Introduction

Anthony Gerbino

This volume explores the mathematical character of architectural practice in diverse pre- and early modern contexts. It draws together two nominally distinct disciplines; the history of architecture is here seen through the prism of the history of science, and one subfield of that discipline in particular. Our theme concerns the role of practice in the scientific revolution. This subject—sometimes expressed in more anachronistic terms as the relationship between science and technology—has burgeoned in recent years, and our contributions here are premised on the results of important recent work in this area.[1] In contrast to the oppositional and hierarchical categories that used to mark the historiography of this subject, scholars now tend to emphasize the jumble of intellectual, scientific, and technical factors associated with various forms of practice and, conversely, how practical and material factors were implicated in the process of actually doing science.

One of the most fruitful innovations of this approach is that it levels the artificial disparity between the mental and the manual, knowledge and know-how, theory and "application". Even where such categories are evident, our challenge is to show how they depend on and reinforce each other, not in a process of top-down "vulgarization" but rather in something like a reciprocal cycle or feedback loop. We emphasize, likewise, a micro-historical focus. In architectural as in scientific practice, various forms of knowledge—whether explicit and codified as in "science" or implicit and tacit as in "craft"—meet, interact, and augment each other in local, embodied ways. Such a focus is perhaps not unfamiliar to architectural historians, who are used to working at a fine-grained level of the individual project. To the extent that our approach entails a change in perspective, it is one that sees the designer's studio, the stone-yard, the drawing floor, and construction site not merely

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as places where the architectural object takes shape, but where knowledge itself is deployed, exchanged, and amplified among various participants in the building process.

Mathematics provides an obvious disciplinary framework for this kind of investigation. In contrast to its academic counterpart, natural philosophy, early modern mathematics was partly defined by its orientation toward practice. In the Aristotelian tradition, this connection was most clearly marked in the "subalternate", intermediary, mixed, or composite sciences, as they were variously known. Those fields—which included optics, astronomy, harmonics, and mechanics—all treated geometrical quantity abstracted from sensible matter. That is, they treated the properties of the physical world with a level of demonstrative rigor comparable to that of "pure" geometry.[2] This classification of mathematical disciplines echoed that of the traditional *quadrivium*, but by the mid-seventeenth century, the mixed sciences had grown immensely in importance. Not only did they provide a strictly mathematical rationale for the investigation of nature, they also served as an umbrella category for newly resurgent technical arts of virtually any geometrical character. Early modern surveyors, cartographers, engineers, instrument-makers, gunners, navigators, and even painters routinely identified their activities with the various "mathematical sciences" and themselves as part of an all-embracing culture of both pure and practical mathematics.[3] Moreover, these practitioners created an important intellectual and technological context for the work of scholars and mathematicians—the kinds of figures whose names and discoveries feature more prominently in the history of early modern science. Galileo stands out here, but many others would also fit the bill. Historians have been increasingly attentive to the way practical and theoretical concerns were imbricated in their work as well.[4]

This volume proceeds from a conviction that architectural history, too, can benefit from an approach that contextualizes design and construction in terms of contemporary mathematical knowledge, attendant forms of mathematical practice, and relevant social distinctions between the mathematical professions. This perspective is intended to respond, in the first place, to the actual character of the art; geometrical and arithmetical operations of some form or another lay at the heart of early modern architectural practice. Indeed, the process of design was largely defined by the application of proportional or compass-based rules. These protocols were more-or-less pervasive, potentially controlling the design in both plan and elevation, from the concept to details. Mathematics was indispensable in other ways. Measurement and scale conversion—particularly important when fitting the proportions of a design to on-site dimensions—surveying, cost estimates, bookkeeping, and even the use of routine graphic techniques all presupposed a certain amount of mathematical training.

Architecture was also connected to learned or theoretical traditions of mathematics, those associated not with the workshop or building site, but rather with the university and the humanist's library. The profession was after all largely shaped by scholar-practitioners working in or alongside a tradition of classical commentary. Leon Battista Alberti is the obvious touchstone here, but it is important to point out how broad and long-lasting this tradition was. Alongside its purely architectural content, Vitruvius's *De Architectura* contains a wealth of information on ancient theories of proportion, an anecdotal knowledge of Greek mathematics, and several chapters on technical and engineering-related subjects.[5] From the earliest printed editions, Renaissance commentators found the text to be a compelling stimulus, both as a source for ancient science and in many areas of recent research. This interest is perhaps epitomized by Daniele Barbaro, whose translation and commentary, appearing in two editions of 1556 and 1567, set new standards for the interpretation of the book's technical and mathematical contents, particularly those parts dealing with astronomy and sun-dialing (Book 9). Barbaro's famous description of the Vitruvian analemma—reconstructed with the help of Federico Commandino's then-recent edition of Ptolemy's *De Analemma* (1562)—was quickly recognized as a milestone both in the understanding of ancient astronomy and in the study of gnomonics.[6] In this respect, Barbaro was an exemplary figure, but not an uncommon one. A similar conjunction of mathematics, technology, and architecture is evident in the work of Bernardino Baldi, Guillaume Philandrier, François Blondel, Teofilo Gallaccini, Nicolaus Goldmann, and Giovanni Poleni.[7]

There was clearly a manifold bond between mathematical and architectural practice, yet historians have only partially explored this relationship. The most relevant research focuses on the design methods of medieval architects and on the proportional and geometrical layout of medieval buildings, an important vein of scholarship that I will have occasion to review below. More recently, there have been a growing number of specialist subfields—in the history of structural mechanics or building technology, for example—that sometimes overlap with architectural history.[8] Our volume builds on these approaches, but it is worth pointing out that they have remained largely peripheral to the discipline. The relative lack of scholarly engagement is not fully explained by the technical nature of the subject. Research into the mathematical basis of architectural design has a long pedigree, even if it has recently fallen off. Nor have other disciplines run afoul of the same hurdles. The historiography of early modern mechanics—to take one example relevant to architecture—has shown how technical content can be fruitfully combined with broader hermeneutic and historical concerns.[9]

The obstacles to further study in this area are several, but among the most challenging—and paradoxical—may be those posed by the physical reality of the building itself. Although mathematical practice was integral to the making of architecture, it is often subsumed and concealed by the finished object. In many cases, the designer's intentions have to be reconstructed, and—in the absence of original drawings or written records—with often partial or even unreliable results. The most valuable scholarship in this area hews closely to the measurements of the built work, to known drawing practices, and to rare written sources about the design process. Even with these controls, however, conclusions can remain speculative, and, too often, scholars have done without such checks. Indeed, older scholarship on design methods—until well past mid-twentieth century—is frequently characterized by fanciful, complex diagrams overlaid onto building plans. Recent research in this field is finally beginning to overcome the stigma of these earlier attempts.[10] Aside from being difficult to recuperate, the designer's point-of-view is only one of the perspectives that informs the history of architecture. Scholars are equally and justifiably attentive to the desires, resources, and input of both patrons and users of a building. Any reconstruction of a project's genesis will naturally involve a host of issues, from planning and style to patronage, iconography, and reception. Mathematics, in other words, is merely one part of a complex and multifaceted process.

The current state of scholarship reflects a normative conception of the design process that sees it solely as the preliminary to the building, but it is worth noting that this view involves some unintended drawbacks. To the extent that the built work remains the privileged object of study, it hinders a fuller understanding of the figure of the architect, particularly those aspects of his intellectual culture and expertise that were non-architectural. Indeed, this approach unintentionally restricts the wide range of mathematical practices that went hand-in-hand with building. Early modern architects built fortifications, drew maps, used instruments, and designed machines, but that is not what they are remembered for. To be sure, practitioners themselves did much to establish architecture as an autonomous discipline. The touchstones provided by Vitruvius and by the built remains of the ancient world allowed early modern architects and theorists to separate themselves very clearly from the wider strata of mathematical practitioners, even when their practical activities were almost indistinguishable. Yet, from a modern disciplinary perspective, the often-uncritical adoption of such a focus is nonetheless distorting, for it relies implicitly on a relatively narrow, twentieth-century definition of what the architect is, one that defines the profession retrospectively in terms of distinct socio-professional boundaries, where none existed at the time.[11]

The alternative is certainly not to ignore the built work, but rather to shift our view slightly. Instead of a restricted focus on buildings as the outcome of the design process, we might rather see design itself as part of a constellation of related activities that were no less central to the architectural culture of the period. We might try to contextualize, in other words, not only the design of specific projects, but rather the process of design itself. That process was and remains, of course, historically contingent. It is subject both to technological constraints and the level of knowledge available at a given time. Such an approach would consider not merely whether a given reconstruction fits the measured dimensions, but also how projects are conceptualized and executed within a horizon of existing practices, abilities, tools, and techniques.

Crucial to any attempt of this sort is to recognize the distance between educated elites and the traditional craft culture of the building trades. The mathematical notation, number systems, and methods of calculation that we take for granted cannot often be assumed for early modern practitioners until well into the period. Building craftsmen appear to have begun receiving education in the *abaco*—commercial arithmetic and simple geometry—only from the Trecento, and such schools were not widespread outside of central Italy. Nor did formal mathematics of this sort always take precedence over the practical training of the traditional apprenticeship, which tended to provide graphical and rule-of-thumb solutions to most problems.[12] In any given case, the most likely design methods were those that produced simple arithmetical and geometrical relationships, generated by physical manipulations of the compass and ruler or, alternatively, of the yardstick or stakes and string. It is not simply that such methods kept calculations as straightforward and unobtrusive as possible. They also reflected the manual and instrumental character of the setting-out process, in which elements of the project are treated not in terms of abstract number or dimension but as linked properties of the full-scale design.[13]

The relationship, in other words, between drawing and building has hardly been constant or universal. Indeed, the recent advent of computer-generated design and the almost complete disappearance of pen-and-paper drawing from architectural offices and schools bring home the mutability of this relationship like nothing else. Architectural practice in the early modern period was no less dependent on changes in contemporary mathematical culture and available technology, and it is this theme that ties together the essays presented in this volume. In the first place, our contributors explore the various *uses* of mathematics by early modern architects. The emphasis here is on practice, on activities as basic to architecture as drafting, calculating, measuring, surveying, composition, and design. In this sense, our papers will present a picture of architects as "consumers" of mathematics, dependent both on the level of mathematical knowledge available at the time and the degree to which they were able to understand and employ it in their own work. We also explore the opposite side of this issue, that is, the extent to which architects were themselves "producers" of mathematical knowledge or the degree to which they collaborated with mathematicians and natural philosophers in the production of such knowledge. These groups had long been associated and we know that in many cases they overlapped, particularly in the seventeenth century. Figures such as Christopher Wren, Robert Hooke, François Blondel, Girard Desargues, and Guarino Guarini spring easily to mind.

As these names suggest, the role of the sciences already colors our understanding of early modern architecture—at least for the Baroque. In fact, the entire period bounded by this volume was one of increasing mathematical and scientific expertise among architects, and this transformation was largely characterized by the reciprocal relationship between the two phenomena described above. On the one hand, mathematical and technological advance in architecture was often frustrated by the limited educational background and conservative practical outlook of the average builder. This "advance" is not a whiggish story of unimpeded progress. Yet, on the other hand, we also see continual instances in which architectural practice was both deepened and enriched by coeval advances in mathematics from both practical and learned spheres. This kind of influence was not unusual and kept the two fields closely connected throughout our period. It accounts for the live curiosity among architects in what mathematicians and mathematical scholars were actually doing and, contrariwise, for the fact that so many scholars found in the art a natural outlet for their own interests.

Scholars and Practitioners at Milan Cathedral

As an introduction to the papers that follow, this chapter presents three examples of the interaction of “high” and “low” mathematics, to illustrate its transformative effects over the period covered in this volume. I mention these instances not to suggest that they are canonical nor to imply any causal relationship between them, but to outline the range of issues inherent in our subject. Indeed, their geographical and chronological separation serves to highlight the evolution of some common themes across several centuries. These include: the interdependence of geometrical design and arithmetic calculation; the embeddedness of architecture in other mathematical disciplines; and the link between drawing and instruments. My examples also serve to illustrate a methodological point: the mathematical content of architectural practice has often been relegated as either peripheral or merely preparatory to the form-generating process of design. Making that content visible requires a change in perspective that places it in its own historical schema.

The late fourteenth-century conferences held by the cathedral workshop of Milan, made famous in a classic article by James Ackerman, still offer a useful baseline from which to compare similar events.[14] The preserved records of these meetings comprise one of our most extensive sources for medieval architectural theory in relation to an actual project. More importantly for our purposes, they also offer telling evidence for the relative level of mathematical knowledge among at least one group of medieval masons. As Ackerman recounts, the Lombard masters had fixed the ground plan and even began construction on the foundations before deciding on what the upper profile of the building was going to look like. They had envisaged an elaborate Gothic decoration for the cathedral—which still sets it apart from other comparably sized Italian churches—and began the work on a much more ambitious scale than they were normally used to working with. As a result, they were forced to call in a succession of outside experts to advise them both about decorative matters as well as the optimal height and form to give to the cross-section of the nave and aisles.

The workshop’s initial intentions for the nave section are preserved in a drawing by a visiting Bolognese architect, Antonio di Vincenzo (Fig. 1). Typically dated to early 1390, some four years after work began, the sketch combines the measured plan of the cathedral with a section of the nave as it was then projected. The plan is presumably based either on a model or on a survey of the cathedral’s rising walls, but the design of the upper parts was still very much in flux. The vertical elements are not drawn to scale, but Di Vincenzo’s annotated dimensions suggest that the section was based on a simple modular schema, in which a basic unit of 10 Milanese *braccia* (about 5.95 m) served to establish the height of the various vertical elements (Fig. 2). In this early scheme, the springing of the outer aisle vaults were to be 30 *braccia* high, that of the inner aisles 50 *braccia* high (including tall capitals of 10 *braccia*), and the springing of the nave vault 60 *braccia* high. The ground plan, in contrast, had been laid out according to a different module. Corresponding to the aisle bays, it formed a square 16 *braccia* to the side. With the nave two modules wide, the total width of the cathedral section was 96 *braccia*.[15]

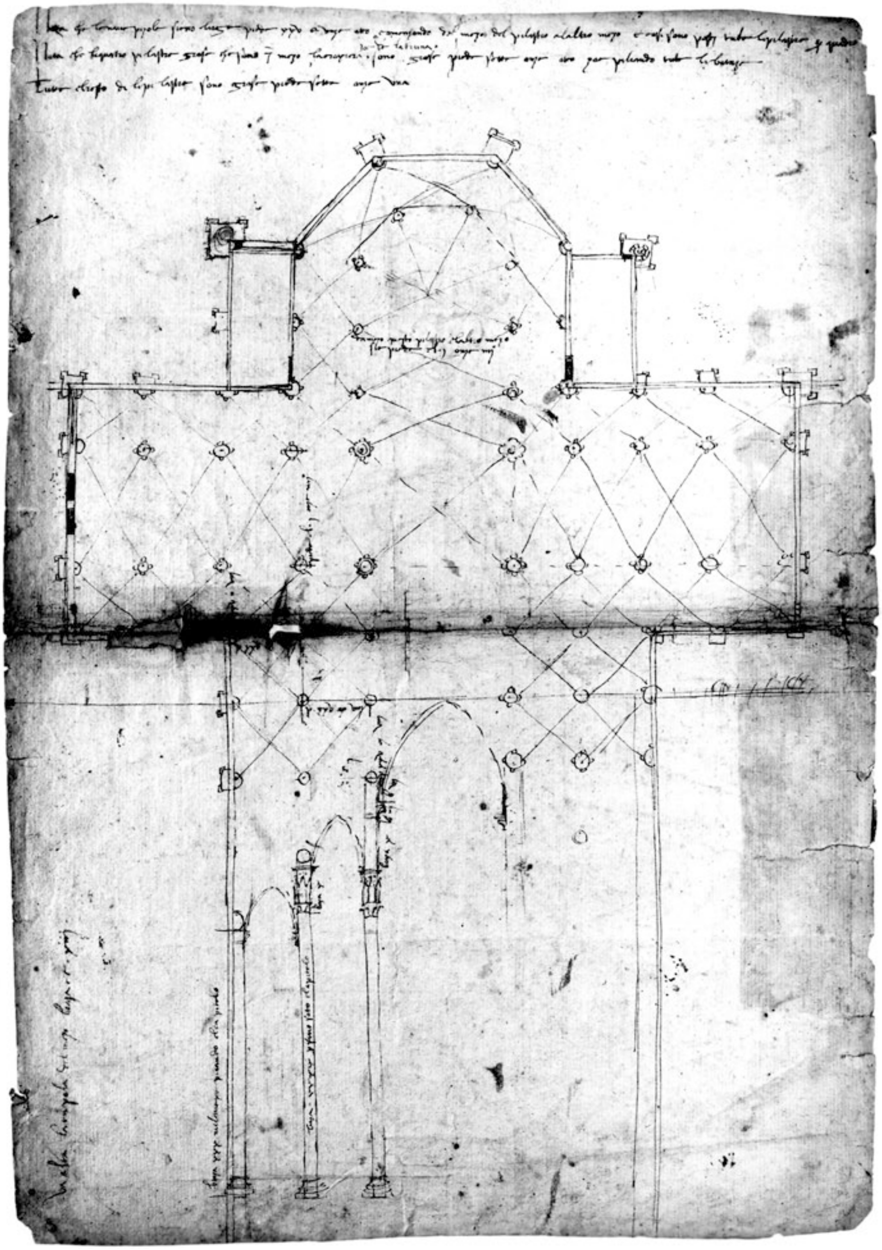
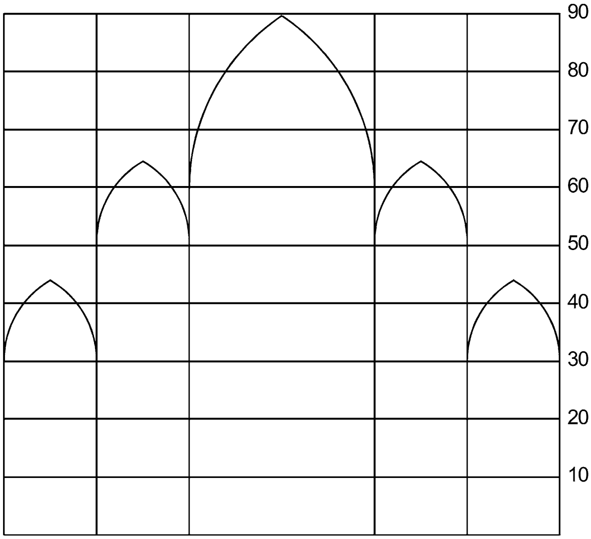


Fig. 1 Antonio di Vicenzo, plan and section of Milan Cathedral, 1390

Fig. 2 Modular schema for the first design of Milan Cathedral



At an early point in the story, sometime after March 1391, the Milanese masters decided to adopt a revised design for the cross-section, based on an equilateral triangle. This idea, put forward by the new consultant engineer, Annas de Firimburg, was presumably intended to lower the profile of the building and to inscribe it in a regular geometric shape, avoiding the somewhat arbitrary relationships between the aisles and the nave that had governed the earlier design. In accepting the new scheme, however, the building council encountered a difficult problem, for they needed somehow to calculate the projected height of the new structure, now incommensurable with its width. The workshop appealed, not to another consulting engineer, but to a mathematician from Piacenza, one Gabriele Stornaloco. Described as an "expert[us] in arte geometriae," he was asked to "discuss the questions concerning the height and about other things with the engineers of the said *fabbrica*," that is, to determine the height and reconcile it with a modular system based on the Milanese *braccio* for the rest of the cathedral.[16]

Stornaloco's solution to the problem is known from the reply that he sent to the committee, which was decoded by Paul Frankl and Erwin Panofsky in another famous article from the 1940s.[17] Panofsky showed that Stornaloco solved the problem by translating it into a four-step algorism—that is, a calculation using Hindu-Arabic numerals—involving the multiplication and division of sums to three and four places. The formula served to approximate the irrational term that was central to the solution of the problem, namely the square root of three.[18] Panofsky surmised that Stornaloco had employed an inherited formula, invented by Leonardo Fibonacci, but in wide use by the late fourteenth century. Adjusting the algorism to suit the circumstances of this particular problem (to express the value in terms of a whole number divisible by 8, that is, half that the module length), Stornaloco was able to determine

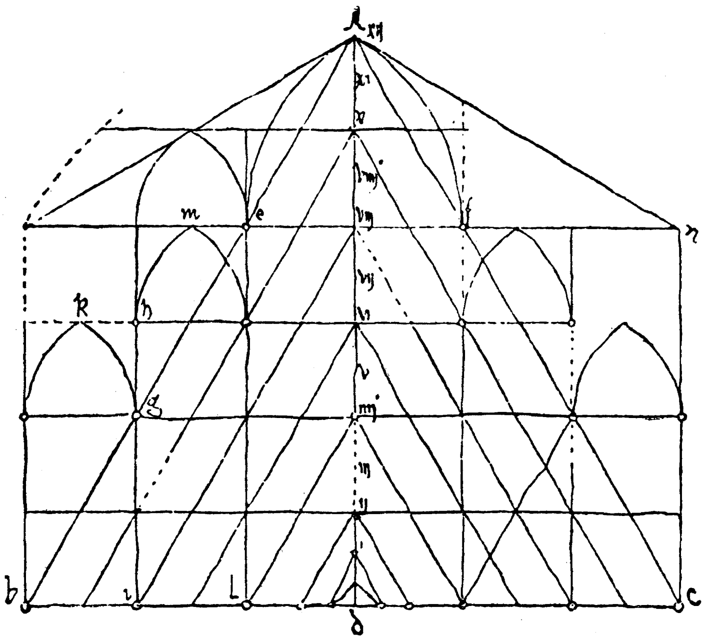


Fig. 3 Stornaloco's design for Milan Cathedral (From Beltrami 1887)

the height of the nave within .04 % of the true value (83.168 *braccia*). Indeed, he went farther. Using 84 *braccia* as a convenient approximation (giving him a module height of 14 *braccia*, as opposed to 16 in width), he was also able to correlate the nave height with the springing of the outer aisle and nave vaults. As Stornaloco pointed out in his letter, the scheme implies a series of concentric, similar triangles that link the module widths at the base of the section with the module heights along the centerline (Fig. 3).[19] The two largest of these inscribed triangles establish the heights of the principal vertical elements, namely the nave and aisle piers.

Stornaloco's solution is characteristic of an academic mathematician, not a practicing builder. Indeed, the strict formal regularity and thorough internal consistency of his design are unparalleled in Gothic architecture. These qualities may partially explain the project's immediate impact. The workshop adopted his proposal not only for the nave height, but for the entire cross-section, using it to direct construction for the next several months. This was enough to dictate the height of the existing outer aisles and to provide the basis for a revised, compromise design for the rest of the work in May 1392.[20]

What does this episode tell us about the mathematical abilities of medieval masons? The answer is not straightforward. The incident is typically seen in terms of the masons' limitations, that is, as an example of the kind of mathematical problem that medieval architects could not solve. While that reading is true in broad terms, the details need some unpacking in light of what we have learned about medieval design since Frankl and Panofsky's day. The relevant questions are not only whether the workshop was able to calculate the height of the nave *à la* Stornaloco—which is unlikely—but also whether the masons were able to determine that dimension using their own methods. In this latter respect, the fact that the workshop felt compelled to call on a mathematician is indeed surprising. Geometrical design was a mainstay of medieval architectural practice, as were the "irrational" relationships it produced. The masons' techniques were largely instrumental, graphic, and empirical—not demonstrative. Yet, combined with on-site testing and verification, they were more than enough to achieve very high levels of building precision. This was true not only for equilateral triangles. Gothic builders appear to have been fully capable of incorporating even more complicated shapes into their buildings: octagons and pentagons were not unusual.[21]

Looking at the problem from a different angle—arithmetically—we might again ask whether the workshop strictly needed an outsider to determine the nave's height. Assuming the cathedral masons could not calculate the solution in the same manner as Stornaloco, is it possible that they could have approximated it with an arithmetic rule-of-thumb, one that could relate the base of the cathedral to its height in terms of a commensurable ratio? Procedures of this kind are believed to have been used widely. In the first place, geometrical relationships were not always possible to set out physically. Existing buildings or other obstructions might easily block the sweep of a long diagonal, and such operations were particularly difficult to perform in elevation, as Frankl himself noted with respect to the problem at Milan. Practical difficulties alone would suggest the occasional need for rational approximations of geometrically derived dimensions, and scholars have, in fact, found some evidence for the use of such ratios by medieval builders. Indeed, recent discoveries by Matthew Cohen have thrown this practice into sharp relief. At Santa Maria del Fiore in Florence, the first bay of the nave arcade, designed by Francesco Talenti around 1357, incorporates a ratio of 29:41 *braccia*. These dimensions approximate the relationship between the side and the diagonal of a square with an error of only .03 %. The same ratio appears in an even more sophisticated form at San Lorenzo, built by Brunelleschi from 1421, where the width of the bays and height of the six westernmost columns measure $9\frac{2}{3}$ and $13\frac{2}{3}$ *braccia* respectively.[22] This approximation and others were known in antiquity. Theon of Smyrna reports an arithmetic method for generating increasingly accurate whole-number approximations for the side-to-diameter ratio of a square, that is, a series that progressively converges toward the square root of two. Although a route of transmission remains elusive, it is possible that this method was known to ancient builders and handed down through the Middle Ages. Indeed, some scholars have gone farther, arguing that medieval builders would have had similar formulas for calculating ratios that progressively converge toward any desired square root, thereby approximating proportions inherent in the equilateral triangle, the root-5 rectangle, and even the "golden section".[23]

These broader claims have not been verified, but in the case of Milan cathedral they are probably not relevant. A simple ratio like 8:7—the same used by Stornaloco in his own design (96×84 *braccia* for the width of the cathedral to its height)—could have been handed down as part of the masons' oral tradition. The workshop would, in any case, have been fully capable of working out an adequate ratio for the cross-section of the church using scaled drawings or cords set out at full scale. What the masons could not have known is the degree of divergence between their physical approximation and the closest possible numerical value, but this level of accuracy was for all practical purposes unnecessary, as Stornaloco himself seems to have recognized. In this light, the decision to call on the mathematician appears less as evidence of an intellectual failing on the part of the masons than as an artifact of the very unusual circumstances at Milan. As historians have long recognized, the Lombard masters were unused to the Gothic design system of their northern consultants. This clash of sensibilities—which would only intensify with Heinrich Parler's arrival in late 1391 and Jean Mignot's in 1399—may have led the two parties to see Stornaloco as a useful go-between.[24]

Even this charitable interpretation, however, should not obscure the essential fact about this incident. As "unnecessary" as Stornoloco may have been, his solution to the problem was nonetheless diametrically opposed to the methods then available to architects and masons. Instead of working out a sequence of increasing number pairs for the sides and height of the triangle (assuming the masons had such a technique), Stornaloco recognized the root that lay at the heart of the problem and expressed it, moreover, in terms suitable to the particular circumstances he found at Milan cathedral. If Panofsky is right—his is still the most convincing explanation—Stornaloco used a formula that expressed the side of the triangle in units divisible by eight (half the module width) and multiplied all the terms by ten to avoid fractional remainders. He then adjusted his approximation for $\sqrt{3}$ to be both easier to manipulate and more accurate (175/101 instead of 173/100).[25] In comparison to Theon's method, which calls for nothing more than basic addition, Stornaloco's is a true algorism that requires multiplication and division of terms to three and four places.

Stornaloco's approach to the problem, in other words, was thoroughly arithmetized, and it is this quality that sets it apart from the masons' techniques. Indeed, his explanatory letter suggests that it may not have been simply the translation of the problem into a formula that lay beyond the capabilities of the builders, but also the manipulation of the numbers themselves. Why else would he have expressed the coefficients used in the solution with Roman numerals, rather than the Hindu-Arabic ones necessary to calculate it? It is also worth noting that Roman numerals predominate in the conference minutes. Calculation using Hindu-Arabic numerals was still not universal in 1391, and Stornaloco may have had some reason to assume that the masons were not familiar or comfortable with them.[26]

These conferences are very well-trodden ground, but the great advantage of this material is that it offers a concrete historical link between the mathematics of the worksite and that of the classroom. Indeed, these conferences remain the only source we have for the direct interaction between medieval mason-practitioners and a university- or *abaco* trained mathematical scholar. At the same time, the Milan affair also makes clear the distance between these two worlds, and in this respect, reinforces the work of other historians in this area. As Lon Shelby has shown, the "geometry" of the medieval mason appears distant from most practical geometrical texts of the time. The few and scattered sources we have of the mason's art—texts by Villard de Honnecourt, Matthias Roriczer, and Hanns Schmuttermayer—reveal that it consisted largely of rule-of-thumb procedures involving the construction and manipulation of simple geometrical forms. Unlike the methods taught in the schoolman's *Practica geometriae*, the mason's techniques were often imprecise and approximate. They utilized no arithmetic calculations of the kind known at the time, nor did they reveal any understanding of the Euclidian theorems and proofs that would justify the operations involved. Although medieval masons certainly possessed a sophisticated intuitive grasp of spatiality and of spatial forms, their design and construction methods appear to have been essentially pragmatic and empirical, not mathematical or analytic.[27] What is immediately striking about the Milan story, particularly about Stornaloco's involvement, is that the two parties involved correspond, almost perfectly, to the two modes of geometrical thought that Shelby describes.

"High" and "Low" Mathematics: Antonio da Sangallo the Younger

Did medieval builders ever benefit from contact with "higher" mathematics, that of the university classroom or the humanist's library? The record is indeed scarce, but historians have unearthed some isolated examples that point to increasing interaction between the two domains. The Florentine new towns of the early fourteenth century, brought to light by David Friedman, provide unique evidence of an entirely innovative and sophisticated application of geometry to the problems of urban design.[28] The street plans of San Giovanni and Terranuova, founded in 1299 and 1337 respectively, are laid out in such a way that the widths of the residential blocks decrease in proportion to the chords of a circle advancing at set intervals. As Friedman points out, these designs presuppose a knowledge of trigonometry that could only have originated in a textbook tradition far removed from the working knowledge of most mason-builders. Matthew Cohen's work on San Lorenzo in Florence has brought to light an analogous example. The nave arcade appears to embody a complex set of interdependent proportional relationships that may incorporate a Boethian number system.[29] Without these examples, we might be entirely justified in believing that the level of mathematical knowledge at Milan cathedral was representative of medieval masons in general. In reality, there may have been particular workshops, cities, or regions, where graphical, numerical, and technical ability were more advanced and where individual craftsmen were more receptive to influences from parallel or analogous fields. Given the prevalence of the *abaco* curriculum in central Italy, the influence of a highly educated merchant culture, and the sophistication of local surveyors, it is perhaps not surprising that innovations of this kind first appear there.

The Stornaloco incident is worth scrutinizing not to belittle medieval masons, but to underline one of the central premises of this volume, namely that Renaissance architectural practice was characterized by a new orientation toward both the speculative tradition of ancient geometry and to the advances in practical mathematics that accompanied its revival. Indeed, Renaissance architects often distinguished themselves from their predecessors on these very grounds. In the opening pages of the *Primo Libro* (1545), for example, Sebastiano Serlio complained of those "who today bear the title, 'architect' but who do not know how to give a definition of a point, a line, a plane or body, or say what correspondence and harmony are."[30] He explicitly identified his approach with the theoretical principles derived from Euclid's *Elements*. This transformation has typically been laid to the influence of Neoplatonism, but it was in fact impelled by a number of factors, including the rediscovery of linear perspective and the general resurgence of the practical, mathematical sciences in the fifteenth and early sixteenth century. This influence is evident in many areas. As several recent studies have affirmed, Renaissance architects show a newfound awareness of the dynamic properties of structure, a greater familiarity with techniques of arithmetic calculation, and a growing interest in new mathematical sciences, such as trigonometry.[31] This change is also reflected in the art's renewed status as an intellectual discipline. Sixteenth-century divisions of knowledge—ramified disciplinary "trees"—often list architecture under the general heading of mathematics, usually alongside mechanics.[32]

It is against the background of this transformation that I want to set a second example, a series of drawings of the 1520s and 1530s from the recently published corpus of Antonio da Sangallo the Younger.[33] Ann Huppert's chapter below discusses these documents at length and in a more specific architectural context. They serve here simply as a contrast with the picture suggested by the Milan cathedral workshop and to illustrate the enormous sea change that was entailed in the transition to the humanist architectural culture of the Renaissance. In that light, the drawings are remarkable, because they show the architect engaged in a purely personal study of just the kinds of problems that appear to have stumped the Milanese masters and that Shelby describes as being outside the normal working methods of medieval masons in general. Indeed, the great fascination of these notes and sketches is that we have very few earlier examples for this kind of interest or ability among architects. It is important to note here that we are not dealing here with an Alberti or a Barbaro, but with a building practitioner trained in a traditional—if not to say medieval—apprenticeship system.[34]

In the first place, the drawings evince an ease and facility with arithmetic calculations, examples of which cover large portions of the sheets concerned. These are all the more striking in light of the Milan episode, since the multiplied sums often include regular fractions in an attempt to find approximate values for square- and cube roots (Fig. 4). In a general sense, the figures show the importance of the *abaco* in the period. Indeed, the editors of the volume make a point of noting Sangallo's mastery of technique.[35] Not only was he apparently taught by such a master, he was also able to think in terms of algorism, adapting it to new problems thrown up by craft practice and his theoretical interests. In this respect, Sangallo was far from alone among contemporary architects. As Ann Huppert remarks in her essay below, his mathematical abilities were matched, if not exceeded, by those of Baldassarre

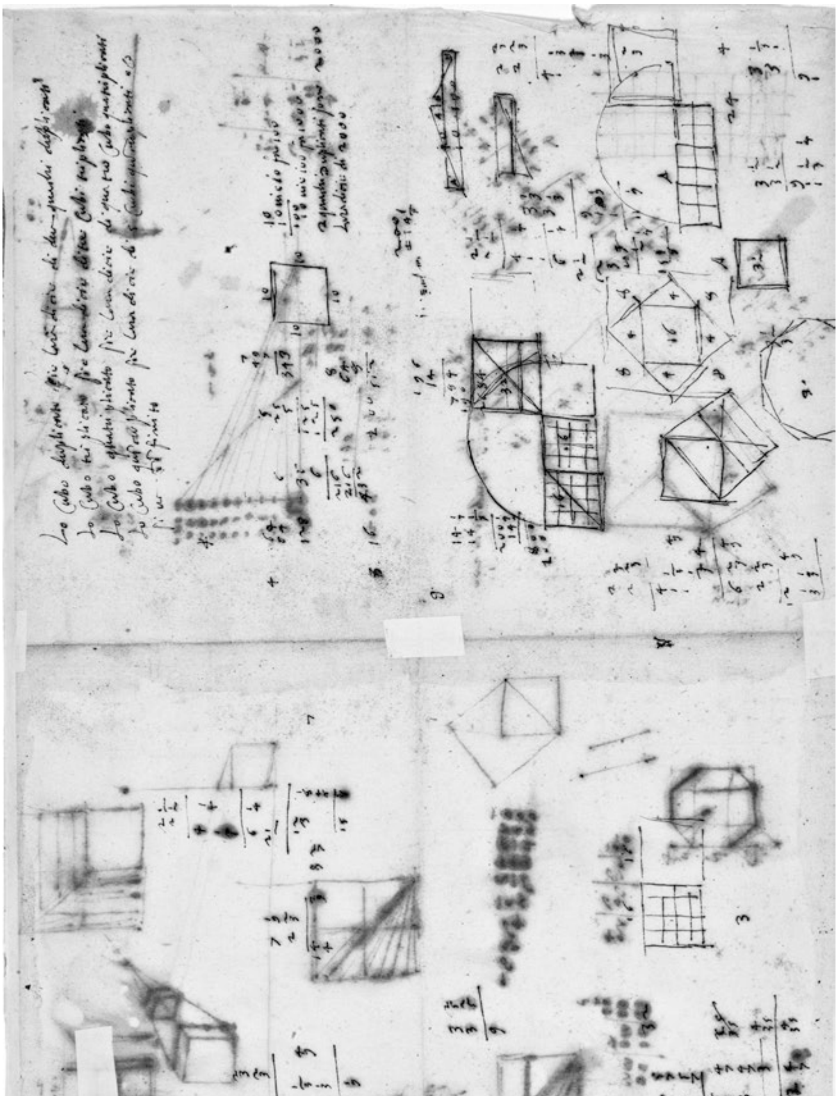


Fig. 4 Antonio da Sangallo the Younger, sheet of calculations, UA 856v

Peruzzi, whose own use of algorism was even more closely intertwined with his architectural practice.

In many respects, these drawings are characteristic of what Carlo Maccagni has described as the intermediary or "vulgar" science of Renaissance artisans, technical experts, and practitioners. Works of this kind typically take the form of *taccuini* and *zibaldone*, written in the vernacular with a mixture of notes and diagrams. Like Leonardo's notebooks—the most emblematic of the genre—they represent an *ad hoc* process of learning adapted from the experience of the workshop, where activity tends to follow not a formal curriculum, but rather the meandering course of an apprenticeship or the unpredictable demands of a clientele. Such works investigate individual problems as they are encountered and worked through on a case-by-case basis. In this respect at least, they tend to mirror the format of contemporary abacist manuals and practical geometries.[36]

Sangallo's drawings illustrate the way in which his *abaco* education provided routes into his theoretical and quasi-scholarly interests. In the case noted above, the architect's calculations seem to have arisen from an exploration of geometric constructions relating to the doubling of the square and the cube, as mentioned in Vitruvius (Book 9, Introduction). Other drawings also seem related to those parts of the *abaco* curriculum concerned with solid geometry. One sheet, for example, reflects attempts to find the volume of pyramids of different form (Fig. 5). This kind of problem was famously and rather more expertly explored by Piero della Francesca and Luca Pacioli in their own *abaco* instruction manuals, but here Sangallo seems to have again been inspired by Vitruvius. As the editors point out, the interspersed diagrams of stepped pyramids may reflect an attempt to reconstruct the Mausoleum of Halicarnassus as described by the Roman author (Book 2, Chapter 8.10–11).[37] The figure in the upper right portion of the sheet, showing the doubling of the square, is likely stimulated by the same source.

The range and variety of Sangallo's graphic *oeuvre* are remarkable. Like the drawings of his older contemporary, Francesco di Giorgio Martini, Sangallo's encompass an array of technical and engineering-related activities that far exceeds modern notions of the architect's role. Drawings of artillery, instruments, and machines of all kinds are perhaps the most unexpected, precisely for their lack of any specific connection with building. Yet, it is also clear that these sketches formed part of a common disciplinary constellation. The same range of interests—especially in the fields of astronomy and cartography—would characterize the principal concerns of mathematical practitioners throughout the sixteenth and seventeenth centuries.

Among the most striking examples of Sangallo's curiosity is a proposal—"my opinion" he calls it—for a system of curved panels or gores for the construction of a globe (Fig. 6). This technique of cartographic projection was relatively new, having been published for the first time by Martin Waldseemüller in 1507. Sangallo might have come across the idea in an intermediary text—several other examples had appeared over the intervening twenty-odd years—but his awareness of an innovation so far outside his own training is nevertheless surprising. The projection is composed of twelve gores, dividing the globe by 30° intervals of longitude. The figure

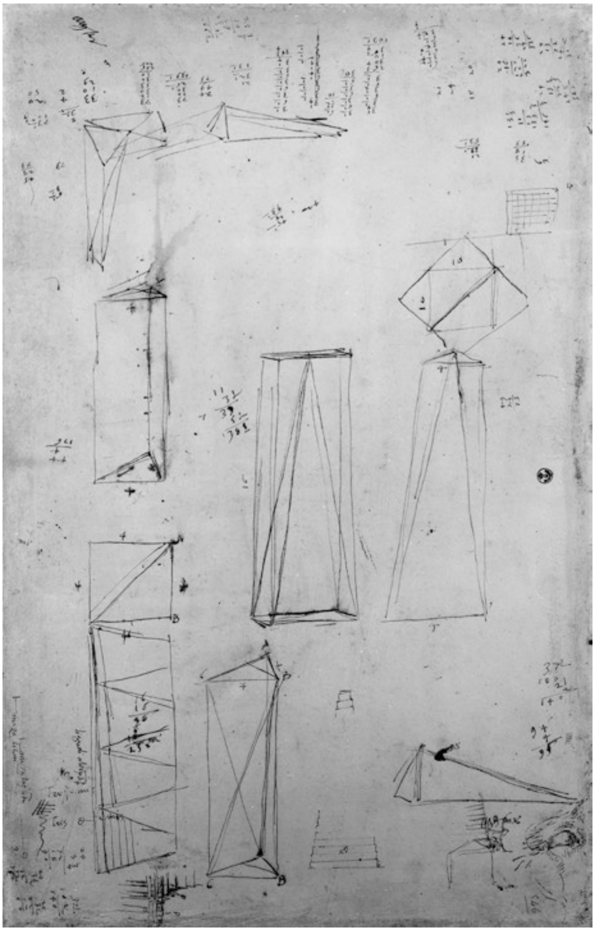


Fig. 5 Antonio da Sangallo the Younger, calculations and geometrical diagrams, UA 857r

1865 is inscribed between each gore, calculated (incorrectly) as one twelfth of the total circumference of the Earth, which Sangallo gives at the top of the sheet as 22,500 miles. As the editors point out, no other world maps are found among the architect's drawings, but that is not to say that the work is entirely isolated. Antonio executed several topographical surveys—typically involving fortifications—and still other drawings related more broadly to cosmography. A geometrical study of the constellations in the northern hemisphere, for example, is conceptually not far removed from Sangallo's globe gores.[38]

Several drawings of mathematical instruments reveal another dimension of this concern for terrestrial and celestial measurement. The most extraordinary of this group—indeed, of the corpus as a whole—is an astonishingly faithful replica of an Arabic astrolabe, complete with its Kufic inscriptions (Fig. 7). Sangallo surely could not read them, but this did not diminish his fascination with the object, as is

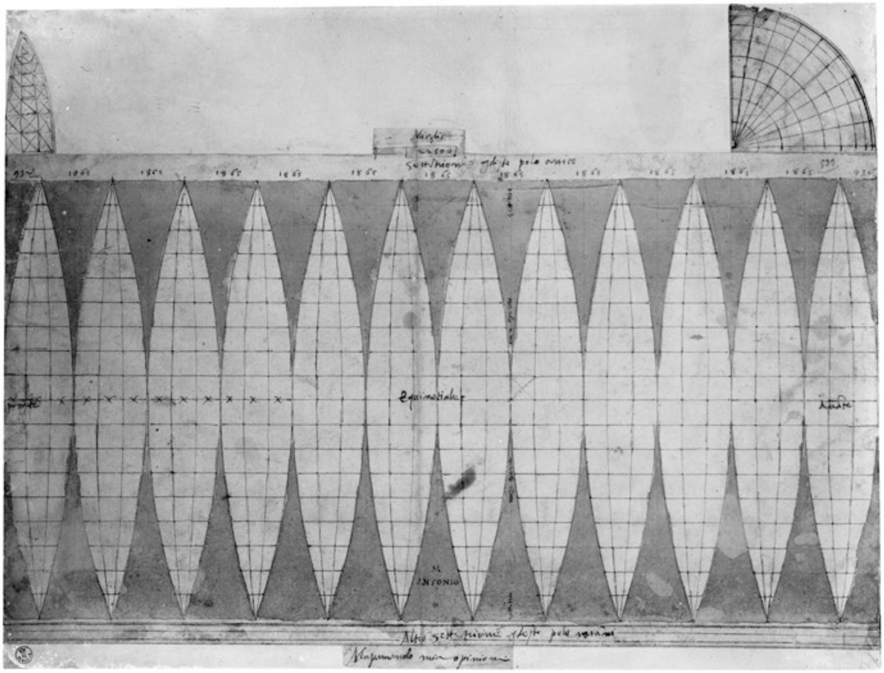


Fig. 6 Antonio da Sangallo the Younger, globe gores, UA 850r

evidenced by his painstaking method of description. The drawing stunned the editors of the volume. "We marvel," writes Gustina Scaglia, "at his determination to make a facsimile of all the letters." Indeed, the transcription is legible enough to determine the maker's signature, as well as the place and rough date of origin: Morocco, probably—on the basis of its similarity to other known instruments—in the tenth or eleventh century.[39] Not only was Sangallo's study precise, it is also systematic. The astrolabe's front and rear faces are juxtaposed, and its component parts—alidade, rete, even a replacement screw and nut—presented separately and distinctly. As Scaglia points out, Sangallo evidently knew something about such instruments. The inscribed notes suggest that he was familiar with the names, arrangement, and function of the parts, including the use of the sight holes in the alidade. He evidently knew how to engrave the scales and geometric constructions on the two faces. Astrolabes featured commonly in medieval and quattrocento practical geometries, where they are typically shown as helpmates for rudimentary surveying problems. Sangallo's knowledge of such instruments seems to have gone much deeper.

One last image from the group reveals Sangallo's more-than-passing interest in instruments. In terms of contemporary advances in mathematical practice, it is one of the most noteworthy of the entire corpus: the earliest known representation of a flat-sided, proportional compass—or sector—with scales incised on the face of each leg (Fig. 8). It is a hinged rule used with dividers to transfer dimensions to and from

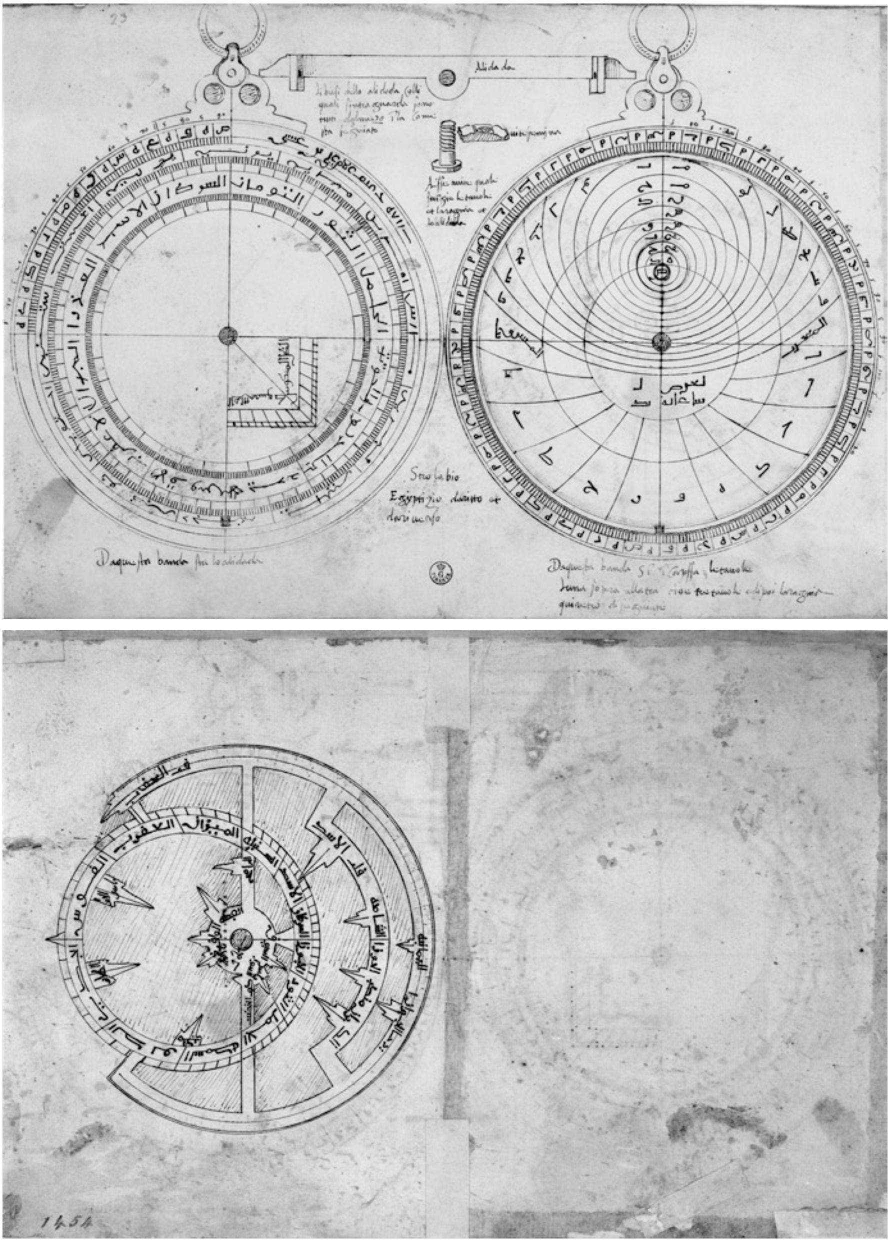


Fig. 7 (**a, b**) Antonio da Sangallo the Younger, front and rear faces of an Arabic astrolabe, with rete below, UA 1454r-v

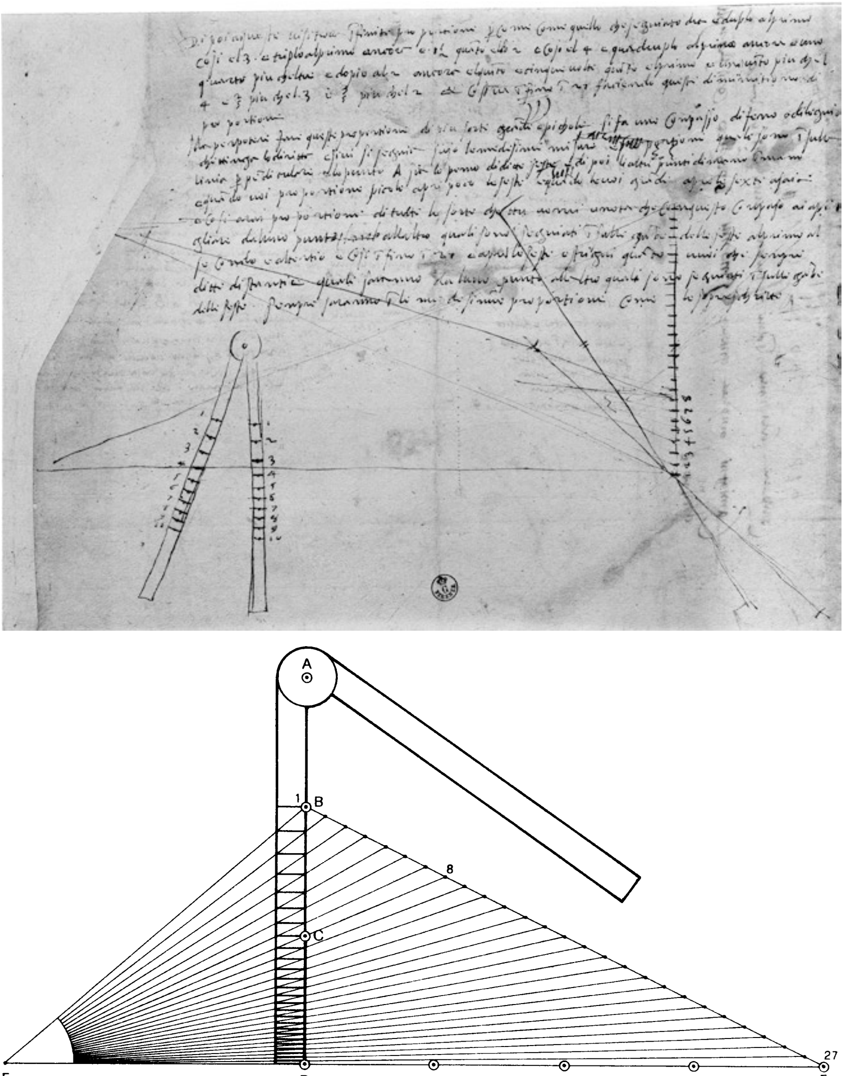


Fig. 8 (**a, b**) Antonio da Sangallo the Younger, sheet of notes for a proportional compass or sector (UA 1491r), with modern reconstruction (from Camerota 2000)

the pairs of identical scales on its two legs. Opened to a given length, the matching scale divisions provide additional dimensions continuously proportioned to that initial length. Sangallo designed the instrument as a rough graphic method of approximating cubes and cube roots. As Filippo Camerota has shown, theorists of perspective—Alberti, Piero, and Dürer, in particular—had earlier demonstrated the basic principle, but it was Sangallo who gave this instrumental form to it. The idea may have been inspired by his study of the problems in Book 9 of Vitruvius; the drawing appears to have been preceded by a number of preparatory studies exploring analogous methods of approximating square and cube roots. In practical terms, the instrument could be used to easily rescale architectural elements, but the important innovation was conceptual: the sector provided a mechanical and geometrical approach to problems that resisted a quick arithmetic solution. The same principle governed the many more sophisticated forms of the instrument that appeared throughout the century, culminating in Galileo's own version, invented in 1597 and published in 1606. That the device had originated in an architectural context had been largely forgotten, but it makes sense in light of later attempts to adapt the sector to the rules of classical design, first by Ottavio Revesi Bruti in the early seventeenth century and later in eighteenth-century England.[40]

Sangallo's mathematical drawings make a blunt, but telling, contrast to the Milan cathedral debates—the Stornaloco episode in particular. In light of the latter, the former reveals the profound transformation that had taken place in the mathematical culture of elite building practitioners during the preceding century. Sangallo's training was traditional, but it was bolstered by a sound mathematical education and by a rich humanistic culture that linked his art with the latest advances in the practical mathematical sciences. Sangallo's inspirations were not only textual, but also personal and professional. The drawings as a group reflect two important influences. The first was the Della Volpaia family of clock- and instrument makers, with whom the Sangallos had been linked since the late fifteenth century. Antonio appears to have been close to Benvenuto della Volpaia, in particular. The two men are believed to have shared drawings of instruments and mechanisms; several duplicates are found among their respective manuscripts. The second—and more important—influence was likely the learned architect, engineer, and antiquarian Fra Giovanni Giocondo of Verona. Among contemporary humanists, Giocondo had a virtually unique ability to marry erudition and practice, an admixture evident both in his groundbreaking 1511 edition of Vitruvius—the first with illustrations—and in his work as a manuscript hunter. Giocondo was an assiduous collector of medieval practical geometries, including several rare French versions, acquired presumably during his time in Paris as the royal engineer to Charles VIII. Sangallo's connection with Giocondo probably dates to sometime after June 1514, when both men were employed at the workshop of St. Peter's. In terms of the direction of influence, we are here on more solid ground. A note on one of the geometrical studies in Antonio's collection attributes it explicitly to the Franciscan friar.[41]

From the Mathematical to the Physical Sciences: Pierre Bullet

It would be difficult to link Sangallo's mathematical notes and drawings directly to the design of any one of his buildings. As Ann Huppert demonstrates, below, they bear rather on the history of design itself. Architects of Sangallo's generation were among the first to incorporate Hindu-Arabic algorithms into the design process,

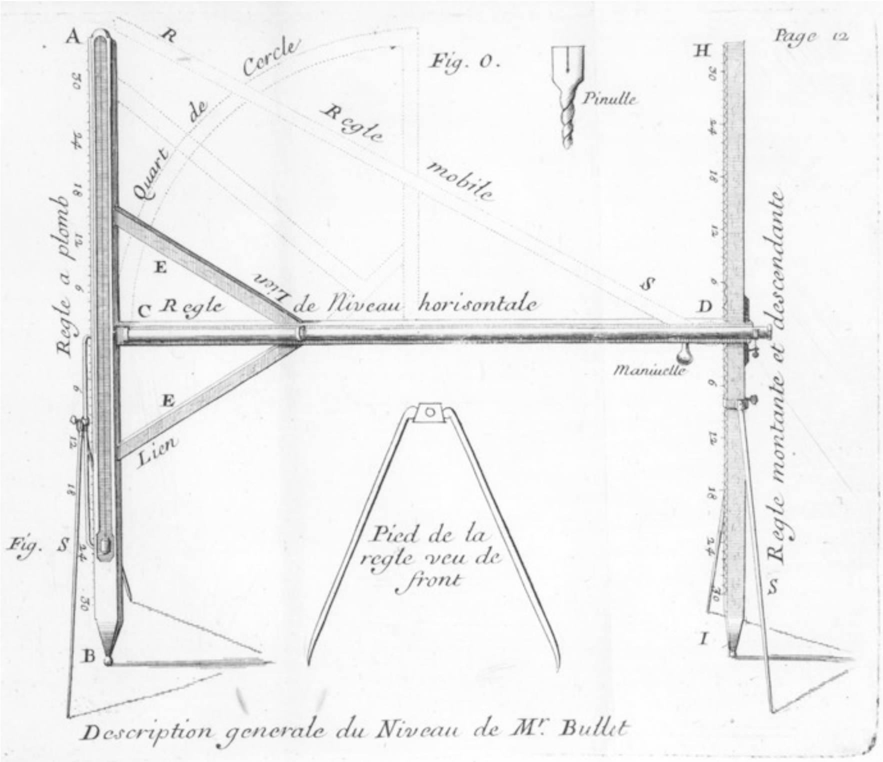


Fig. 9 Long-distance level (from Bullet 1688)

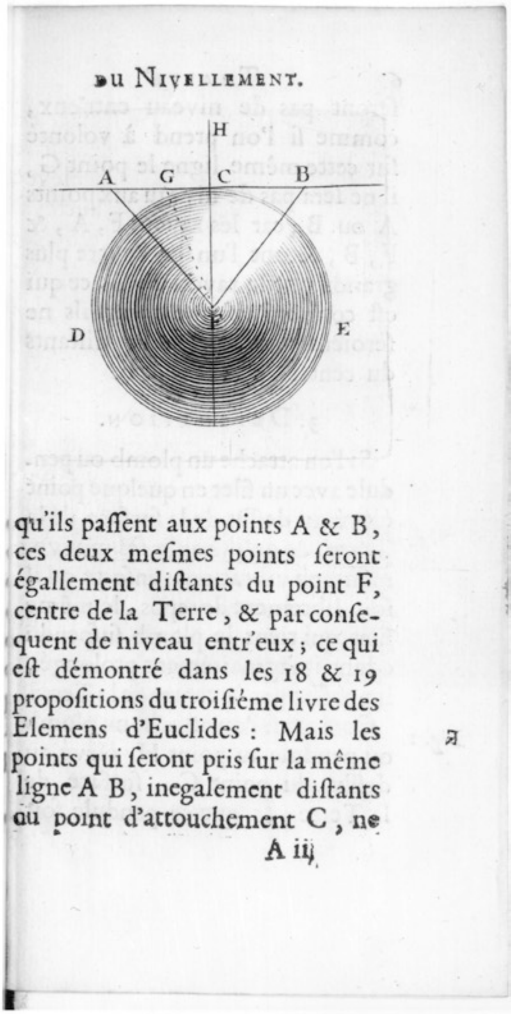
with all the advantages that entailed for cost estimates and bookkeeping, not to mention site surveying and the ability to build from accurate drawings.[42] Sangallo's notes also say something about his broader intellectual culture, particularly his receptivity to forms of mathematics that lay, strictly speaking, outside his craft. In this respect, the influence of figures like Giocondo must have been decisive. This kind of encounter is also significant in a longer historical perspective, for it was typical of the kind of intellectual relationships that architects would continue to exploit throughout the early modern period.

It is with this idea in mind that I want to adduce a third case-study. This one comes from a later moment, in 1688, toward the end of our period. In that year, the architect Pierre Bullet published a treatise on the long-distance level (Fig. 9).[43] This was an instrument used to determine precise gradients over large tracts of land, to cut slopes and terraces, for example, or to align canals or divert waterways. The principle was simple enough. One team would stabilize the device horizontally and aim it into the distance. Communicating with visual signals, a second team at the other end of the area to be surveyed would raise or lower a cardboard marker until it met the sightline. Changes in elevation were determined simply by subtracting the height of the eyepiece from the height of the marker or vice versa. Over longer distances, the cumulative rise or fall of the terrain was calculated by adding or subtracting the shifts recorded with each measurement.

The success of such operations often hinged on the precision with which they were conducted. Water diversion projects were especially tricky, as they often involved extremely small variations in height over large areas, and the distances involved would considerably magnify any small mistakes or inaccuracies in measurement. Bullet's instrument was intended to respond to this need. His level took the form of a wooden H-shaped frame, with either open sights or—more innovatively—a telescope fixed to the crossbar. The long surveying distances assumed throughout the book make it clear that Bullet favored the latter option. One upright was hollowed out to contain a plumb line, while the other could be raised or lowered by a crank to keep the crossbar level. More than earlier treatises, Bullet detailed the technique and materials to be used in constructing the instrument, an indication of the kind of audience he envisioned for the book. As his thorough instructions suggest and as Bullet himself asserted, "I have used it on several occasions where I've needed it."[44]

As is evident, the substitution of the telescope for sight vanes increased the level of precision by an order of magnitude. Bullet himself refers to the span of 400 m for a typical observation, a distance that far exceeded the effective range of more common instruments. This innovation in what is more or less a standard leveling instrument suggests some knowledge of geodesy and cartography, but in fact there are several other indications of Bullet's more-than-passing interest in these fields. He began, for example, by distinguishing the true level from the apparent. Few practitioners were aware, he complained, that the sight line produced by a telescopic level does not follow the curved surface of the earth but actually forms a tangent to that surface (Fig. 10). The longer the distance surveyed, the more the visual ray actually rises away from the true level, which follows a circular arc with the radius toward the center of the earth. He then provided a rule-of-thumb to compensate for the deviation. In distances over 200 m (100 *toises*) the apparent level has to be lowered by about 21 mm (1/12 *pouce*) to remain accurate. After 600 m, it has to be lowered by 190 mm, and so on. More accurate values for the deviation, Bullet noted, could be obtained by using sine- or log tables. In contrast, his rule was "more mechanical but also quicker and easier to understand." [45]

That a guild-trained architect would be familiar with the basic trigonometry of the problem is surprising enough. More impressive is the fact that the effective value that he employed for the radius of the Earth was taken from the most up-to-date work of "les astronomes modernes." That was not all. Bullet also warned of the refraction of the visual ray caused by water vapor in the air. "This," he explained to his readers, is "what mathematicians call parallax." In describing the phenomenon, Bullet articulated a fully-fledged mechanical theory of evaporation, noting that the rays of the sun on the surface of the earth "excite and cause large particles of water to rise with the more subtle matter."[46] His method of correcting such errors called for two leveling teams, each compensating for the discrepancy in the other's measurement. The technique was borrowed directly from astronomical practice for verifying the built-in deviation caused by non-centered lenses.


DU NIVELLEMENT.

qu'ils paſſent aux points A & B, ces deux meſmes points ſeront égallement diſtants du point F, centre de la Terre, & par conſequent de niveau entr'eux; ce qui eſt démontré dans les 18 & 19 propoſitions du troiſiéme livre des Elemens d'Euclides: Mais les points qui ſeront pris ſur la même ligne A B, inegalement diſtants du point d'attouchement C, ne

A iij

Fig. 10 The apparent level (from Bullet 1688)

Bullet's source for this information is not difficult to find. His interest in the subject of large-scale leveling followed a number of recent advances in surveying by the Académie royale des sciences, in particular by the astronomer Jean Picard, as part of his research in geodesy and large-scale cartography. As reported in his official account, Picard's procedure involved establishing an arc of meridian between two distant localities by connecting between them a series of triangles formed by prominent landmarks—typically hilltops and church steeples. Triangulation had been used for this purpose since the early sixteenth century, but Picard revolutionized the technique. By incorporating telescopes into his surveying device—an astronomical quadrant adapted to take horizontal measurements—he greatly expanded the distance between stations (Figs. 11 and 12). The resulting meridian ran through

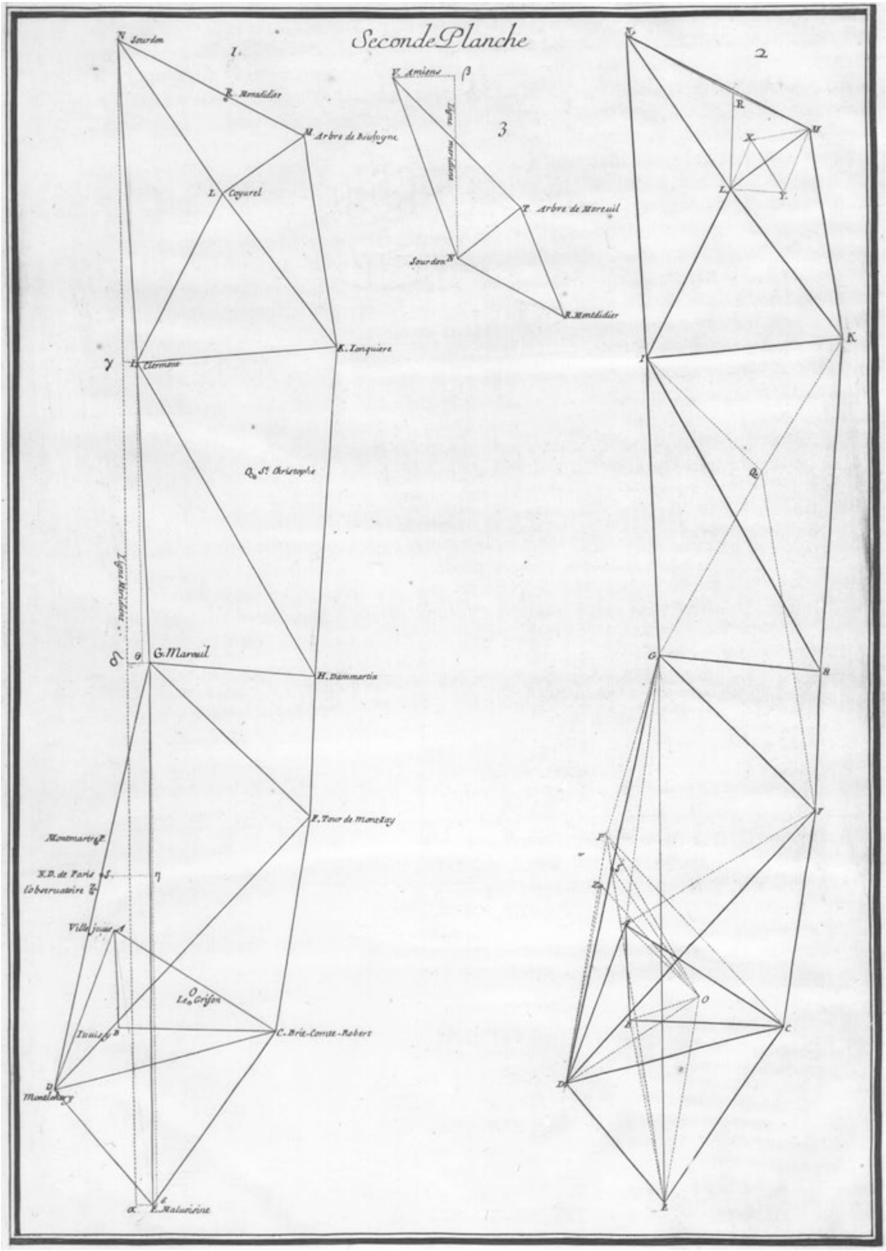


Fig. 11 Triangulation of the meridian between Malvoisine and Amiens (from Picard 1671)

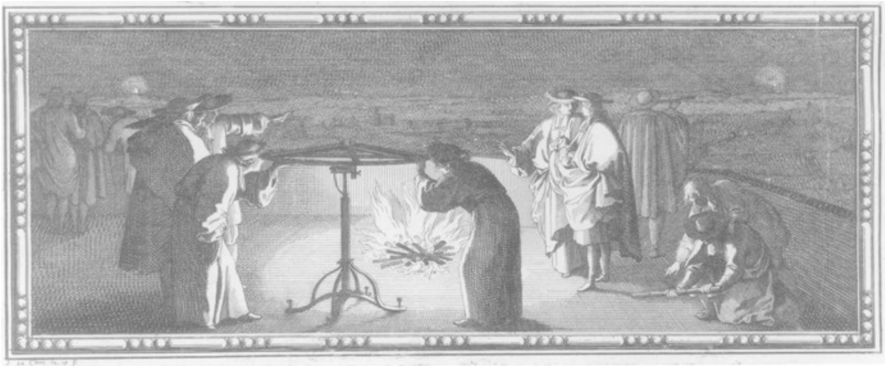


Fig. 12 Triangulation of the Paris meridian, showing a night-time observation (from Picard 1671)

Paris between Amiens and Malvoisine, a distance of about 154 km, calculated to the nearest third of a meter. By comparing the latitude at both ends of the meridian, Picard used this measure to determine the length of a terrestrial degree. His results for the Earth's dimensions were more accurate than any previously attained and were cited well into the eighteenth century. In fact, they have been shown to be within 0.15 % of the latest values for this latitude (111.38 km).[47]

As a by-product of his meridian project Picard developed a second instrument: a telescopic level—the progenitor of Bullet's—for measuring changes in elevation over long distances (Fig. 13). Unlike the telescopic quadrant, Picard's level played no role in the construction of the meridian, which was established using only angular, horizontal measurements.[48] The level was, therefore, purely a practical tool, brought about as an incidental outcome of the meridian project. Yet it was precisely this connection that legitimized it, for the results of the operation made it possible to utilize the device with previously unimaginable precision. As Picard pointed out, the newly derived value for the radius of the Earth now made it possible to calculate the rise of the apparent level over the true level, that is, the rate at which the visual ray of the telescope diverged from the curved surface of the globe. Anticipating Bullet, he even provided a table of measurements to compensate for that deviation in distances beyond 100 m. The telescope also prompted another important "philosophical" consideration: the problem of water in the atmosphere, which subjected the visual ray to downward refraction as it passed from thinner to denser air, thus raising the apparent level. In this case, rather than tinkering with readings Picard recommended the use of middle stations to avoid the problem altogether.[49] What Bullet took from Picard's work was not merely a newfangled instrument, but an optical-physical theory—partly adopted from astronomical practice—to explain and justify its use.

Picard's *Mesure de la terre*—prestigious as it was—would hardly have been enough to give the telescopic level a broad appeal. People did notice, however, what

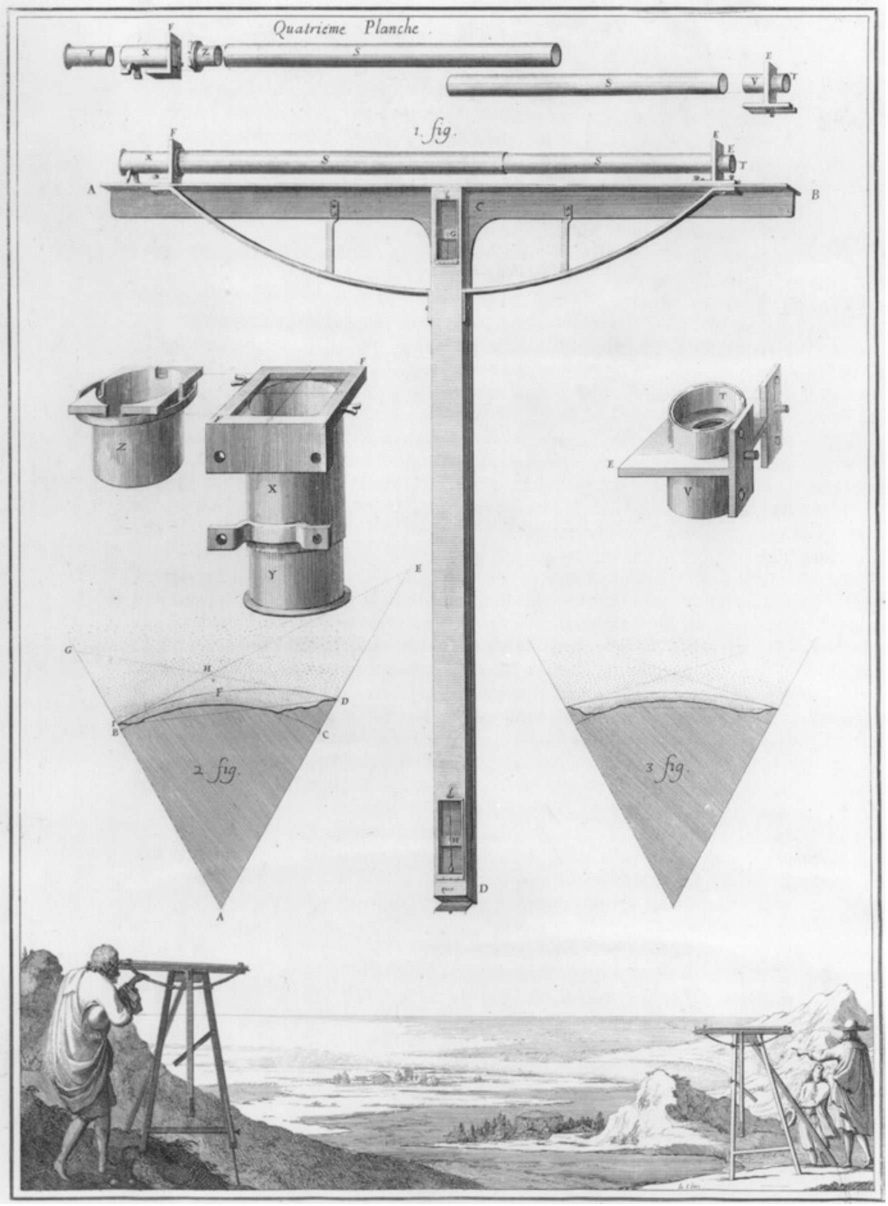


Fig. 13 Telescopic level (from Picard 1671)

the academicians did with it. In spring 1668, the academy began a long involvement with the gardens of Versailles, when they used the new instrument to establish the level of the Grand Canal. In the early 1670s, Picard and his colleagues systematically surveyed the terrain around the chateau and, from October 1677, developed the first branches of an extensive rainwater collection system to feed the garden's many fountains. Picard himself set out the channels and determined their rate of descent. Six years later, the marquis de Louvois commissioned Philippe de La Hire to extend Picard's system to the west, by damming several additional plains at consecutively higher elevations and linking them to the already existing conduits. In 1684–1685, the monumental—if ultimately aborted—project for the canal de l'Eure offered the Académie even greater scope for their abilities.[50] By the time Bullet published his own treatise, the telescopic level had been used to reshape the area around Versailles for 20 years.

The academicians knew that their work in this area had a potentially large and untapped audience. In 1684, La Hire edited and published Picard's manuscript treatise, *Traité du nivellement*, and in 1689, he wrote his own work dealing substantially with the subject, *L'Ecole des arpenteurs*. Unlike Picard's sumptuous folio volume, *Mesure de la Terre*, these were inexpensive, pocket-sized books oriented explicitly toward practitioners. The Académie's efforts to publicize their work paid off. Indeed, the added combination of efficacy and glamour associated with such devices could hardly fail to have had a wider effect. References to them began to appear in the *Journal des savants* from 1677 and in popular books on practical geometry from 1685.[51]

Bullet may have learned about the instrument from one of these texts, but a direct route is more likely; as a royal architect, he travelled in circles close to the Académie. The architect-academician François Blondel, with whom Bullet had collaborated since the late 1660s, is one possible source. From 1672, the two men worked together on a new urban plan of Paris, the first since 1652 to coordinate the streets and landmarks in a rigorous geometrical survey. For an architect interested in the latest advances in surveying and cartography, the Académie's work in this area would have been a natural focus of curiosity. More direct knowledge probably dates to 1685, during the planning of the canal de l'Eure. In spring of that year, Louvois summoned the Académie royale d'architecture to submit proposals for the aqueduct at Maintenon. A long elevation for the project is held among Bullet's papers in the Stockholm Nationalmuseum.[52] Whatever the ultimate source of Bullet's knowledge, it is the result of this interaction that stands out. His treatise represents a serious effort to respond to the Académie's discoveries in terms of everyday use. He was the first practitioner to digest these developments systematically and re-present them in a manner suitable for professional gardeners and architects.

A comparison between Bullet and Sangallo—artificial though it may be—serves to highlight some salient themes. Both men subscribed to a disciplinary identity that transcended their background in the building trades and that linked architecture with analogous—but quite distinct—forms of mathematical practice. Indeed, one of the striking similarities of this comparison is how prominently geography and astronomy figure in the work of each. Mathematics provided not only the basis of the craft, but also the means by which the architect might exceed the limitations of the builder's traditional background. New instruments were a particular focus of interest, both for the routes that they opened in to more prestigious, "theoretical" fields and for the greater control and effectiveness that they promised in everyday practice. The ambitions and motivations of both men, finally, were stimulated by personal and intellectual relationships, formed in active communities of both scholars and "enlightened" practitioners.

The comparison suggests a broad continuity of mathematical expertise and interests between elite building practitioners in the early sixteenth and late seventeenth century, but some significant differences are also worth noting. The first is the role of institutional authority in fostering a culture of both "high" and "low" mathematics. Not only did the crown sponsor Picard's high-status work in geodesy, the commissions for Versailles also spurred the Académie to develop the practical benefits of the research. The court provided, moreover, a pervasive patronage context that served to drive interest in the telescopic level. The instrument was effective and it had prestigious scientific origins, but it also contributed dramatically to the self-presentation of the king in an area that was both deeply important to him personally and central to contemporary notions of absolutist monarchy. The gardens at Versailles represented, above all, the control of nature as an expression of royal power. This patronage context—fully attuned to the material and political value of "utility"—contrasts markedly with that of Sangallo, who was able to pursue a broader interest in mathematical practice only in the shadow of his "official" architectural work.

A second difference involves the role of print. Whereas Sangallo's mathematical interests were sustained by a humanistic and erudite print culture—witness his continued engagement with Vitruvius—manuscripts were evidently still an important source of information. The extent to which he traded, copied, and collected drawings underlines both the personal nature of his collaborations and their origins in the workshop. Bullet, in contrast, had recourse to a stream of inexpensive publications, directed towards like-minded practitioners. Indeed, Bullet himself participated in this extended community, publicizing the new surveying techniques to a wider audience of architects, builders, and gardeners.

The third and most important difference has to do with the broadening scope of mathematical practice in the late seventeenth century, transformed not only by new inventions and techniques, but by novel physical claims about the natural world. These arguments—like Picard's about the accurate size of the Earth—were both derived from instrumental practice and consequently fed back again into it. For Bullet, the telescope made the "theory" of surveying—that is, its optical and physical content—integral to its use. As a result, his directions for using the instrument were replete with "scientific" content wholly novel for a surveyor's manual. What is perhaps most unexpected is the role that architects played in this cycle. Although mathematical instruments do not feature in the scholarship on early modern architecture, they were nevertheless central to the profession. They also show how the art was connected to the broader intellectual currents of the early modern period, indeed, to the scientific revolution itself.

Comparing Bullet's example to that of Stornaloco turns us back 180 degrees. The later episode—in which theory and practice directly informed each other—is the inverse of the earlier. By the late seventeenth century, Shelby's distinction between the schoolman's textual understanding of practical geometry and the mason's empirical, "constructive" approach no longer held. Indeed, they two traditions of mathematics had folded into one another.

Aims and Scope of the Volume

In addition to this long narrative trajectory, the three examples adumbrated above also suggest something of the diversity inherent in early modern architectural and mathematical practice. In the Milan cathedral episode, we see how mathematical knowledge—unequally distributed across disparate mathematical communities—informed the design of an actual project. In Sangallo's notes and drawings, we see mathematics shaping the wide-ranging interests of an individual architect, while Bullet's publication on the long-distance level reveals how innovations in mathematical and instrumental technology drove architectural practice. All three cases involve the interaction—sometimes incidental, sometimes concerted—of scholars and practitioners engaging in, for want of a less value-laden distinction, "high" and "low" forms of mathematical practice. Juxtaposing these case studies also serves a valuable end, for it evokes themes that broadly represent the historical evolution of this relationship. These transitions, though blunt, do highlight a number of broad realignments in the period covered by this volume. These include a marked increase in mathematical competence among practitioners, the shift from a culture of manuscript to one of print, and the transformation in institutional and patronage structures.

The essays in this volume may also be considered as case studies of a kind. As discrete examples of the use of mathematics in architectural practice and discourse, they reveal the diverse forms this relationship took, while also greatly expanding on the issues presented above. Setting these historical episodes against each other is likewise intended to generate questions about development and change. Our contributions are arranged chronologically and thematically into sections that follow a familiar subdivision into four broad historical "moments": Antiquity, Renaissance, Baroque, and Enlightenment. We use these terms not to reify them, but merely as convenient shorthand, to group papers into coherent sections, while also marking out some basic historical shifts. Although the contributions cover a wide time span, they are linked by a basic premise: the use of mathematics was a defining feature of early modern architectural practice, one that both characterizes the period as a whole and helps to explain developments within it.

Notes

1. See, for example, Long (2001, 2011); Ash (2004); Smith (2004); Roberts et al. (2007); Smith and Schmidt (2007); Ash (2010) and Smith et al. (2014). My thanks to Suzy Butters for alerting me to this last volume.
2. This classification originated in Aristotle's discussion of demonstrative science in the *Posterior Analytics*. See Apostle (1952, 131–136) and McKirahan (1978). On the medieval evolution of the mixed sciences, see Weisheipl (1965); Gagné (1969); and Laird (1983). For the seventeenth century, see Lennox (1986); Dear (1995, 34–40); and Remmert (2009). For the eighteenth century, see Brown (1991).
3. For an overview, see Bennett (1986, 2006). For a now-classic study of mathematical professions in Italy, see Biagioli (1989). For a recent monograph on an exemplary mathematician-practitioner, see Marr (2011). On classical geometry in practical situations, see Camerota (2006) and Brioist (2009).
4. On the practical background to Galileo's mathematics, see Lefèvre (2000); Renn et al. (2000); Renn and Valleriani (2001); and now Valleriani (2010). Fokko Jan Dijksterhuis teases out the mix of "manipulation and contemplation" in the seventeenth-century study of dioptrics. See Dijksterhuis (2007).
5. On ancient technical culture as reflected in Vitruvius, see Romano (1987, 195–219) and Rowland (2002). On his Renaissance "rediscovery", see Pagliara (1986). Also see Bernard Cache's contribution in this volume (Proportion and Continuous Variation in Vitruvius's *De Architectura*) and the Introduction to Part I (Part I: Foundations).
6. The literature on Barbaro has grown considerably in recent years. For his translation of and commentary on Vitruvius, see Barbaro (1997). On Barbaro's intellectual context, see Tafuri (1987, 1989, 114–38). On Barbaro's reconstruction of the Vitruvian analemma, see Losito (1989a, b). More recently, see Mitrovic (2004); Cellauro (2004); and Howard (2011).
7. On Alberti, see Wassell et al. (2010); Carpo and Furlan (2007); and March (1998). On Baldi, see Serrai (2002); Becchi (2004); and Nenci (2005). On Philandrier, see Lemerle (2000, 2011). On Gallaccini, see Payne (2012). On Goldmann, see Goudeau (2005, 2006–07). On Blondel, see Gerbino (2010). On Poleni, see Lenci (1975) and Soppelsa (1988).
8. See, for example, the work of the *Avista Forum*, the *Nexus Network Journal*, the *Associazione Eduoardo Benvenuto*, and the *Construction History Society*. Each of these bodies promotes research that falls partially under the rubric adopted here.
9. See, for example, Keller (1976); Gabbey (1993); and Meli (2006). For examples of the impact of mechanics in architecture, see Schlimme (2006) and Gargiani (2008).
10. On the abuse of proportions in the nineteenth and early twentieth century, see Wittkower (1960). Also see the prefatory remarks in Wilson Jones (2000, 1–6) and Bork (2011, 11–20). For some methodological guidelines, see Fernie (1990, 2002). For an exemplary recent study, which promises to set new methodological standards for the subject, see Cohen (2008), now expanded in Cohen (2013).

11. Early modern architects also worked within other broad disciplinary matrices. Engineering, for example, involved a similar combination of drawing, mathematics, print culture, antiquarian study, and large-scale construction. For different approaches to this tradition, see Vérin (1993); Conforti (2002); Long (2008); and Maffioli (2010). As an actors' category, the concept of *disegno* is also relevant here, particularly for the way it united different art forms, including architecture, with the practice of drawing and mathematics. See Barzman (2000, 143–80) and Marr (2011, 167–76).
12. On the *scuole dell'abaco*, see Goldthwaite (1972). On the implications of the new system, see Swetz (2002). On the mathematical education of craftsmen, see Zervas (1975) and Shelby (1970, 1972, 1977).
13. On classical and medieval design methods respectively, see Jones (2000) and Bork (2011). For recent scholarship on drawing conventions and setting-out, see Toker (1985); Ousterhout (1999, 58–85); Davis and Neagley (2000); Wu (2002); Carpo (2003); Rossi (2004); Gerbino and Johnston (2009, 17–44); Hadjitryphonos (2009); and, most recently, Yeomans (2011). On the relationship between architectural practitioners and scholars in an Islamic context, see Özdural (1995).
14. See Ackerman (1949), reprinted in Ackerman (1991) and Romanini (1964, vol. 1, 351–415). There is still some disagreement about whether the Lombards' discomfort with the Gothic design system was characteristic of Italian architects in general. For two recent interpretations, see Ascani (1997, 115–27) and Bork (2011, 411–20).
15. Older literature concentrates on the metrical content of the two superimposed drawings and their role in the project's history. See Ghisalberti (1994), with a round-up of earlier studies, and Ascani (1991). More recently, scholars have become attuned to the drawings' unusual visual conventions. See Sakarovitch (1998, 45); Ackerman (2002, 45–46); and Bork (2011, 413–14).
16. "Deliberaverunt, quod discreto viro Gabrieli Stornalocho de Placentia experto in arte geometriae, pro quo missum fuit parte deputatorum dictae fabricae juxta deliberationem in consilio dictae fabricae factam die 24 septembris p.p. et Mediolanum venit… causa discutendi cum inzingneriis dictae fabricae de dubbis altitudinis et aliorum de quibus dubium erat inter dictos inzignerios… dentur… dono pro recognitione et recumpensatione expensarum per eu factarum veniendo…" Cantù (1877–85, vol. 1, 55) (13 October 1391), quoted in Ackerman (1949, 90).
17. Frankl and Panofsky (1945).
18. Stornaloco's cryptic, but crucial, sentence reads "Erit […] altitudo summitatis eclexie radix dc dcc mxx XXVII/[sesquialtera] quia tregesime, quod est aliquid minus de LXXXIIII." Including emendations and in modern notation, Panofsky's paraphrase would read, "the maximum height of the church will be [a] root [approximated by] $600 \times 700 \div 1010$." This gives 415.842, which is 10 times the height divided by two, or 83.168. The actual height of an equilateral triangle with a side of 96 is 83.138, which diverges from Stornaloco's result, as Panofsky pointed out, by less than .04 %. The second part of the sentence can be paraphrased "[the height is also approximated by] $10 \times (27 \times 1½)$, because

[27] is one thirtieth [of ten times the height], which is somewhat less than 84...." The result, 405, is only roughly approximate to 415.841, but if 28 represents the upper whole-integer limit for "one thirtieth" of 10 times the height, 27 is the lower limit for that value. See Frankl and Panofsky (1945, 61–64).

19. The drawing accompanying the letter no longer survives, but it is known from later copies, including a version made by Cesare Cesariano, which he published in his 1521 edition of Vitruvius. A simplified, nineteenth-century copy by Luca Beltrami, shown here, is generally accepted as an adequate substitute. See Frankl and Panofsky (1945). Beltrami's diagram was originally published in Beltrami (1887), reprinted in Ramelli (1964, 37–91, esp. 73).
20. Ackerman (1949, 93–95).
21. Although the methods used to set out such shapes at full scale remain unknown, surviving architectural drawings often show a very close match to the built work. See Bork (2011) for several examples, including the equilateral triangles in the nave and façade of Strasbourg cathedral (64–76) and the octagons in the section of Prague cathedral (205–14). For the use of pentagonal geometries, see his remarks on 32–33.
22. See Cohen (2010) and, now, Cohen (2013). For an exploration of similar issues in a northern context, see Nussbaum (2011).
23. Peter Kidson, in particular, has long argued for the advanced numeracy of medieval masons on just these grounds. He has pointed to metrical evidence, for example, to suggest that various local measures were related to Roman ones as more-or-less accurate approximations of $1{:}\sqrt{2}$ and of the "golden section" (in geometrical terms, the ratio of the length of a 2:1 rectangle to the sum of its diagonal and its width, or $1{:}\ (1+\sqrt{5})/2$). See Kidson (1990). He has also argued for similar relationships in medieval buildings. His analysis of Salisbury Cathedral, based on a measured survey, emphasizes dimensions that appear to be related in ratios of 4:9 and 5:11 (both for $1{:}\sqrt{5}$), as well as 5:8 (for the golden section). See Cocke and Kidson (1993, 62–82). More recently, see Kidson (2008).
24. Peter Kidson has offered an analogous explanation: that Stornaloco was called not to provide the cathedral workshop with a solution to the problem, but merely as a security measure. In this interpretation, someone in authority felt it necessary to call on an "expert" to confirm a decision that had already been made. The same author also proposes an alternative interpretation of Stornaloco's letter (see below, note 25): Kidson (1999). For similar skepticism about Stornaloco's role, see Beaujouan (1975).
25. See Frankl and Panofsky (1945). Peter Kidson's analysis of Stornaloco's letter—which differs considerably from Panofsky's—proposes that the mathematician's formula was based implicitly on the ratio 256/153, as an approximation for $\sqrt{3}$. However, this interpretation, while ingenious, is not as convincing as the one it is intended to replace. Panofsky's has several advantages: it accounts for the lack of decimal notation in the period; it is more clearly related to the wording in Stornaloco's description; it provides several parallels in contemporary mathematics; and it is generalizable for any given input (as Panofsky was

careful to point out, the algorism needed to be formulated in terms of a any given module). See Kidson (1999).

26. For the conference minutes for this period, see the transcription in Ackerman (1949, 108–11). Documents of the *Opera* of Santa Maria del Fiore in Florence show that Hindu-Arabic numerals begin to overtake Roman ones from 1411. See Cohen (2010, 16, 2013, 244). On Renaissance architects' knowledge of arithmetic, see Carpo (2003) and Ann Huppert's contribution in this volume (Practical Mathematics in the Drawings of Baldassarre Peruzzi and Antonio da Sangallo the Younger).
27. See Shelby (1972). On the medieval *Practica geometriae*, see Saint-Victor (1991) and Victor (1979). For two recent attempts to challenge Shelby's rather strict distinction between mathematical and masonic practices, see Zenner (2002) and Liefferinge (2010).
28. Friedman (1988).
29. Cohen (2008).
30. Serlio (1996–2001, vol. 1, 5). Although purely practical in aim, the *Primo Libro* is nevertheless organized according to a deductive, "Euclidean" structure, in which preceding constructions furnish the concepts necessary to complete subsequent ones. Alberti begins *De pictura* with a similar formulation. This method contrasts with Albrecht Dürer's *Underweysung der Messung* (Nuremberg, 1525), which is otherwise taken as Serlio's model. See Lorber (1989).
31. On Renaissance innovations in the theory and practice of structural design, see Sanabria (1982, 1989); and Betts (1993). On the rise of numeracy and arithmetic calculation, see Carpo (2003). For examples of the application of trigonometry to town planning, see Friedman (1988, 117–48) and Jäger (2004).
32. There were ancient and medieval precedents for such classifications. For one Hellenistic classification system, see Downey (1948). For medieval schemes and early Renaissance schemes involving fortification, see Wilkinson (1988). Alina Payne mentions several sixteenth-century examples in Payne (1999). For the English context, see Bennett (1993, 23–30).
33. The drawings are held in the Gabinetto dei Disegni e Stampe degli Uffizi, Florence, Architettura (hereafter UA). See Frommel and Adams (1994–2000).
34. See the still-useful overview by Ackerman (1954, 3–11). For a basic biography, see Bruschi (1983). On the role of drawings in Antonio's career, see Frommel and Adams (1994–2000, vol. 1, 1–60).
35. Pier Nicola Pagliara and Gian Luca Veronese, "U 856A *recto*" and "U 856A *verso*," in Frommel and Adams (1994–2000, vol. 1, 156–58). William E. Wallace, however, notes that there is also a degree of literalism in Sangallo's arithmetic, which appears rather alien to modern modes of calculation. See Wallace (1995).
36. See Maccagni (1993, 1996). An earlier example of such a manuscript from the hand of an architect is Francesco di Giorgio Martini's practical geometry, focusing largely on surveying problems. See Martini (1970).
37. Pier Nicola Pagliara and Gian Luca Veronese, "U 857A *recto*," in Frommel and Adams (1994–2000, vol. 1, 158). Also see Veronese, "U 851A *verso*," and "U

1478 *recto*," in Frommel and Adams (1994–2000, vol. 1, 154, 240–41). On Piero's mathematics, specifically his *Trattato d'abaco*, see Davis (1977) and Field (2005, 6–32, 119–28). On Pacioli, See Jayawardene (2008) and Baldasso (2010). On Sangallo's study of Vitruvius, see Pagliara (1986, 46–55, 1988).

38. Nicholas Adams, "U 850A recto" in Frommel and Adams (1994–2000, vol. 1, 153–54). On the star chart, see Maria Losito, "U 1459A *recto* and *verso*," in Frommel and Adams (1994–2000, vol. 1, 231–32). Also see Snyder (1993, 40–43) and Shirley (2001).
39. In addition to recording instruments himself—see his similarly detailed study of a trecento quadrant (U 1455A recto)—Antonio also collected such drawings. Scaglia cites anonymous reproductions of two other Moorish astrolabes among the architect's former papers. See Gustina Scaglia, "U 1454A *recto* and *verso*," in Frommel and Adams (1994–2000, vol. 1, 227–29). Sangallo's transcription skills appear to go well beyond those of his contemporaries. On Italian artists' fascination with Kufic script, see Mack (2002).
40. See Nicholas Adams, Pier Nicola Pagliara, and Gian Luca Veronese, "U 1491A *recto and verso*," in Frommel and Adams (1994–2000, vol. 1, 246–47). For Sangallo's preparatory studies, see 856A*r*, 1456A*r*, 1457A*r*, 1466A*r*, 1499A*r*, 1500A*r*, 3949A*r*. On the history of the proportional compass, see Camerota (2000, 5–19). On Galileo's sector, see Drake (1978). On the later adaption of the sector to architecture in the seventeenth and eighteenth century, see Bruti (1627) and Gerbino and Johnston (2009, 111–51).
41. See Gustina Scaglia, "1463A recto and verso," in Frommel and Adams (1994–2000, vol. 1, 233–34). For the most recent biography, see Pagliara (2001). On Giocondo's Vitruvius, see Juřen (1974), Ciapponi (1984), and Rowland (2011). On his mathematical manuscripts, see Tura (2008).
42. On this subject, also see Thoenes (1990) and Carpo (2003, 463, 468–9). On Sangallo's drawing conventions, see Lotz (1977) and Lefèvre (2004).
43. Bullet (1688). More broadly on this subject, see Gerbino (2008).
44. Bullet (1688, Unpaginated Preface).
45. Bullet (1688, 29–35).
46. Bullet (1688, 60–65); Gerbino (2008, 89).
47. Picard (1671). See also Taton (1987). For the comparison of modern results with Picard's, see Levallois (1987). For a longer perspective, see Gallois (1909). More recently, see Pelletier (2002).
48. Changes in elevation between stations were disregarded, as these were considered to be minimal in relation to the circumference of the Earth. Picard (1671, 16).
49. Picard ignored refraction while establishing the meridian, as this phenomenon affected the direction of the visual ray only on the vertical plane, leaving the angular measurements between stations unaltered. Picard (1671, 27–28).
50. For Picard's work on the reservoir system, see Loriferne (1987). For a still-useful overview of the whole water management system, see Barbet (1907). For the Académie's later involvement, see Gerbino (2008).

51. *Journal des sçavans* (1677, 227–28); (1678, 441–43); (1679, 215–16); (1680, 21–24, 174–76, 275–76). Also see Deshayes (1685, 9); Du Torar (1688, 182–85); Clermont (1693, 112); and Ozanam (1693, 241–44). On the reception of the instrument, see Gerbino (2008).
52. On Bullet's collaboration with Blondel, see Gerbino (2010, 71–117). On his later involvement at Versailles, see Lemonnier (1911–1929, vol. 2, 71–91). For the aqueduct drawing, see Walton (1985, 52–53).

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Archivio della Fabbriceria di San Petronio: Fig. 1
Author: Fig. 2
Beltrami (1887, 73): Fig. 3
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Filippo Camerota: Fig. 10
Bibliothèque Nationale de France : Figs. 11–13

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