Interrelationship of Fuzzy Decision System Parameters

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Abstract. The fuzzy decision systems contain fuzzified input and outputs parameters sequentially in premises and consequences of the decisions rules. The interrelationship of those can be very different from the basic relations described with if... then rules till the correlations of input parameters described with cognitive maps. The paper gives a summary of the fuzzy parameters' inter-relationship investigated by the authors in recent years based on fundamental results published by Imre J. Rudas, or investigated in joint works with him.

Keywords: fuzzy decision making, distance based operators, AHP, risk management.

1 Introduction

The interrelationship of the system parameters of fuzzy decision making system can be very different from the basic relations between parameters of premises and parameters of consequences of the if... then rules till the correlations and quantitative representation of relationships of input parameters from the set of premises.

Fuzzy decision making systems can be constructed from if A then B types of rules, where A and B are fuzzified system parameters and the mathematical background of the calculation is based on the definition of fuzzy relations and basic binary opera-tions of t-norms and conorms [9].

In the last few decades in fuzzy control systems the Mamdani type of decision model is widely used, and beyond the min and max operators others are also investigated in theoretical and practical environment in order to increase the efficiency of the system operation.

The basic interrelationship of the fuzzy system parameters is this one, the relationship between the t-norm or conorm operators or other aggregation operators applied in fuzzy based approximate reasoning methods. Uninorms, and especially generalized distance based operators, introduced by Rudas [1] continue to bring new possibilities in fuzzy systems models. Distance-based operators as the basic operators applied in approximate reasoning resulted investigation of similarity measures of fuzzy sets and residuum of those special operators [2].

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Other investigations are focused on the representation of distance based operators in the group of uninorms and general aggregation operators. Considering that the uninorms are parameter-dependent norms, it is possible to investigate the behavior of the fuzzy decision making or control system by changing or sliding the parameter values [3].

In complex systems it is very important to recognize the measure of interaction and the measure of importance of the system parameters. This knowledge can help one to construct the structure of the decision system. The models investigated by the authors and summarized in this paper are related to the risk management systems, and the studied methods are the hierarchical construction of decision making model and the AHP model [4].

2 Distance Base Operators

2.1 Definition and Special Properties

The distance-based operators can be expressed by means of the min and max operators as follows [5]:

- the maximum distance minimum operator with respect to $e \in [0, 1]$ is defined as

$$T_e^{\max} = \max_e^{\min} = \begin{cases} \max(x, y) & \text{if } y > 2e - x \\ \min(x, y) & \text{if } y < 2e - x \\ \min(x, y) & \text{if } y = 2e - x \end{cases} \quad (1)$$

- the minimum distance minimum operator with respect to $e \in [0, 1]$ is defined as

$$T_e^{\min} = \min_e^{\min} = \begin{cases} \max(x, y) & \text{if } y > 2e - x \\ \min(x, y) & \text{if } y < 2e - x \\ \max(x, y) & \text{if } y = 2e - x \end{cases} \quad (2)$$

- the maximum distance maximum operator with respect to $e \in [0, 1]$ is defined as

$$S_e^{\max} = \max_e^{\max} = \begin{cases} \max(x, y) & \text{if } y > 2e - x \\ \min(x, y) & \text{if } y < 2e - x \\ \max(x, y) & \text{if } y = 2e - x \end{cases} \quad (3)$$

- the minimum distance maximum operator with respect to $e \in [0, 1]$ is defined as

$$S_e^{\min} = \min_e^{\max} = \begin{cases} \min(x, y) & \text{if } y > 2e - x \\ \max(x, y) & \text{if } y < 2e - x \\ \max(x, y) & \text{if } y = 2e - x \end{cases} \quad (4)$$

The modified distance based operators described above are changed in the boundary condition for neutral element e:

- the maximum distance minimum operator and the maximum distance maximum operator with respect to $e \in]0, 1]$,

- the minimum distance minimum operator and the minimum distance maximum operator with respect to $e \in [0,1[$.

The distance-based operators have the following properties :

- $\max_e^{\min}$ and $\max_e^{\max}$ are uninorms,
- the dual operator of the uninorm $\max_e^{\min}$ is $\max_{1-e}^{\max}$, and
- the dual operator of the uninorm $\max_e^{\max}$ is $\max_{1-e}^{\min}$.

Based on results from [6] we conclude:

Operator $\max_{0.5}^{\min}$ is a conjunctive left-continuous idempotent uninorm with neutral element $e \in]0,1]$ with the super-involutive decreasing unary operator

$$g(x) = 2e - x = 2 \cdot 0.5 - x \quad \Rightarrow \quad g(x) = 1 - x.$$

Operator $\min_{0.5}^{\max}$ is a disjunctive right-continuous idempotent uninorm with neutral element $e \in]0,1]$ with the sub-involutive decreasing unary operator [2]

$$g(x) = 2e - x = 2 \cdot 0.5 - x \quad \Rightarrow \quad g(x) = 1 - x.$$

2.2 Distance-Based Group of Operator in Fuzzy Inference Mechanism

In control theory much of the knowledge of a controller can be stated in the form of if-then rules, involving some variables. The fuzzy theory and fuzzy logic control has been carried out searching for different mathematical models in order to supply these rules. The Mamdani type of decision model is widely used in control problems. In this model the IF x is A THEN y is B rule is modeled just as an connection between so called rule premise: x is A, and rule consequence: y is B, where A and B are fuzzy sets, and sequentially x is the rule input variable from the universe X, and y is the rule output variable from universe Y. The connection is represented by t-norm types of operators. From set of if ... then ... rules the rule base system is constructed describing the system behavior. When the system works, the influence of the system input is investigated based on the given rule base. This influence is represented by the system output. The algorithm and mathematical calculation of the actual system output is the inference mechanism. One of the widely used methods for inference calculation in fuzzy control theory is the generalized modus ponens (GMP). The system output y is B' (similar to rule output) is obtained when the proposition are: the rule

IF x is A THEN y is B,

and the system input x is A' (similar to rule premise).

In Mamdani type of inference the general rule consequence for the i-th rule from the rule base system is obtained by

$$B_i'(y) = \sup_{x \in X} \Big(T1\Big(A'(x), T2\big(A_i(x), B_i(y)\big)\Big)\Big), \quad x \in X, \quad y \in Y. \tag{5}$$

The connection $T1$ and $T2$ are generally defined, and they can be some type of fuzzy conjunctive operators.

If we use the same T operator instead of $T1$ and $T2$ operators, based on the t-norm operators' properties, from the above expression follows

$$B_i'(y) = \sup_{x \in X} \Big(T\Big(A'(x), T\big(A_i(x), B_i(y)\big)\Big)\Big). \tag{6}$$

Generally speaking, the consequence (rule output) is given with a fuzzy set $B'(y)$, which is derived from rule consequence $B(y)$, as a cut of the $B(y)$. This cut,

$$\mathrm{DOF}_i = \sup_{x \in X} T\big(A'(x), A_i(x)\big) \tag{7}$$

is the generalized degree of firing level of the rule, considering actual rule base input $A'(x)$, and usually depends on the covering over $A(x)$ and $A'(x)$. But first of all it depends on the sup of the membership function of $T\,(A'(x), A(x))$. Rule base output B'_{out} is an aggregation of all rule consequences $B_i'(y)$ from the rule base. As aggregation operator a disjunctive operator (conorm) is usually used.

$$B'_{\text{out}} = S\Bigg(B'_n(y), S\Big(B'_{n-1}(y), S\Big(\ldots S\big(B'_2(y), B'_1(y)\big) \ldots \Big)\Big)\Bigg). \tag{8}$$

If in the application a crisp output y_{out} is needed, it is constructed as a crisp value calculated with a defuzzification method, from rule base output, for example with the center of gravity method, given by

$$y_{\text{out}} = \frac{\int\limits_Y B'_{\text{out}} \cdot y \, \mathrm{d}y}{\int\limits_Y B'_{\text{out}} \, \mathrm{d}y}. \tag{9}$$

It can be concluded, that in decision making approximate reasoning the (T, S) pair of operators are used.

Instead of the operators T and S an operator from the group of distance-based operators can be chosen. Considering the structure of distance based operators, namely that they are constructed by the min and max operators; it was worth trying to move away from the strictly applied max (disjunctive) and min (conjunctive) operator pair in approximate reasoning. Therefore, in a simulation systems different operators from the group of distance based operators were applied as disjunctive and conjunctive ones. Moreover, the distance based operators are parameterized by the parameter e, therefore the program, which performs the task of decision making in the simulation system, has global, optional, variables (Con, Dis, e), where Con is the operator applied by GMP, and the Dis is the aggregation operator for the calculation of the B'_{out}.

The neutral element of the Con operator is the parameter e, and the neutral element of the Dis operator is the parameter $1 - e$. Details about the simulation results can be found in [3]. Hence and because by the simulation the triple (Con, Dis, e) can be chosen by even running of the simulation system, it enables the parameters to be set at every running of the system in order to achieve greater efficiency.

Although the minimum plays an exceptional role in fuzzy control theory, there are situations requiring new models. In system control one would intuitively expect: to make the powerful coincidence between fuzzy sets stronger, and the weak coincidence even weaker. The distance-based operators group satisfy these properties, but the covering over $A(x)$ and $A'(x)$ are not really reflected by the sup of the membership function for example if we use $\min_e^{\max}$ to calculate degree of firing as $\min_e^{\max}\big(A'(x), A_i(x)\big)$.

Hence, and because of the properties of distance-based operators, it was unreason-able to use the classical degree of firing (7), to give expression of the coincidence of the rule premise (fuzzy set A), and system input (fuzzy set A'), therefore a Degree of Coincidence (Doc) for those fuzzy sets has been initiated. This is actually the propor-tion of area under membership function of the distance-based intersection of those fuzzy sets, and the area under membership function of their union (using max as the fuzzy union).

$$\mathrm{Doc}_i = \frac{\int\limits_X \min_e^{\max}\big(A_i(x), A'(x)\big)\,\mathrm{d}x}{\int\limits_X \max\big(A_i(x), A'(x)\big)\,\mathrm{d}x} \tag{10}$$

This definition has two advantages:

- it considers the whole measure of coincidence of A_i and A', and not only the "height", the sup of the coincidence, and
- the rule output is weighted with a measure of coincidence of A_i and A' in each rule.

How to get the rule output?

The rule output can be the cut of the rule consequence, in this case

$$B_i'(y) = \min\big(\mathrm{Doc}_i, B_i(y)\big). \tag{11}$$

Despite the fact that Mamdani's approach is not entirely based on compositional rule of inference, it is nevertheless very effective in fuzzy approximate reasoning. Because of this it is possible to apply several t-norms, or, as in considered case, distance based operators. This leads to further tasks and problems. The problem of the measurement of covering over of the rule premise and rule input is partly solved with the degree of coincidence. But in any case there must be a system of conditions that is to be satisfied by the new model of inference mechanism in fuzzy systems [8].

For a given input fuzzy set $A'(x)$, in a mathematical-logical sense, the output fuzzy set $B_i'(y)$ in one rule, can be generated with the expression

$$B_i'(y) = \max\left(B_i(y), \sqrt{1-\mathrm{Doc}_i}\right). \tag{12}$$

It is easy to prove, that $\mathrm{Doc}_i \in [0,1]$, and $\mathrm{Doc}_i = 1$ if A_i and A' cover over each other, and $\mathrm{Doc}_i = 0$ if A_i and A' have no point of contact.

Several pairs of distance-based operators have been tried out in a simulation system for a control problem, with special emphasis on the pairs $\left(T_e^{\max}, S_{1-e}^{\max}\right)$ and $\left(T_e^{\min}, S_{1-e}^{\min}\right)$.

The choosing of pairs $\left(T_e^{\max}, S_{1-e}^{\max}\right)$ and $\left(T_e^{\min}, S_{1-e}^{\min}\right)$ by the simulation, using the same e value, gives results with negligible difference. So it was sufficient trying out the pairs $\left(T_e^{\max}, S_{1-e}^{\max}\right)$ for example. The choosing of the pair $\left(T_e^{\max}, S_{1-e}^{\max}\right)$, where e is near zero, return in short time the desired state of the system, but it is not stable. If e is near 1, the situation is known, because it develops to choosing of pair $(\min, \max)$. The desired state is obtained easier, and the systems stay stable. It can be observed, that continual sliding of e from zero to 1 results continual improvement in stability, and continual increasing time of obtaining desired state in the system. The choosing of pair $(T_{0.5}^{\max}, S_{0.5}^{\max})$ gives acceptable result by both criteria [7].

3 Interrelationship of the Input Parameters in Complex Systems

A risk model is a multi-parameter and multi-criteria decision making system. The complexity of the systems increases the runtime factor by the decision, and the large system parameter set has not a user-friend transparency. The traditional well-known models work without management of the uncertainties. The complexity and uncertainties in those systems raise the necessity of soft computing based models. The use of fuzzy sets to describe the risk factors and fuzzy-based decision techniques to help incorporate inherent imprecision, uncertainties and subjectivity of available data, as well as to propagate these attributes throughout the model, yield more realistic results. The structural modeling of risk and disaster management is case-specific, but the hierarchical model is widely applied. The system characteristics are as follows: it is a multi-parametrical, multi-criteria decision process, where the input parameters are the measured risk factors, and the multi-criteria rules of the system behaviors are included in the decision process. The Analytical Hierarchy Process (AHP) expands this complex system with the pairwise comparison of the factors' importance and interaction [10].

The techniques used in risk management have been taken from other areas of system management. The first step is the identification of risks and potential risks to the system operation at all levels. Evaluation, the measure and structural systematization of the identified risks, is the next step. Measurement is defined by how serious the risks are in terms of consequences and the likelihood of occurrence. It can be a qualitative or quantitative description of their effects on the environment. Plan and control are the next stages to prepare the risk management system. This can include the development of response actions to these risks, and the applied decision or reasoning method. Monitoring and review will ensure that the risk management process is dynamic and continuous, with correct verification and validity control.

Generally, the risk management system in its preliminary form is a knowledge-based model, where objective and subjective knowledge is included in the decision process. Considering the possible uncertainties and imprecision, and the large number or quantitative description of the parameters, we can conclude that the fuzzy set theory extended with the AHP matrixes manage complexity [11].

Fuzzy-based risk management models assume that the risk factors are fuzzified (because of their uncertainties or linguistic representation); furthermore the risk man-agement and risk level calculation statements are represented in the form of *if premises then conclusion* rule forms, and the risk factor calculation or output decision (summarized output) is obtained using fuzzy approximate reasoning methods. Considering the fuzzy logic and fuzzy set theory results, there are further possibilities to extend fuzzy-based risk management models modeling risk factors with type-2 fuzzy sets, representing the level of the uncertainties of the membership values, or using special, problem-oriented types of operators in the fuzzy decision making process [4].

The hierarchical or multilevel construction of the decision process, the grouped structural systematization of the factors, with the possibility of gaining some subsystems, depending on their importance or other significant environment characteristics or on laying emphasis on risk management actors, is a possible way to manage the complexity of the system [12].

4 The Present and Planned Further Works

The recent works of the authors of this paper are related to the interrelationship of fuzzy decision system parameters based on the fuzzy cognitive maps of them. It is obvious that the AHP matrix and weights of fuzzy cognitive maps have a similar role in the relationship description of the system parameters, but the determination, and later the tuning of them offers new challenges for as the experts. Current active fields of investigation regarding interrelationship description of the fuzzy system parameters includes risk management, medical diagnostic problems and the student work evaluation.

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