem that involved finding the diameter of a column from a fragment. Pappus's solution, however, is not general and supposes that four of the five points are found on two parallel lines.[8] In fact, the construction of a conic section from five arbitrary points derives from a theorem of projective geometry that was not formulated explicitly until the nineteenth century.[9] Despite its color of practical usefulness, the study of conics does not seem to have elicited any direct application, either in perspective or gnomonics. To take one striking example, the concept of the visual cone formed by rays from the eye or from a specific object is well attested in ancient times, but its consequences—for a system of representation consisting in the intersection of the cone by a "picture plane"—are nowhere picked up. Euclid himself, who is reported to have written his own treatise on conic sections, describes the image of chariot wheels viewed obliquely as oblong, not as ellipses.[10]

These considerations must necessarily invalidate Hallier's conjectures. The historical problem posed by Vitruvius's text involves not the construction of a curve from given points or lines, but rather the determination of fractional values in series in a way that happens to approximate a certain curve. It follows, too, that these ratios cannot have originated as a hyperbola. Despite the seeming accuracy of Choisy's formulas, Greek mathematical thought did not provide the techniques necessary to model such complex curves arithmetically. These objections, however, do not fundamentally alter the fact that we are dealing with a second order variation, that is to say, a variation in proportional relationships where the coefficient $L{:}A$ is itself depending on the variable A, expressed in increments and interpolable,

Table 2 Vitruvius's proportional series *L:A* (columns 1 and 2), as interpreted by Herman Geertman (columns 3 and 4) (to be read with Fig. 4)

Atrium length	Ratio *L:A as recommended by Vitruvius*	Geometrical series	Approximation used
From 30 to 40 ft	1:3 = 2/6 (0.333)	$1{:}2\sqrt{2}$ (0.354)	$\sqrt{2} \approx 15/10$
From 40 to 50 ft	1:3.5 = 2/7 (0.289)	$1{:}2\sqrt{3}$ (0.289)	$\sqrt{3} \approx 17.5/10$
From 50 to 60 ft	1:4 = 2/8 (0.250)	$1{:}2\sqrt{4}$ (0.250)	$\sqrt{4} \approx 20/10$
From 60 to 80 ft	1:4.5 = 2/9 (0.222)	$1{:}2\sqrt{5}$ (0.224)	$\sqrt{5} \approx 22.5/10$
From 80 to 100 ft	1:5 = 2/10 (0.200)	$1{:}2\sqrt{6}$ (0.204)	$\sqrt{6} \approx 55/10$

moreover, in a continuous form. More importantly, as Hallier shows, the ratios seem to correspond to archaeological reality, falling within a cluster of points produced by the analysis of the remains of roughly 100 Roman villas.[11]

This formulation of variation clearly goes well beyond the simple concept of proportion, strictly speaking. That is not to say that Vitruvius somehow anticipates the modern concept of "function", which would only appear in the seventeenth century. For this, Vitruvius would have had to overcome a deeply rooted epistemological obstacle to the concept of a change *in* change. To Aristotle, for example, change was an irreducible category belonging to the order of the pure event. It is worth noting that Galileo himself did not go so far as to elaborate a concept of acceleration.[12] This Vitruvian variation is, instead, best seen as one of many incremental steps necessary for the formation of the concept of the continuous mathematical function. One of the interests of the *De Architectura* lies precisely in this and other such contributions to the archeology of the modern sciences.

For his part, Herman Geertman has developed a competing interpretation of the Vitruvian ratios of the atrium. Geertman sees the ratios as an attempt to simplify and approximate not a curve, but a diminishing geometric series defined by the ratio $1{:}2\sqrt{n}$. This interpretation has a very different orientation in that it focuses not on an implied continuity, but on the discontinuity resulting from the approximate roots of a series of consecutive integers.[13] Vitruvius's instructions, as interpreted by Geertman, appear in Table 2.

This interpretation has a number of strengths. In the first place, it is based on a geometric pattern conceivably rooted in an ancient design technique, namely lengthening a given rectangle by means of its diagonal. At full-scale, such a procedure would have made use of stakes and string (Fig. 4). Moreover, similar ratios appear in other passages of the text. Geertman notes that Vitruvius recommends apparent approximations of $1{:}\sqrt{5}$ for the width and height of doors in Doric temples and of $1{:}\sqrt{6}$ for Ionic temples (Book 4, Chapter 6). Finally, Geertman's interpretation rests on methods of approximating square roots that would conceivably have been codified at least in the fourth century BCE. As Geertman and others have argued, Vitruvius may have inherited standard approximations for such values from Hellenistic mathematical texts.[14]

The main weakness of the hypothesis, however, relates to this last point: Geertman's series relies on at least one rather imprecise approximation, in

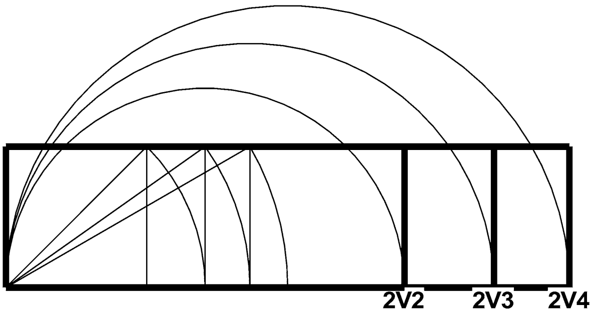

Fig. 4 Generation of rectangles of the ratio $2\sqrt{n}$

particular, that of $\sqrt{2}$ to 15/10 or 3/2. This would have been among the least accurate of the available approximations for this value, differing from the next closest (7/5) by more than 6 %. It also requires explaining why Vitruvius, a few lines above, where he lists three types of atrium by length to width, would have distinguished the ratio 3/2 from the geometrical process leading to the ratio $\sqrt{2}/1$. This divergence may spring from an inaccuracy in the manuals or graphical constructions that the author relied on, but it is nevertheless jarring, given his earlier instructions. Perhaps the most incongruous aspect of this hypothesis is that it ignores the straightforward and consistent series that Vitruvius himself provides, to replace it with a conjectural and more complicated one.

Tablinium Width to Atrium Width, *T:a*

In this regard, the case of the atria is certainly exemplary. For if we continue the examination of the other elements, namely the *tablinium* and *fauces*, we find the same characteristic approach. Regarding the *tablinium*, Vitruvius says explicitly:

> For smaller atria cannot have the same principles of symmetry that larger ones do. If we use the proportions of larger atria in the design of smaller ones, the *tablinum* and the *alae* will be too small to be functional. If, on the other hand, we use the proportional systems of smaller atria to design the larger ones, the dependent rooms will seem vacant and oversized. Therefore I thought that the principles for the dimensions of *atria* should be recorded precisely in the interests of function and appearance (Book 6, Chapter 3.5).[15]

On this basis, the architect explains that the ratio *T:a*, which determines the width of *tablinium* as a function of the width of the atrium, will be 2/3 for *atria* 20 ft wide, 1/2 for *atria* 30–40 ft wide, and 2/5 for those between 40 and 60 ft wide. Note here that the author provides three increments rather than five. This reduction in the number of variables reflects a different approach to dealing with the subsidiary spaces of the atrium, also evident in the rules for dealing with entryways, or *fauces*, below. For the moment, it is worth noting the mathematical consequences of this change. Although three increments might still plausibly correspond to points on a continuous curve, they alone cannot provide the construction of the curve itself, at least for a conic section. This consideration, in case any more were needed, further weakens the hypothesis that the architect had conceived of these points as a hyperbola.

Fauces Width to Tablinium Width, *f:T*

The same reasoning that characterizes the discussion of the *alae* and *tablinium* also applies to the *fauces*, but with a still further reduction in the number of increments. For these spaces, Vitruvius declares simply: "The entryways for smaller atria should be determined by the width of the *tablinum*, minus one-third; those of the larger *atria* should be one-half (Book 6, Chapter 3.6)"[16] This formulation, reduced now to only two values, indicates that Vitruvius intended his readers to adopt a different approach in determining the dimensions of this room. Rather than moving abruptly between only two ratios, it is more likely that he expected practitioners to gradually interpolate the proportions for *atria* of intermediate size, even according to a linear variation, as suggested by the two extreme values of the ratio of the *fauces* to *tablinium f/T*. In the absence of explicit rules, Vitruvius seems to be recommending a trial-and-error process of interpolation, reminiscent of the notion of "correction". This idea, mentioned throughout *De Architectura*, is always described with a combination of two words, *adiectio*/*detractio*, as though to suggest that the method proceeds by estimation, sometimes by adding, sometimes by taking away. In such cases, Vitruvius implicitly calls on the architect to exercise his own qualities of *ingenium* and *acumen*, talent and skill.

Although often discussed in relation to the use of optical refinements, the dual concept *adiectio*/*detractio* is not confined to that field. The terms appear, in fact, in the introduction to the chapters on the atrium, in a general formulation that relates only partially to the visual appearance of a building. Here, *adjectio*/*detractio* appear as an *ad hoc* method of fine-tuning a given proportional system:

> Thus, once the principle of the symmetries has been established and the dimensions have been developed by reasoning, then it is the special skill [*acuminis*] of a gifted architect to provide for the nature of the site, or the building's appearance, or its function, and make adjustments by subtractions or additions, should something need to be subtracted from or added to the proportional system, so that it will seem to have been designed correctly with nothing wanting in its appearance (Book 6, Chapter 2.1).[17]

This explanation for correcting a set of "symmetries" seems to point to a visual, or more specifically, a graphical method of interpolation. To determine the correct ratio *f/T* between two extreme values requires that it be visually calibrated according to the length of the atrium, which is itself situated between the larger and the smaller *atria*. In addition, the coefficients of proportionality governing the relationships of the *tablinium* to the atrium and of the *alae* to the atrium themselves vary depending on the length of the atrium. These intermediate cases, defined only by a limited set of values, would be difficult to determine without the aid of an elementary diagram.

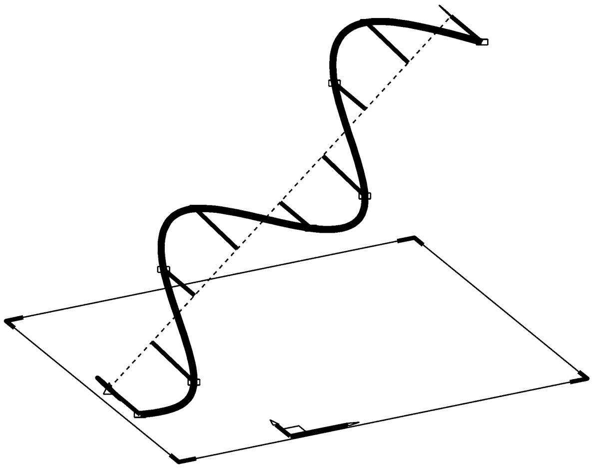

Fig. 5 Interpolation of an Archimedean screw using splines, from *De Architectura*, Book 10, Chapter 6.2

We know of similar graphical procedures in ancient design and construction. Aristotle—an unusual source in this context—speaks of a flexible, leaden rule used to replicate molding profiles.[18] The *De Architectura* itself provides other examples. Like the passage on the *scamili impares*, they appear to relate to the point-by-point construction of curves. In the chapter on baths, Vitruvius describes how to hang a plaster ceiling from metal arcs suspended from rafters in order to mimic a curved vault (Book 5, Chapter 10.3).[19] In explaining the construction of the water screw, the author gives explicit instructions for wrapping strips of willow or chasteberry around a beam so as to build up a helicoid (Fig. 5). This lattice of lateral and longitudinal strips forms a cylindrical graph, on which one literally plots the path of the spiral: "Where the lines have been drawn along the length, the transverse scorings create intersections, and these intersections determine specific points (Book 10, Chapter 6.1)." These supple branches, coated with pitch, constitute the physical equivalent of our contemporary curve-approximating software for Beziers, splines or NURBS. The word "spline" derives, in fact, from a craft context of just the sort Vitruvius describes, to designate flexible strips forced to pass through specified points. We can imagine an analogous attempt to regulate the proportions of the atrium by virtue of drawn plans. In some respects, these would follow a pre-established proportional or schematic logic, but in others, they would have to be estimated more-or-less faithfully by the eye. Indeed, Vitruvius emphasizes the role of visual judgment in this process, "so that [the whole] will seem [*videatur*] to have been designed correctly with nothing wanting in its appearance [*in aspectu*]."[20]

Whatever the tools used to achieve it, it is evident that Vitruvius's conception of the atrium possesses a high degree of elaboration. Taken as a whole, his instructions clearly form a system or, more precisely, a variational one. The consistency of the system is not always easy to achieve, but it is described well enough that we can construct an organizational diagram for it—the kind required, incidentally, in computer-aided design and manufacturing (Fig. 6). We see, in this case, that two of the interrelated variables—the length of the atrium, and the ratio of its length to its

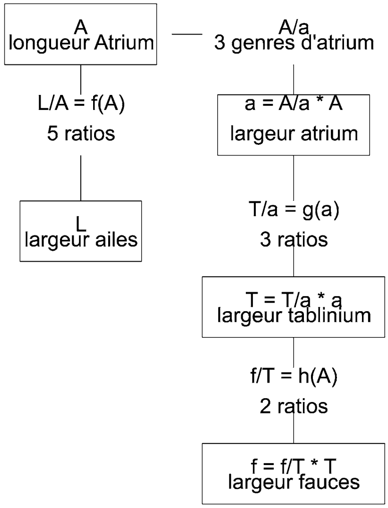

Fig. 6 Organizational diagram for the composition of the atrium, based on *De Architectura*, Book 6, Chapter 3.3 [The *boxes* denote entities to be dimensioned, while the *unframed terms* refer to the variable ratios that govern them.]

width—launch the two deductive chains that determine the dimensioning of the wings as well as that of the *fauces* and *tablinium*. Everything therefore depends on the first two decisions regarding the length and type of the atrium.

That Vitruvius's ratios for the atrium cannot be related to modern continuous functions, such as a hyperbola, should come as no surprise. Yet, it is also clear that the proportional series varies in a way that seems to imply some sort of interpolated continuity. This is what I have termed—for want of a better expression—a "second order" variation. To be sure, modern mathematicians would have a totally different notion of continuity, but it is enough only to open an up-to-date CAD-CAM package to see that Vitruvius's methods are in other ways not far from our own. To describe a continuous variation, all that is necessary is to input a set of values and let the software interpolate the resulting curve.

Notes

1. The explanation given by Vitruvius here closely reflects the definitions that Varro gives for the words *domus*, *aedes*, *cavum*, *aedium*, *impluvium*, and *atrium*. See Varro (1977–1979, 1st ed. 1938, 151–53) (Book V, 160–161). Further on the *tablinium*, see Riposati (1939, Book I, 29).
2. The curve corresponds to the equation $L{:}A = 1/9 + 70/9\ (1/A)$. His values for all five points come within three decimal places of Vitruvius's fractions. Solving for the atrium width L, reduces this to the linear equation $L = 1/9\mathrm{A} + 7.77$ ft. Choisy (1909, vol. 1, 230–36; vol. 4, pl. 62, Fig. 3).
3. Choisy (1909, vol. 1, 149–156; vol. 2, pl. 30, 31).
4. See the very detailed commentary in Vitruvius (1990, 139–145), which provides several interpretations for controlling the curvature of the stylobate.

For the current state of the question, including recent archaeological discoveries, see Bankel (1999) and Haselberger (1999).
5. Hallier (1989).
6. This method, known from Arabic sources and the surviving writings of the agrimensor Hyginus, derives the meridian from any three shadows made during the day. See Neugebauer (1975, vol. 2, 840–43). My thanks to Bernard Vitrac for bringing this important work to my attention.
7. See Heath (1896, cli–clvi).
8. See Pappus of Alexandria (1982, Book VIII, chapter 16). Also see Heath (1921, vol. 2, 434–437).
9. The theorem was discovered independently by William Braikenridge and Colin Maclaurin c. 1733. See Coxeter (1964, 85).
10. See, for example, Euclid, *Optics*, see Definition 2 and Proposition 46. On the concept of the visual cone, with reference to Roman sources, see Haselberger (1999, 57–58).
11. Hallier (1989, 199).
12. Panza (1989, Chapter 2).
13. Geertman (1984).
14. See Heath (1921, vol. 1, 60–63; vol. 2, 323–24). Also see Gros (2006 [1976]).
15. Vitruvius (1999, 79).
16. Vitruvius (1999, 79).
17. Vitruvius (1999, 78).
18. Aristotle describes this building tool in terms of a metaphor for laws that are applicable only to particular situations. "In fact this is the reason why all things are not determined by law, that about some things it is impossible to lay down a law, so that a decree is needed. For when the thing is indefinite the rule also is indefinite, like the leaden rule used in making the Lesbian moulding; the rule adapts itself to the shape of the stone and is not rigid, and so too the decree is adapted to the facts." Aristotle (1925, 1137b).
19. Vitruvius (1999, 72).
20. "…uti id videatur recte esse formatum in aspectuque nihil desideretur." Vitruvius (1999, 78).

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