Chapter 9 Manfred R. Schroeder: A Personal Memoir, Optimizing the Reflection Phase Grating

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I did not have the privilege of working directly with Manfred Schroeder, but rather I was motivated and inspired by the development of the reflection phase grating in the late 1970s. This research was the spark that ignited my passion for acoustics and converted my avocation for music and recording into a second vocation and the birth of RPG Diffusor Systems, Inc.

This story begins in 1980, in the conference room of the Laboratory for the Structure of Matter at the Naval Research Laboratory (NRL) in Washington, DC, Fig. [9.1](#page-1-0), where I was employed as a diffraction physicist. Knowing my interest in music, a colleague handed me the latest issue of Physics Today, Fig. [9.2,](#page-2-0) with a cover photo of Manfred Schroeder seated in an anechoic chamber. This was my first virtual meeting with Manfred Schroeder. The article suggested using number theoretic diffusors in concert halls to provide lateral reflections. While my interest at the time was not in concert halls, and in fact at this time my only link to the field of acoustics was a love of composing, recording and performing music, I became fascinated with the thought of using these diffusors in a renovation of Underground Sound, Fig. [9.3](#page-3-0), a private recording studio I originally built in 1972 with Jerry Ressler, a colleague and fellow musician. The acoustic renovation utilized a new concept called Live End Dead End proposed by Don and Carolyn Davis, of Synergetic Audio Concepts ["The LEDE concept for the control of acoustic and psychoacoustic parameters in recording control rooms," J. Audio Eng. Soc., 28, 585–95 (1980)].

At NRL, I was examining the three-dimensional structure of matter in various phases, using electron, X-ray, and neutron diffraction techniques, Fig. [9.4.](#page-3-0) I shared the article with John Konnert, a colleague in my group, and it became apparent that the "reflection phase gratings" suggested by Schroeder were in effect,

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Fig. 9.1 U.S. Naval Research Laboratory, Washington, DC

two-dimensional sonic crystals, which scatter sound in the same way that threedimensional crystal lattices scatter electromagnetic waves, Fig. [9.4.](#page-3-0) Since the diffraction theory employed in X-ray crystallographic studies were applicable to reflection phase gratings, it was straightforward for us to model and design the reflection phase gratings. The central theoretical connection was the coherent diffraction equation developed by Sir Lawrence Bragg, shown in Fig. [9.4](#page-3-0), where the product of two times the periodicity repeat, d , and the sin of the diffraction direction, θ , equaled the wavelength, λ , times and integer *n*.

Having scientific backgrounds, John Konnert and I approached acoustics as we did the field of diffraction physics and began researching and publishing findings in the scientific literature. The Audio Engineering Society and Syn-Aud-Con offered a unique forum and community for discussing the research. In October 1983, at the 74th AES Convention in New York, I presented our research on the Schroeder diffusor in the Studio Design Session C, shown in Fig. [9.5,](#page-4-0) with a bit of intimidation, because Manfred Schroeder was the lead-off invited speaker. As part of this presentation, I described an Apple II program, shown in Fig. [9.6](#page-5-0), which allowed acousticians to design these phase gratings, as well as plot their diffraction patterns. Following the session, Manfred and I had our first personal meeting where he enlightened me on the use of the Chinese Remainder Theorem, which enabled the creation of a two-dimensional primitive root sequence from a longer one-dimensional sequence, maintaining the beneficial Fourier property of a flat power spectrum. In Fig. [9.7,](#page-6-0) I show his handwritten notes in the 74th Technical Meeting & Professional Exhibits AES Program Oct. 8–12, 1983, illustrating the diagonal filling process, making use of periodicity, for several primes, N, for which $N-1$ could be factored into two relative coprimes, i.e., it cannot be used for $N=5$.

Fig. 9.2 Cover of Physics Today, October 1980, where I first was introduced to Manfred Schroeder

Two periods of a reflection phase grating, with $N = 17$ divided wells and well width W, are shown in Fig. [9.8](#page-7-0). The angles of incidence and diffraction are α_i and α_d , respectively. There are three aspects of the phase grating that are important, namely the number of periods, the number of wells, and their relative depth with respect to a reference surface plane. Any periodic surface, a diffraction grating, a crystal or a reflection phase grating, scatters sound coherently when a certain condition is satisfied, as described in Fig. [9.9](#page-7-0). Incident ray AB is reflected as BD. Incident ray EG is reflected as ray GH. When the difference in path length BC-FG is equal to an integral number of wavelengths, mλ, coherent scattering occurs in the diffraction direction α_d , according to Eq. 1, where N is an odd prime and W is the width of a well.

$$
\sin \alpha_{\rm d} = \frac{m\lambda}{NW} - \sin \alpha_{\rm i} \tag{9.1}
$$

In far field theory, sin α_i is assumed to be 0. The second and third aspects pertaining to the number of wells and their relative depths, is where Schroeder's

Fig. 9.3 Underground Sound Recording Studios, where my interest in diffusion was born

Fig. 9.4 A collage of matter in gas, crystalline, and amorphous phases that I was studying in my work as a diffraction physicist at NRL, in a group headed by Nobel Laureate Dr. Jerome Karle. Bragg's Law was directly transferrable from crystallography to the reflection phase grating

insight and genius came into play. Being a proficient mathematician, he was very aware of the power and "magic" of prime numbers. He often marveled at the unreasonable effectiveness of number theory in science and communication and described numerous applications in his books. Much of this early research was done by Carl Friedrich Gauss in the eighteenth century.

Fig. 9.5 Program for the Studio Design Session C, at the 74th AES Convention in New York, where I first met Manfred Schroeder

I would like to digress a bit to present a statement made by the New York Times writer John Tierney in which he stated that "No matter how its practitioners of mathematics try to deliberately ignore the physical world, they consistently produce the best tools for understanding it." A few examples help to emphasize this idea. The Greeks decide to study a strange curve called an ellipse and 2000 years later astronomers discover that it describes the orbits of the planets. In 1854 Bernhard Riemann conjectured that it's not possible to draw two parallel lines ad infinitum and described curved space, which 60 years later Einstein announced as the shape of the universe. In the eighteenth century in Gottingen, Carl Friedrich Gauss

Fig. 9.6 Apple II program presented at the Studio Design Session C, which calculated the well depths and the diffraction pattern of a QRD

discovered quadratic residues, quadratic reciprocity and much more, with no application in mind. In 1975 Schroeder introduced number theory into the world of room acoustics from simple binary m-sequences to multivariate sequences, one of which being the quadratic residue sequence, with good autocorrelation properties and broader bandwidth. In his 1987 Rayleigh Lecture, the topic was "The Unreasonable Effectiveness of Number Theory in Science and Communication." Schroeder pointed out that in wave interference it is not the path differences that determine the interference pattern, but the residues after dividing by the wavelength.

The reflection phase grating has two fundamental properties, the incident sound is scattered into diffraction directions determined by the width of the period, NW, and the energy in the diffraction directions is equal, because the exponentiated well depths have a flat power spectrum. The first property of grating lobes from periodic surfaces is well established in optics, i.e., diffraction gratings. However, the second property, namely the uniformity of the energy in the diffraction lobes, is where Schroeder made one of his brilliant realizations. Flat surfaces reflect energy preferentially in one direction, the specular direction. Schroeder realized that one way to scatter sound uniformly into all of the diffraction lobes was to create a periodic scattering surface consisting of divided wells whose depths were based on the number theory sequences that Gauss developed, e.g., the quadratic residue sequence. These reflection phase gratings, based on quadratic residue sequences shown in Fig. [9.10](#page-7-0), have the unique property that the energy in the seven diffraction

Fig. 9.7 Schroeder's hand written notes in the back of the 74th Technical Meeting & Professional Exhibits Program describing the Chinese Remainder Theorem to me

directions shown is equal. In Fig. 9.11 , d_n are the well depths based on the sequence values S_n , the wavelength λ and the prime N. k is the wavenumber, α_i and α_d are the angles of incidence and diffraction, $R(x)$ is the reflection factor, and the Fourier transform of the reflection factor yields a constant energy, $|p(k)|^2$ equal to 1/N for all of the diffraction orders. Another way to think of this is that the autocorrelation of the sequence values is zero except for the zero shift modulo N. And it is well known that the spectrum of a two valued autocorrelation is flat. In this case, the frequencies are spatial frequencies or directions.

Fig. 9.8 Two periods of a QRD $N = 17$ diffusor showing incident and diffracted waves

Fig. 9.9 Construction illustrating constructive interference condition in which BC-FG = $m\lambda$

Fig. 9.10 Left: commercial QRD $N = 7$. Right: diffraction pattern at 3,000 Hz for 50 periods where the energy is concentrated in the diffraction directions (Taken from "Acoustic Absorbers and Diffusers: Theory, Design and Application", T.J. Cox and P. D'Antonio, Taylor & Francis, 2nd Edition (2009))

Fig. 9.11 Equations describing the well depths, d_n , wavenumber, k, reflection factor, $R(x)$, and scattered pressure, $p(k)$, along with the diffraction patterns for several prime numbers, N, and repeats, P

Following my presentation and meeting with Schroeder, I met Bob Todrank at an evening reception. Bob was designing a new studio for the Oak Ridge Boys in Hendersonville, TN and was interested in utilizing these new acoustical surfaces. The Oak Ridge Boy's Acorn Sound Recorders project, Fig. [9.12](#page-9-0), was celebrated with a Syn-Aud-Con control room design workshop in 1984. This project was a resounding success and turned out to be a harbinger of many exciting things to come. It also led to many other projects and collaborations with a growing community of new studio designers. Use in recording studios soon led to broadcast studios, high end listening rooms, worship spaces, and eventually to performance spaces and schools.

In 1983, I carried out the first measurements of quadratic residue and primitive root diffusors with a TEF 10 analyzer at a Syn-Aud-Con seminar in Dallas, Texas, with the assistance of Don Eger of Techron, shown in Fig. [9.13](#page-10-0). In 1984, an intensive measurement program was carried out using Richard Heyser's time delay spectrometry. Farrell Becker was very helpful in the initial evaluation of these exciting new surfaces. Not having access to an anechoic chamber, a boundary measurement technique was developed. These measurements were initially carried out at full scale in large spaces, like open fields and parking lots, eventually moving indoors to a sports arena, a motion picture sound stage, and a local high school gymnasium. The measurements enabled the theories to be validated.

Fig. 9.12 First application of commercial diffusors at Acorn Sound Recorders, Hendersonville, TN in 1984

It was clear that to properly evaluate these surfaces, a standard needed to be created. This turned out to be a 28-year process! The diffusion coefficient is now standardized as ISO 17497-2. The goal was to measure the scattered polar responses and extract from these data a diffusion coefficient, which was a measure of how uniformly these surfaces scattered sound versus frequency, as a complement to the absorption coefficient. In the early 1980s polar response measurements were made by measuring impulse responses one at a time, as shown in the left panel of Fig. [9.14](#page-11-0), from a loudspeaker at a given angle of incidence to 37 microphones separated by 5° . This was an incredibly laborious process, but yielded polar responses that allowed evaluation of these early surfaces. As RPG began to grow from a cottage industry, it became necessary to make these measurements routinely, so a 1:5 scale boundary plane measurement goniometer was built, using a microphone switcher, shown in the right panel of Fig. [9.14](#page-11-0). Under computer control, the TEF analyzer emitted 37 sequential MLS test signals and the switcher automatically switched to adjacent microphones. This was a great time savings and eliminated the need to constantly find large open spaces to make full scale measurements. As computer hardware evolved, it became possible in 2011 to measure all of the observation positions, for a given angle of incidence, simultaneously with one MLS test signal. This setup is shown in Fig. [9.15](#page-11-0), using

Fig. 9.13 First measurements of QRD and PRD commercial diffusors, using the TEF Analyzer in the Winter 1984 issue of the Syn-Aud-Con Newsletter

Fig. 9.14 Left: the first goniometer measurements were made full scale one at a time with a TEF analyzer with a microphone radius of 5 m and a speaker radius of 10 m. $Right$: in 1994 a 1:5 scale goniometer was built with a 1 m mic radius and a 2 m speaker radius

Fig. 9.15 Top: original TEF and microphone switcher followed by the 32 MOTU preamps; Bottom: Reaper screen illustrating the MLS Stimulus at the top and a few of the 32 recorded reflected signals below

32 microphones. Each microphone was connected to a MOTU preamp and the Firewire output was sent to a computer, which recorded the scattered MLS signals on hard disk. The scattered signals were deconvolved, using the MLS test stimulus, to obtain the impulse responses. The impulse responses for the 32 direct sounds and scattered reflections are shown in Fig. 9.16, along with the room sound interference. The scattered sound was extracted via a multistep process illustrated in the left panel of Fig. [9.17](#page-13-0), in which a background response, $h2(t)$, with no sample present, is subtracted from the full impulse response, $h_1(t)$, to minimize the direct sound and interfering room reflections. $h2(t) - h1(t)$ is then deconvolved with the loudspeaker/microphone response, $h3(t)$, to yield $h4(t)$ and windowed to isolate the scattered impulse response. In the right panel of Fig. [9.17,](#page-13-0) we show the entire process leading to the diffusion coefficient. (A) the goniometer with a speaker at 150 degrees, (B) the total impulse response at one microphone position, with the scattered sound outlined, (C) the isolated impulse responses at all microphone positions, (D) 5 selected Fourier transforms of the scattered impulse responses and three selected 1/3rd octave polar responses, (E) the diffusion coefficient obtained from the circular autocorrelation of these polar responses, without and with normalization. To remove edge diffraction, the diffusion coefficient of the sample is normalized by the diffusion coefficient of the reference reflector. In Fig. [9.18](#page-14-0), we show a photo of a test sample, in this case, three hemicylinders, the diffusion coefficient for the sample and reference reflector and the normalized diffusion coefficient for normal incidence. Below we show the 1/3rd octave polar responses for the sample (red) and the reference reflector (blue).

While the QRD was revolutionary, there were three aspects that we investigated to improve performance, shown in Fig. [9.19.](#page-15-0) These included extending the bandwidth, minimizing the effect of grating lobes, i.e., making the response uniform, and lastly

Fig. 9.17 Left: data reduction procedure to extract scattered sound at a given microphone position from a given angle of incidence; Right: complete process to determine the diffusion coefficient from the scattered impulse responses (Taken from "Acoustic Absorbers and Diffusers: Theory, Design and Application", T.J. Cox and P. D'Antonio, Taylor & Francis, 2nd Edition (2009))

eliminating the quantized well depth effect, which results in a specular reflection at the frequency where all wells scatter in phase. For a QRD, these frequencies occur at integer multiples of the prime multiplied by the design frequency.

When considering how to expand the bandwidth, I was intrigued by the idea of the self-similarity of fractals and proposed nesting, scaled versions of the QRD forming a self-similar design, in which each generation of nesting would cover different frequency ranges [P. D'Antonio, "A new 1 or 2-dimensional fractal sound diffusor," J. Acoust. Soc. Am., Suppl. 1, 87, S10]. On my way to an Acoustical Society meeting at Penn State, I accidentally met Manfred Schroeder in the Philadelphia airport and we flew together to the meeting. During the flight we discuss many things, including how his diffusors were being accepted, the success of RPG and the diffusing fractal, now called a Diffractal, which fascinated him. He then contacted Freeman, the publisher of his forthcoming book, Fractals, Chaos,

Fig. 9.18 Top: photo of 3 hemicylinders, diffusion coefficient of the sample (red) and reference reflector (blue), normalized diffusion coefficient of sample (black). Bottom: third octave polar responses of the sample (red) and reference reflector (blue)

Fig. 9.19 Three problems that were mitigated to optimize the phase grating

Power Laws: Minutes from an Infinite Paradise, and asked them to include the statement at the bottom of Fig. [9.20](#page-16-0) prior to publication. During a subsequent Audio Engineering Society Convention in NY, Manfred Schroeder visited me at the RPG booth and he can be seen pointing to a Diffractal with product literature in hand.

The second problem is associated with grating lobes and is very ironic, because the QRD is based on the concept of periodicity, using number theory sequences which insure equal energy in the diffraction directions. Yet to achieve uniform scattering in all directions, a way had to be found to minimize grating lobes! The scattered polar responses in Fig. [9.18](#page-14-0) are dominated by grating lobes generated by the fact that the diffusors are periodic. The lobe energy may be constant, but there are large minima between the lobes, except at high frequencies when the number of lobes becomes very large. For this reason, significantly better performance can be obtained if the periodicity lobes can be removed by making the diffusor aperiodic or increasing the repeat distance. It seemed the QRD was cursed by periodicity. James Angus came up [J.A.S. Angus, "Large area diffusors using modulated phase reflection gratings," Proc. 98th Convention Audio Eng. Soc., Preprint 3954, D4 (1995)] with a solution in a series of papers outlining methods for using two phase grating base shapes in a modulation scheme to minimize periodicity. Another approach Trevor Cox and I developed is to form an asymmetric QRD sequence and instead of repeating it periodically, one would follow the prescription of an optimal binary sequence whose aperiodic Fourier transform is as flat as possible.

Fig. 9.20 Manfred Schroeder and Peter D'Antonio in the RPG booth at AES in New York in front of a Diffractal along with a mention in his new book with reference [D'An 90]

That is, if the binary sequence is a zero, the base shape is used, if the sequence value is one, the asymmetric QRD is flipped. In this way an aperiodic modulation is formed and grating lobes are minimized.

The last problem deals with specular scattering at a frequency where all of the wells scatter in phase. For the QRD, this frequency is equal to integer multiples of the prime times the design frequency. This occurs because the well depths are integer multiples of one another. To minimize these flat plate frequencies an optimization program was created, in collaboration with Trevor Cox, that combined boundary element prediction, multidimensional optimization techniques, and the diffusion coefficient. It is an iterative program which cycles until the shape produces a desired diffusion coefficient. When used for divided wells or nondivided steps, the goal is to find non-integer related wells or steps, thus avoiding the flat plate frequencies. It can also be used to define a wide range of curvilinear shapes which can complement contemporary architecture, shown in Fig. [9.21.](#page-17-0)

Thus by utilizing a variety of techniques including, boundary element prediction, multidimensional minimization, defining a diffusion coefficient, fractal geometry, optimal aperiodic modulation, etc. we have been able to optimize Schroeder's seminal idea of an acoustical reflection phase grating. This research has yielded essentially three types of diffusive surfaces, optimized and modulated reflection

Fig. 9.21 A wide variety of curvilinear diffusive shapes obtained with the Shape Optimizer software

phase gratings, planar binary absorption-reflection amplitude gratings, and optimized curvilinear shapes, seen in Fig. [9.22](#page-18-0).

It was a great pleasure to meet Manfred Schroeder a few additional times. One was in Rome at the ICA, where Michael Vorlander convened a special session on diffusors. It was a great privilege for me to present a paper outlining the progress we had made in optimizing his Schroeder diffusor. Unfortunately he was recovering from a stroke, but was still in good spirits. Our last meeting was at an ASA meeting in Paris, where we shared some drinks in the hotel lobby and met again briefly in the Louvre.

As a small way to thank Manfred Schroeder for inspiring us and launching our careers in acoustics, Trevor Cox and I dedicated our book Acoustic Absorbers and Diffusers: Theory, design and application to him. His thank you letter and a photo of the Second Edition are seen in Fig. [9.23.](#page-18-0)

Manfred Schroeder had many hobbies and cycling was one of them. For comic relief, I compiled a collage of Trevor Cox, James Angus, Manfred Schroeder, and myself entitled Diffusor Docs, in Fig. [9.24](#page-19-0). As a fitting tribute to Manfred Schroeder, the RPG Diffusor was inducted into the music industry's Technology Hall of Fame, in 2013. My closing sentiments are expressed in Fig. [9.25](#page-19-0).

Fig. 9.22 Three types of diffusors that evolved from Schroeder's seminal suggestion. Left to right: a family of 1D and 2D reflection phase grating diffusors; a family of 2D binary amplitude diffusor-absorbers (diffsorbers); and a family of wood and glass reinforced gypsum curvilinear optimized diffusors

Fig. 9.23 A photo of a book written by Trevor J. Cox and Peter D'Antonio, with a dedication to Manfred Schroeder, along with his thank you letter

Fig. 9.24 Diffusor Docs: Top left clockwise, Manfred Schroeder, Trevor Cox, James Angus, and Peter D'Antonio

Fig. 9.25 Vielen Dank und Abschied

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Biography

Dr. Peter D'Antonio was born in Brooklyn, NY in 1941 and is Founder/Chairman of RPG Diffusor Systems, established in 1983. He has co-authored Acoustic Absorbers and Diffusers: Theory, Design and Application, 2nd Edition, Taylor & Francis 2009 and contributed several chapters to the Master Handbook of Acoustics, 5th Edition, McGraw Hill 2009. Dr. D'Antonio served as Chairman of the AES SC-04-02 and is a member of ISO 17497-1 and ISO 17497-2. He is an adjunct professor of acoustics at the Cleveland Institute of Music, since 1991 and a Fellow of the Acoustical Society of America and the Audio Engineering Society.
