Chapter 2
The Early History of Ray Tracing in Acoustics

Asbjørn Krokstad, U. Peter Svensson, and Svein Strøm

Abstract Manfred Schroeder and Bishnu Atal are behind two seminal conference presentations on room acoustics, one in 1962 and the other in 1967. In the first, computerized auralization was outlined remarkably early, long before the term auralization was introduced. In the second, a ray tracing technique in room acoustics was presented. Independently, Asbjørn Krokstad and colleagues in Norway worked on the ray tracing technique, which lead to a journal paper in 1968. Here we describe some developments that lead up to these two breakthroughs for the use of computers in room acoustics.

2.1 Introduction

Bishnu Atal and Manfred Schroeder presented a paper titled "Study of sound decay using ray-tracing techniques on a digital computer" at the 73rd Meeting of the Acoustical Society of America in New York, April 1967, but only an abstract was published for this presentation [1]. In 1968, a similarly titled paper, "Calculating acoustical room response by use of a ray tracing technique" was published by Asbjørn Krokstad, Svein Strøm, and Svein Sørsdal in the relatively recent Journal of Sound and Vibration [2]. The latter was submitted in November in 1967, but no reference to the Atal and Schroeder presentation was made—since neither of the authors went to the ASA meeting. At the same time, Krokstad remembers the great inspiration from another conference paper by Schroeder, Atal, and Bird at the ICA congress in Copenhagen in 1962 on "Digital computers in room acoustics" [3]. Schroeder did eventually, in 1970, publish a journal paper titled "Digital simulation of sound transmission in reverberant spaces" in the Journal of the

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Acoustical Society of America, submitted in February 1969, which described the ray-tracing technique but which did not cite the paper by Krokstad et al. [4]. Curious as this may seem to the researcher generation who has grown up with access to most publications right at the computer, in the times before efficient computer search engines it was quite a tedious work to look for all potentially relevant references.

Being the first published full paper, the paper by Krokstad et al. has become the standard work to cite for later studies on computerized prediction in room acoustics and has been a remarkably relevant paper to cite throughout more than 40 years. It was the first journal paper that presented the computerized ray-tracing technique for finding the impulse response, or rather echogram, in any three-dimensional model of a room. The reason that it has stayed relevant is that computerized prediction techniques still is an active research area, and such prediction still relies on the ray-tracing technique to a large extent. As an example, the two leading commercial softwares that are used by practitioners today, CATT Acoustic [5] and Odeon [6], use variants of the ray-tracing technique. In this paper, we will try to present some of the work in Trondheim and other places that lead up to the publishing of this paper. It will be a mix of a view from the inside (by the first and third author) and from the outside (by the second author). We will also see that the development was progressing in quite natural steps, yet at one point the development was perceived to have reached a mature enough stage that the "definitive" citation resulted. We will also see that in Trondheim there was an ongoing development of, and practical use for, the ray-tracing technique from the mid-1960s and onwards. The initial work by Atal and Schroeder, on the other hand, wasn't followed up to any larger degree, simply because there were several other highly significant developments going on, such as the linear prediction speech coder [7].

2.2 First Studies by Allred and Newhouse in 1958: Mean Free Path Calculations

In the late 1950s, researchers started to use computers quite widely for solving demanding numerical problems. The computers offered a new and more efficient tool to solve an existing problem, or to get more accurate results than had been possible before.

The field of room acoustics gives an interesting example of how the computers offered a straightforward improvement to existing numerical problems. Sabine's famous equation from 1898 states that the sound level in a room decays linearly with time, that is, an exponential decay for the sound pressure amplitude. The decay rate, or rather reverberation time, T_{60}, was determined by a very simple relationship, the first equality below,

$$T_{60} = 0.163\left[\mathrm{m}^{-1}\mathrm{s}\right]\frac{V}{\overline{\alpha}S} = 0.041\left[\mathrm{m}^{-1}\mathrm{s}\right]\frac{l_m}{\overline{\alpha}} \tag{2.1}$$

where V is the room volume (in m^3), $\overline{\alpha}$ is the average absorption coefficient, and S is the total surface area of the room (in m^2). Researchers have been kept busy ever

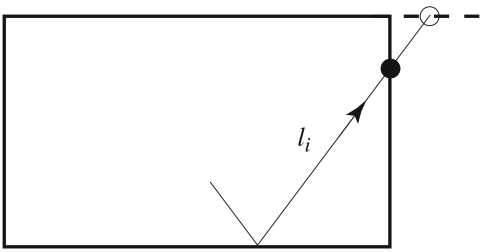

Fig. 2.1 Illustration of the Allred and Newhouse algorithm from 1958 [9]. Their paper studied three-dimensional shoe-box-shaped rooms; here a two-dimensional version is drawn. Each ray was reflected specularly and the next hit point was chosen as the closest of the three (two for the two-dimensional version) possible cross-points with other planes. The data sought were the lengths of the free paths, l_i

since studying to which degree this simple formula is true in various room geometry and absorption distribution cases. This relationship could also be written on a form which involves the so-called mean free path length in a room, that is, the average length that a sound wave can travel between wall hits, l_m (in m), and that is the second equality above.

This mean free path length had been established theoretically for some simple room shapes but novel approaches were used to find the mean free path in rooms of other shapes. In his book from 1932, *Architectural Acoustics*, Vern Knudsen described experiments where they used light rays in a scale model to figure out where sound waves would hit during several consecutive specular reflections [8]. By marking hit points on walls and measuring the corresponding lengths, he could collect data on these path lengths. Obviously, the accuracy was quite limited but he did find that quite different room shapes seemed to give the same average path length, or "mean free path." Such optical techniques in scale models continued to be in use, but also drawings, a pen and a ruler offered similar possibilities.

A first step towards computerized ray tracing was taken by collecting data on the path lengths and the collision probabilities of the walls using a computerized Monte Carlo technique by Allred and Newhouse in 1958 [9]. They studied rectangular/shoe-box-shaped rooms and Fig. 2.1 illustrates that the algorithm was quite straightforward. The algorithm generated rays in a number of randomized directions from one source position. For each ray, its path was followed through a succession of specular reflections. Data on path lengths and which reflecting walls that were hit were stored. Allred and Newhouse did not specify what kind of computer they used, or calculation times, but they could follow ten consecutive reflections for each ray, and they used 150 rays that were emitted in random directions.

This computerized calculation method gave accurate values for the mean free path in shoe-box rooms of different room dimension aspect ratios, and showed that there will be different reverberation times in rooms with different ratios. Two follow-up papers pointed out two errors in the original paper: the authors themselves found a programming error that gave around 10 % too long rev. times in the first paper [10]. Then Hunt, in 1964, showed that they had used a randomization of the source emission directions which did not correspond to an isotropic sampling of the sound field [11].

Allred and Newhouse did outline an extension of their work in [9]:

> The Monte Carlo method of machine computation is shown to be applicable to the evaluation of room acoustics. Its extension to the study of reverberation time is straightforward, and air absorption, as well as frequency dependence of air and absorbers, can also easily be taken into account. .... It would be a simple matter also to vary the absorption coefficient as a function of position, either discretely or continuously, ...
>
> The method is also applicable to the study of coupled rooms and rooms of irregular shape, such as auditoria. The complications are geometrical, and are of degree rather than kind as referred to our parallelopiped calculations.

Apparently, they did not follow this path of development, and the reverberation time was their focus also for future directions of the research.

2.3 Schroeder at ICA in 1962: The Path Towards Auralization

A visionary conference paper by Schroeder, Atal, and Bird was presented at the ICA in Copenhagen in 1962 titled "Digital computers in room acoustics" [3], as mentioned above. That paper was remarkable in that it laid out the methodology for what much later was to be called auralization, that is, the technique for creating audible computerized simulations of the sound in a room [12]. The auralization process was still in the early 1990s a computationally very demanding process but Schroeder and colleagues had identified the necessary steps:

The use of computerized convolution between a (simplified) impulse response and anechoic music.

The cross-talk cancellation technique for the presentation of binaural sound over two loudspeakers in an anechoic room, which can give the impression of sound incidence from any direction.

The development of a natural sounding reverberation unit without needing to carry out a convolution with a huge number of discrete reflections.

Interestingly, Schroeder mentioned that discrete echoes can be handled by discrete delay units but otherwise did not describe the computation of a detailed impulse response or how these discrete echoes should be found. Rather more focus was given to the use of computers for implementing Schroeder's earlier (1954) seminal work on the statistics of frequency response functions [13].

2.4 Developments in Oslo 1965–1967: Diffuseness of Reverberation Rooms with Hanging Reflectors

At the University of Oslo, Department of Physics, there were two professors who were both interested in room acoustics, but using different approaches: Johan Holtsmark, who published a paper titled "Reverberation as a stochastic process"

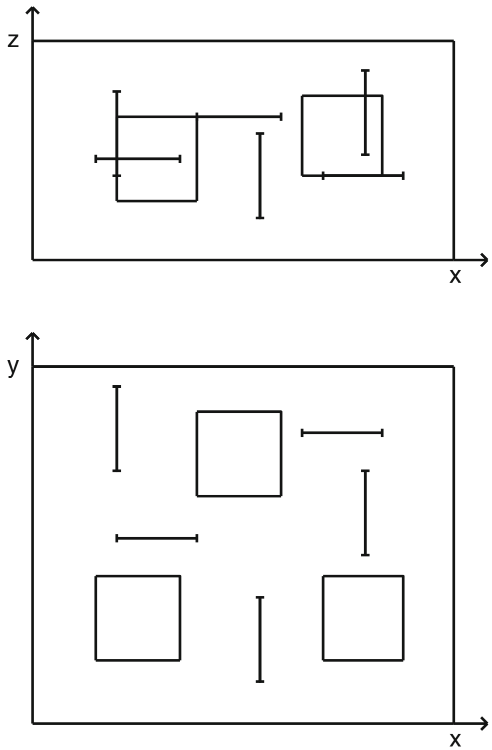

Fig. 2.2 Illustration of a shoe-box-shaped room with free-hanging reflectors. Reproduction from [15] where reflectors parallel to the walls were used

in 1966 [14]; his colleague Wilhelm Løchstøer was also interested in the reverberation process and supervised a Master's student Nikolai Stenseng who published his master thesis in January 1965 titled "The influence of diffusers on sound absorption measurements of a small volume" (translation of the Norwegian title) [15]. This thesis by Stenseng was a gradual development from the previous studies at that time in that it implemented a ray-tracing technique for a 3D room, albeit shoe-box-shaped, with free-hanging diffusors. These diffusers were always parallel to one of the walls, which made the computation of obstruction checks somewhat easier. Since reverberation rooms typically have smooth and flat walls and reflectors, only specular reflections were implemented and the need for something else was not discussed either. The program was written in the language Fortran IV for the computer UNIVAC 1107 (which had been delivered to the Norwegian Computing Center in Oslo in August 1963, after having been introduced in the USA in October 1962). An illustration from the thesis by Stenseng shows the parallel diffusers, see Fig. 2.2. Stenseng had spent a period at the NATO underwater research centre in La Spezia in Italy in the early 1960s, and remember being introduced to the power of the computers for the first time during that visit. When he returned to Oslo, he took one of the first programming courses offered, and when he started his master thesis, the idea to use a computer to simulate the propagation of light rays came quite naturally.

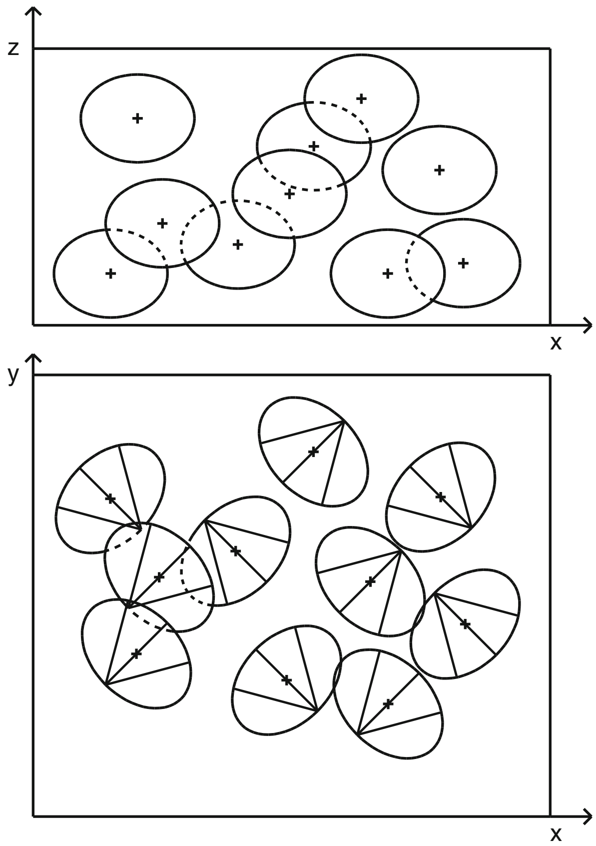

Fig. 2.3 Illustration of room with oblique reflectors (reproduced from [16])

An extension of this study by Stenseng was the master thesis by Svein Strøm in 1967, on “The diffusivity in small reverberation rooms with and without diffusers” (translation of the Norwegian title), also under Løchstøer in Oslo [16]. Strøm expanded Stenseng’s study by implementing oblique reflectors, and also spherical reflectors. Furthermore, Strøm also computed actual decay curves, rather than collecting statistics data on path lengths. The purpose was still, however, to study the reverberation time. Figure 2.3 shows an example of a room with reflectors from [16].

2.5 Developments in Trondheim, Leading up to the Paper in 1968

Asbjørn Krokstad was hired as a research assistant at the Norwegian Technical Institute in Trondheim in 1956, and got the task to manage the specification of a new anechoic chamber, which was finished in 1958.[1] Soon after that, he got the further task to evaluate the performance of this new anechoic chamber. Since the chamber was rather small, compared to the recently finished one in Lyngby, Denmark, the classical free-field distance law criterion was difficult to apply, and Krokstad wanted to study the influence of the first-order reflections represented by image sources. The Institute got its first computer, a "GIER" from Denmark, in late 1962. Krokstad took the first course offered in Trondheim on the ALGOL programming language in 1963 (500 students and employees took this first course on computer programming!). During the course he started to use computer calculations to quantify the influence of reflections. The work on evaluating the anechoic chamber [17] later became part of his Ph.D. degree in 1963, which had Wilhelm Løchstøer and Gordon Flottorp, both from Oslo, as opponents (the supervisor of Krokstad was Reno Berg, associate professor in acoustics in Trondheim since 1935). As mentioned above Løchstøer was associate professor at the University of Oslo, Department of Physics, specializing on acoustics, whereas Flottorp was chief audiologist at the Oslo University Hospital. Curiously, this trio of supervisor and opponents were the three founders of the Acoustical Society of Norway in 1955.[2]

Krokstad got inspired by the successful application of the first-order image source method to the anechoic room and wanted to extend this to more general room shapes (and less absorbing ones too). Beranek had published his classical book "Music, acoustics & architecture" in 1962, and the book contained simplified plans for 100 of the most famous concert halls in the world [19]. This book gave the possibility to get input data for further studies, and furthermore, a possibly crucial event took place in Norway around this time: a new concert hall, "Grieghallen," was being planned in Bergen. An architectural bidding process was launched with a deadline in February 1965 and the winning design was made by the Danish architect Knud Munk. Krokstad naturally got very interested in studying the suggested design of the new hall.

The acoustics group in Trondheim, which had a brand new laboratory, opened in March 1965 [20], with Krokstad as director, hired Svein Sørsdal as a computer-savvy summer student, and the two of them tried to implement an image source

[1] The anechoic chamber in Trondheim still, in 2014, stands as it did in 1958 (but is rather ready for renovation). It has the quite unusual feature of graphite-covered wool wedges in order to improve the anechoic properties for electromagnetic waves.

[2] Reno Berg was also one of the founders of the *Nordic Acoustical Society*, founded on 19 June 1954 which happened to be the day of the 25th anniversary of the Acoustical Society of America. A telegram was sent by the "newly born son" to the "grandfather" ASA, as referred to in [18].

visibility test for general 3D geometries on the GIER computer that they both had experience with. This failed, however, since the visibility tests turned out to be too complex. They changed horses, so to say, aiming at using ray tracing instead and readily got in touch with Stenseng, and later Strøm, who were both in Oslo. At the same time, the Institute got their next type of computer, a UNIVAC 1107, in the end of 1965, after the GEIR computer had been running non-stop for a few years.

Strøm moved to Trondheim after he had finished his master thesis in early 1967 and joined the simulation project. His Fortran implementation of a 3D visibility test turned out to be exactly what was needed, and was combined with the ALGOL program developed earlier. Sørsdal implemented the main program structure, routines for reading geometry data, as well as for post-processing the result for each ray. A visualization technique was implemented where each ray's hit point in the audience area was plotted, with a little tail indicating the incidence angle onto a receiving surface. A Kingmatic plotter had been acquired by the Institute in 1966 and it made such visualizations possible.

Strøm carried out most of the actual calculations, and used the program as the Trondheim group were acoustics consultant in a sequence of projects: the "Hjertnes Kino og Kulturhus" in Sandefjord, opened in 1975; Grieghallen in Bergen, opened in 1978; and Oslo concert hall, opened in 1977. Prior to these projects, several of the halls in Beranek's book were studied and published in a report (in Norwegian) in 1971 [21]. The Grieghallen project had got delayed quite severely after the design competition had been settled in 1965. Locally in Bergen, Helmer Dahl, research director at the Christian Michelsen Research Institute, had got involved in the plans to build the new concert hall, and had tried to use the old lightbeam method of Vern Knudsen for studying different room shapes. These attempts had failed and Dahl therefore had contacted Krokstad who had the ray-tracing program up and running.

Svein Strøm describes that the small Trondheim group was hired as advisors for the Grieghallen project in Bergen [22]. The result was that the architect had to accept the suggestions from Trondheim for the orchestra enclosure, the ceiling shape, the sidewall reflectors, and for large sound reflecting objects that were hung from the ceiling along the sidewalls (Fig. 2.4). The exciting trial concert on the 12th of May, 1978, showed that the project was successful. Ten years later the local newspaper, *Bergens Tidende*, confirmed this in an article about the hall.

The journal paper was finished and submitted in November 1967. In order to develop a functioning engineering tool, they had taken several steps from the previous state of the art where ray length histories could be collected. In addition to the handling of arbitrary 3D geometries (admittedly with small numbers of polygons: up to 99 corners and 50 polygons per symmetric half of a room; and typically with around 2,000 emitted rays [21]), Krokstad et al. introduced some refinements [23]:

1. Some areas were marked as audience surfaces, and ray histories were collected for these. Also smaller sub-areas were introduced in order to study the sound field distribution over the audience area, see Fig. 2.5 (from [2]).

Fig. 2.4 Grieghallen in Bergen, one of the first concert halls where computerized ray tracing was used as a design tool

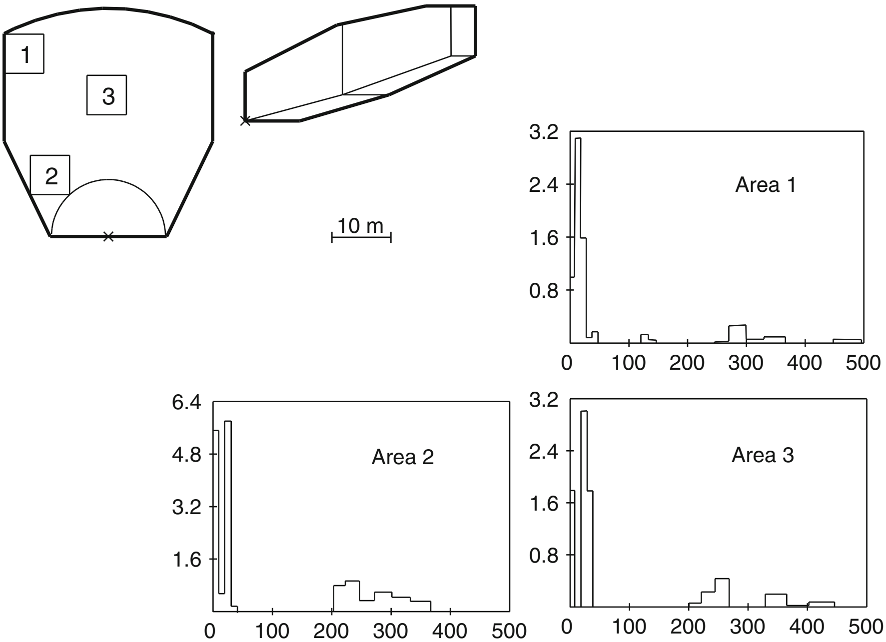

Fig. 2.5 Illustration of an irregularly shaped room, and computed echograms for three different receiver patches (reproduced from [2])

2. A kind of echogram was collected for each such audience surface, see Fig. 2.5. These resulted, however, simply from counting the number of hits per discrete time interval.
3. Different absorption coefficients were introduced—sort of. Surfaces were classified as totally reflecting, totally absorbing, or audience (which were also considered as totally absorbing). The motivation was that the room shape determined, to a large degree, the amount of early energy and its spatial distribution.

There were clearly some simplifications compared to what is common today, but some of these were commented by the authors in [2].

On the topic of specular reflections: "However, in the actual design of a hall, one may also wish to know the effects of changes in details, such as the introduction of diffusing wall elements. Even if such details can be included in the mathematical model of a hall without causing excessive computing time, the validity range of the results may be difficult to state." It was, however, not indicated how to handle diffuse reflections.

On the topic of absorption: "As mentioned in Sect. 2.2, the surfaces of the room are considered to be either totally reflective or totally absorbent. Other absorption characteristics of the surfaces, together with air damping, may readily be included in the computations. The authors believe that these factors are of minor interest in view of the building materials commonly used in concert halls."

2.6 Schroeder at the ASA Meeting in 1967: Computing Decays in Generally Shaped Convex Rooms

After the Allred and Newhouse study, and some subsequent theoretical discussion papers on mean free paths, other studies were looking into the computation of reverberation time (rather than mean free path and collision frequencies). At each reflection, the ray's energy was decreased by the wall reflection factor. Then, each ray gave a stepwisely decreasing contribution, and averaging across many of them gave a smooth exponentially decaying curve. This extension to the ray-tracing technique was presented by Atal and Schroeder at the ASA meeting paper in April 1967 [1], completely independently of the work in Norway. ASA meetings don't publish any papers, so all that is available is an abstract, plus some example results in two later papers by Schroeder in 1969 and 1970 [4, 24]. It did seem like Atal and Schroeder had a functioning program for general geometries, see Fig. 2.6, however, only for two dimensions.[3]

[3] In a paper by Wayman and Vanyo in 1977 [25], the quite straightforward extension of Schroeder's approach from two to three dimensions was presented, with no reference to the 1968 paper by Krokstad et al.

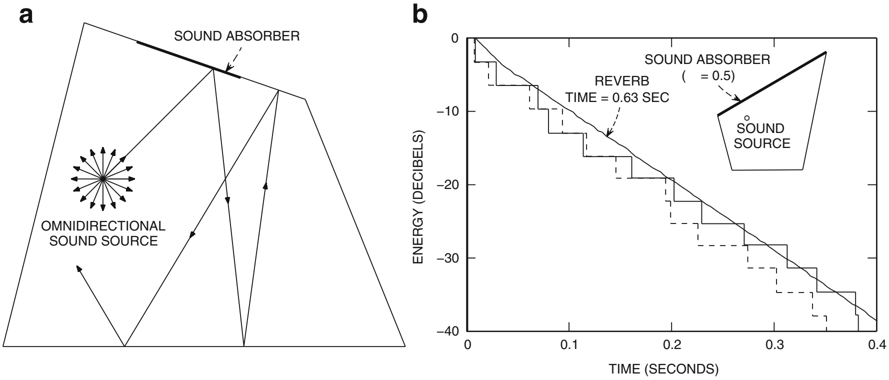

Fig. 2.6 Illustration of how the ray-tracing technique was used to gather individual ray-decay curves, which could be averaged to a smooth exponential decay (reproduced from [24])

The Atal and Schroeder algorithm was presented in the JASA paper in 1970 and their implementation clearly handled various degrees of absorption coefficients, as opposed to the approach by Krokstad et al. Furthermore, Atal and Schroeder specified the handling of diffuse reflections: "These rays are traced on the computer and are reflected from the walls either specularly or according to some specified random law." It seems to have been implemented as well, according to a further comment in the paper: "The discrepancies found for rooms with randomly reflecting walls and 'suspended' diffusing elements were generally, although not always, somewhat smaller."

On the other hand, Atal and Schroeder did not study the distribution of received sound across any audience surface; instead the global decay curve was studied, see Fig. 2.6. An interesting detail of the 1970 JASA paper by Schroeder is its reference number 16, to a paper by Atal and Schroeder in JASA which is marked as "To be published," but it never seems to have been published.

2.7 Later Developments of Ray Tracing in Room Acoustics

After the paper by Krokstad et al. was published in 1968, it appeared as if the paper was unknown for some time, at least to American researchers. In 1973, a paper published by Haviland and Thanedar outlined a method to compute the detailed response in one specific receiver location using ray tracing—but only for rectangular rooms [26]. This paper cited the Schroeder 1967 ASA abstract but not the 1968 paper by Krokstad et al. Another paper, published in 1977, that also seemed to be unaware of the Krokstad paper, has been mentioned above [25]. On the other hand, citations of the Krokstad paper appeared in 1971 [27], 1973 [28], and later. Krokstad et al. summarized the work in Trondheim in the paper titled "Fifteen years' experience with computerized ray tracing" [29].

Of special relevance is the paper by Kuttruff in 1971 [27]. That paper presented an integral equation formulation (later, independent versions of this became known as the radiosity method), and a discrete Monte Carlo-type solution where narrow beams where traced until they hit a wall, and then they were reflected in a direction generated by two random numbers so as to simulate a Lambert-like reflection. Probably, the beam was represented by a single center ray. From the paper: “In der so festgelegten Richtung läuft das Teilchen also wieder in den Raum, wird erneut an einer Wand gestreut usw., bis es schliesslich einmal auf die absorbierende Wand trifft, die seiner Wanderung ein Ende setzt.”, or (our translation) “The narrow beam thus propagates through the room in the determined direction, hits another wall where it is scattered, etc., until it finally hits the absorbing wall, which will end its propagation.”

A lot of research has been done to develop the ray-tracing technique further. Much work has studied accuracy issues since ray tracing is inherently a stochastic process (as long as there is at least one partly diffusing surface area in a room). In addition, there are surface-sampling issues for the deterministic part of the process. The so-called cone tracing was presented by Van Maercke et al. in 1986, replacing thin rays (hitting a receiver sphere) by propagating cones (hitting a receiver point) [30]. An important development was presented by Vorländer in 1989 with the so-called hybrid technique: ray tracing was used to find possible reflection paths but in a subsequent phase, the ray-tracing-identified specular reflection paths were replaced by their image source equivalents. An advantage was that the image source method gives exact reflection paths, which can be used for improved accuracy in the early path of the impulse response [31]. This linking between ray tracing and the image source methods was also explored through the beam-tracing technique presented by Walsh already in 1980, where ray bundles were treated as coherent “beams” [32]. This beam tracing was shown by Stephenson to facilitate the inclusion of diffraction and developments to avoid its computation time explosion [33]. Also, Funkhouser et al. have demonstrated very efficient implementations of beam tracing via special geometry structures [34].

A step towards avoiding the “either specular or diffuse reflection” approach in ray tracing was taken by Dalenbäck where a method let reflections generate both specular and diffuse reflections while avoiding an exponential growth in path number tracing as function of reflection order [35]. The ray-tracing technique was combined with another algorithm, radiosity, by Lewers [36]. Later, the Odeon software employed components from radiosity [37] in their implementation. In fact, ray tracing in a room with only diffusely reflecting walls, is a discretized/ Monte Carlo-type solution of the integral equation in radiosity, which was presented by Kuttruff in 1971 as described above [27].

2.8 Ray Tracing in Other Fields

So, was room acoustics the pioneer field for the computerized ray-tracing technique? Not really; as early as 1954 examples from optics were published. A paper titled “Ray tracing on the Manchester university electronic computing machine” by

Black was published in the British Proceedings of the Physics Society [38]. This was not so surprising—the term ray tracing does after all come from optics, and ray tracing in optics is typically used to study the refraction through optical lenses. Compared to room acoustics the "direct wave" is dominating, and studied in detail, while reflections might be ignored. On the other hand, the medium is refractive so that the rays do not travel along straight lines.

In underwater acoustics, early work was apparently done using ray tracing in the US Navy. The abstract in a 1956 report by Anderson and Peterson claims that "This report discusses: (1) an acoustic intensity program for estimating convergence zone propagation loss using ray theory" [39]. More publically, in an abstract from an ASA Meeting in November 1961, Norris claimed that "A ray-tracing program which has been developed for the IBM 704 and 7090 computers yields results which show excellent agreement with experimental data. The program permits ray computations for arbitrary source and receiver depths, bottom profiles, and velocity structures. The tabulated results give actual ray trajectories as well as the geometrical spreading associated with each path." [40].

In underwater acoustics, a central problem is that the medium is not homogeneous, which leads to that straight-line rays cannot be used (similar to in optics). On the other hand, geometries are practically always modeled as 2D, with a flat top surface (the sea surface) and a deterministically or stochastically shaped bottom profile. Multiple reflections are studied but with more restricted geometries and lower orders of reflection. Of curious interest is also early ray-tracing work on so-called analog computers—electronic circuits that performed calculations such as integrations and differentiations, by, e.g., Graber et al. in 1961 [41].

Ray tracing has more recently been developed heavily in computer graphics. A paper by Whitted in 1980 is considered as an important foundation of ray tracing in computer graphics [42], but an earlier paper from IBM in 1969 is considered as the original citation [43]. This kind of ray tracing was similar to the one in acoustics, except that rays were "shot" from a receiver's eye, through a grid of pixels that represent the display, reflected off surfaces, and finally reaching light sources. The more acoustics-like approach of emitting rays from (light) sources is usually called "global illumination" in graphics. Because of the generally large interest in computer graphics, many researchers are active in this field and the techniques are developed rapidly. In addition, the hardware for computer graphics generation (graphics processing units, or GPUs) has recently been developing at a faster pace than general CPUs for computers. This has lead to that so-called general-purpose GPUs (GPGPU) can be used as numerical coprocessors for many kinds of numerical calculations with processing power many times higher than CPUs [44]. Acoustical ray tracing has been demonstrated on such GPGPUs [45, 46].

A last field where ray tracing has become a common tool is the study of radio wave propagation, indoors as well as outdoors in city environments. This application applies specular reflections (as in room acoustics) as well as transmission, and of multiple orders. An early paper on this technique was published in 1991 [47]. It could be noted that diffraction over edges has been employed in radio propagation studies for a long time, either based on classical Fresnel diffraction, or based on the high-frequency asymptotic geometrical theory of diffraction (GTD) that was presented by Keller in 1962 [48].

2.9 Concluding Remarks

The longevity of the ray-tracing technique, and the number of fields that it is applied to, indicates a very flexible method which can handle many phenomena. Ray tracing is based on geometrical acoustics, and equivalent approximations to true wave fields. Consequently, there will always be wave-related aspects that are more or less difficult to represent but the concept of edge diffraction has been implemented to some degree.

This overview has clearly had a focus on the development in Norway, which we know the best, and on the early work by Atal and Schroeder. Early attempts that have been published through more or less easily available channels might certainly have slipped our attention, and we humbly apologize if this is the case. We think that the development of the ray-tracing technique in room acoustics is a nice example of quite a gradual development process where several steps have been taken by different researchers, at different places, and thereby reached a kind of breakthrough.

Acknowledgements The authors are grateful to discussions with Svein Sørsdal, Nikolai Stenseng, and Prof. Bishnu Atal for sharing their memories from their early work on ray tracing.

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Biography

Dr. **Asbjørn Krokstad** is a professor emeritus at the Norwegian University of Science and Technology, Trondheim, Norway, since 1998. He has a Ph.D. from the Norwegian Institute of Technology, Trondheim, in 1962, and was professor from 1970. He did research on computational room acoustics, with the first paper on ray tracing in 1968, reverberation enhancement systems, and was behind the development of one of the world's first digital hearing aid prototypes in 1989, and holds several patents related to this. He was president of the Norwegian acoustical society 1994–1998, and the general chairman of the ICA in Trondheim 1995. He has been an active musician and conductor throughout his career.

Dr. **U. Peter Svensson** is a professor of electro-acoustics at the Norwegian University of Science and Technology, Trondheim, Norway, since 1999. He has a Ph.D. from Chalmers University of Technology, Gothenburg, Sweden, in 1994. His research has dealt with auralization, especially computational methods involving diffraction modeling and loudspeaker reproduction techniques. He has also worked on beamforming techniques for microphone arrays, measurement techniques, reverberation enhancement, and interaction over Internet/video conferencing. He has been on the boards of the acoustical societies of Sweden, Norway, and the European Acoustics Association.

Svein Strøm is a senior adviser at COWI AS, Trondheim, Norway. He has an M.Sc. in physics from the University of Oslo in 1967. He worked at the ELAB/SINTEF in Trondheim with room acoustics research together with Dr. Krokstad until 1994. He did his M.Sc. thesis on the ray-tracing technique, which paved the way for the paper by Krokstad, Strøm, and Sørsdal in 1968. Strøm has worked with room acoustical design throughout Norway, including many examples of passive and active variable acoustics installations. Together with Prof. Krokstad, he was the acoustical consultant for the concert hall in Trondheim, Olavshallen, which was opened in 1992.