Bi-level Sensor Planning Optimization Process with Calls to Costly Sub-processes

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Abstract. While there is a variety of approaches and algorithms for optimizing the mission of a sensor, there are much less works which deal with the implementation of several sensors within a human organization. In this case, the management of the sensors is done through at least one human decision layer, and the sensors management as a whole arises as a bi-level optimization process. The following hypotheses are considered as realistic: Sensor handlers of first level plans their sensors by means of elaborated algorithmic tools based on accurate modelling of the environment; Higher level plans the handled sensors according to a global observation mission and on the basis of an approximated model of the environment and submit its plan to a costly assessment by the first level. This problem is related to the domain of experiment design. A generalization of the Efficient Global Optimization method (Jones, Schonlau and Welch) is proposed, based on a rare event simulation approach.

Keywords: sensor management, Efficient Global Optimization, rare event simulation, non-Gaussian/non-linear models, experiment design.

1 Introduction

The main background of this paper is the optimal planning of sensors in the context of an acquisition mission. Typically, the acquisition mission may result in the localisation of a target, with the final purpose of intercepting this target. In this work, we focus especially on dealing with the modelling errors of the sensor planning problem. Then, the question of interest is: *how to spend resources optimally in order to reduce the model errors, and how does that affect the sensor planning problem?*

Sensor planning, especially in order to localize a target, has been thoroughly studied in the literature. First works in this domain track back to the works of Koopman during World War II [1, 2]. This seminal works has been extended in various manner, so as to take into account motion models [3, 4], or reactive behaviours of the target [5, 6]. Sensor planning now deals with the general domain of search and surveillance [7, 8]. The combination of multiple sensors with their constraints is addressed by some works and in various application contexts:

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optimizing the performance of a sensor network [9, 10]; optimizing the tasks-to-sensors affectation in the context of an intelligence collection process [11–14]. Another major issue in sensor planning is also to maximize the positive effect of subsequent data processing in regards to mission objectives. For example, entropic-based criterion is used in order to take into account optimal post-processing (data fusion) of the collected information[15]. A more direct approach has also been addressed by means of Partially Observable Markov Decision Processes [16, 17]. From this last point of view, sensor planning is clearly related to the domain of robotic.

Thus, a variety of approaches have been investigated for many contexts of the sensor planning. Nevertheless, there is not as much works dedicated to the question of modelling the sensor planning. In their inspiring work[18, 19], Koopman addressed initially this formalisation, priorly to sensor planning problem. Le Cadre studied various practical case of use of the model of Koopman, and deduced related parametrization of the models[20]. Whatever, it appears that a minimal effort is necessary for acquiring a good estimation of the parameters modelling our sensor planning. In the case of a reproducible scenario, it is possible to learn such parameters.

However, there are cases where a prior learning of the parameters is clearly impossible. Such cases hold typically when the planning team has a limited control on the sensors, and relies on sub-processes or on sub-teams in order to implement the sensors or compute their performance parameters. Especially, military organizations are characterized by a hierarchical structure, where decisions are made through at least two human-driven levels. In practice, *accurate models of the sensors and mission contexts are only available to the first, close-to-sensor, level.* The coordination level only works on the basis of approximated models; Accurate parameters learning or acquisition are generally not possible at this level, since the request to sub-processes resources are costly *and* restricted.

The following hypotheses are thus considered as realistic:

- Sensor handlers of the first level plans their sensors by means of elaborated algorithmic tools based on accurate modelling of the environment,
- Higher level plans globally a variety of sensors according to a global observation mission on the basis of an approximated model; at this stage, the global plan has to be confirmed by the first level,
- In order to assess the global plans and to enhance their accuracy, higher level may request the first level. Each request to the first level is costly: it implies communication procedures, as well as the parametrization and execution of the algorithmic tools by the sensors handlers.

This bi-level problem is formalized very generally as the maximization of a function, defined with a *prior* model noise. Each actual evaluation of the function increases the knowledge about the function, and subsequently the efficiency of the maximization. The issue is to optimize the sequence of value to be evaluated, in regards to the evaluation costs: this problem is related to the domain of *experiment design.* Jones, Schonlau and Welch proposed a general method, the

Efficient Global Optimization (EGO) [21, 22], for solving this problem in the case of additive functional Gaussian law. In this work, a generalization of the EGO is proposed, based on a rare event simulation approach. This simulated approach makes possible the implementation of non-Gaussian functional law, and even of simulated functional law. It is applied to the aforementioned bi-level sensor planning.

In the first section 2 of this paper, we propose a general formalisation of the sensor planning with experiment sub-processes, and its description as an abstract problem. In section 3, a rare-event simulation approach is proposed for solving this bi-level sensor planning. Section 4 presents a scenario and numerical results. Section 5 concludes.

2 Sensor Planning with Experiment Sub-processes

2.1 A Model-Noised Sensor Planning

The purpose of this paper is to solve the planning of a set of sensors in order to answer to a set of requests and on the basis of a noisy prior knowledge of the environment. More precisely, the problem is characterized as follows:

- M requests characterized by their locations $z[m]$ with $m = 1 : M$,
- K sensors with indices $k = 1 : K$:
  - Starting/ending location of sensor k is $s[k]$,
  - Maximum autonomy of sensor k (maximum cumulative cost performed by k) is $\gamma[k]$,
- A noisy map μ, which describes the difficulty of the ground: from this map is computed the minimum cost $c[z_1, z_2; \mu]$ for moving from z_1 to z_2 . The exact map is not known from the planner: only the prior law p_μ on μ is known,
- A true map $\widehat{\mu}$ which is unknown to the planner and only known by the monitoring teams which process the sensors,
- Moving constraints: a sensor travels from starting/ending point through some request locations and back to starting/ending point. The trip of sensor k is denoted $\tau[k] = s[k]z[m_1^k] \cdots z[m_{i_k}^k]s[k]$. Moreover the cumulative cost for a trip is smaller than the sensor autonomy:

$$\text{If } \tau[k] \text{ is a valid trip, then } C(\tau[k]; \mu) \leq \gamma[k] \ , \tag{1}$$

  where:

$$C(\tau[k]; \mu) = c[s[k], z[m_1]; \mu] + c[z[m_1], z[m_2]; \mu] + \cdots + c[z[m_{i_k}^k], s[k]; \mu] \ . \tag{2}$$

- Criterion to maximize:

  Priority 1: Maximize the number of requests performed by the sensors:

$$G[\tau; \mu] = \text{card}\left(\{m = 1 : M \,/ \exists k = 1 : K,\ m \in \tau[k] \text{ and } \tau[k] \text{ is valid}\}\right) \ , \tag{3}$$

Priority 2: Minimize the global cost:

$$C[\tau;\mu] = \sum_{k=1:K} C(\tau[k];\mu) \ . \tag{4}$$

Since the environment is known with noise, the global plan τ does not always fit the actual constraints, and *has to be evaluated by the first planning level.* It is assumed that the proposed plan is truncated by the first planning level, in order to fit the accurate models:

$$\tau[k] \text{ is truncated to: } \widehat{\tau}[k] = s[k]z[m_1^k]\cdots z[m_{n_k}^k]s[k] \ , \tag{5}$$

where:

$$n_k = \arg\max\{n = 1 : i_k / C(s[k]z[m_1^k]\cdots z[m_n^k]s[k];\widehat{\mu}) \leq \gamma[k]\} \ . \tag{6}$$

The first planning level also provides an actual evaluation of the (truncated) plan in regards to the true map:

$$\widehat{G}[\tau] = G[\widehat{\tau};\widehat{\mu}] \text{ and } \widehat{C}[\tau] = C[\widehat{\tau};\widehat{\mu}] \ . \tag{7}$$

These information imply an improvement of the knowledge of the map, but it is costly. In order to optimize the plan, the global planner has to tune between the optimization of the plan and the actual evaluation requests.

2.2 The Theoretical Problem

A refined theoretical formulation of the planning problem of section 2.1 is now made with the following meaning: f is the evaluation of the plan, x is a plan, ν is the noisy map. Then, the optimization problem is characterized by means of a noisy criterion function:

$$f : (x,\nu) \in X \times N \mapsto f(x,\nu) \ , \tag{8}$$

where:

$$x \in X \text{ is a parameter to be optimized} \ , \tag{9}$$

$$\nu \in N \text{ is a model noise} \ , \tag{10}$$

$$p_\nu \in \mathcal{P}(N) \text{ is a known probabilistic noise prior} \ . \tag{11}$$

and by an unknown *actual* model noise:

$$\widehat{\nu} \in N \text{ is the actual value of the model noise} \ . \tag{12}$$

The noise on f is a model noise and *it is always possible to evaluate the actual criterion $f(\cdot,\widehat{\nu})$ for any specific actual parameter $\widehat{x}$.* Then the purpose is not to optimize a mean criterion, but rather to choose a *good* sequence of actual parameters $\widehat{x}$ so as to approximate an optimum for the actual criterion $f(\cdot,\widehat{\nu})$. It comes that each evaluation of the actual criterion is costly, while, in comparison,

the evaluation of the modelled criterion $f(\cdot,\nu)$ is considered free for any noise hypothesis ν. Since each evaluation of the actual criterion provide also some knowledge about the actual model noise $\widehat{\nu}$, the issue is to balance optimally between actual evaluation and model-based optimization, so as to find a near optimal solution to the actual criterion.

So as to deal with this problem, Welch proposed[21] the famous Efficient Global Optimization method, which is based on an iterative optimization maximizing the Expected Improvement. More precisely, Welch considered the case of a (spatial) Gaussian noise combined with a linear model, and derived exact computation of the sequence. Our main contribution is to extend Welch algorithm to any cases by means of simulation approaches. Rare event simulation methods are quite instrumental here.

From a general point of view, Welch approach takes the form of the following recursive computation:

[Expected Improvement Maximization (EIM)]

1. Set $n=0$,
2. Repeat:
   (a) Compute $\widehat{x}_{n+1}$, the next candidate for an actual evaluation:

$$\widehat{x}_{n+1} \in \arg\max_{x\in X} \int_{\nu\in N} p_\nu[n](\nu) f[n](x,\nu)\,\mathrm{d}\nu\ , \tag{13}$$

$$\text{where:}\qquad p_\nu[n](\nu) = p_\nu\big(\nu \bigm| \forall k=1:n,\ f(\widehat{x}_k,\nu)=\widehat{y}_k\big)\ , \tag{14}$$

$$f[n](x,\nu) = \max\left\{ f(x,\nu), \max_{k=1:n} \widehat{y}_k \right\}\ . \tag{15}$$

   (b) Request the actual evaluation of $\widehat{x}_{n+1}$ $\widehat{y}_{n+1} = f(\widehat{x}_{n+1},\widehat{\nu})$,
   (c) Set $n \leftarrow n+1$,
   until the convergence of $(\widehat{x}_{1:n},\widehat{y}_{1:n})$ is sufficient.
   [Output:] The sequence $(\widehat{x}_{1:n},\widehat{y}_{1:n})$ and model noise estimation $p_\nu[n]$.

The function $f[n]-\max_{k=1:n}\widehat{y}_k$ evaluates the improvement of f at step n. The conditional probability $p_\nu[n]$ is the posterior knowledge of ν obtained after the n first measurements.

From these considerations, it appears that we need to:

- Evaluate the conditional probability $p_\nu[n]$,
- Compute the optimal parameter $\widehat{x}_{n+1}$.

We will see that both tasks are performed by rare event simulations. We will also propose a combined approach for performing these tasks at same time.

3 Rare Event Simulation

A rare event is an event with very small probability. In this section, the following notations are considered:

- Ω is a probabilistic space,
- $p_\omega \in \mathcal{P}(\Omega)$ is a probabilistic distribution on Ω,
- $\phi : \omega \in \Omega \mapsto \mathbb{R}$ is a measurable function,

We are considering events the form $\phi^{-1}([\gamma, +\infty[)$. Then $\phi^{-1}([\gamma, +\infty[)$ is a rare event, if $\int_{\omega\in\Omega} I[\phi(\omega) \geq \gamma] p_\omega(\omega)\, \mathrm{d}\omega \ll 1$, where $I[\text{true}] = 1 - I[\text{false}] = 1$. The two following subsection explain how conditional sampling and optimization may be solved by simulating a rare event. Third section explains the *cross-entropy method* as a general process for rare event simulation.

3.1 Conditional Sampling and Rare Event Simulation

It is possible to sample the conditional law $p_\omega(\omega|\phi(\omega) \geq \gamma)$ by sampling the law p_ω and rejecting all samples such that $\phi(\omega) < \gamma$. In the case where $\phi^{-1}([\gamma, +\infty[)$ is a rare event, a direct Monte-Carlo approach is not feasible, but dedicated rare event simulation approaches may be used.

As a specific case, the simulation of conditional law $p_\nu[n]$ may be approximated by $p_\omega(\omega|\phi(\omega) \leq \epsilon)$, where $\omega = \nu$ and:

$$\phi(\nu) = \sum_{k=1:n} \left(f(\widehat{x}_k, \nu) - \widehat{y}_k\right)^2 . \tag{16}$$

In such approach, *the threshold ϵ is a measure of the quality of the conditional sampling.*

3.2 Optimization and Rare Event Simulation

Actually, the set of maximizers of a function could be defined as a limit of rare events. More precisely:

$$\arg\max_{\omega\in\Omega} \phi(\omega) = \bigcap_{\gamma<\max\phi(\Omega)} \phi^{-1}([\gamma, +\infty[) . \tag{17}$$

Then, it comes naturally that the optimization of a function may be obtained by simulating an arbitrarily rare event. Especially, the Cross-Entropy simulation method presented subsequently has been applied to the optimization of functions. Such approaches may be compared to population-based metaheuristics (*eg.* genetic algorithm).

3.3 The Cross-Entropy Method

The cross-entropy method (CE) has been pioneered by Rubinstein [23], and was initially settled for the simulation of rare event. It is based on a recursive importance sampling driven by a family of sampling densities:

$$\pi(\cdot|\Theta) = \left(\pi(\cdot|\theta)\right)_{\theta\in\Theta} \tag{18}$$

Without loss of generality, it is assumed that there is $\theta^o \in \Theta$ such that $p_\omega = \pi(\cdot|\theta^o)$. By denoting N_t the number of samples ω_t^i generated at step t, $R_t : \varphi \in \mathbb{R} \mapsto R_t(\varphi) \in [0,1]$ a selective function for the samples (typically, a quantile-based selection) and $\alpha_t \in]0,1]$ a smoothing parameter, the CE simulation may be defined as follows:

[CE simulation]

1. Set $t = 0$ and $\theta_0 = \theta^o$,
2. Repeat until the convergence is sufficient:
   (a) Generate the samples $\omega_t^i \in \Omega$, for $i \in \{1 : N_t\}$, according to the probabilistic density function (pdf) $\pi(\cdot|\theta_t)$,
   (b) Compute the evaluations $\phi(\omega_t^i)$ of the samples for $i \in \{1 : N_t\}$,
   (c) Compute the selective parameters:

$$\rho_t[0] = 1 - \alpha_t \ , \tag{19}$$

$$\rho_t[i] = \alpha_t \frac{R_t(\phi(\omega_t^i))}{\sum_{i=1:N_t} R_t(\phi(\omega_t^i))} \times \frac{\pi(\omega_t^i|\theta^o)}{\pi(\omega_t^i|\theta_t)} \ , \quad \text{for all } i \in \{1 : N_t\} \ , \tag{20}$$

   (d) **[Update]** Update the importance sampler by maximizing the cross-entropy with the selected samples:

$$\theta_{t+1} \in \arg\max_{\theta \in \Theta} \int_\Omega \left(\rho_t[0]\pi(\omega|\theta_t) + \sum_{i=1:N_t} \rho_t[i]\delta[\omega = \omega_t^i] \right) \log\left(\pi(\omega|\theta)\right) \, \mathrm{d}\omega \ ,$$

   where: $\delta[\omega = \omega_t^i]$ is the Dirac distribution on ω_t^i , (21)

   (e) Set $t \leftarrow t + 1$,

[Output:] The importance sampler $\pi(\cdot|\theta_t)$ and likelihood ratio $\dfrac{p_\omega}{\pi(\cdot|\theta_t)}$.

The criterion for convergence may be, as in the classical CE [23], achieved when a sufficient ratio of samples is within the rare event $\phi^{-1}([\gamma, +\infty[)$.

4 Practical Implementation and Numerical Results

4.1 Definition of the Maps, Costs and Plans

The map is defined by mapping from a vector parameter to a matrix of practicability level: $Z \in \mathbb{R}^p \mapsto \mu[Z] \in \mathbb{R}_+^{[0,1]\times[0,1]}$. In practice, Z combines the positions of threats and $\mu[Z]$ is computed as distances from these threats. The practicability level $\mu[Z](x)$ infers a local cost to any sensor which moves on position z: the cost $c[z, z']$ of a trajectory $z \to z'$ is obtained by integrating the local cost. The threat vector Z is known with a Gaussian noise, the model noise of our problem.

4.2 Generating Laws for Conditional Sampling and Optimization

The map is sampled as a real-valued vector. The plan is obtained by sampling mission to sensor mapping and a priority order between the mission. Both are easily derived from a real-valued vector by mean of a surjective discrete mapping. Gaussian laws are thus considered for both conditional sampling and optimization. The update step (21) is quite easy for such laws family and is typically derived from the empirical mean and covariance.

4.3 Numerical Results

Settings. The considered scenario is characterized by 5 sensors and 20 missions:

- Sensor position: $(1,1)$, $(1,1)$, $(9,1)$, $(9,1)$, $(5,1)$
- Sensor autonomy: 1, 2, 1, 2, 2
- Missions: 20 missions chosen uniformly on $[1,10] \times [1,10]$

and by a map of threats characterized by 4 threats:

- Theoretical threat position, with noise $\nu_i \sim N(0, \mathrm{diag}(2,2))$:

$$\mu = \big\{(2,3)+\nu_1, (5,4)+\nu_2, (4,7)+\nu_3, (2,3)+\nu_4\big\}$$

- Actual threat position: $\widehat{\mu} = \big\{(1,1),(4,6),(3,7),(1,4)\big\}$.

The cost inferred by the map is computed as follows:

- The local cost $c[z] = 1/(1+d(z,T)^2)$ decrease with the distance to the set of threats,
- The cost of a path $z \to z'$ is computed by integrating on the interval $[z,z']$, *ie.* $C[z,z';\mu] = \int_{\omega\in[z,z']} c[\omega]\,\mathrm{d}\omega$.

A Sequence of Run. First at all, a reference plan is optimized on the basis on the true map:

iter	1	20	40	60	80	100	120	140	160	180	200
opt	11.6	13.3	14.3	14.8	15.1	15.5	16	16.6	17.3	17.7	17.9
$\int$ *samp*	100	$2K$	$4K$	$6K$	$8K$	$10K$	$12K$	$14K$	$16K$	$18K$	$20K$

In this table, *iter* is the number of iteration in the CE algorithm, *opt* is the reached maximal value and $\int$ *samp* is the cumulative number of generated samples. This optimization required 200 iteration and 20000 samples for convergence. The following optimized plan is sampled be means of the sampler $\pi(\cdots|\theta_{200})$ after the last iteration:

Sensor	planned trajectory	corrected trajectory
0	{3}	{}
1	{2, 10, 12, 16}	{2, 10, 12, 16}
2	{4, 0, 18}	{4, 0}
3	{13, 6, 5, 15, 7, 8}	{13, 6, 5, 15, 7, 8}
4	{17, 1, 9, 19, 14, 11}	{17, 1, 9, 19, 14, 11}

Now, the following sequence of actual evaluations is obtained by applying the CE-based EIM algorithm:

ϵ	NaN	0.07	0.96	0.72	0.8	2.6
y	15.7	17	17.7	17.7	17.1	NaN
$\widehat{y}$	14.9	13.9	11.9	15.92	15.93	NaN

The quality of the conditional estimation is evaluated by means of the feasible threshold ϵ, as defined in section 3.1, obtained after CE convergence. It is of course undefined at step 0 (there is no conditioning). The estimation is good at step 1, rather good from step 2 to step 4. The estimation is bad at step 5. The optimized actual evaluation $\widehat{y}$ is 14.9 at step 0 and it is 15.93 at step 4, and only 4 evaluation is needed in order to reach 15.92. At step 5, the conditioning is bad, and the CE-based EIM fails.

Obviously, the failure of CE-based EIM is related to the quality of the conditional estimation. In fact, the conditional law is obtained from nonlinear constraints. As a consequence, it is multi-modal and cannot be efficiently sampled by means of Gaussian sampler. In order to enhance the algorithm, it will be necessary to consider mixtures of laws, or multi-modal law by construction.

5 Conclusion

In this paper we considered the problem of planning a set of sensors in the presence of model noises. The following hypotheses were considered: the planner only knows the law of the model noise, but he can request an actual but costly evaluation of a solution. In such case, each actual evaluation of the criterion function increases the knowledge about the model, and subsequently the efficiency of the plan optimization. The issue was to optimize the sequence of value to be evaluated, in regards to the evaluation costs. In our work, we defined a generalization of the Efficient Global Optimization (EGO) algorithm, based on a rare event simulation approach. The results are promising, and the algorithm produced good plans while requesting quite few sub-process calls. It appeared that this optimization was limited by the Gaussian approximation of potentially multi-modal conditional law. Future works will consider mixtures of laws for approximating the conditional law.

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